

Global Positioning System

張子明

物四甲

491712270

GPS 工作原理

總共**24**個衛星

週期為**12**小時

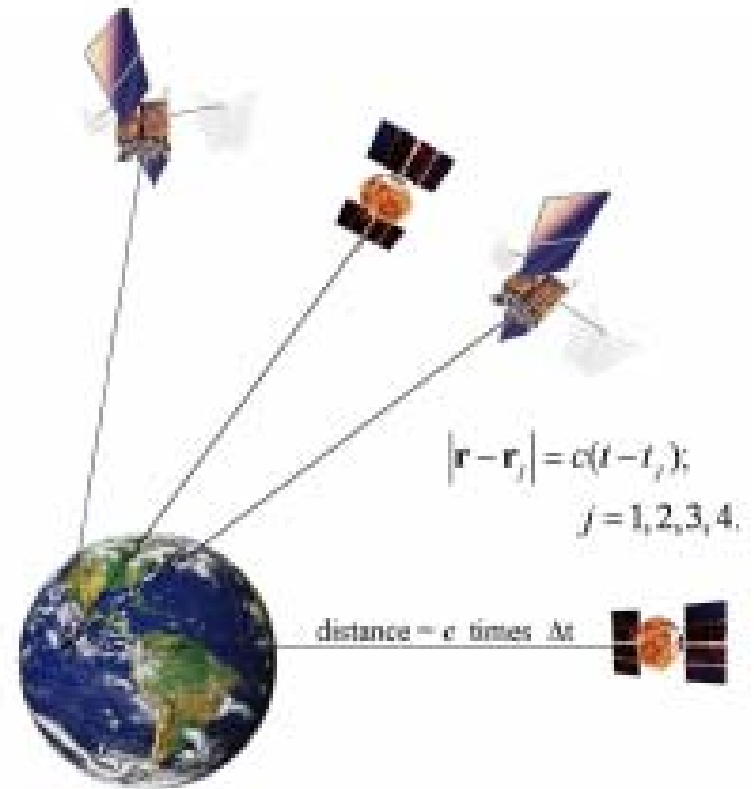
平均分為**6**軌道面

衛星上有**atomic clock**，記錄的是當時時間，以及當時位置，能發射微波訊號到地表。

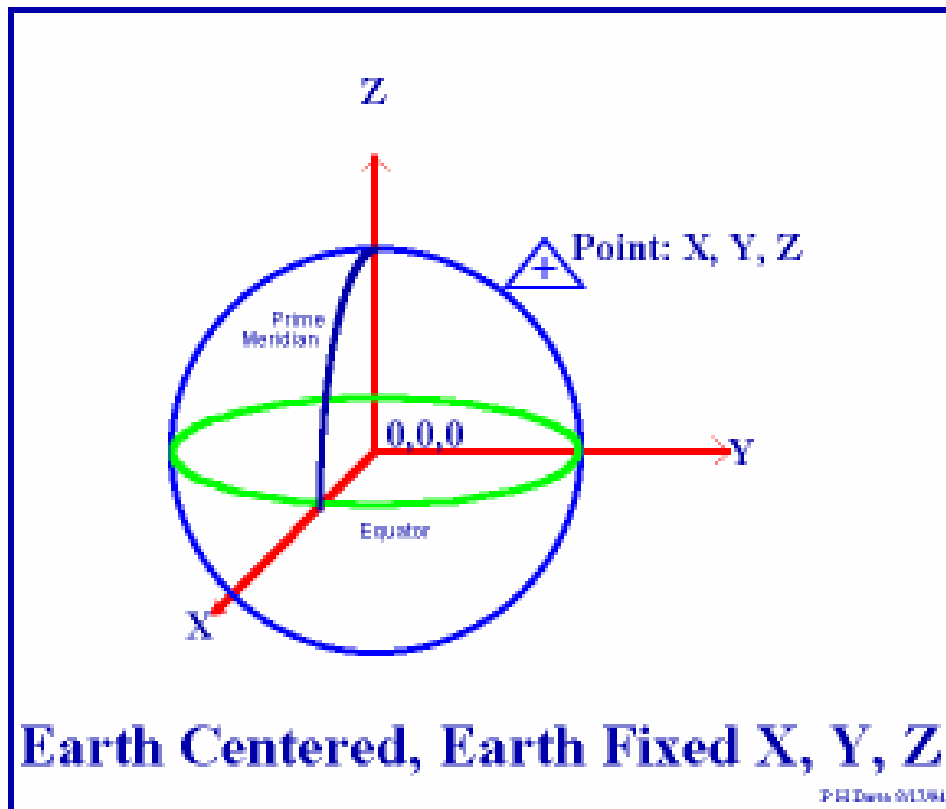


GPS 工作原理

訊號以光速行進，所以只要以光速 c 乘以訊號傳遞的所需時間，就可以算出“接收器--衛星”的距離

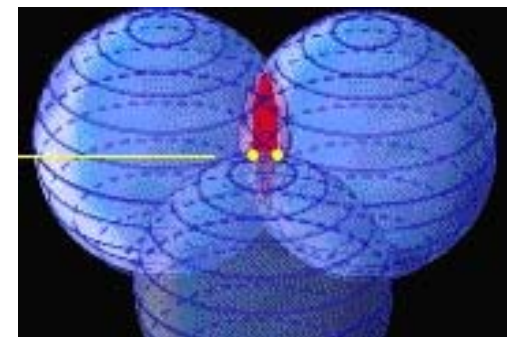
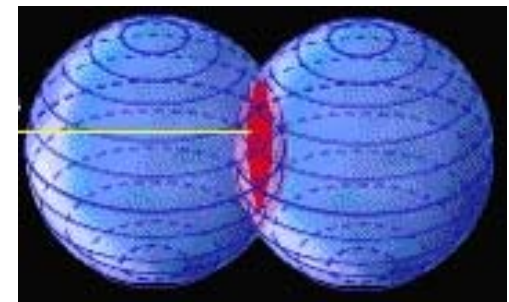
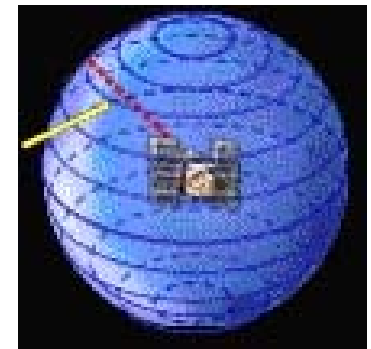


GPS 工作原理




使用ECEF座標作標準

接收器中有micro-processor，當接收到這些訊號後便能算出位置




GPS in General Relativity

Schwarzschild metric : $d\tau^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2M}{r}} - r^2 d\phi^2$




$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{2M}{r}\right) - r^2 \left(\frac{d\phi}{dt}\right)^2$$



$$\left(\frac{d\tau}{dt_{\text{衛星}}}\right)^2 = \left(1 - \frac{2M}{r_{\text{衛星}}}\right) - v_{\text{衛星}}^2$$

$$\left(\frac{d\tau}{dt_{\text{地球}}}\right)^2 = \left(1 - \frac{2M}{r_{\text{地球}}}\right) - v_{\text{地球}}^2$$



$$\left(\frac{dt_{\text{衛星}}}{dt_{\text{地球}}}\right)^2 = \frac{\left(1 - \frac{2M}{r_{\text{衛星}}}\right) - v_{\text{衛星}}^2}{\left(1 - \frac{2M}{r_{\text{地球}}}\right) - v_{\text{地球}}^2}$$

GPS in General Relativity

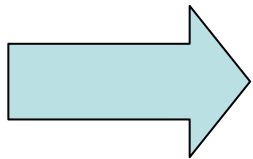
Taking approximation :

$$\left(\frac{dt_{\text{衛星}}}{dt_{\text{地球}}} \right)^2 = \frac{\left(1 - \frac{2M}{r_{\text{衛星}}} \right) - v_{\text{衛星}}^2}{\left(1 - \frac{2M}{r_{\text{地球}}} \right) - v_{\text{地球}}^2}$$

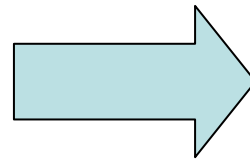
$$(1 + d)^n \approx 1 + nd$$

$$\frac{dt_{\text{衛星}}}{dt_{\text{地球}}} = \left[1 - \frac{2M}{r_{\text{衛星}}} - v_{\text{衛星}}^2 \right]^{1/2} \left[1 - \frac{2M}{r_{\text{地球}}} - v_{\text{地球}}^2 \right]^{-1/2} \approx \left[1 - \frac{M}{r_{\text{衛星}}} - \frac{v_{\text{衛星}}^2}{2} \right] \left[1 + \frac{M}{r_{\text{地球}}} + \frac{v_{\text{地球}}^2}{2} \right]$$

$$\approx 1 - \frac{M}{r_{\text{衛星}}} - \frac{v_{\text{衛星}}^2}{2} + \frac{M}{r_{\text{地球}}} + \frac{v_{\text{地球}}^2}{2}$$

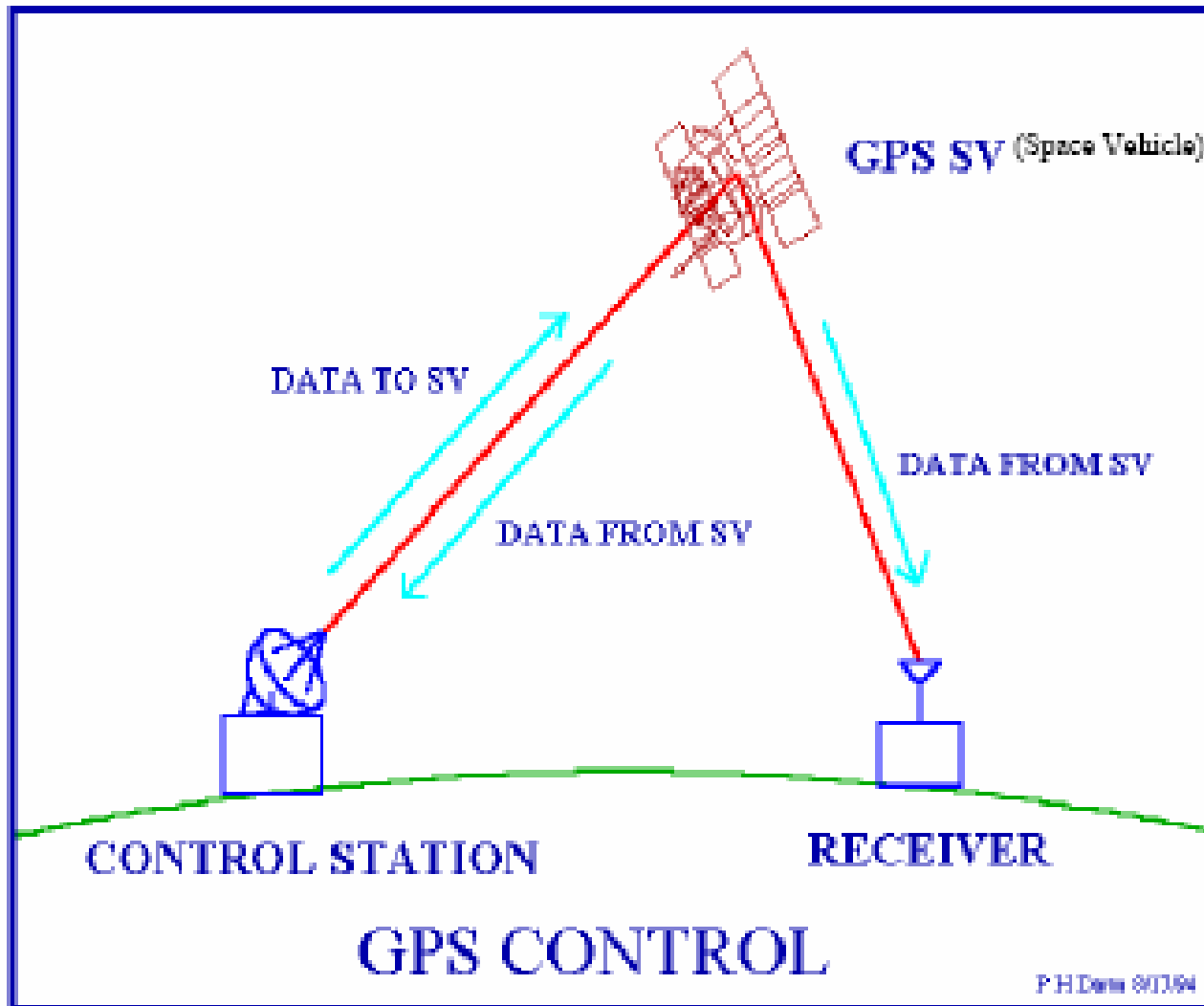


$$\frac{dt_{\text{衛星}}}{dt_{\text{地球}}} \approx 39,000 \text{ ns} / \text{day}$$



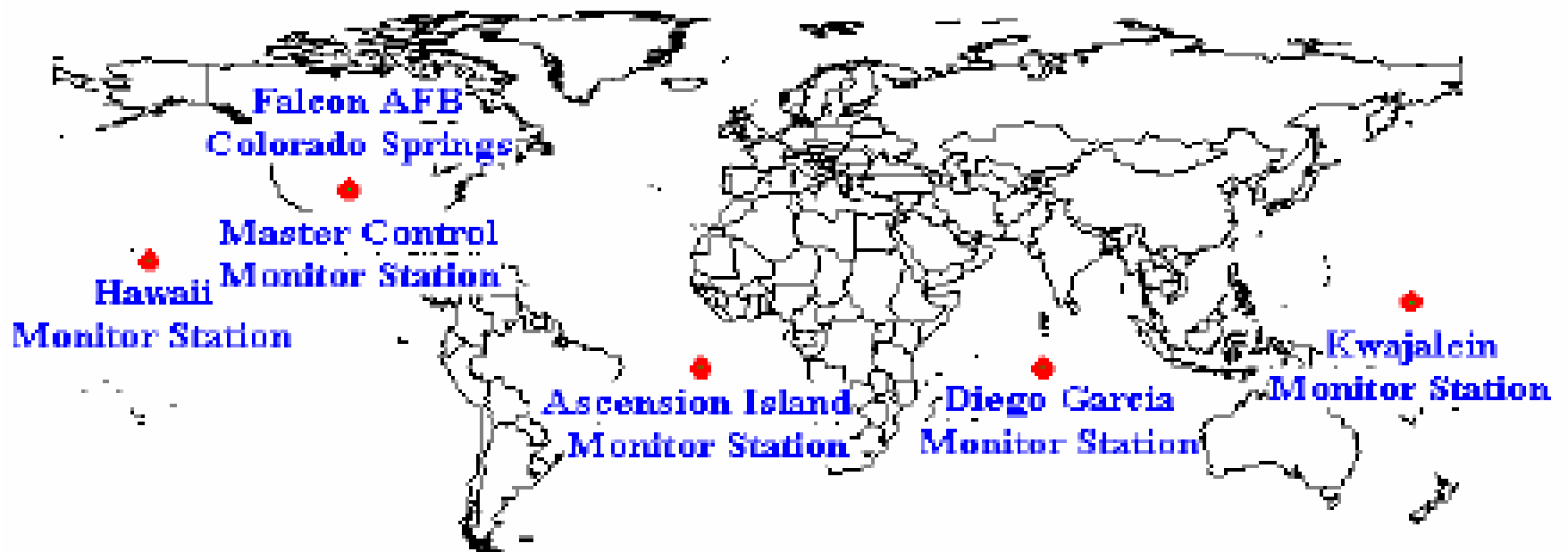
位置的誤差會在 10^2

How to solve ?



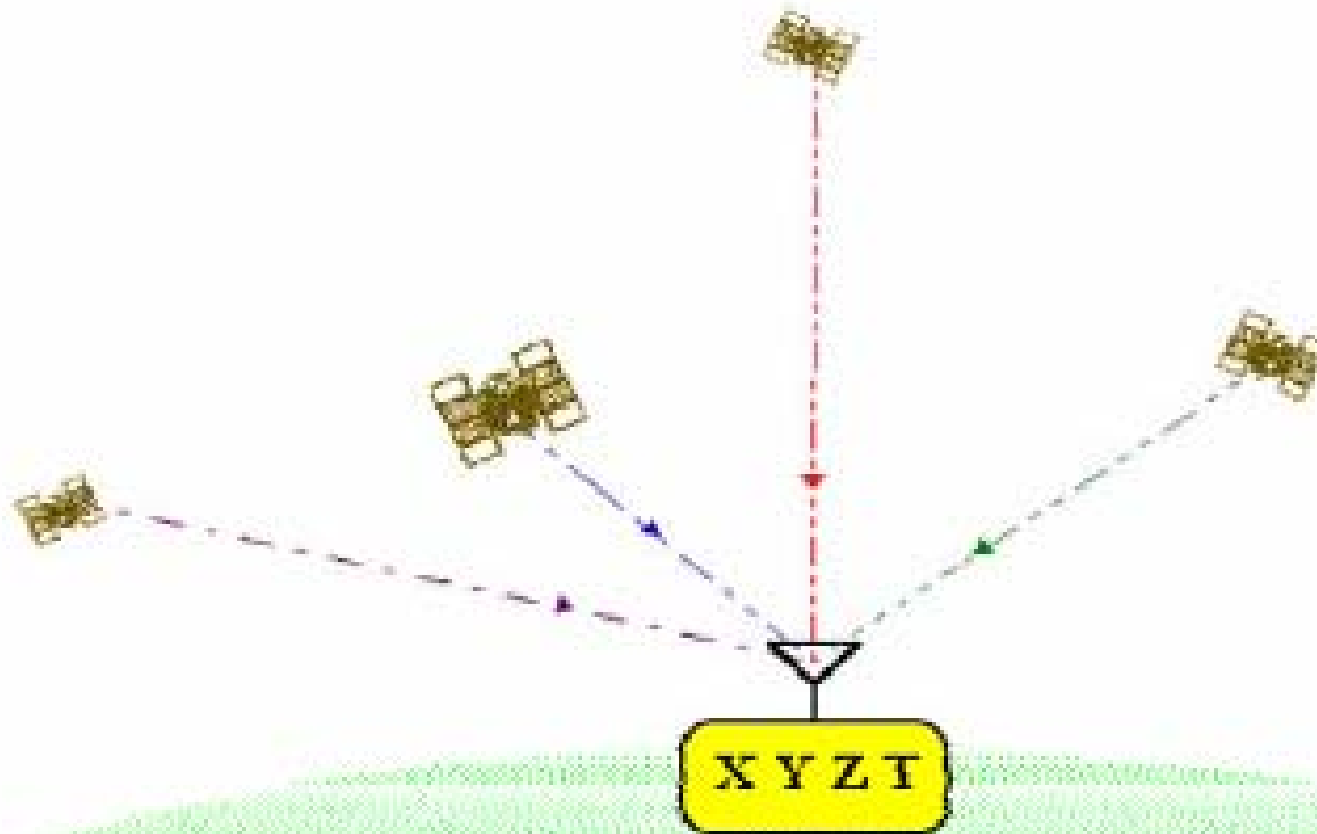
How to solve ?

Peter H. Dana 5/27/95



Global Positioning System (GPS) Master Control and Monitor Station Network

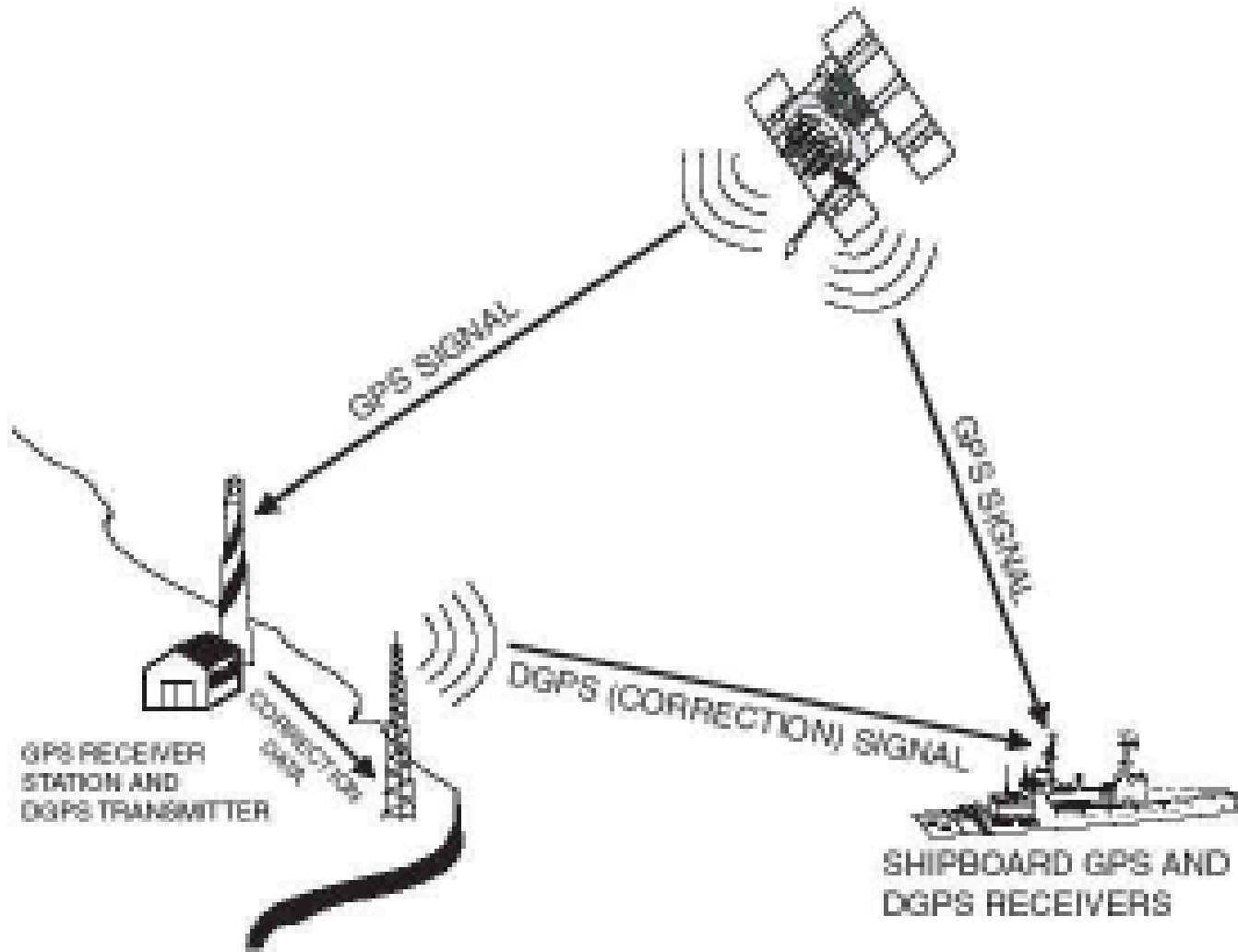
How to solve ?



The Global Positioning System

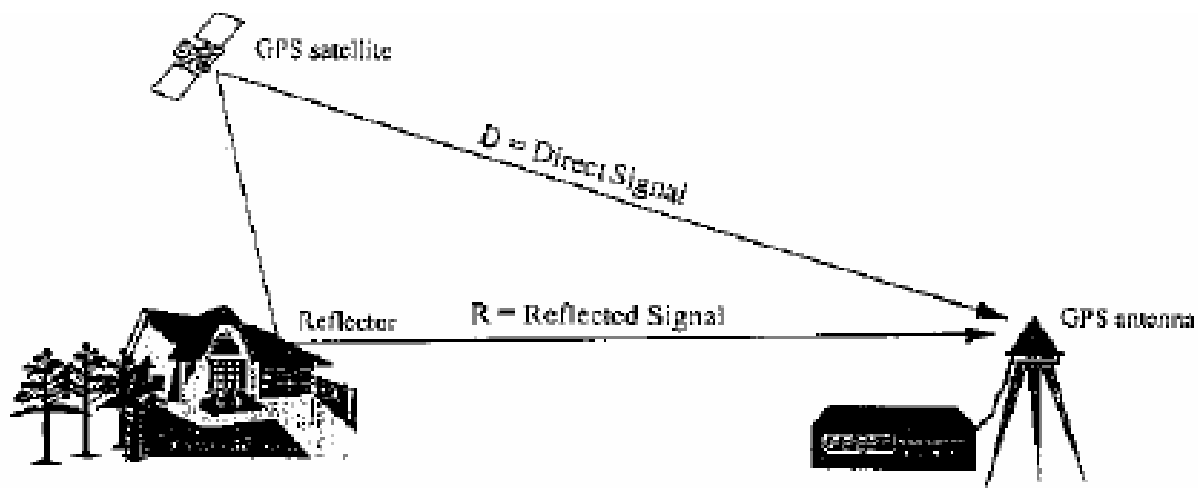
Measurements of code-phase arrival times from at least four satellites are used to estimate four quantities: position in three dimensions (X, Y, Z) and GPS time (T).

How to solve ?



影響GPS的其他因素

- Refraction
- Noise
- Doppler effect
- Multi-path



Applications



For Geodesy



For Geodesy



For Geodesy



For GIS



References

Exploring Black holes—introduction to general relativity

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Atmospheric humidity retrieval with GPS

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Relativistic Effects in the Global Positioning System

Neil Ashby