

Deterministic Dense Coding and Faithful Teleportation with Multipartite Graph States

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Abstract

We proposed novel schemes to perform the deterministic dense coding and faithful teleportation with multipartite graph states. We also find the sufficient and necessary condition of a viable graph state for the proposed scheme. That is, for the associated graph, the reduced adjacency matrix of the Tanner-type subgraph between senders and receivers should be invertible.

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The discoveries of dense coding and teleportation [1, 2], two impossible tasks in classical information theory, launched the extensive explorations and studies on quantum information science. For more than a decade, people have been searching for the deeper connection between quantum physics and information science. Wherein, as an intriguing feature of quantum physics, quantum entanglement has been exploited as the physical resource both in quantum communication and quantum computation. Different entangled states are requested for different quantum information processing. For example, to perform either dense coding or teleportation, the entangled states are always employed as quantum channel. In particular, the graph states play an eminent role in many applications of quantum information such as the scalable measurement-based quantum computation [3–7], the additive and non-additive quantum error-correction codes [8, 9]. As for their physical realization, the graph states can be scalably generated based on the realistic linear optics [10]. Recently, the six-photon graph state has been experimentally demonstrated [11]. Also, it has been reported that graph states can be also effectively prepared using cavity QED [12].

In the original proposals, the deterministic dense coding and faithful teleportation require two-qubit maximal entangled states, which, in fact, are equivalent to the simplest connected two-qubit graph states under local operations. Lee *et. al.* firstly showed the possibility of teleporting two-qubit state using a four-qubit entangled state [13]. Rigolin studied the dense coding and teleportation using multipartite entangled states [14–16]. Recently, Yeo and Chua proposed the schemes of teleportation and dense coding with a genuine four-qubit entangled state [17], which has been verified equivalent to some specific graph states under local operations [18]. In addition, very recently, Wang and Ying proposed the deterministic distributed dense coding and perfect teleportation schemes with stabilizer states [19, 20]. Eventually, any stabilizer state is equivalent to a graph state under local Clifford operations [21–23]. Therefore, the graph states are implicitly regarded as the physical resource for the deterministic dense coding and faithful teleportation.

In this letter, we study the deterministic dense coding and faithful teleportation with the graph states associated with the simply connected graphs, which are nontrivial multipartite entangled states. We propose the generalized schemes of the many-to-one dense coding and the one-to-many teleportation, respectively. In the proposed scenario, the $2n$ -qubit graph state, $|G\rangle$, which comprises qubits $1, 2, \dots, 2n$, is initially prepared. The associated $2n$ -vertex graph is denoted as $\Sigma_G = (V(\Sigma_G), E(\Sigma_G))$. Therein, the qubit i is associated

with the vertex i , and, moreover, each vertex i is associated with a stabilizer generator, $g_i = X_i \prod_{j \in N(i)} Z_j$ (Here we denote the Pauli matrix σ_x , σ_y , and σ_z acting on i -th qubit by X_i , Y_i and Z_i , respectively.). The neighboring set of vertex i is denoted as $N(i) = \{\text{vertex } j \mid (i, j) \in E(\Sigma_G)\}$. The state $|G\rangle$ fulfills the stabilizer condition $g_i |G\rangle = |G\rangle, \forall i \in \{1, \dots, 2n\}$. In other words,

$$X_i |G\rangle = \prod_{j \in N(i)} Z_j |G\rangle. \quad (1)$$

Obviously,

$$\{Z_i, g_i\} = 0, \quad (2)$$

where $\{, \}$ is anti-commutator. In fact, Eqns. (1) and (2) play a key role throughout.

Before further processing, some notations are introduced here. The vertex set $V(\Sigma_G)$ is decomposed as two subsets $V_S = \{\text{vertex } i \mid 1 \leq i \leq n\}$ and $V_R = \{\text{vertex } i \mid n+1 \leq i \leq 2n\}$. The edge set $E(\Sigma_G)$ is decomposed into three subsets $E_{SR} = \{(i, j) \mid \text{for the edge with } i \in V_S \text{ and } j \in V_R\}$, $E_S = \{(i, j) \mid i, j \in V_S\}$, and $E_R = \{(i, j) \mid i, j \in V_R\}$. The graph is called the Tanner-type graph if $E_S = \emptyset$ and $E_R = \emptyset$. In general, a graph Σ_G can be decomposed as $\Sigma_G^T \oplus E_R \oplus E_S$, where the Tanner-type subgraph $\Sigma_G^T = (V(\Sigma_G), E_{SR})$.

Here we define the following $n \times n$ sub-adjacency matrices: (i) the reduced adjacency matrix Γ_T of Σ_G^T , where the entry $(\Gamma_T)_{ij} = 1$ if $(i, n+j) \in E_{SR}$ and 0 otherwise; (ii) the adjacency matrix Γ_S , where for $1 \leq i, j \leq n$ the entry $(\Gamma_S)_{ij} = 1$ if $(i, j) \in E_S$ and 0 otherwise; (iii) the adjacency matrix Γ_R , where for $n+1 \leq i, j \leq 2n$ the entry $(\Gamma_R)_{ij} = 1$ if $(i, j) \in E_R$ and 0 otherwise. Moreover, for later usage it is convenient to define $\Gamma_{S,R}^<$ and $\Gamma_{S,R}^>$ which are the lower and upper triangular part of $\Gamma_{S,R}$, respectively. Note that $(\Gamma_{S,R}^>)^T = \Gamma_{S,R}^<$ as can be easily seen by definition.

Moreover, given a graph state $|G\rangle$ the set of states $\{|\vec{k}\rangle := |(k_1, \dots, k_{2n})\rangle \prod_{i=1}^{2n} Z_i^{k_i} |G\rangle\}$ forms an orthogonal measurement basis so that $g_i |\vec{k}\rangle = (-1)^{k_i} |\vec{k}\rangle$. (Hereafter, we will use the short-handed notation $Z^{\vec{k}} |G\rangle$ for $\prod_{i=1}^{2n} Z_i^{k_i} |G\rangle$)

Now we state the main results of the letter as follows, and then we will investigate the deterministic dense coding and faithful teleportation in details, respectively.

Main results : For the deterministic dense coding and faithful teleportation with the graph state $|G\rangle$, the sufficient and necessary condition is that the reduced adjacency matrix Γ_T of the Tanner-type subgraph Σ_G^T must be invertible.

Deterministic many-to-one dense coding : In the proposed dense coding scenario, the qubit $1, \dots, n$ are distributed among n distant senders, where the i -th sender holds the qubit i . The other n qubits $n+1, \dots, 2n$ are held by the receiver, Bob. For clear illustration, the graph state $|G^T\rangle$ with the associated Tanner-type graph, Σ_G^T , is initially prepared. In the encoding phase, to send Bob two classical bits a_i and b_i , $a_i, b_i \in \{0, 1\}$, the i -th sender performs the local operation $X_i^{a_i} Z_i^{b_i}$ on the qubit i . Then all qubits at senders' hands are delivered to Bob. Notably, the encoded state now becomes $|G_D^T\rangle := \prod_{i=1}^n X_i^{a_i} Z_i^{b_i} |G^T\rangle$. For further procedure, define the n -bit binary message vectors \vec{a} and \vec{b} with the i -th components being a_i and b_i , respectively. According to the (1), the encoded graph state can be also written as $|G_D^T\rangle = \prod_{i=1}^n Z_i^{b_i} Z_{n+i}^{a_{n+i}} |G^T\rangle$. As a result, according to the (2), $g_i |G_D^T\rangle = (-1)^{b_i} |G_D^T\rangle$ and $g_{n+i} |G_D^T\rangle = (-1)^{a_{n+i}} |G_D^T\rangle \forall i \in \{1, \dots, n\}$. Similarly, we define two n -bit binary vectors \vec{a}' and \vec{b}' , where the i -th components are a'_{n+i} and b'_i respectively.

In the decoding phase, Bob is firstly to find all components of \vec{a}' and \vec{b}' . That is, he is to measure the eigenvalues of all stabilizer generators using quantum circuits. Such task is analogue to finding the syndromes of the stabilizer quantum error-correction codes. Or, Bob performs the orthogonal measurement using the orthogonal basis $\{|\vec{k}\rangle\}$. In this way, suppose the post-measurement state is $|\vec{k}\rangle$, then $\vec{k} := (\vec{b}', \vec{a}')$.

On the other hand, after some calculation we have

$$\vec{b} = \vec{b}', \quad (3)$$

and $\vec{a}' = \Gamma_T \vec{a}$, or equivalently,

$$\vec{a} = \Gamma_T^{-1} \vec{a}'. \quad (4)$$

That is, Bob can verify the message vectors \vec{b} and \vec{a} using Eqns. (3) and (4), respectively. Notably, to guarantee the deterministic decoding, the map $\vec{a}' \rightarrow \vec{a}$ must be *bijective* (one-to-one and onto). From (4) this requires the reduced adjacency matrix Γ_T of Σ_G^T must be invertible.

Now we turn to the deterministic dense coding using the general $2n$ -qubit graph state $|G\rangle$ with non-empty sets E_R and E_S of the associated graph Σ_G . Note that, by definition, the adjacency matrix Γ_T is irrelevant of E_R and E_S . That is, the (4) is unchanged for any associated graph Σ_G . With straight calculation, the (3) should be revised as

$$\vec{b} = \vec{b}' + \Gamma_S \vec{a}. \quad (5)$$

Lastly, the nonempty E_R will only complicate the quantum circuits to extract the eigenvalues \vec{a}' and \vec{b}' , and can be taken care appropriately in designing the decoding circuits. To sum up, in decoding phase Bob firstly measures the eigenvalues of stabilizer generators to derive \vec{a}' and \vec{b}' , respectively. Then he decodes the message vector \vec{a} using (4). Once \vec{a} is identified, the map $\vec{b}' \rightarrow \vec{b}$ is also bijective. Finally he can determine the message vector \vec{b} using (5).

Notably, two graph states $|G\rangle$ and $|\tilde{G}\rangle$ are locally equivalent if there is a local unitary $U \in [U(2)]^{2n}$, such that $|\tilde{G}\rangle = U|G\rangle$. The induced transformation between the corresponding graphs Σ_G and $\Sigma_{\tilde{G}}$ is called local complementation [3, 23]. On the other hand, the capacity of dense coding is invariant under local operation. Therefore, if $|G\rangle$ can be exploited for the deterministic dense coding, $|\tilde{G}\rangle$ will also do for the same task. In other words, the invertibility of the reduced adjacency matrix Γ_T of the Tanner-type subgraph Σ_G^T is preserved under local complementation.

Faithful one-to-many teleportation : In the proposed teleportation scenario, the sender, Alice, is to teleport the unknown qubits i' to the i -th distant receiver $\forall i \in \{1, 2, \dots, n\}$. Similarly, the qubit i' is associated with the vertex i' , and we denote the vertex set as $V_{S'} = \{\text{vertex } i' | 1 \leq i \leq n\}$. Without loss of generality, the density matrix of the n -qubit unknown state is denoted by ρ_u and

$$\rho_u = \sum_{z_1, \dots, z_n, x_1, \dots, x_n, =0}^1 \lambda_{\vec{z}, \vec{x}} \prod_{i=1}^n Z_{i'}^{z_i} X_{i'}^{x_i}, \quad (6)$$

where the information about the unknown state is encoded in $\lambda_{\vec{z}, \vec{x}}$, and $\vec{z} = (z_1, \dots, z_n)$ and $\vec{x} = (x_1, \dots, x_n)$. Hereafter, we will use the short-handed notation $\sum_{\vec{x}}$ for $\sum_{x_1, x_2, \dots, x_n=0}^1$.

To achieve the teleportation task, the $2n$ -qubit graph state, $|G\rangle$ is initially prepared as addressed before. Wherein, n qubits $\{n+1, \dots, 2n\}$ are at Alice's hand, and the qubit $i \in V_S$, $1 \leq i \leq n$, is held by the i -th receiver. Then Alice performs the $2n$ -qubit joint measurement with the orthogonal measurement basis [25]

$$\{|\vec{k}\rangle := |(k_1, \dots, k_{2n})\rangle | \prod_{i=1}^n Z_{i'}^{k_i} Z_{n+i}^{k_{n+i}} |G'\rangle\}, \quad (7)$$

where the graph state $|G'\rangle$ is the $2n$ -qubit state which comprise qubits $\{1', \dots, n', n+1, \dots, 2n\}$ and is identical to $|G\rangle$ if the qubit i' is replaced by the qubit $i \forall i \in \{1, \dots, n\}$. Similarly, the corresponding graph is denoted by $\Sigma_{G'}$ and its vertex set is decomposed as

two subsets $V_{S'}$ and V_R . The density matrices of $|G\rangle$ and $|G'\rangle$ are denoted as follows by ρ_G and $\rho_{G'}$ respectively,

$$\rho_G := |G\rangle\langle G| = \frac{1}{2^{2n}} \sum_{\vec{j}} \prod_{i=1}^{2n} g_i^{j_i} \quad \text{and} \quad \rho_{G'} := |G'\rangle\langle G'| = \frac{1}{2^{2n}} \sum_{\vec{j}'} \prod_{i=1}^{2n} (g'_i)^{j'_i}. \quad (8)$$

Wherein, g'_i is the stabilizer generator of $|G'\rangle$ and can be derived via g_i with the local operator on the qubit i' instead of the qubit i , where $1 \leq i \leq n$. Since these two corresponding graphs, Σ_G and $\Sigma_{G'}$ of the states $|G\rangle$ and $|G'\rangle$, are identical except with two different n -vertex label sets $(1, \dots, n)$ and $(1', \dots, n')$. Therefore, these two graphs form a mirror pair with respect to the vertex set V_R as illustrated in Fig. 1, and their associated adjacency matrices such as Γ_T and Γ'_T (Γ_S and Γ'_S) as previously mentioned, are exactly equal.

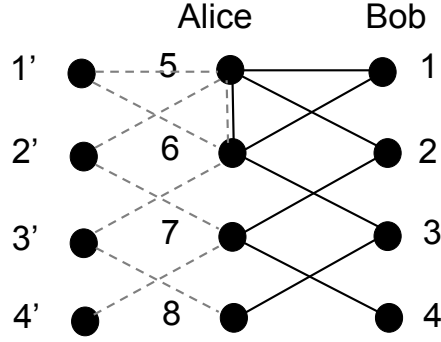


FIG. 1: Sketch of a mirror pair of graphs

Let the post-measurement state be $|\vec{k}\rangle$ after Alice's measurement, then she announces the $2n$ -bit binary vector $\vec{k} = (k_1, k_2, \dots, k_{2n})$ of the orthogonal measurement, which will be exploited to reconstruct the unknown states. Moreover, the corresponding density matrix of the post-measurement state with qubits $\{1, \dots, n\}$ held by n distant receivers is denoted by $\rho_n := \text{tr}_{2N}(\rho_G \otimes \rho_u \otimes Z^{\vec{k}} \rho_{G'} Z^{\vec{k}})$. Here tr_{2N} denotes the tracing-out of qubits $\{1', \dots, n', n+1, \dots, 2n\}$. After collecting the corresponding factors we arrive

$$\rho_n = \sum_{\vec{x}, \vec{z}, \vec{j}, \vec{j}'} (-1)^{\vec{x} \wedge \vec{k} < + \vec{j} > \wedge [\vec{k} > + \Gamma_R^> (\vec{j} > + \vec{j}' >)] + \vec{x} \wedge \Gamma_S^< \vec{j}' < + \vec{j} < \wedge \Gamma_S^< \vec{j} <} \lambda_{\vec{z}, \vec{x}} \times \prod_{i=1}^n Z_i^{\sum_{p \in N(i)} j_p} X_i^{j_i} \text{tr}_{2N} \left[Z_{i'}^{z_i + \sum_{p \in N(i')} j'_p} X_{i'}^{x_i + j'_i} Z_{n+i}^{\sum_{p \in N(n+i)} (j_p + j'_p)} X_{n+i}^{j_{n+i} + j'_{n+i}} \right]. \quad (9)$$

For conciseness, in the above we have defined $\vec{j}_< := (j_1, j_2, \dots, j_n)$ and $\vec{j}_> := (j_{n+1}, j_{n+2}, \dots, j_{2n})$, similarly for $k_< := (k_1, k_2, \dots, k_n)$ and $k_> := (k_{n+1}, k_{n+2}, \dots, k_{2n})$. In fact, the expression (9) can be read directly from the graph Σ_G and $\Sigma_{G'}$. Recall the stabilizer $g_i := X_i \prod_{j \in N(i)} Z_j$ so that the exponent of $X_i(X_{i'})$ in (9) is just the sum of $j_i(j'_i)$ and x_i , and similarly the exponent of $Z_i(Z_{i'})$ is the sum of z_i and $j_i(j'_i)$ of its associated neighboring vertices. In collecting the above exponents one should interchange X and Z at the same vertex, and using the fact $Z^b X^a = (-1)^{a \wedge b} X^a Z^b$ it results in the phase factor in Eq (9). More specifically, (a) collecting the exponents associated with vertices in $V_{S'}$ yields the phase factor $(-1)^{\sum_{i=1}^n x_i \wedge (k_i + \sum_{p \in N(i), p < i} j'_p)}$; (b) collecting the exponents associated with vertices in V_R yields the phase factor $(-1)^{\sum_{i=1}^n j_{n+i} \wedge (k_{n+i} + \sum_{p \in N(n+i), p > n+i} j'_p + \sum_{p \in N(n+i), p < n+i} j_p)}$; and (c) collecting the exponents associated with vertices in V_S yields the phase factor $(-1)^{\sum_{i=1}^n j_i \wedge \sum_{p \in N(i), p > i} j_p}$. Finally, to arrive the compact form of the phase factor in (9) we have used the following identities

$$\sum_{p \in N(i), p < i} j'_p = \sum_{q=1}^n (\Gamma_S^<)_{i,q} j'_q, \quad \sum_{p \in N(i), p > i} j_p = \sum_{q=1}^n [(\Gamma_T)_{i,q} j_{n+q} + (\Gamma_S^<)_{i,q} j_q] \quad (10)$$

and

$$\sum_{p \in N(n+i), p > n+i} j'_p = \sum_{q=1}^n (\Gamma_R^>)_{i,q} j'_{n+q}, \quad \sum_{p \in N(n+i), p < n+i} j_p = \sum_{q=1}^n [(\Gamma_T)_{q,i} j_q + (\Gamma_R^>)_{i,q} j_{n+q}]. \quad (11)$$

Taking the trace in (9) results in the following $4n$ Kronecker deltas

$$\delta(j_{n+i} + j'_{n+i}) \quad (12)$$

$$\delta\left(\sum_{p \in N(n+i)} (j_p + j'_p)\right) \quad (13)$$

$$\delta(x_i + j'_i) \quad (14)$$

and

$$\delta\left(z_i + \sum_{p \in N(i')} j'_p\right) \quad (15)$$

for $i = 1, \dots, n$. The symmetric form between j_p and j'_p in (12)-(13) is due to the fact that graphs Σ_G and $\Sigma_{G'}$ form a mirror pair. These $4n$ Kronecker deltas will be used to *completely* eliminate the sum over the dummy vectors \vec{j}, \vec{j}' in (9) if they are all linearly independent,

which is also the condition to guarantee the faithful teleportation. The sufficient and necessary condition of the $4n$ linearly independent Kronecker deltas turns out to be the same as the one for the deterministic dense coding, namely, the reduced adjacency matrix $\Gamma_T(\Gamma'_T)$ of the Tanner-type subgraph $\Sigma_G^T(\Sigma_{G'}^T)$ is invertible. This can be seen as follows.

After imposing the conditions (12) in the arguments of Eq. (13), the n Kronecker deltas (13) can be reduced to

$$\delta\left(\sum_{\ell=1}^n (\Gamma_T)_{\ell,i}(j_\ell + j'_\ell)\right) \quad (16)$$

which is associated with each vertex $n+i \in V_R$. Therefore, the linear independence of the n Kronecker deltas (13) is the same as the one for (16), which is equivalent to the invertibility of the reduced adjacency matrix Γ_T . Once the linear independence is guaranteed, it is easy to see that (12) and (13) are reduced to $\delta^{(2n)}(\vec{j} + \vec{j}')$, which also yields the other $2n$ linearly independent Kronecker deltas $\delta(x_i + j_i)$ and $\delta(z_i + \sum_{p \in N(i)} j_p)$ obtained from (14) and (15). Using the Kronecker deltas $\delta^{(2n)}(\vec{j} + \vec{j}')$ and $\delta^{(n)}(\vec{x} + \vec{j})$ one can reduce the phase factor in (9) into $(-1)^{\vec{x} \wedge \vec{k} < + \vec{j} > \wedge \vec{k} >}$. Besides, these Kronecker deltas also help to turn the factor $Z_i^{\sum_{p \in N(i)} j_p} X_i^{j_i}$ in (9) into $Z_i^{z_i} X_i^{x_i}$. Moreover, using (10) and the above relations the n Kronecker deltas $\delta(z_i + \sum_{p \in N(i)} j_p)$ are reduced to $\delta^{(n)}(\vec{j} > + \Gamma_T^{-1}(\vec{z} + \Gamma_S \vec{x}))$ which are linearly independent if Γ_T is invertible. One can then use these Kronecker deltas to solve the dummy vector $\vec{j} >$ in terms of \vec{x} and \vec{z} to reduce the phase factor in (9) further. The phase factor now becomes

$$(-1)^{\vec{x} \wedge \vec{k} < + \Gamma_T^{-1}(\vec{z} + \Gamma_S \vec{x}) \wedge \vec{k} >} = (-1)^{\vec{x} \wedge [I + (\Gamma_T^{-1} \Gamma_S)^T] \vec{k} + \vec{z} \wedge (\Gamma_T^{-1})^T \vec{k}} := (-1)^{\vec{x} \wedge \vec{c}_x + \vec{z} \wedge \vec{c}_z} \quad (17)$$

Therein, the i -th components of \vec{c}_x and \vec{c}_z , $c_{x,i}$ and $c_{z,i}$ are functions of \vec{k} . As a result, the i -th receiver can derive the values of $c_{x,i}$ and $c_{z,i}$ since \vec{k} has been publicly announced by Alice.

Consequently, as long as Γ_T is invertible we can arrive

$$\rho_n = \sum_{\vec{x}, \vec{z}} (-1)^{\vec{x} \wedge \vec{c}_x + \vec{z} \wedge \vec{c}_z} \lambda_{\vec{z}, \vec{x}} \prod_{i=1}^n Z_i^{z_i} X_i^{x_i}. \quad (18)$$

Finally, in the correction phase, the i -th receiver performs $Z_i^{c_{x,i}} X_i^{c_{z,i}}$ on the qubit i to recover ρ_u faithfully.

Discussion: It should be emphasized that the viable graph for our proposed scheme should have full rank reduced adjacency matrix Γ_T of the Tanner sub-graph. As an example, in the

four-qubit case, there are two inequivalent graph states. One is the four-qubit Greenberger-Horne-Zeilinger (GHZ) state associated with the star graph, the other is the cluster state associated with the linear cluster graph. Consequently, the cluster state rather than GHZ state can be exploited for deterministic dense coding and faithful teleportation. So is the $2n$ -qubit GHZ states which is known to be associated with the star graph and cannot be exploited in our proposed schemes.

Moreover, the rank of Γ_T is related to the Schmidt measure of the graph state with respect to the bi-partition into V_S and V_R [7], which is kind of the channel capacity for the quantum communication between Alice and Bob. This somehow explains why the GHZ state is not viable here since its Schmidt measure is not maximal though the state itself is maximally entangled. It is hoped that the above Schmidt-type measure is related to the recently proposed negative quantum conditional entropy, which indicates the potential to “receive future quantum information for free” via teleportation and some other ways [24].

It is known that the resultant graph under the local complementation (LC) corresponds to the local unitary operation acting on the original graph state [3], therefore, it will not affect the dense coding and teleportation as shown in [18] for the four-qubit case. This also holds true for more general graphs considered in our case, and can be understood as follows. By definition, the LC on the vertex i is to complement the edges associated with vertices in $N(i)$. It is then easy to see that the above LC action corresponds to add the column (or row) vector associated with vertex i in Γ_T to the other columns (or rows) associated with the vertices in $N(i)$. Therefore, LC will not change the rank of Γ_T , that is, all the LC-equivalent graphs have the same viability for the deterministic dense coding and faithful teleportation in our proposed scheme.

Finally, although so far we focus on the two-level multipartite graph states, the generalization to the dense coding and teleportation with multi-level graphs states is just straightforward.

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