



OpenAI

OpenAI ‘agent’ hacked an Australian health service website

AI lab took months to spot the problem, which Prime Minister Anthony Albanese called ‘obviously unacceptable’

3 HOURS AGO



US-China relations

Xi arrives for high-stakes summit with Trump as leaders extend trade truce

Screenshot

The first publicly reported hack of a government website by an “agent” — bots that can perform complex tasks independently based on human instructions — is likely to heighten anxiety that technological advances are rapidly reaching a point where AI breaks free from human control.

OpenAI said the agent that breached Medicare “took actions we did not intend”. It spotted the problem during a review of its agents’ behaviour in internal training exercises, launched after its agents hacked the start-up Hugging Face in late July.

The Hugging Face incident, which OpenAI [took more than a week to detect](#), involved hundreds of agents that organised in secret within the lab’s system, gained access to the internet and broke into the start-up while trying to complete a cyber security test.

That hack sparked global scrutiny of AI safety issues, which intensified when Anthropic researcher Jacob Coxon resigned earlier this month with grave warnings about advanced models wiping out humanity.

Meta Platforms

+ Add to myFT

Mark Zuckerberg unveils AI 'charm' device that can fit on a keychain

Social media chief says the Muse personal assistant is the 'centrepiece' of his AI vision after launch success



Meta's Mark Zuckerberg used the company's Screenshot event to announce Muse Charm © AP Photo/Jeff Chiu



Muse Charm is a small pendant-like device that fits on a keychain and is activated by a fingerprint sensor

The release of a breakout product to Meta's 3bn users caps a turbulent two



Thursday, September 24, 2026

Today's Paper

The New York Times

Dow -0.68%

U.S. World Business Arts Lifestyle Opinion Video Shows Games Cooking Wirecutter The Athletic

These top stories are **free to read**.

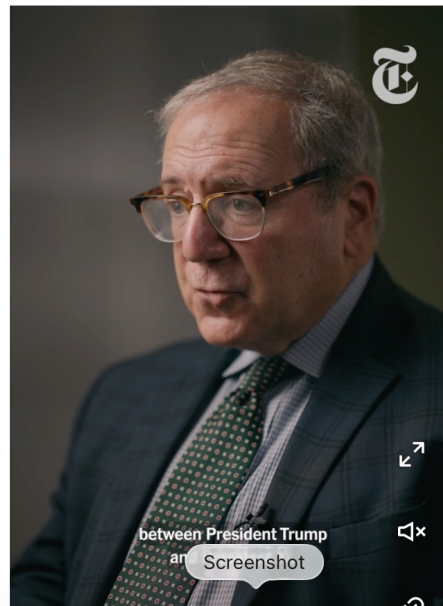
As Xi Visits Washington, Trump Plays Down China's Actions Against U.S. Interests

Despite the rivalry between the U.S. and China, President Trump has repeatedly found ways to give President Xi Jinping a pass, portraying him as a friend.

179

U.S. and China Agree to Extend Trade Truce by 2 Months, Bessent Says

4 MIN READ



Trump Greet Xi for State Visit Amid Tensions

The two-day visit comes as the U.S. and China jostle over trade and artificial intelligence.

See more updates >

More Americans Have a Positive View of China, Polls Show

365

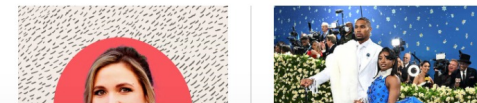


Sela Shiloni for The New York Times

The Lucas Museum's Architect Wants to Help You Escape from Reality

Ma Yansong, China's reigning starchitect, lets his buildings wrapped in nature do the shouting.

8 MIN READ





ALMA, the Atacama Large Millimeter Array
電磁波偵測天線陣列

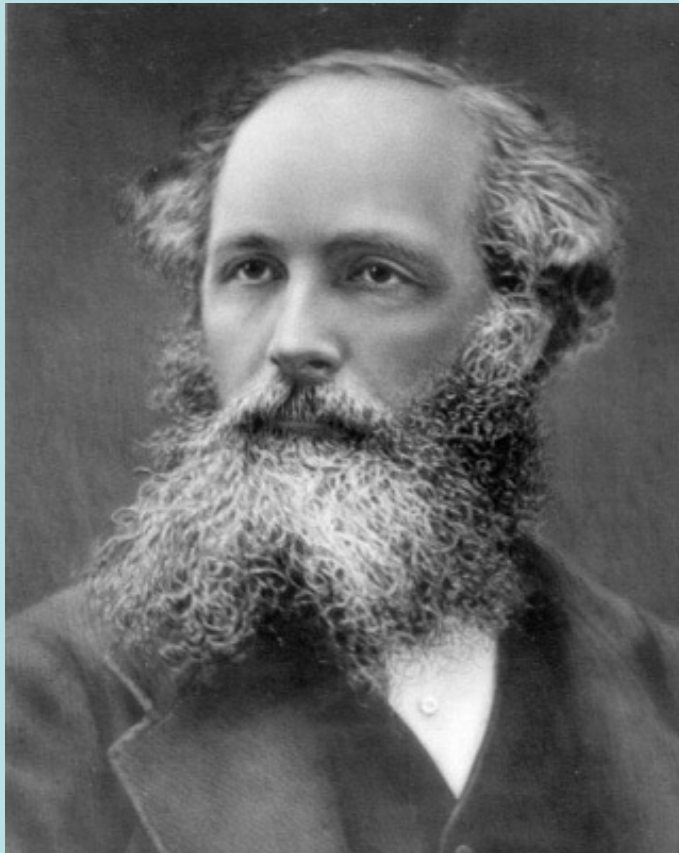


The Antennae Galaxies
(also known as NGC 4038
and 4039) are a pair of
distorted colliding spiral
galaxies about 70 million
light-years away, in the
constellation of Corvus
(The Crow)

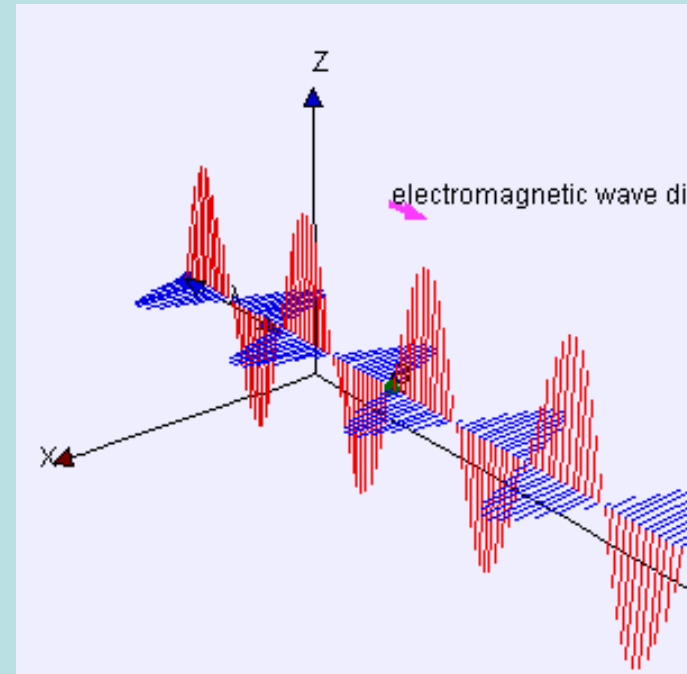
電磁波孤獨旅行了七千萬光年的空間，七千萬年後才到達地球，
可以想見，此時，發出這些電磁波的星體，可能已不存在了。
是甚麼樣的機制，維持這些電磁波可以單獨旅行這麼久這麼遠？

電磁波：電與磁的自動化

馬克思威爾及當時的科學家



James Clerk Maxwell (1831–1879)



many thanks to Fu-Kwun
Hwang

If $\vec{\nabla}$ acts like a vector, combining two $\vec{\nabla}$'s as inner product generates a scalar.

$$\vec{\nabla} \cdot \vec{\nabla} = \left(\frac{\partial}{\partial x} \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \frac{\partial}{\partial z} \right) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \equiv \nabla^2$$



This is an operation to be acted on functions.

$$\vec{\nabla} \cdot \vec{\nabla} U = \nabla^2 U \equiv \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

This operation is called Laplacian ◦

$$\vec{E} = \vec{\nabla} V$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Plug the first formula into the second :

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot (\vec{\nabla} V) = \nabla^2 V = \frac{\rho}{\epsilon_0}$$



Pierre-Simon Laplace 1749-1827

Electric Potential satisfies Laplacian Equation!

$$\nabla^2 V = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\rho}{\epsilon_0}$$

Using differential Maxwell Eq, EM wave equation is easy to derive.



Maxwell Equations in vacuum

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Spatial change of electric field is interrelated to temporal change of magnetic field.

$$\nabla \times (\vec{\nabla} \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

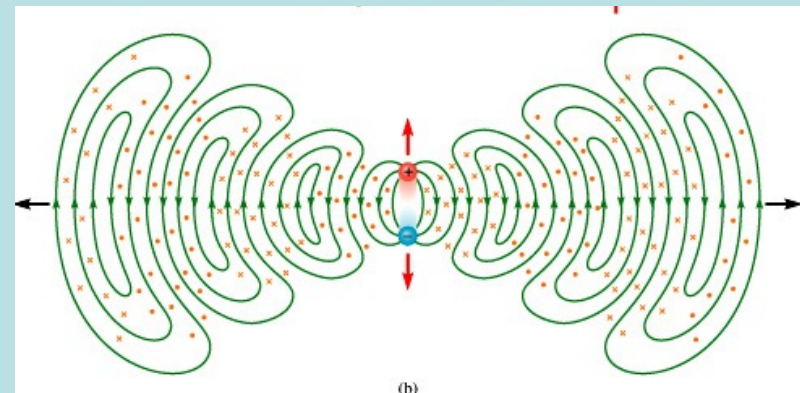
$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$$

Spatial change of magnetic field is interrelated to temporal change of electric field.

$$\nabla(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial(\nabla \times \vec{B})}{\partial t} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

Wave Equation for Electric Field



$\vec{A} \times (\vec{B} \times \vec{C})$ Triple Product



Calculate the components directly:

$$[\vec{A} \times (\vec{B} \times \vec{C})]_z = A_x (\vec{B} \times \vec{C})_y - A_y (\vec{B} \times \vec{C})_x$$

$$= A_x (B_z C_x - B_x C_z) - A_y (B_y C_z - B_z C_y)$$

$$= B_z (A_x C_x + A_y C_y) - (A_x B_x + A_y B_y) C_z$$

$$= B_z (A_x C_x + A_y C_y + A_z C_z) - (A_x B_x + A_y B_y + A_z B_z) C_z$$

$$= B_z (\vec{A} \cdot \vec{C}) - (\vec{A} \cdot \vec{B}) C_z$$

Similar Formula is true for the other two components of $\vec{A} \times (\vec{B} \times \vec{C})$.

$$[\vec{A} \times (\vec{B} \times \vec{C})]_{x,y} = B_{x,y} (\vec{A} \cdot \vec{C}) - (\vec{A} \cdot \vec{B}) C_{x,y}$$

Write the expression in the notations of vectors.

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - (\vec{A} \cdot \vec{B}) \vec{C}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) - (\vec{\nabla} \cdot \vec{\nabla}) \vec{F} = \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F}$$

► 3. TRIPLE PRODUCTS

There are two products involving three vectors, one called the triple scalar product (because the answer is a scalar) and the other called the triple vector product (because the answer is a vector).

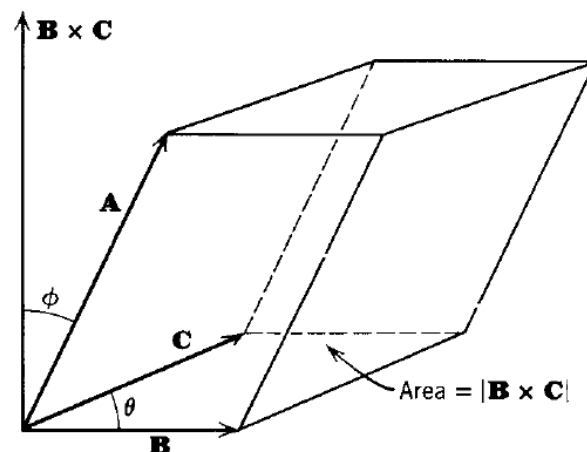


Figure 3.1

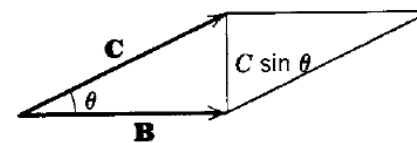


Figure 3.2

Triple Scalar Product This is written $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$. There is a useful geometrical interpretation of the triple scalar product (see Figure 3.1). Construct a parallelepiped using $\mathbf{A}, \mathbf{B}, \mathbf{C}$ as three intersecting edges. Then $|\mathbf{B} \times \mathbf{C}|$ is the area

of the base (Figure 3.2) because $|\mathbf{B} \times \mathbf{C}| = |\mathbf{B}| |\mathbf{C}| \sin \theta$, which is the area of a parallelogram with sides $|\mathbf{B}|, |\mathbf{C}|$, and angle θ . The height of the parallelepiped is $|\mathbf{A}| \cos \phi$ (Figure 3.1). Then the volume of the parallelepiped is

$$|\mathbf{B}| |\mathbf{C}| \sin \theta |\mathbf{A}| \cos \phi = |\mathbf{B} \times \mathbf{C}| |\mathbf{A}| \cos \phi = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}).$$

If $\phi > 90^\circ$, this will come out negative, so in general we should say that the volume is $|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})|$. Any side may be used as base, so, for example, $\mathbf{B} \cdot (\mathbf{C} \times \mathbf{A})$ must also be either plus or minus the volume. There are six such triple scalar products, all equal except for sign [or twelve if you count both the type $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ and the type $(\mathbf{B} \times \mathbf{C}) \cdot \mathbf{A}$].

Triple Vector Product This is written $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$. Before we try to evaluate it, we can make the following observations. $\mathbf{B} \times \mathbf{C}$ is perpendicular to the plane of $\mathbf{B}$ and $\mathbf{C}$. $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ is perpendicular to the plane of $\mathbf{A}$ and $(\mathbf{B} \times \mathbf{C})$; we are particularly interested in the fact that $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ is perpendicular to $(\mathbf{B} \times \mathbf{C})$.

Now (see Figure 3.4) *any* vector perpendicular to $\mathbf{B} \times \mathbf{C}$ lies in the plane perpendicular to $\mathbf{B} \times \mathbf{C}$, that is, the plane of $\mathbf{B}$ and $\mathbf{C}$. Thus $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ is *some* vector in the plane of $\mathbf{B}$ and $\mathbf{C}$, and can be written as some combination $a\mathbf{B} + b\mathbf{C}$, where a and b are scalars which we want to find. (See Chapter 3, Section 8, Problem 5.) One way to find a and b is to write out $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ in component form. We can simplify this work by choosing our coordinate system carefully; recall that a vector equation is true independently of the coordinate system. Given the vectors $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, we take the x axis along $\mathbf{B}$, and the y axis in the plane of $\mathbf{B}$ and $\mathbf{C}$; then $\mathbf{B} \times \mathbf{C}$ is in the z direction. The vectors in component form relative to these axes are:

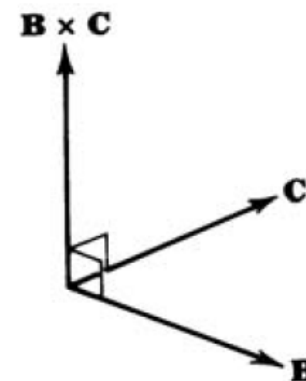


Figure 3.4

$$\begin{aligned}
 \mathbf{B} &= B_x \mathbf{i}, \\
 \mathbf{C} &= C_x \mathbf{i} + C_y \mathbf{j}, \\
 \mathbf{A} &= A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}.
 \end{aligned}
 \tag{3.4}$$

Using (3.4) we find

$$\begin{aligned}
 \mathbf{B} \times \mathbf{C} &= B_x \mathbf{i} \times (C_x \mathbf{i} + C_y \mathbf{j}) = B_x C_y (\mathbf{i} \times \mathbf{j}) = B_x C_y \mathbf{k}, \\
 \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= A_x B_x C_y (\mathbf{i} \times \mathbf{k}) + A_y B_x C_y (\mathbf{j} \times \mathbf{k}) \\
 &= A_x B_x C_y (-\mathbf{j}) + A_y B_x C_y (\mathbf{i}).
 \end{aligned}
 \tag{3.5}$$

We would like to write $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ in (3.5) as a combination of $\mathbf{B}$ and $\mathbf{C}$; we can do this by adding and subtracting $A_x B_x C_x \mathbf{i}$:

$$(3.6) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = -A_x B_x (C_x \mathbf{i} + C_y \mathbf{j}) + (A_y C_y + A_x C_x) B_x \mathbf{i}.$$

Each of these expressions is something simple in terms of the vectors in (3.4):

$$(3.7) \quad \begin{aligned} A_x B_x &= \mathbf{A} \cdot \mathbf{B}, & A_y C_y + A_x C_x &= \mathbf{A} \cdot \mathbf{C}, \\ C_x \mathbf{i} + C_y \mathbf{j} &= \mathbf{C}, & B_x \mathbf{i} &= \mathbf{B}. \end{aligned}$$

Using (3.7) in (3.6), we get

$$(3.8) \quad \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}.$$

This important formula should be learned, but not memorized in terms of letters, because that is confusing when you want some other combination of the same letters. Learn instead the following three facts:

$$(3.9) \quad \text{The value of a triple vector product is a linear combination of the two vectors in the parenthesis } [\mathbf{B} \text{ and } \mathbf{C} \text{ in (3.8)}]; \text{ the coefficient of each vector is the dot product of the other two; the middle vector in the triple product } [\mathbf{B} \text{ in (3.8)}] \text{ always has the positive sign.}$$



$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

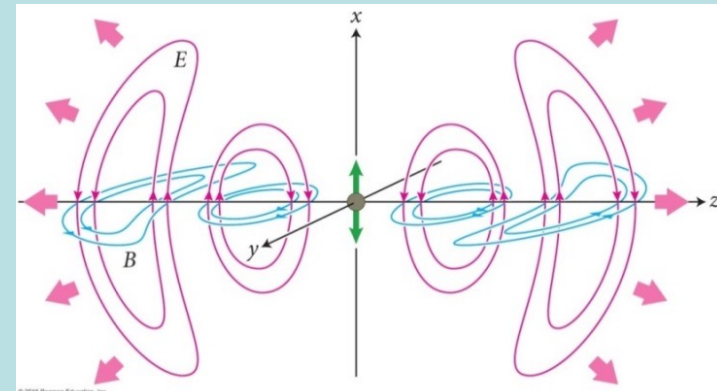
$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times (\vec{\nabla} \times \vec{B}) = \mu_0 \epsilon_0 \nabla \times \frac{\partial \vec{E}}{\partial t}$$

$$\nabla(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial(\nabla \times \vec{E})}{\partial t} = -\mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

Wave Equation for Magnetic Field



In vacuum, electromagnetic field only come from induction.

$\vec{E}, \vec{B}$ both satisfy Wave Equation (no trace of charge and current)!



$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

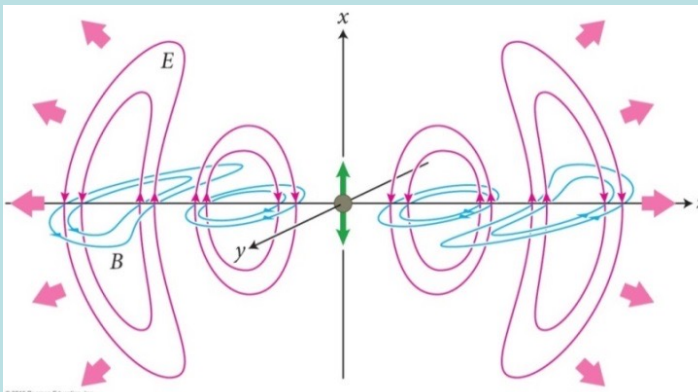
$$\nabla^2 \vec{B} = \frac{\partial^2 \vec{B}}{\partial x^2} + \frac{\partial^2 \vec{B}}{\partial y^2} + \frac{\partial^2 \vec{B}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

3D version of wave equation

The equations mean all the components of $\vec{E}$ and $\vec{B}$ satisfy wave equation, such as:

$$\nabla^2 E_x = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

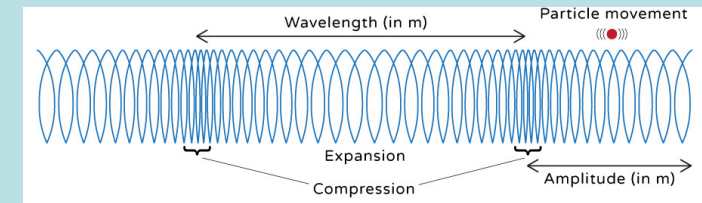
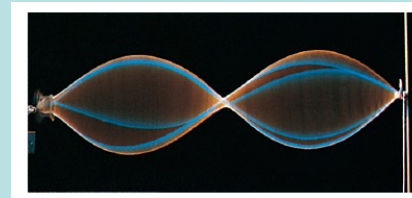
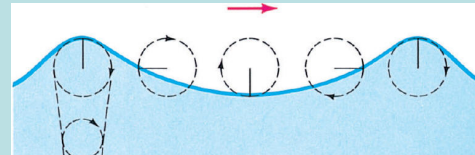
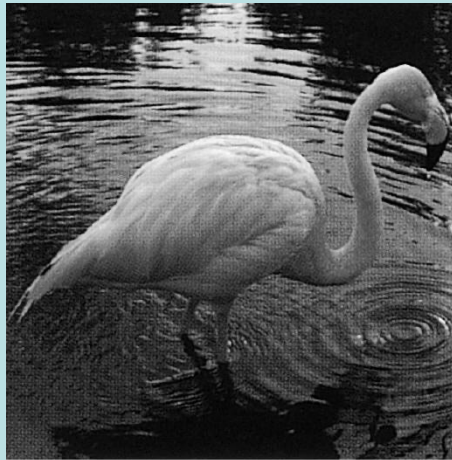
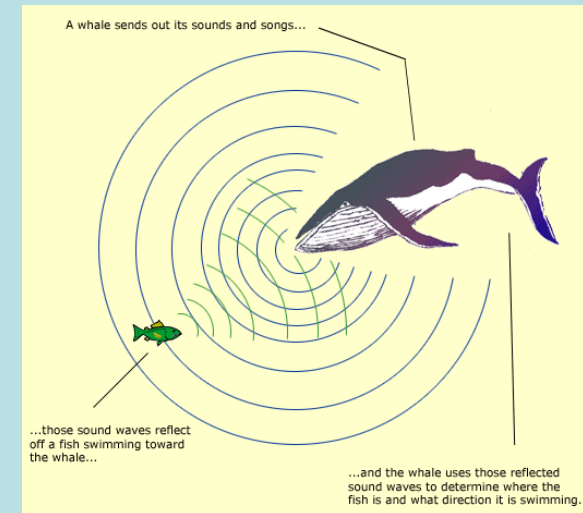
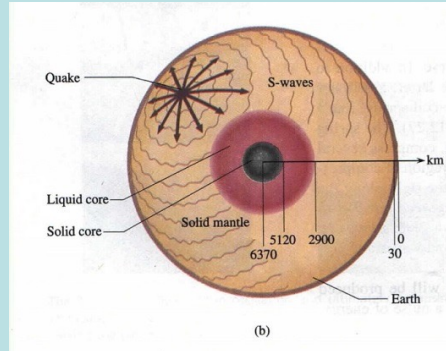
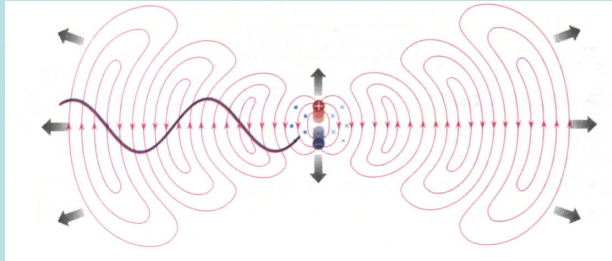
It could describe, for example, EM wave radiated from a dipole antenna.



波方程式 **Wave Equation** Equation of motion satisfied by all wave phenomena.

$$\nabla^2 y = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{3D version}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{1D version}$$



The above derivation is one of the greatest scientific achievements of all times. The electric and magnetic field can exist in vacuum without charge or current and they satisfy the same wave equation as string wave, sound wave and earth wave. The insights we learn from those waves will apply to electromagnetic wave as well.



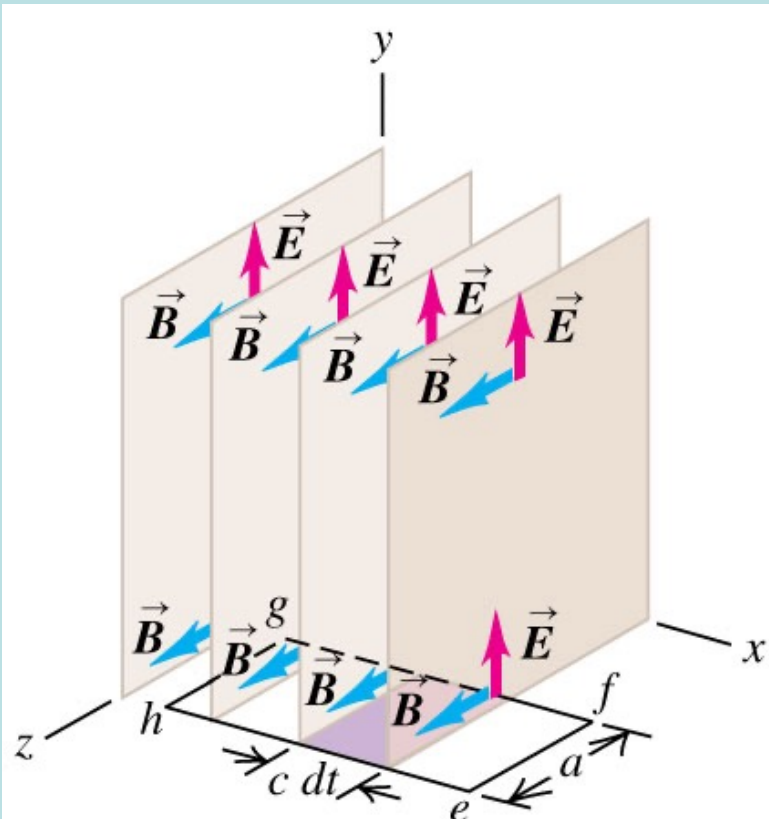
We will concentrate on the 1D version (not really in 1D).

Consider a special version of EM wave with $\vec{E}$ in y and $\vec{B}$ in z directions.

This wave will move in x direction, and we allow $\vec{E}, \vec{B}$ to be **only x dependent**.

$$E_y(x, t), B_z(x, t)$$

They are constant on the $y - z$ plane, and this wave is called **Plane Wave**.



$$\nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

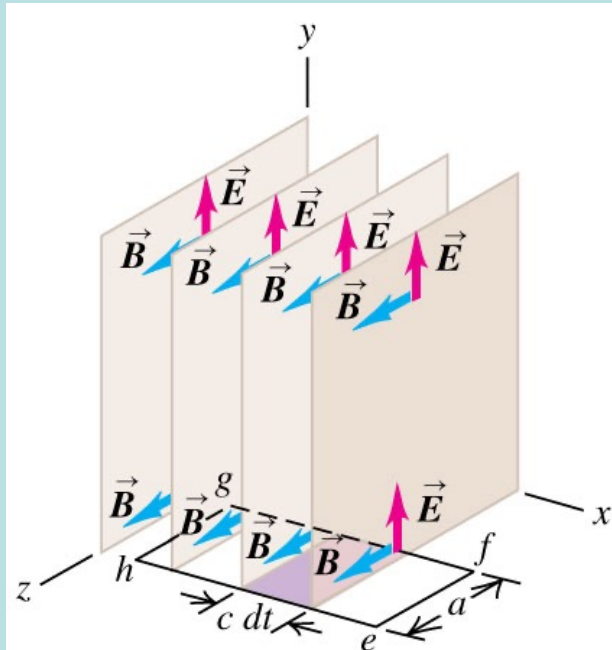
$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\nabla^2 \vec{B} = \frac{\partial^2 \vec{B}}{\partial x^2} + \frac{\partial^2 \vec{B}}{\partial y^2} + \frac{\partial^2 \vec{B}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$



$$E_y(x, t), B_z(x, t)$$



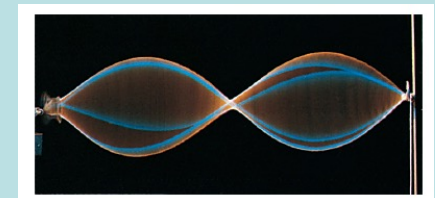
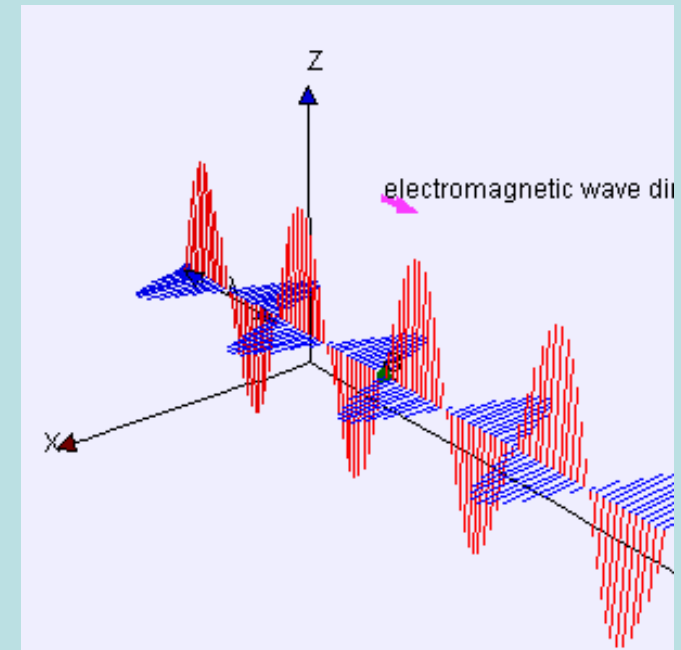
$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$



$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad v \text{ is the wave speed.}$$



Electric field and Magnetic field satisfy wave equation and will wave like all waves!

EM Wave speed is determined by two constants in Maxwell equations.

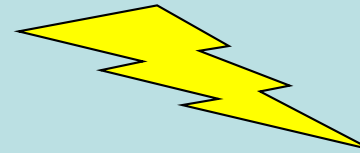


$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$= \frac{1}{\sqrt{1.26 \times 10^{-6} \cdot 8.85 \times 10^{-12}}} \sim 2.99 \times 10^8 \text{ m/s}$$

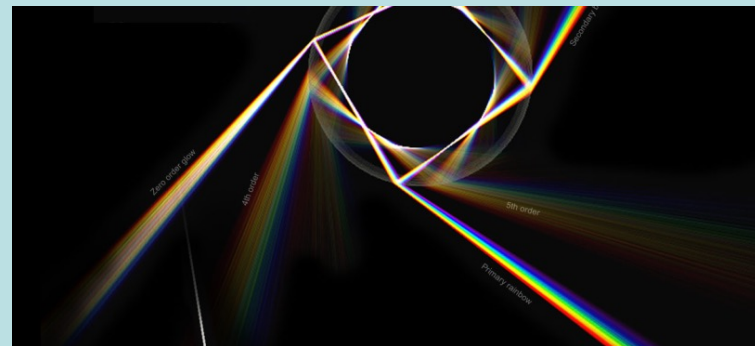
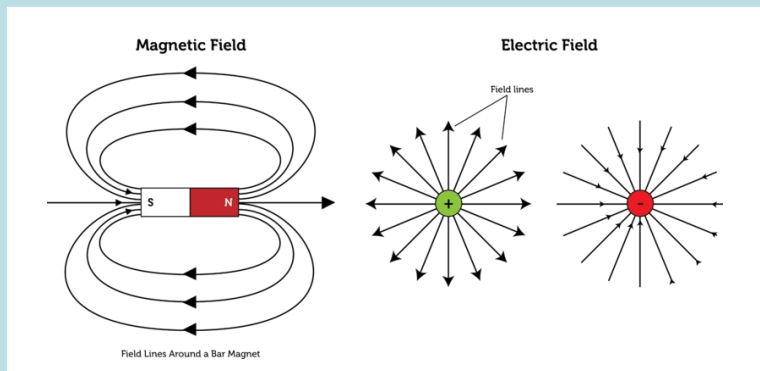
$$v = c$$

光速！！



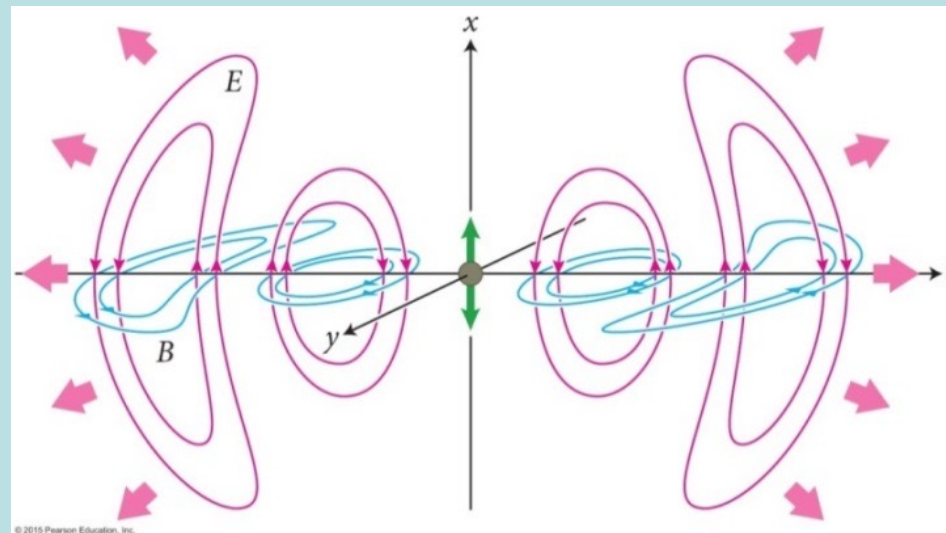
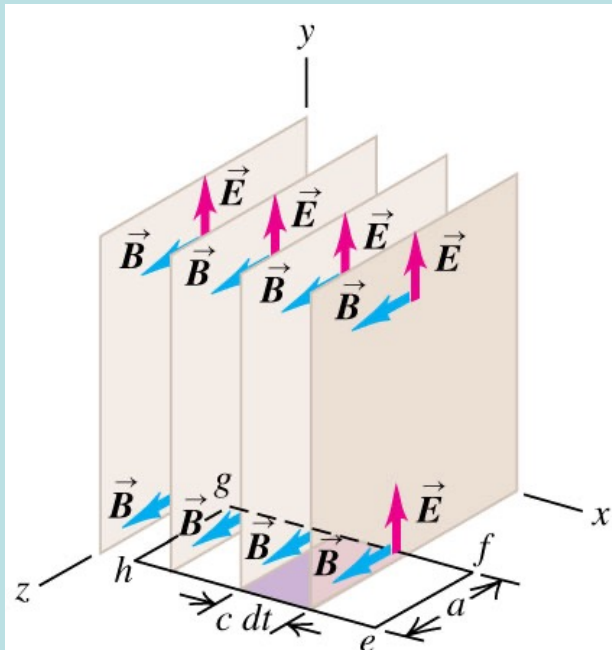
Light is an electromagnetic wave.

馬克斯威爾統一了電磁學與光學！





Plane wave would be a good approximation of 3D EM wave on horizontal plane.



PART VI.

ELECTROMAGNETIC THEORY OF LIGHT.

(91) At the commencement of this paper we made use of the optical hypothesis of an elastic medium through which the vibrations of light are propagated, in order to shew that we have warrantable grounds for seeking, in the same medium, the cause of other phenomena as well as those of light. We then examined electromagnetic phenomena, seeking for their explanation in the properties of the field which surrounds the electrified or magnetic bodies. In this way we arrived at certain equations expressing certain properties of the electromagnetic field. We now proceed to investigate whether these properties of that which constitutes the electromagnetic field, deduced from electromagnetic phenomena alone, are sufficient to explain the propagation of light through the same substance.

$$x = \kappa t, \quad y = \kappa t, \quad z = \kappa t, \quad \dots \quad (66)$$

These electromotive forces are due to the variations either of the electromagnetic or the electrostatic functions, as there is no motion of conductors in the field; so that the equations of electromotive force (D) are

$$\left. \begin{aligned} P &= -\frac{dF}{dt} - \frac{d\Psi}{dx}, \\ Q &= -\frac{dG}{dt} - \frac{d\Psi}{dy}, \\ R &= -\frac{dH}{dt} - \frac{d\Psi}{dz}. \end{aligned} \right\} \dots \dots \dots (67)$$

(94) Combining these equations, we obtain the following:—

$$\left. \begin{aligned} k\left(\frac{dJ}{dx} - \nabla^2 F\right) + 4\pi\mu\left(\frac{d^2 F}{dt^2} + \frac{d^2 \Psi}{dx dt}\right) &= 0, \\ k\left(\frac{dJ}{dy} - \nabla^2 G\right) + 4\pi\mu\left(\frac{d^2 G}{dt^2} + \frac{d^2 \Psi}{dy dt}\right) &= 0, \\ k\left(\frac{dJ}{dz} - \nabla^2 H\right) + 4\pi\mu\left(\frac{d^2 H}{dt^2} + \frac{d^2 \Psi}{dz dt}\right) &= 0. \end{aligned} \right\} \dots \dots \dots (68)$$

If we differentiate the third of these equations with respect to y , and the second with respect to z , and subtract, J and Ψ disappear, and by remembering the equations (B) of magnetic force, the results may be written

$$\left. \begin{aligned} k\nabla^2 \mu\alpha &= 4\pi\mu \frac{d^2}{dt^2} \mu\alpha, \\ k\nabla^2 \mu\beta &= 4\pi\mu \frac{d^2}{dt^2} \mu\beta, \\ k\nabla^2 \mu\gamma &= 4\pi\mu \frac{d^2}{dt^2} \mu\gamma. \end{aligned} \right\} \dots \dots \dots (69)$$

(95) If we assume that α, β, γ are functions of $lx + my + nz - Vt = w$, the first equation becomes

$$k\mu \frac{d^2 \alpha}{dw^2} = 4\pi\mu^2 V^2 \frac{d^2 \alpha}{dw^2}, \quad \dots \dots \dots (70)$$

or

$$V = \pm \sqrt{\frac{k}{4\pi\mu}}. \quad \dots \dots \dots (71)$$

The other equations give the same value for V , so that the wave is propagated in either direction with a velocity V .

(95) If we assume that α, β, γ are functions of $lx + my + nz - Vt = w$, the first equation becomes

$$k\mu \frac{d^2\alpha}{dw^2} = 4\pi\mu^2 V^2 \frac{d^2\alpha}{dw^2} \dots\dots\dots (70),$$

or

$$V = \pm \sqrt{\frac{k}{4\pi\mu}} \dots\dots\dots (71).$$

The other equations give the same value for V , so that the wave is propagated in either direction with a velocity V .

This wave consists entirely of magnetic disturbances, the direction of magnetization being in the plane of the wave. No magnetic disturbance whose direction of magnetization is not in the plane of the wave can be propagated as a plane wave at all.

Hence magnetic disturbances propagated through the electromagnetic field agree with light in this, that the disturbance at any point is transverse to the direction of propagation, and such waves may have all the properties of polarized light.

(96) The only medium in which experiments have been made to determine the value of k is air, in which $\mu = 1$, and therefore, by equation (46),

$$V = v \dots\dots\dots (72).$$

By the electromagnetic experiments of MM. Weber and Kohlrausch *,

$$v = 310,740,000 \text{ metres per second}$$

The velocity of light in air, by M. Fizeau's* experiments, is

$$V=314,858,000 ;$$

according to the more accurate experiments of M. Foucault †,

$$V=298,000,000.$$

The velocity of light in the space surrounding the earth, deduced from the coefficient of aberration and the received value of the radius of the earth's orbit, is

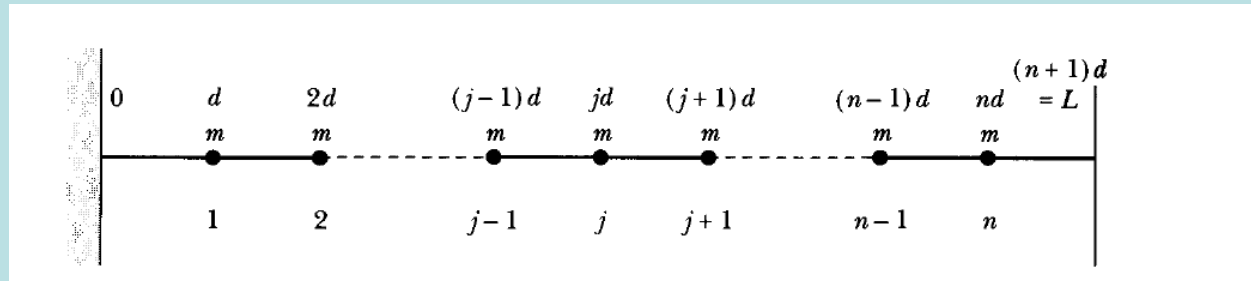
$$V=308,000,000.$$

(97) Hence the velocity of light deduced from experiment agrees sufficiently well with the value of v deduced from the only set of experiments we as yet possess. The value of v was determined by measuring the electromotive force with which a condenser of known capacity was charged, and then discharging the condenser through a galvanometer, so as to measure the quantity of electricity in it in electromagnetic measure. The only use made of light in the experiment was to see the instruments. The value of V found by M. Foucault was obtained by determining the angle through which a revolving mirror turned, while the light reflected from it went and returned along a measured course. No use whatever was made of electricity or magnetism.

The agreement of the results seems to shew that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.

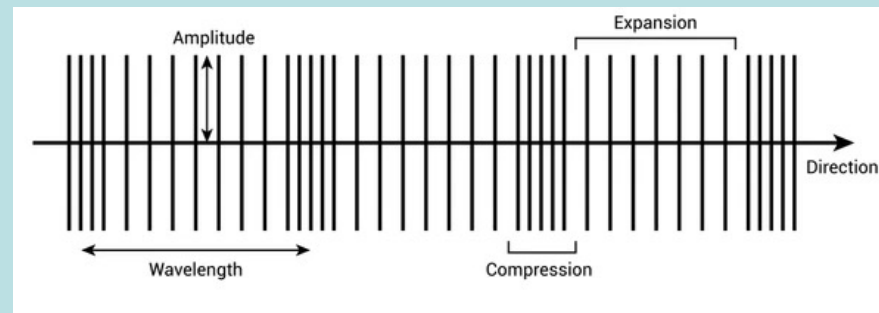
Consider a large number $N \rightarrow \infty$ of particles coupled by springs.

We will show equation of motion for coupled springs becomes wave equation.

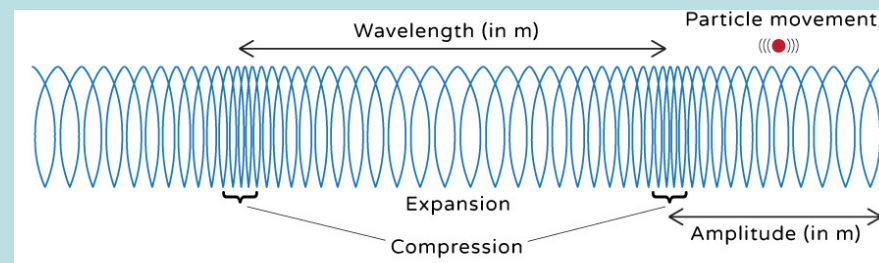


$$N \rightarrow \infty, d \rightarrow 0$$

Continuous limit



Elastic continuous medium

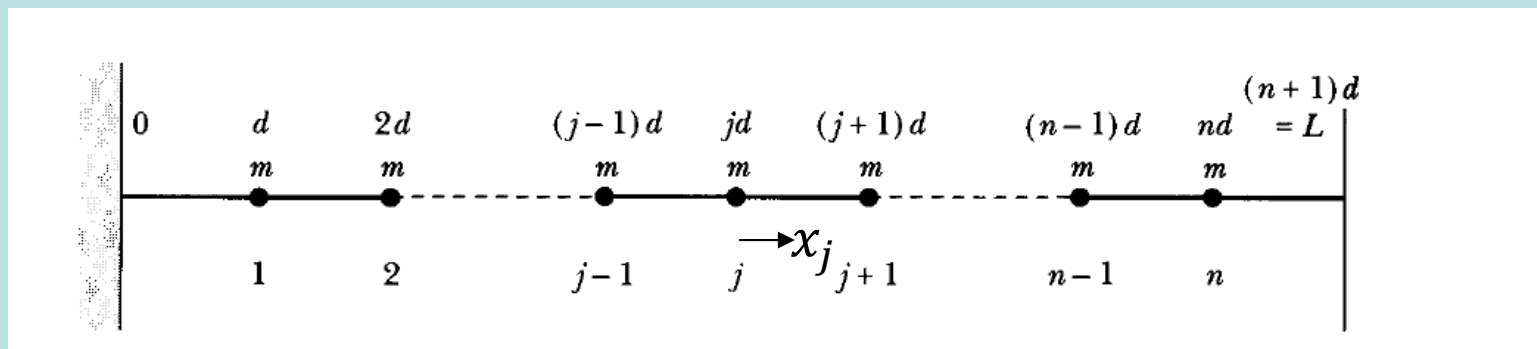


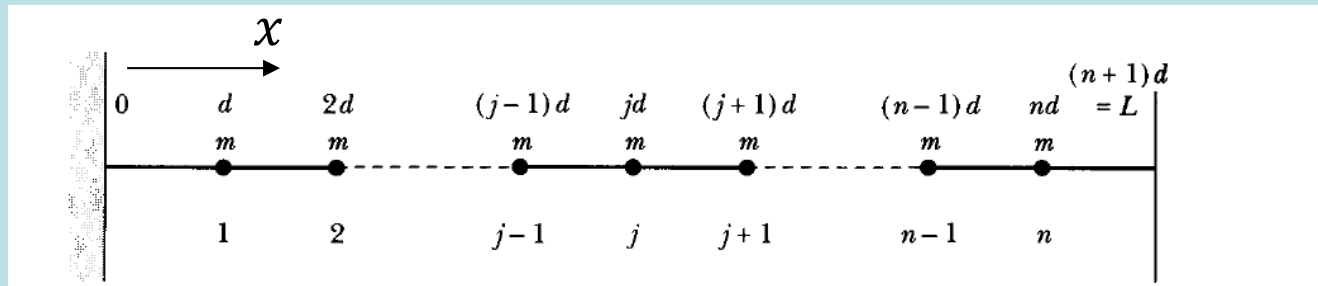
For large number N coupled springs: Equation of motion:

$$m \frac{d^2 x_j}{dt^2} = -k(x_j - x_{j-1}) + k(x_{j+1} - x_j)$$

$$\frac{d^2 \mathbf{X}}{dt^2} = -\mathbf{A} \cdot \mathbf{X} = -\omega_0^2 \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_N \end{pmatrix}$$

x_j is the displacement of j th particle away from its original positions jd .





$$\mathbf{X} \equiv \begin{pmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_N \end{pmatrix}$$

When the number N of particles is huge, column vector is no longer convenient. We assign displacement x_j of j th particle as a function of its original positions jd :

$$\phi(jd) \equiv x_j, \quad j = 1 \cdots N$$

When $N \rightarrow \infty, d \rightarrow 0$, discrete locations become continuous variable x :

$$jd \rightarrow x$$

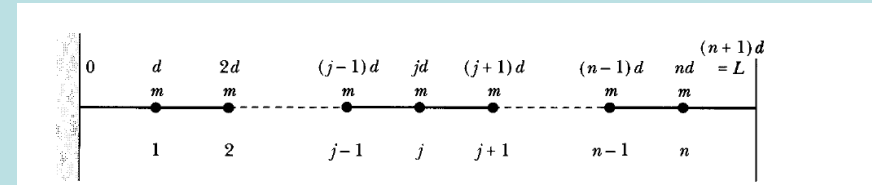
$$\phi(jd) = x_j \rightarrow \phi(x)$$

Now $\phi(x)$ is displacement of the particle whose equilibrium position is x .

It is also time dependent: $\phi(x, t)$.

We get a two-variable function, called wavefunction.

$$m \frac{d^2 x_j}{dt^2} = +k(x_{j+1} - x_j) - k(x_j - x_{j-1})$$



From the equation of motion of coupled springs,
we plug in the new notation: $\phi(dj) \equiv x_j$

$$m \frac{d^2 \phi(dj)}{dt^2} = +k(\phi(dj + d) - \phi(dj)) - k(\phi(dj) - \phi(dj - d))$$

Divide the difference of displacements ϕ by the separation d .

$$= +kd \frac{\Delta \phi(dj)}{d} - kd \frac{\Delta \phi(dj - d)}{d} \quad \text{Become derivatives as } d = \Delta x \rightarrow 0.$$

$$\rightarrow kd \left[\frac{d\phi}{dx}(dj) - \frac{d\phi}{dx}(dj - d) \right] \quad \text{Difference of derivatives at neighboring sites.}$$

$$= kd^2 \frac{\Delta \frac{d\phi}{dx}}{\Delta x} \quad \text{It equals the derivative of derivative.}$$

$$= kd^2 \frac{d^2 \phi}{dx^2}(dj) \quad \text{Finally} \quad \frac{d^2 \phi}{dt^2}(x) = \frac{kd^2}{m} \frac{d^2 \phi}{dx^2}(x)$$

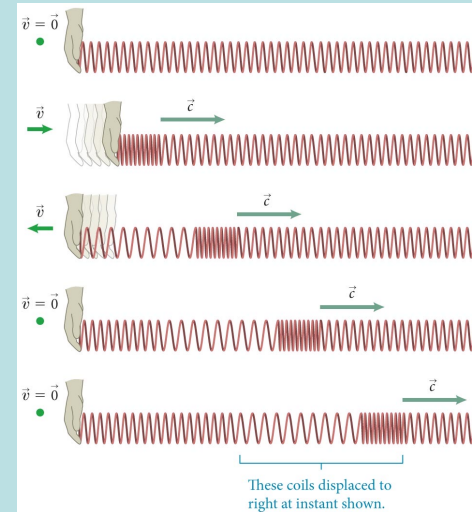
$$v \equiv \sqrt{\frac{kd}{m/d}} \equiv \sqrt{\frac{\tau}{\mu}}$$

$\frac{d^2 \phi}{dt^2}$ is differentiated as $x = dj$ is fixed, being partial differentiation $\frac{\partial^2 \phi}{\partial t^2}$.

$$\text{Same for } \frac{d^2 \phi}{dx^2}, \text{ differentiated with fixed } t. \quad \rightarrow \quad \frac{\partial^2 \phi}{\partial t^2}(x, t) = v^2 \frac{\partial^2 \phi}{\partial x^2}(x, t)$$

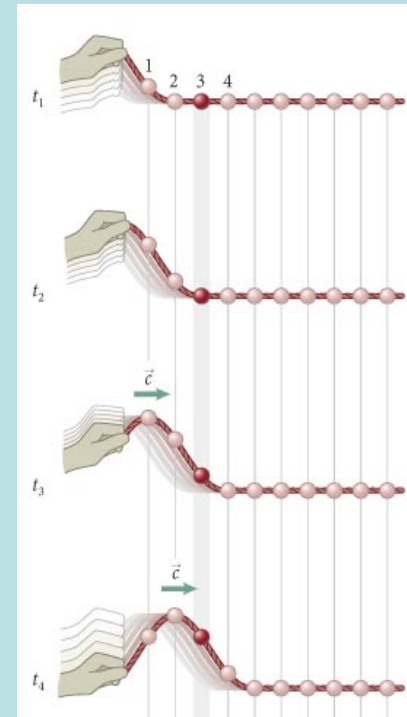
彈性介質縱波的波方程式

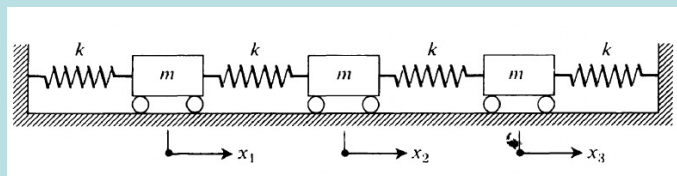
$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$



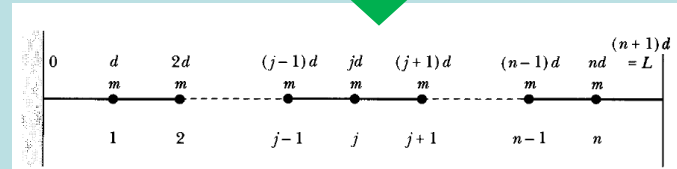
彈性介質橫波的波方程式

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

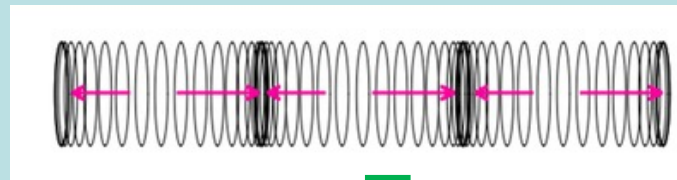




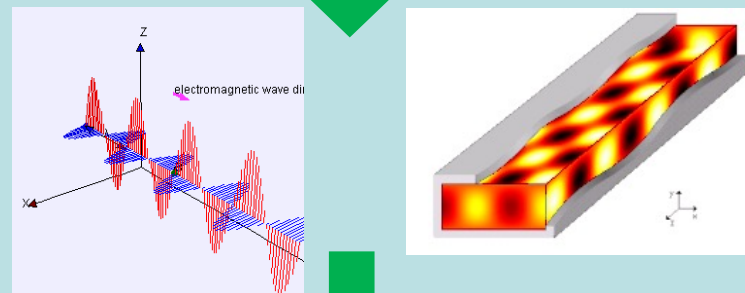
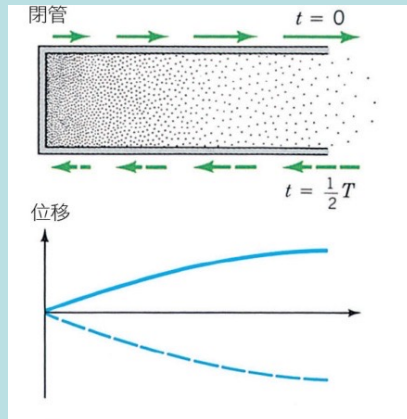
Coupled Springs



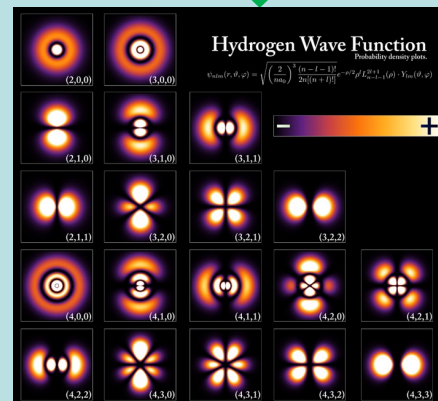
Continuous limit $N \rightarrow \infty, d \rightarrow 0$



Continuous medium wave



Electromagnetic Wave
and all other waves



Schrodinger Electron Wave



Solve Wave equation to find its solutions.

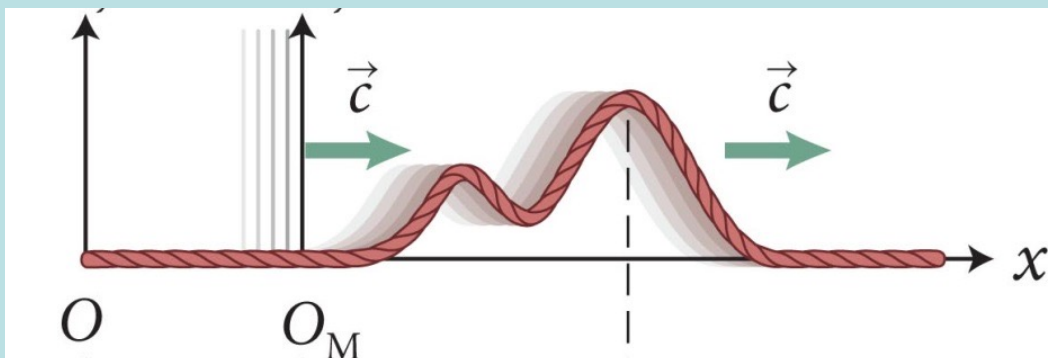
If nature follows wave equation, its solution describe natural phenomenon.

The first (easiest) of the two ways to solve wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

以下證明：這個方程式有以下的解：

$$f(x - vt)$$



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad \text{波方程式}$$



Plug in the expression, which means it is only function of $x' \equiv x - vt$.

$$y(x, t) = f(x - vt) = f(x') \quad x' \equiv x - vt$$

$$\frac{\partial y}{\partial x} = \frac{df}{dx'} \cdot \frac{dx'}{dx} = \frac{df}{dx'} \quad \text{fix } t \text{ and use chain rule.} \quad \frac{dx'}{dx} = 1 \quad \text{with } t \text{ fixed}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{d}{dx} \frac{df}{dx'} = \frac{dx'}{dx} \cdot \frac{d}{dx'} \frac{df}{dx'} = \frac{d^2 f}{dx'^2}$$

$$\frac{\partial y}{\partial t} = \frac{df}{dx'} \cdot \frac{dx'}{dt} = \frac{df}{dx'} \cdot (-v) \quad \text{fix } x \text{ and use chain rule.} \quad \frac{dx'}{dt} = -v \quad \text{with } x \text{ fixed}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{d}{dt} \frac{df}{dx'} \cdot (-v) = \frac{dx'}{dt} \cdot \frac{d}{dx'} \frac{df}{dx'} \cdot (-v) = \frac{d^2 f}{dx'^2} \cdot (-v)^2$$

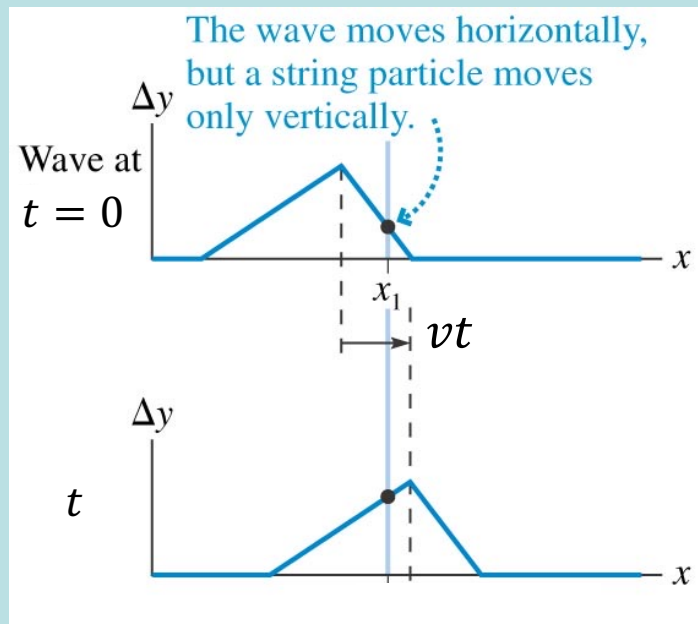
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Indeed, what we guessed is a solution of wave equation !

x' 將時間與空間連鎖在一起，於是時間微分與空間微分也是連鎖的。

Function $f(x)$ is $t = 0$ wave form at $t = 0$ ◦

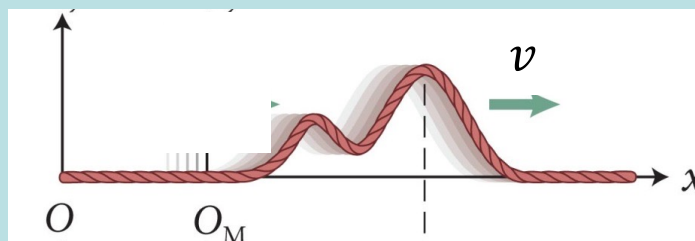
$$y(x, t = 0) = f(x - v \cdot 0) = f(x)$$

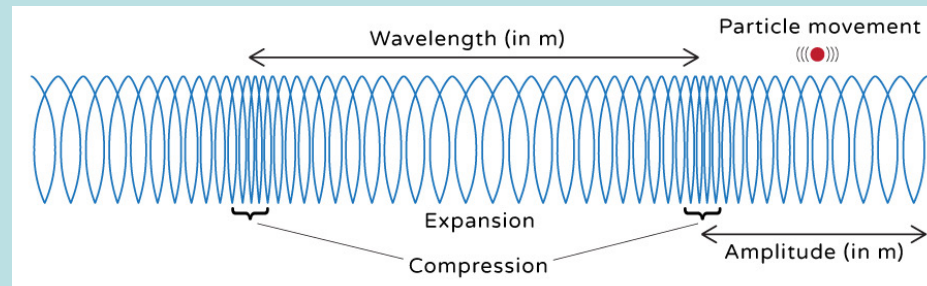


$$y(x, t) = y(x - vt, 0) = f(x - vt)$$

Later time t , wave at x equals wave at $t = 0$ at location $x - vt$.

Hence $f(x - vt)$ means wave form $f(x)$ is moving at speed v as we expected.





Longitudinal and Transverse Waves (HD)
Animations for Physics and Astronomy

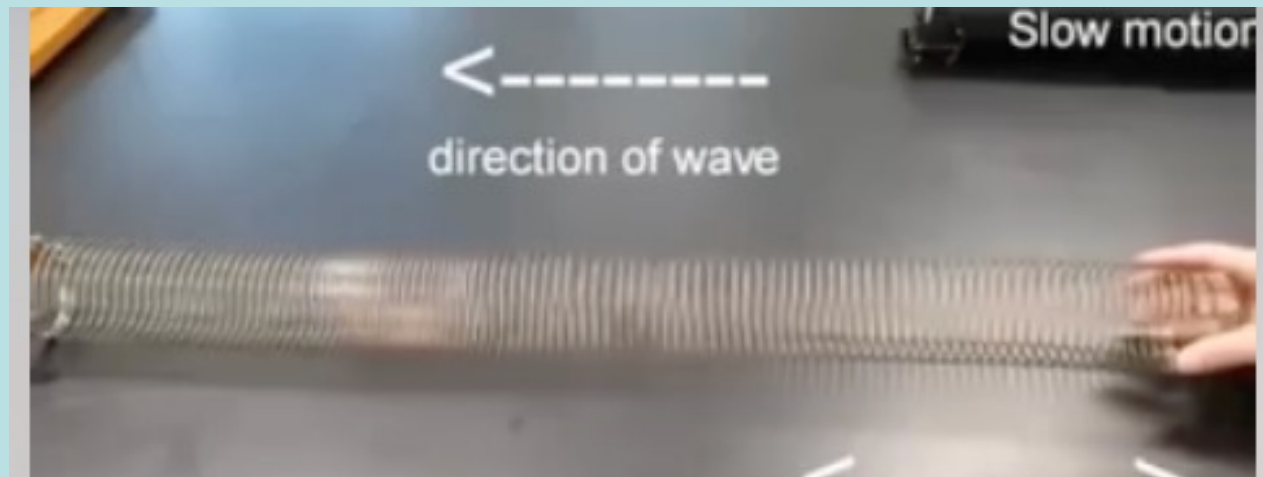
0:00 / 0:24

Longitudinal Wave

More videos

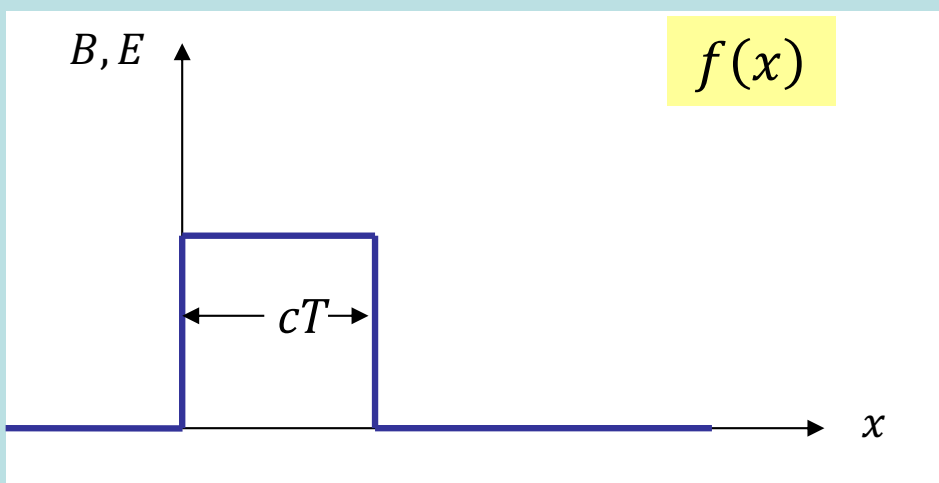
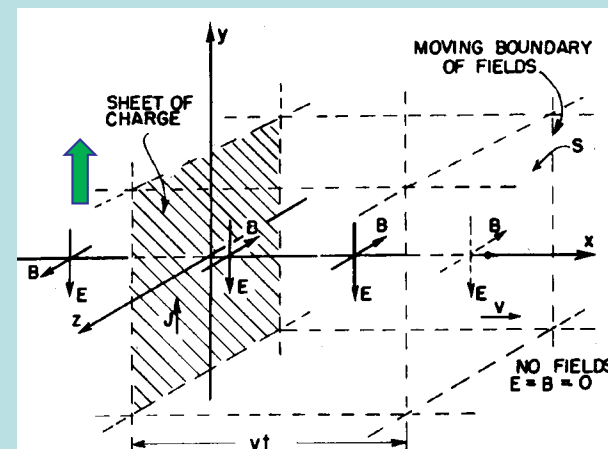
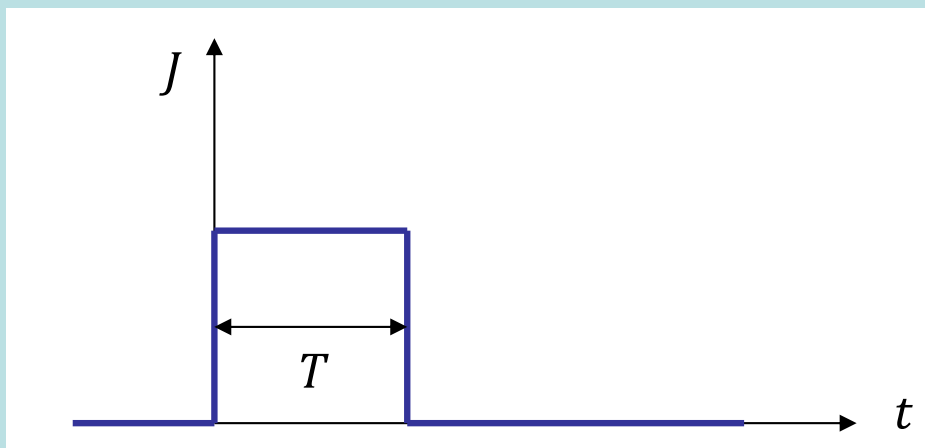
YouTube

The video player interface shows a dark blue background. At the top, there is a title bar with the video title and channel name, along with volume, closed captions, and settings icons. The main content area displays a 3D animation of a coiled spring with a yellow handle on the left. A red double-headed arrow indicates the longitudinal oscillation of the spring. A white play button is centered over the spring. The bottom of the player features a progress bar, a share icon, and a 'More videos' button with a YouTube logo.

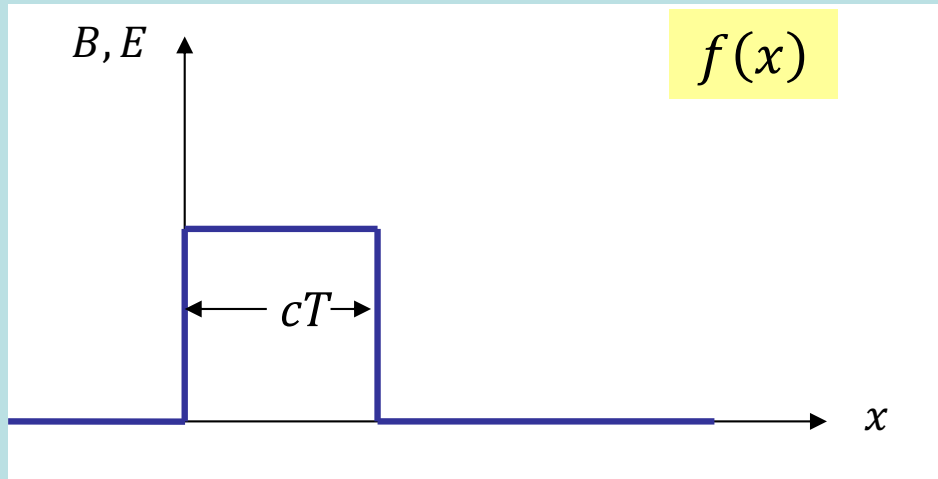




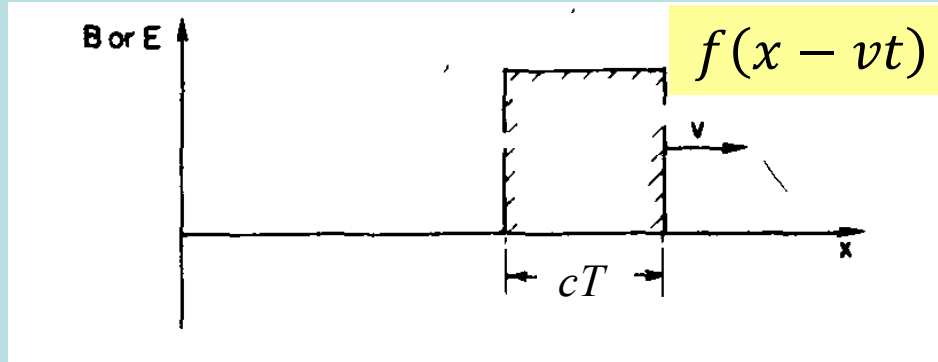
方塊波：若電流突然產生，一段時間 T 後突然停止：



瞬間電流產生一個方塊狀波形的脈衝電磁場。



瞬間電流產生一個方塊狀波形的脈衝電磁場。

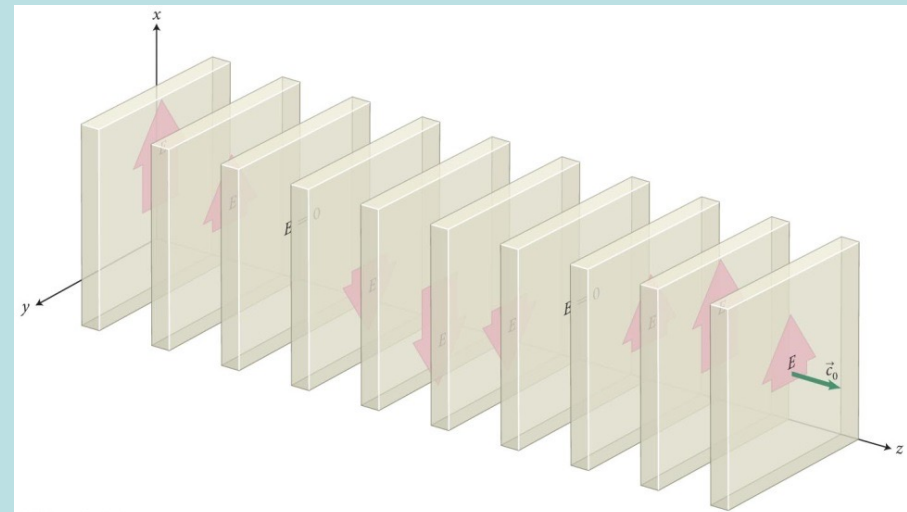
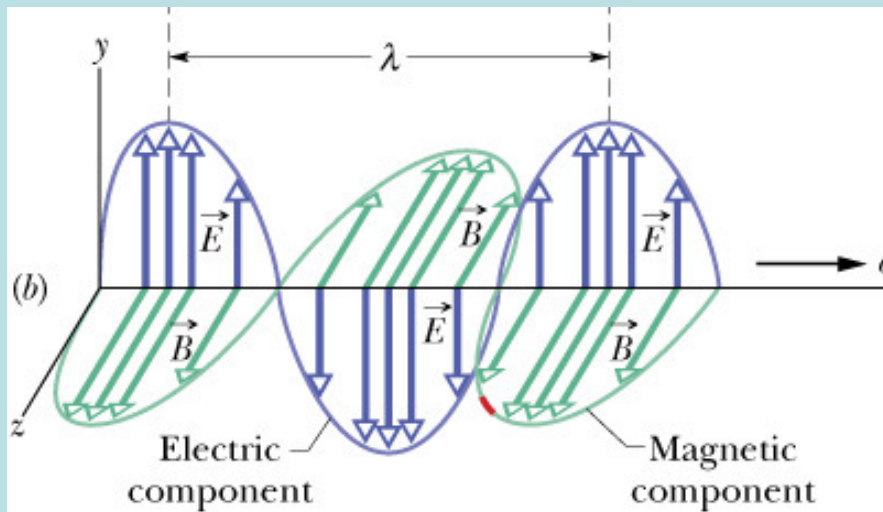


此脈衝方塊波形的電磁場（前後都為零）將以定速 c 向前孤獨地傳播。

此脈衝在電流消失後還會在遠方繼續傳播！

此脈衝的電場與磁場是獨立於產生它們的電流而存在的！

正弦電磁波



$$E(x, t) = E_m \sin[k(x - vt)] = E_m \sin(kx - \omega t)$$

$$f(x) = E_m \sin kx$$

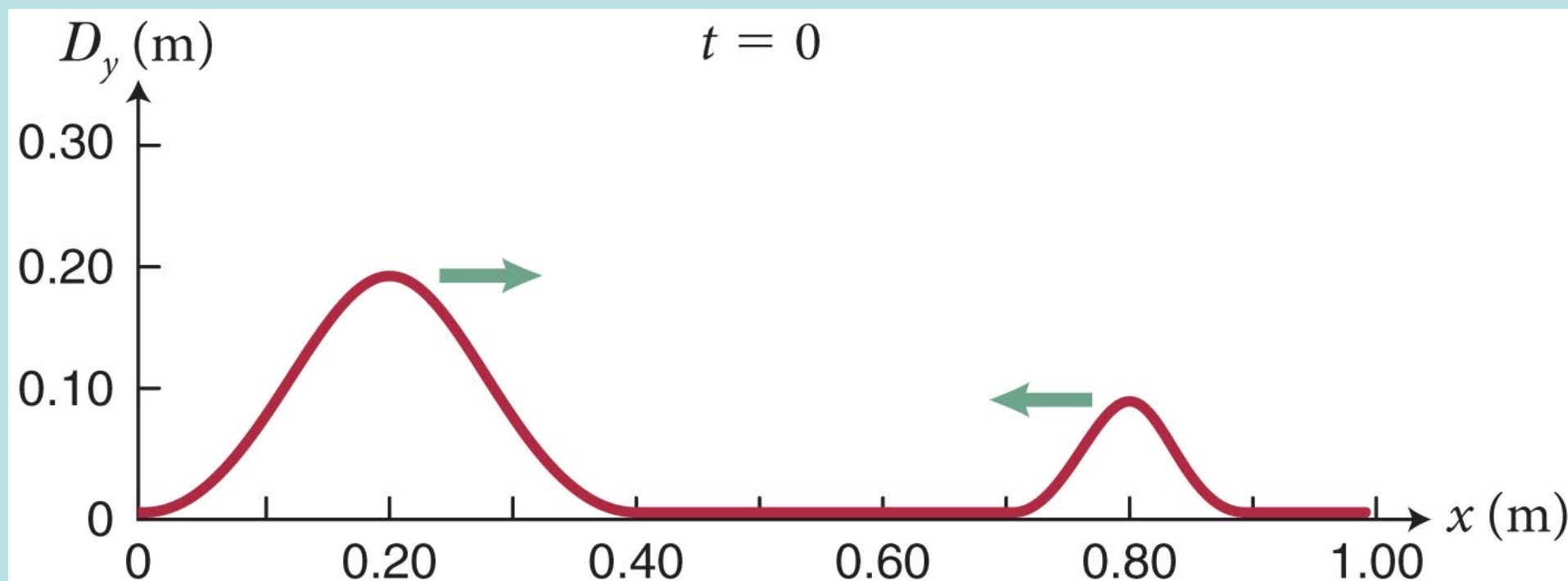
$$E(x, t) = cB(x, t)$$

$$\frac{\omega}{k} = f\lambda = c$$

正弦電磁波中電場與磁場方向垂直，大小隨時隨地都成正比！



向 $-x$ 移動的波，將 v 以 $-v$ 取代即可



$$y(x, t) = g(x + vt)$$

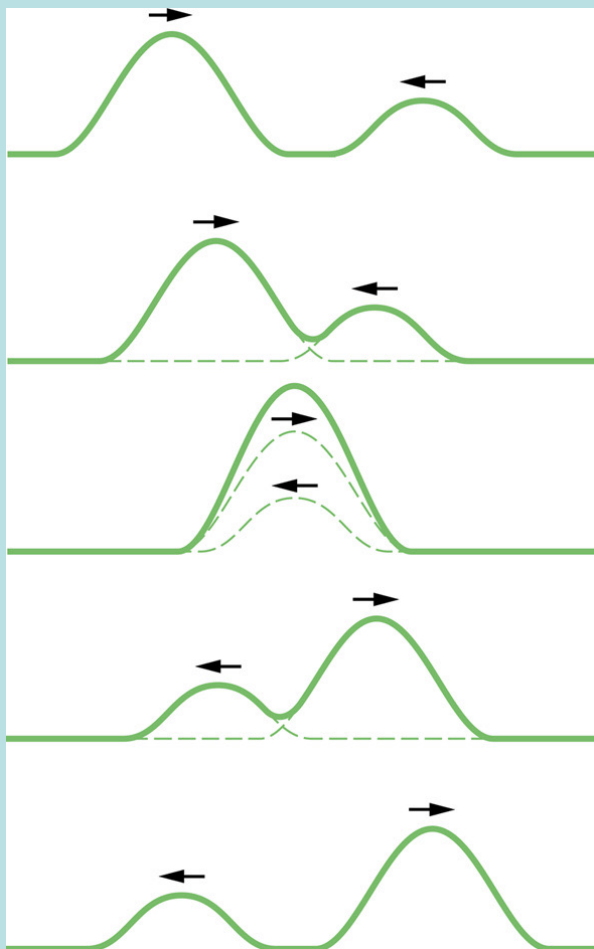
疊加定理

波方程式是線性方程式：兩個波方程式的解的和依舊是波方程式的解：

$$\frac{\partial^2 y_1}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y_1}{\partial t^2} \quad \frac{\partial^2 y_2}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y_2}{\partial t^2} \quad \rightarrow \quad \frac{\partial^2 (y_1 + y_2)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 (y_1 + y_2)}{\partial t^2}$$

$y_1 + y_2$ 依舊是波方程式的解：

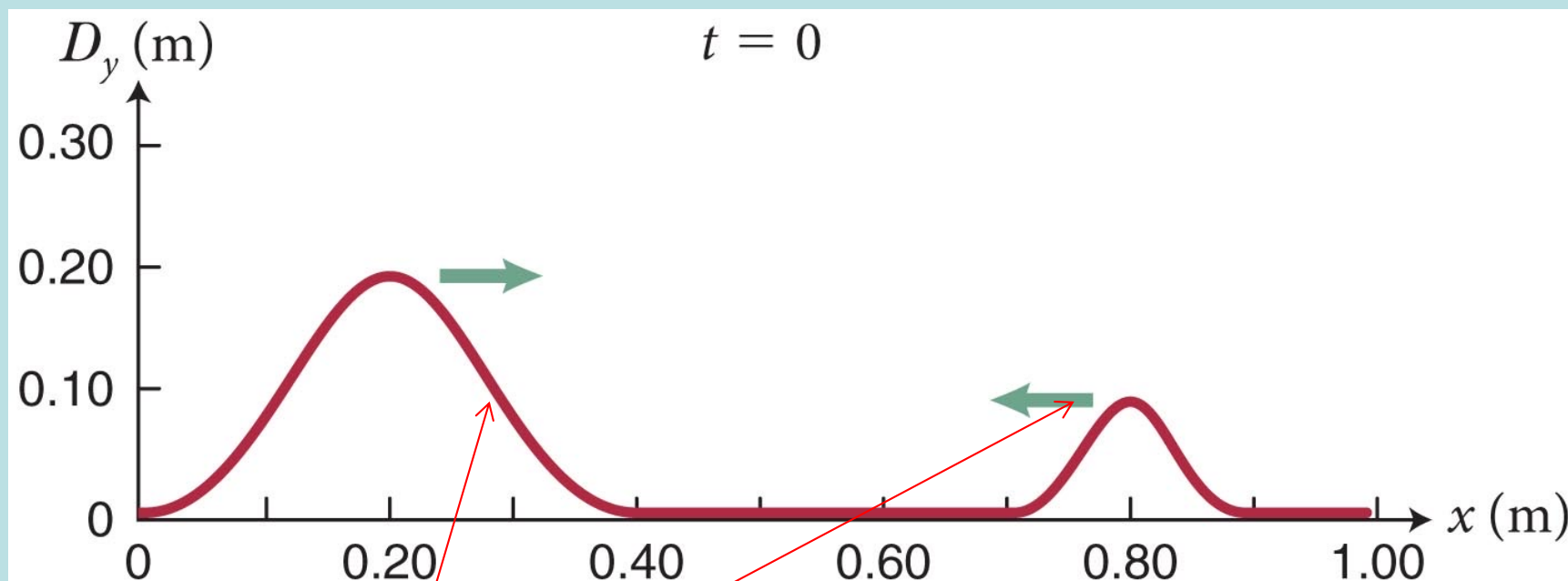
兩個分立的波重疊時，只要將兩個波函數相加即可。
之後若又分立，原來重疊前的波型不變。





弦波波函數的最普遍解d'Alembert solution :

$$y(x, t) = f(x - vt) + g(x + vt)$$



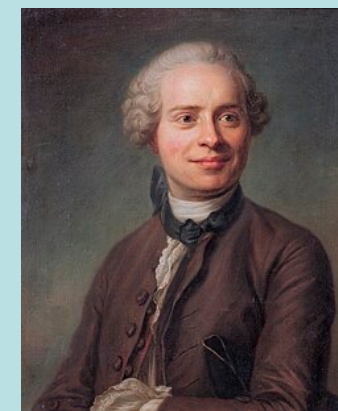
而 $f(x)$ ($g(x)$) 即為向右 (左) 傳播的波的瞬間波型

The wave form propagates at constant wave speed.

The wave form does not change during propagation.

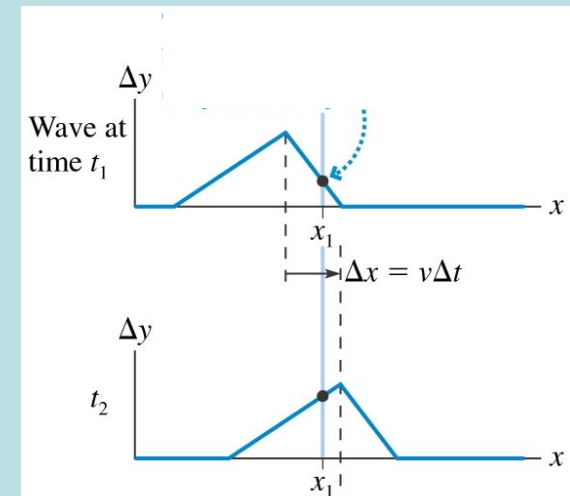
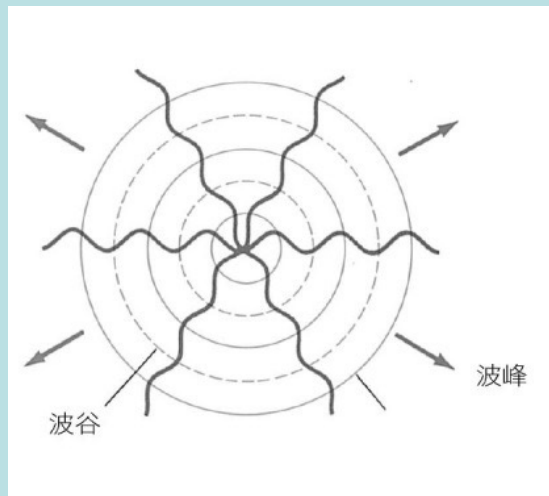
疊加定律

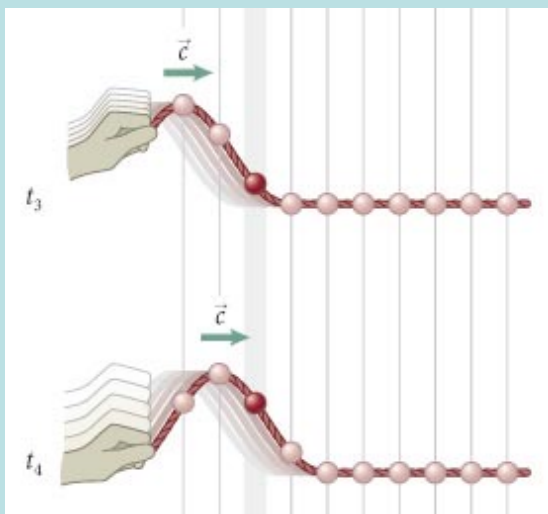
以上結果適用於任何滿足波方程式的波動現象！



Jean le Rond d'Alembert 1717-83

Propagating wave





波方程式其實是一群粒子的運動方程式。

它的唯一解是由所有粒子的初位置及初速度兩個起始條件完全決定。

在此這就對應：起始的弦位移 $y(x, 0)$ ，起始的弦垂直方向速度 $\frac{\partial y}{\partial t}(x, 0)$ 。

由 $y(x, 0)$ 及 $\frac{\partial y}{\partial t}(x, 0)$ 正好可以解出這兩個函數：

$$y(x, 0) = f(x) + g(x)$$

$$y(x, 0), \frac{\partial y}{\partial t}(x, 0) \leftrightarrow f(x), g(x)$$

$$\frac{\partial y}{\partial t}(x, 0) = v[f'(x) - g'(x)]$$

因此由 $f(x), g(x)$ 得到滿足方程式及起始條件的唯一解：

$$y(x, t) = f(x - vt) + g(x + vt)$$