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Make Mathematical Physics really physics and really mathematics.

Not just solution manual.

能力

大學學習的重點是能力，不是知識

知識已經太過複雜，但容易取得，
而且許多人未來的發展還有各式的可能！

知識

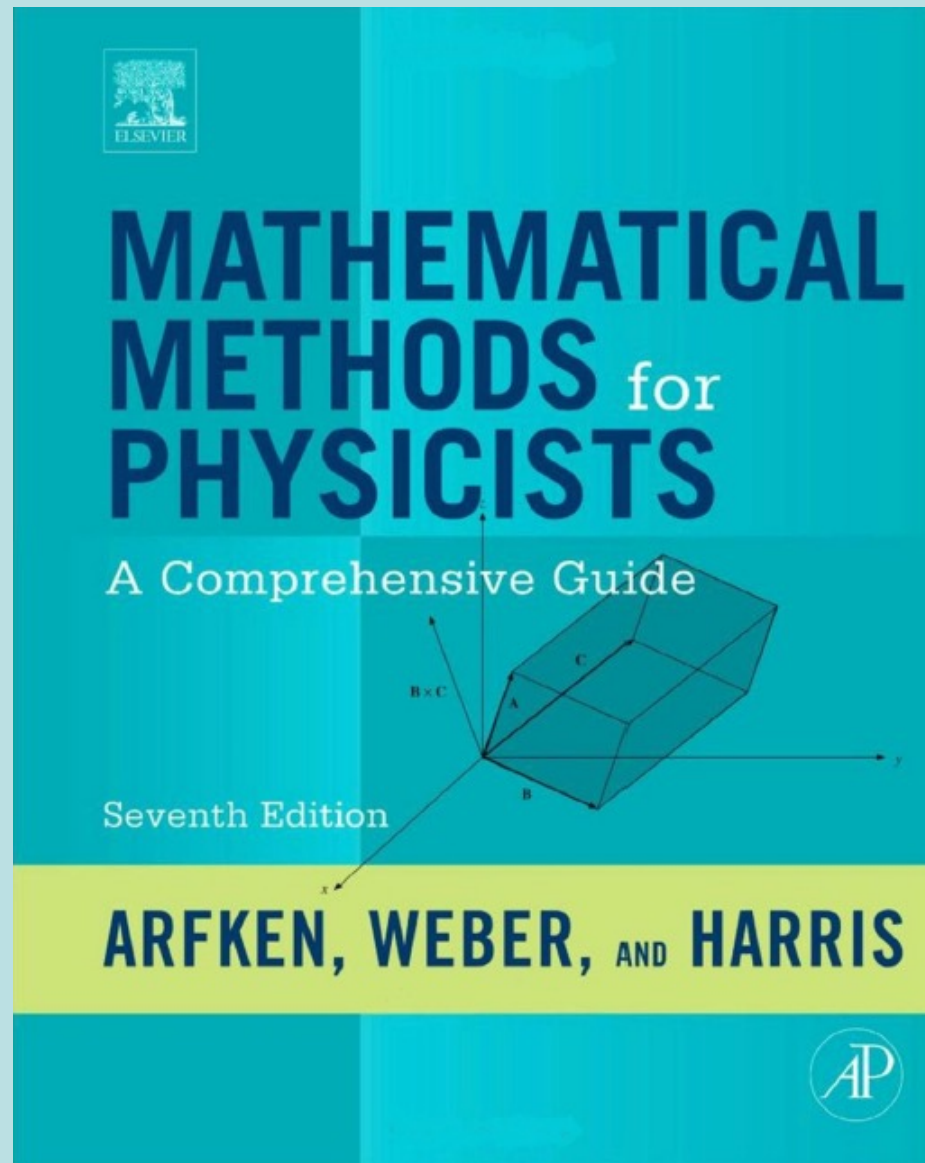
但培養能力，必須透過知識的學習

只有透過知識的**學習、思考、討論、掙扎、挫折**，
才能培養思考與解決問題的能力！

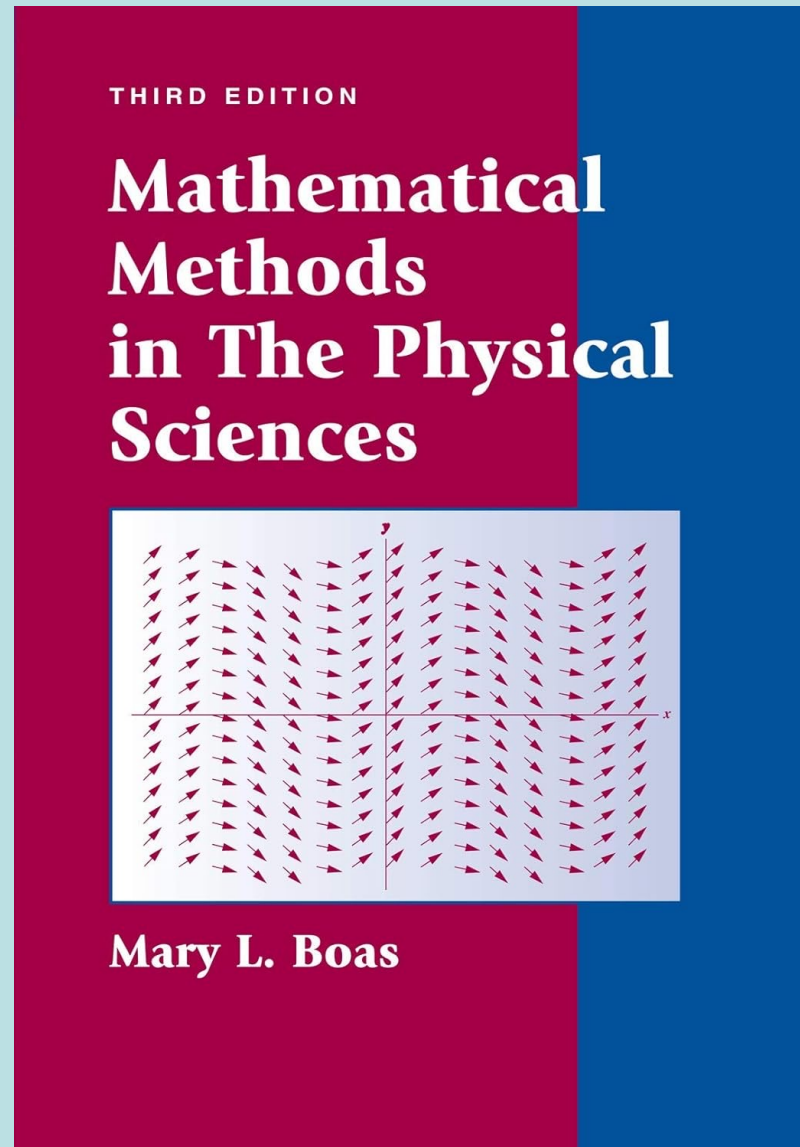
思考力的重訓！



with everything



Easiest to approach



Mathematical methods for application to problems in physics.

數學物理 Mathematical Physics 典型的三大數學主題：

向量微積分 Vector Calculus

偏微分方程式 Partial Differential Equation 粒子與場的運動方程式

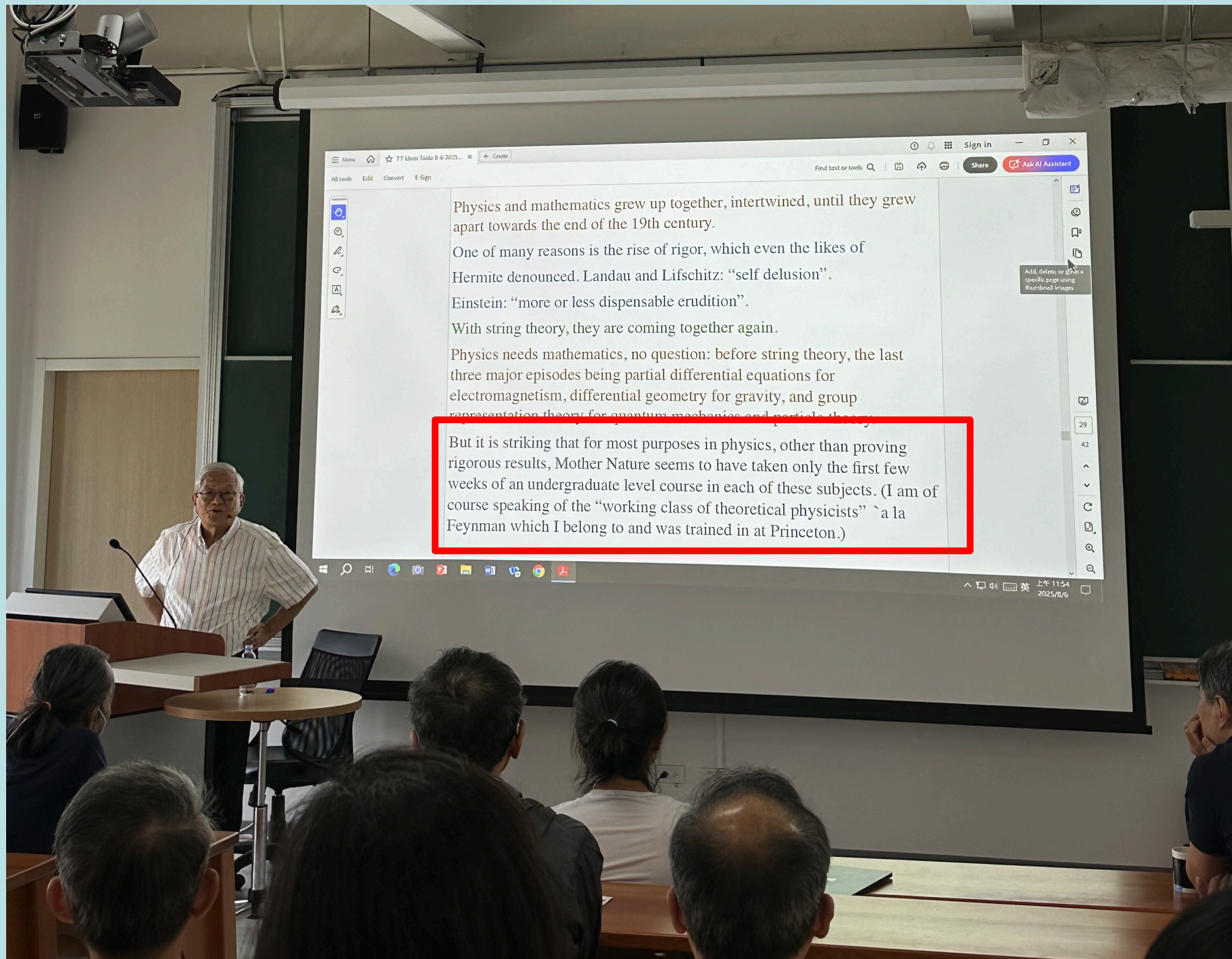
線性代數 Linear Algebra 向量空間的代數

複變函數 Functions of complex variables

We will not insist on covering these mathematical topics comprehensively.

But we will insist on covering their physics as comprehensively as allowed.

This is a physics centric course and is to show how mathematics can help us.





But it is striking that for most purposes in physics, other than proving rigorous results, Mother Nature seems to have taken only the first few weeks of an undergraduate level course in each of these subjects. (I am of course speaking of the “working class of theoretical physicists” `a la Feynman which I belong to and was trained in at Princeton.)



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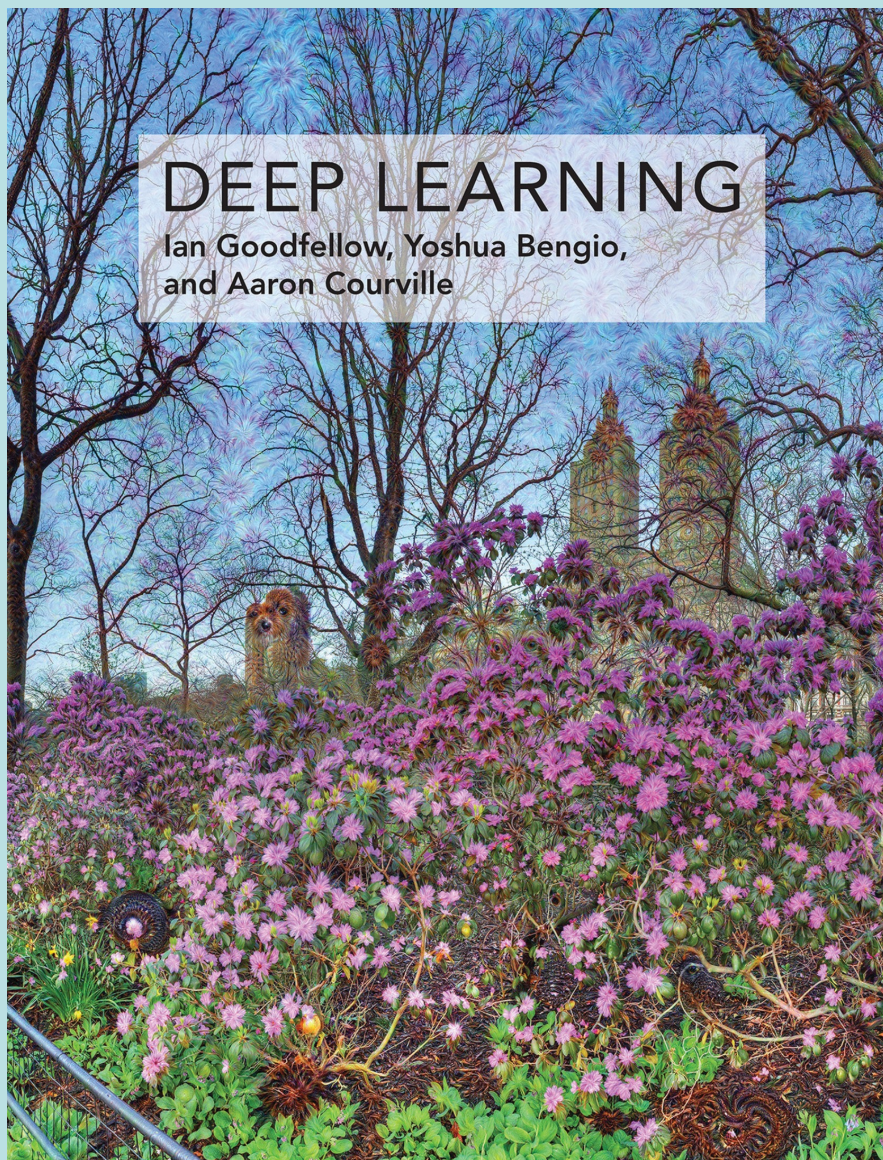
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線性代數竟然是AI深層學習的數學基礎



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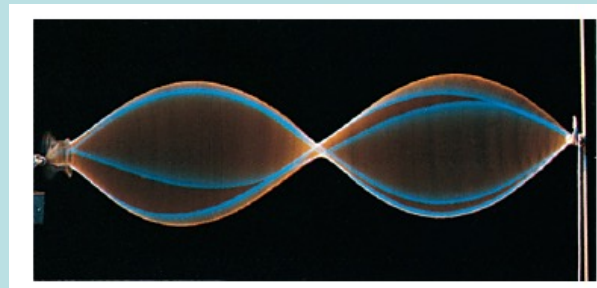
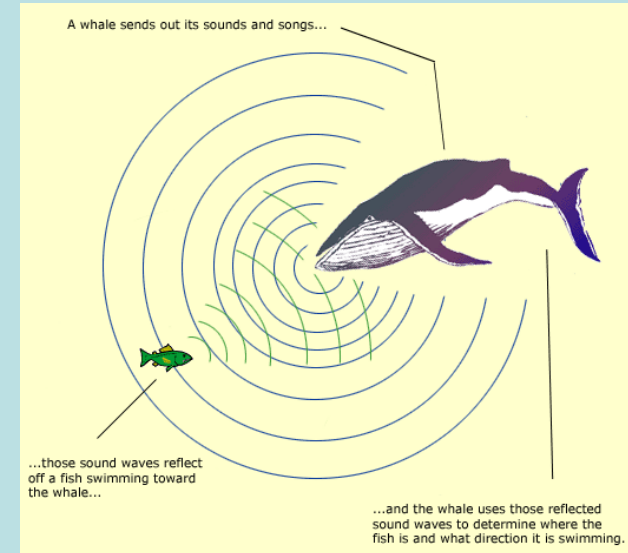
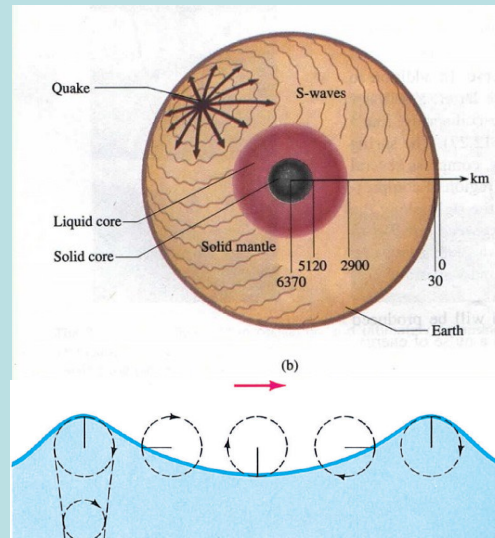
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波方程式 Wave Equation will be the first organizing topic.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$



This is the identical equation of motion of all wave phenomena.

Identical equation leads to identical solution. All wave systems have identical solutions.

Wave Equation without no boundaries

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Solution : $f(x - vt) + g(x + vt)$

波動的特徵皆來自此方程式：

波型以定速傳播

波型在傳播過程中不變形

疊加定律

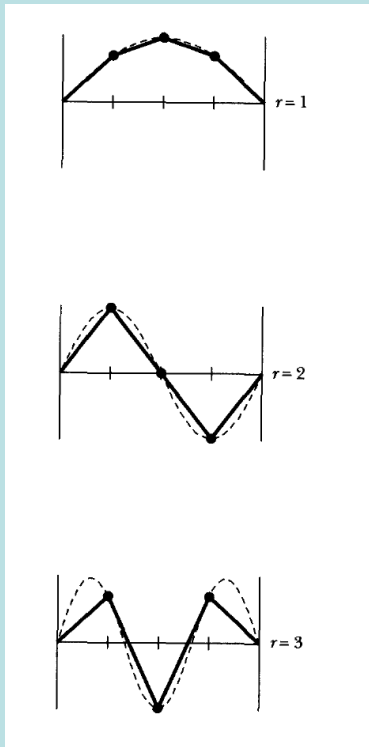


If medium size is limited, it resemble coupled spring system.

To solve coupled spring equation of motion is to solve matrix eigenvector problem.

One eigenvector corresponds to one independent oscillation mode.

General solutions are linear combinations of oscillation modes.



$$\mathbf{a}^{(1)} \sim \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\mathbf{a}^{(2)} \sim \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\mathbf{a}^{(3)} \sim \begin{pmatrix} 1 \\ \frac{1}{\sqrt{2}} \\ -1 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Schedule:

1st week Gamma Function

2nd week Review of partial differentiation and vector calculus. Derivation of EM wave equation. Partial differential equation!

Review of Coupled Springs and continuous limit of the equations of motion as classical Wave Equation.

3rd week Solving equations of motion of coupled springs by oscillation modes.

Solving classical wave equation by the method of separation of variables.

ODE for space part solution as an eigenvalue and eigenfunction problem. |

The importance of wave equation goes beyond the above wave phenomena.
到此都是古典物理。

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這是一個量子世界！

3

THE WORLD IS QUANTUM

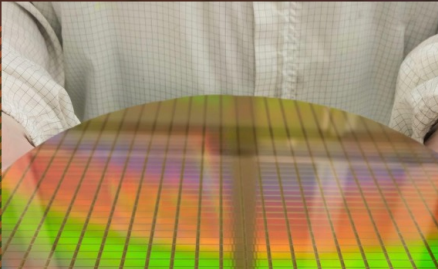
Mysterious and mystifying

Those who are not shocked when they first come across quantum theory cannot possibly have understood it.¹

沒有量子物理就沒有半導體！你天天都在做實驗，證實量子力學是對的！

 發現 節目 有聲書 課程 節目主持人/有聲書主播 訂閱說明 APP下載

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節目 知識好好玩

EP08 | 台灣這美麗的矽島——聊一聊半導體物理

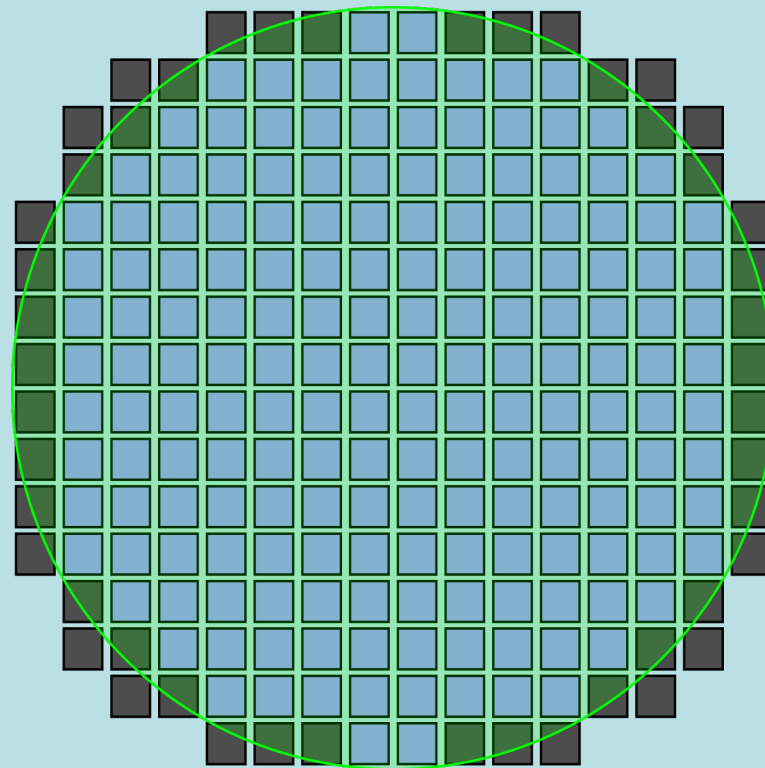
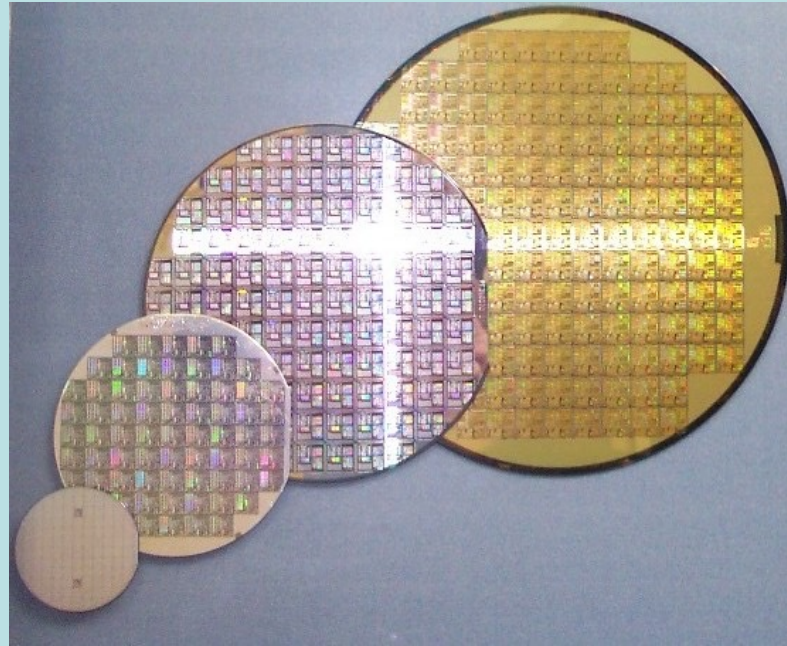
主持人 | 張嘉泓

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開始播放

以上所描述的電晶體配置有一個很長的名字：金屬氧化物半導體、場效型電晶體 MOSFET，比起傳統的雙極型電晶體，場效電晶體更適合於平面的半導體大量製造，而且體積可以極度縮小。如上所述，場效電晶體的主體是由p型與n型半導體分布組成，這個分布可以規劃在二維平面上，於是由電晶體所組成的電路就可以在一片平板上展開。製作時會以矽為晶片基板，第一步先全部摻雜為p型半導體。接著先設計好藍圖，在平面上安排出n型半導體的位置。第三步，在基板上塗一層感光的光阻劑，然後將設計畫在一片光罩上，當光罩放置於基板上時，光罩的圖案會遮蓋規劃為p型半導體的區域。接著以紫外線曝照，消除掉光罩圖案未遮蓋處的光阻劑，於是規劃為n型半導體區域的基板就曝露在外。最後，將此曝露的部分滲進適當雜質，就得到所設計的n型半導體分布，而光阻劑覆蓋的部分則維持原來的p型。利用這樣的製程就可以在晶片上建構出你所設計的分布，其他如絕緣層、導電線路與電極也可以用類似的方式往上堆疊建構。

上一段描述是不是讓你感覺很枯燥、很機械化？那就對了，一個接著一個的步驟，有條不紊，如此，就可以大量機械化生產。而且平面化的設計，可以將電路非常節省空間地集中在一塊小晶片上，這就稱為積體電路。它的最大好處在於，只要你的藍圖光罩夠精細，電晶體的大小幾乎可以無限地縮小。如此我們才能把驚人的計算能力，置於一個日常生活能夠輕鬆攜帶的裝置。積體電路在1960年代出現，大概從



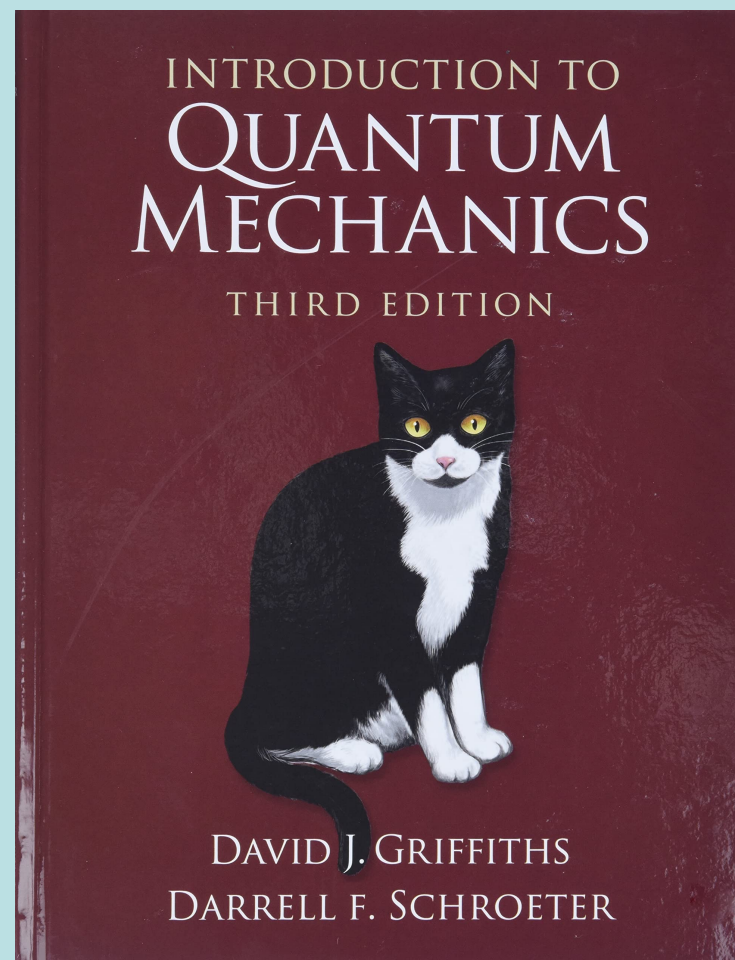
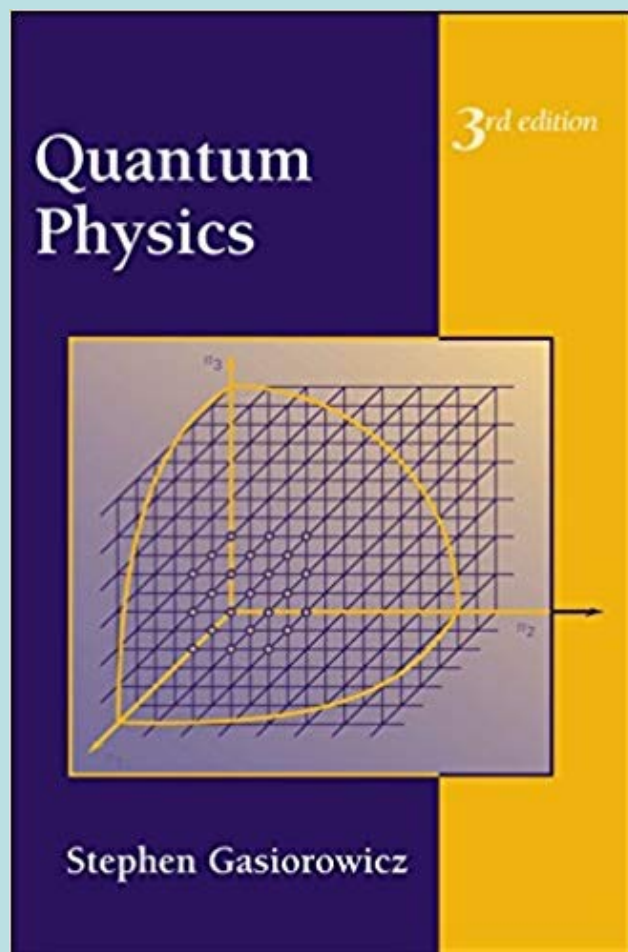






EUV stands for extreme ultraviolet light, which produces the circuitry.

這堂數學物理的課其實是要偷渡量子物理到大二。



本課程將以近代量子物理面對的問題為核心，介紹量子力學基本概念，闡述解決這些問題的數學方法。

$$x(t)$$



$$\psi(x, t)$$



$$m \frac{d^2 x}{dt^2} = - \frac{\partial V}{\partial x}$$



$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$



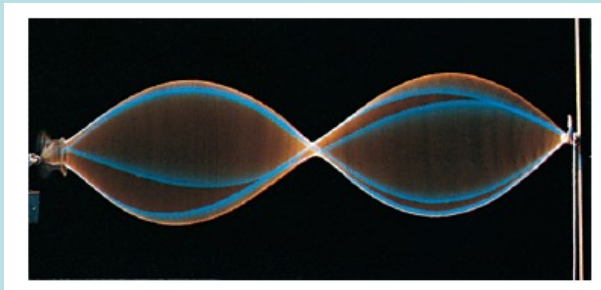
這是一個嶄新的時代開始！描述微觀世界的方法不同了！

It is the beginning of a new age!

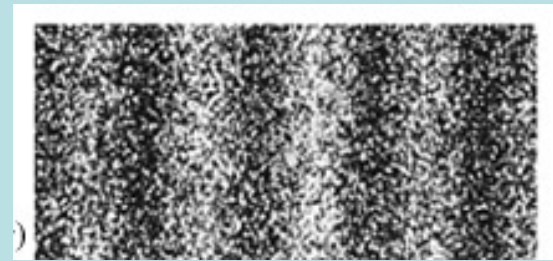
The way quantum world works is totally different.

此波方程式與古典波方程式，同是偏微分方程，式在數學上有許多相似之處！

Schrodinger Wave Equation is a Partial Differential Equation very similar to classical wave equation.



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$



$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

The technique to solve them and their solution are very similar, too.

The technique: Separation of variables and Eigenvalue problem.

Eigenvalue problems 這個字再次重複出現！

數學物理的最重要洞見就是：這兩個解物理問題的絕招：

Differential Equation (Ordinary and Partial) 微分方程式

Vector and Matrix 向量與矩陣

微分方程式與矩陣，竟然有一個共通的數學結構：

Eigenvalues and Eigenfunctions 本徵值與本徵函數

這個課的課名應該叫：“本徵值問題”

求解偏微分方程式、線性代數方程式與矩陣，竟然有共通的數學結構：

Eigenvalues and Eigenfunctions 本徵值與本徵函數，

這個課的課名應該叫：“本徵值問題”

Quantisation as an eigenvalue problem;
by E. Schrödinger*

Linear Algebra 線性代數與偏微分方程式的本徵值問題。

THE WORLD IS QUANTUM

4th week Bohr Atom Model and Atomic Spectrum. Electrons as Waves. De Brogie and Schrödinger. Electron is a particle and a wave.

5th week Probability Interpretation of Wave. Schrödinger vs Max Born.

Uncertainty Principle. Heisenberg vs Schrödinger. Copenhagen Interpretation.

6th week Schrödinger Wave Equation for 1D infinite well potential.

Method of separation of variable for solving Schrödinger Wave Equation



7th week Fourier Series

8th week Simple harmonic oscillator and Energy Quantization.

9th week Midterm exam

10th week Sturm-Liouville eigenvalue problem

11th week Delta function and Fourier Transformation.

12th week | Curvilinear coordinates. Schrödinger Wave Equation for 3D Hydrogen Atom.

13th week Differential Legendre and Laguerre Equations and Series solution.

14th week Special Functions

15th week Functions of Complex Variables

16th week Final Exam