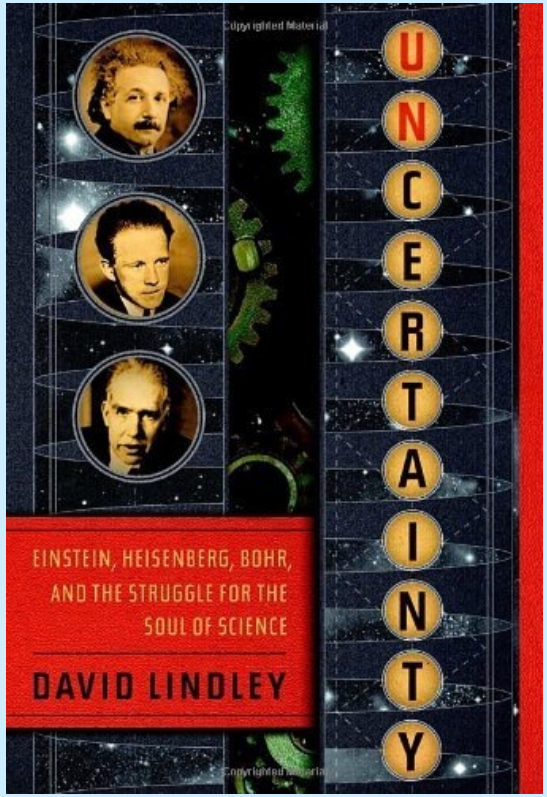
1926，量子物理發展關鍵的一年



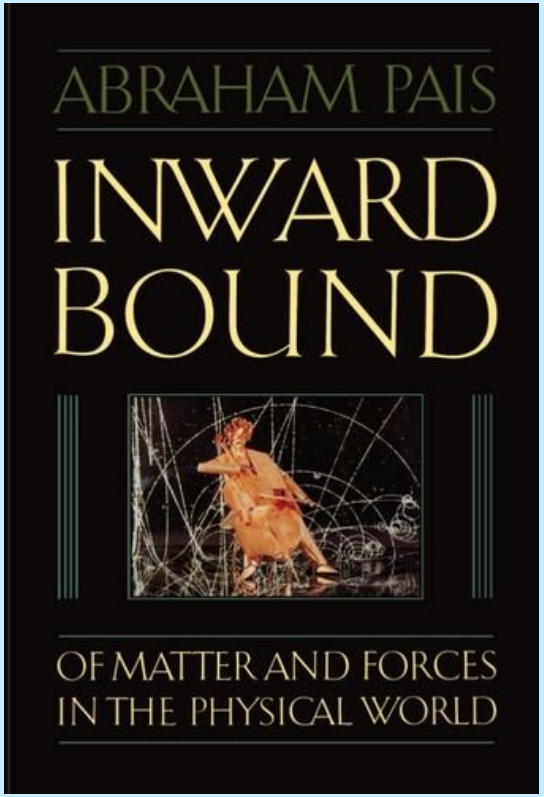
談波恩的機率解釋

揭開電子的真面目

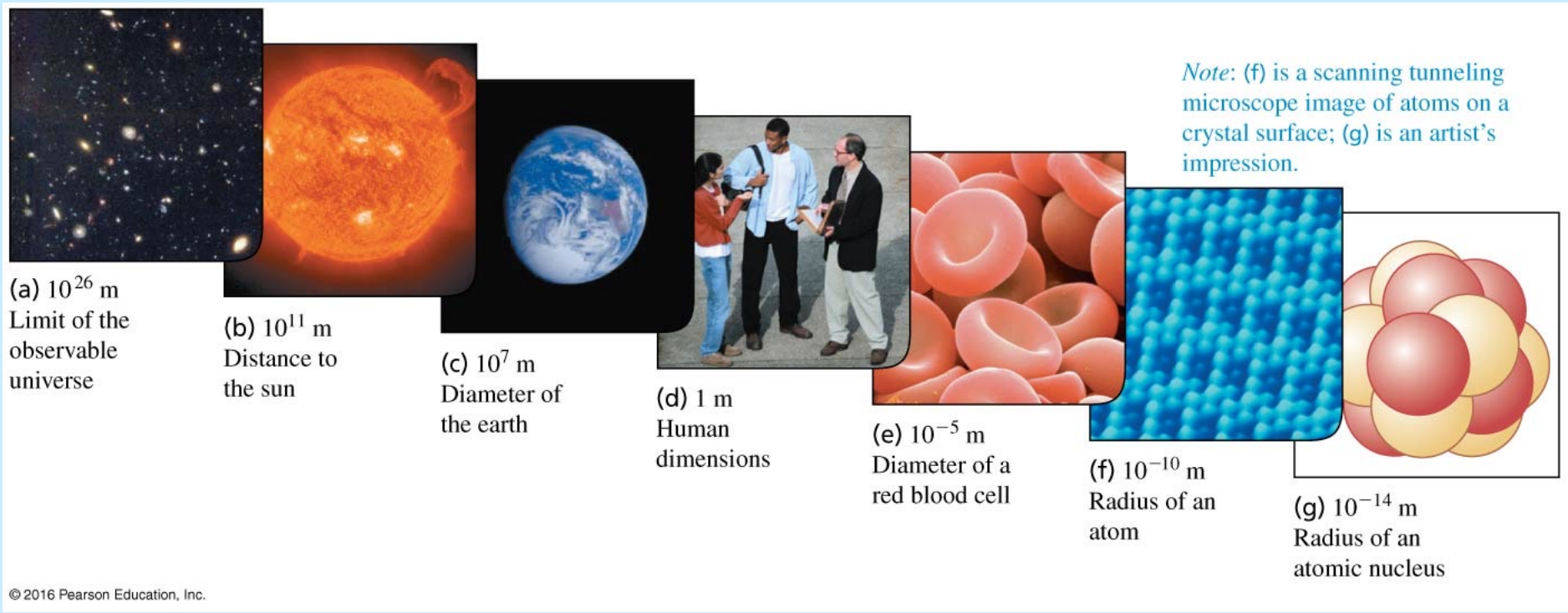




Uncertainty: Einstein, Heisenberg, Bohr, and the Struggle for the Soul of Science
David Lindley



Inward Bound: Of Matter and Forces in the Physical World
Abraham Pais



自然現象似乎可以依參與物體的典型大小來分類！

較大尺寸的現象似乎可以以較小尺寸的物理來了解與解釋！

於是人開始想……..

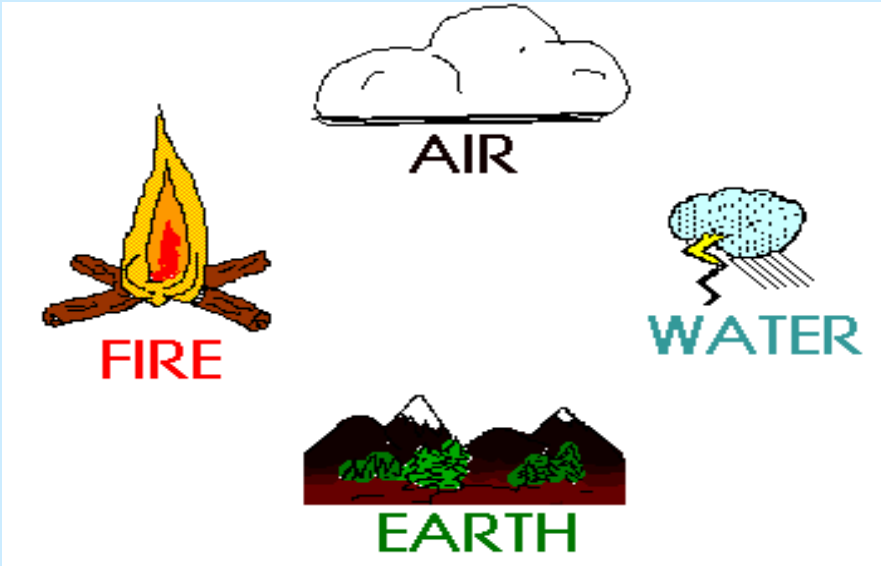


了解基本的組成成分就了解整個宇宙

人類開始思索物質的基本組成成分。

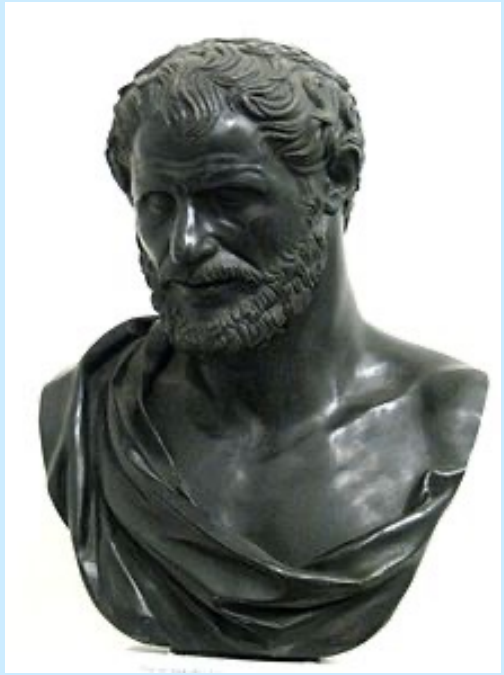






希臘人以為四種元素組成所有物質。

了解這四種元素的性質，就了解整個世界。



Born 460 BC Abdera, Thrace
Died 370 BC (aged approximately 90)

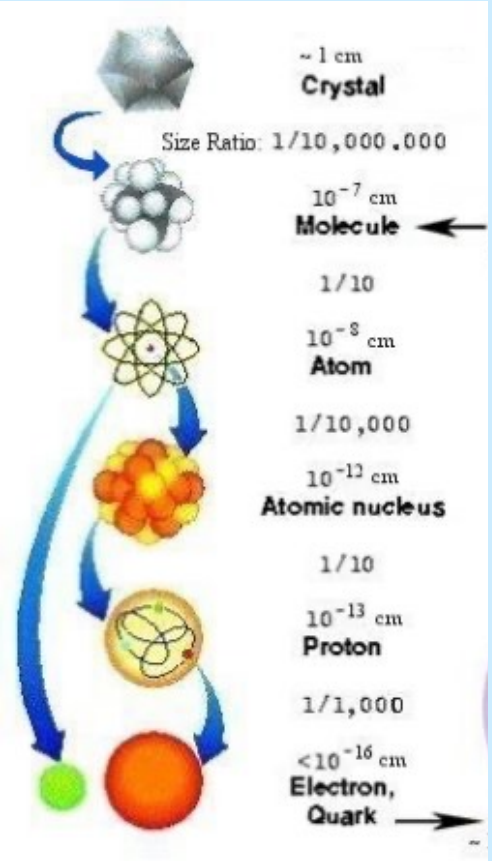
西元前五世紀， Democritus 和 Leucippus 提出：

所有物質分割到最後，將只有微小的、永恆不可分割的粒子。

他們叫這粒子Atom—原子，意思就是無法再分割indivisible。

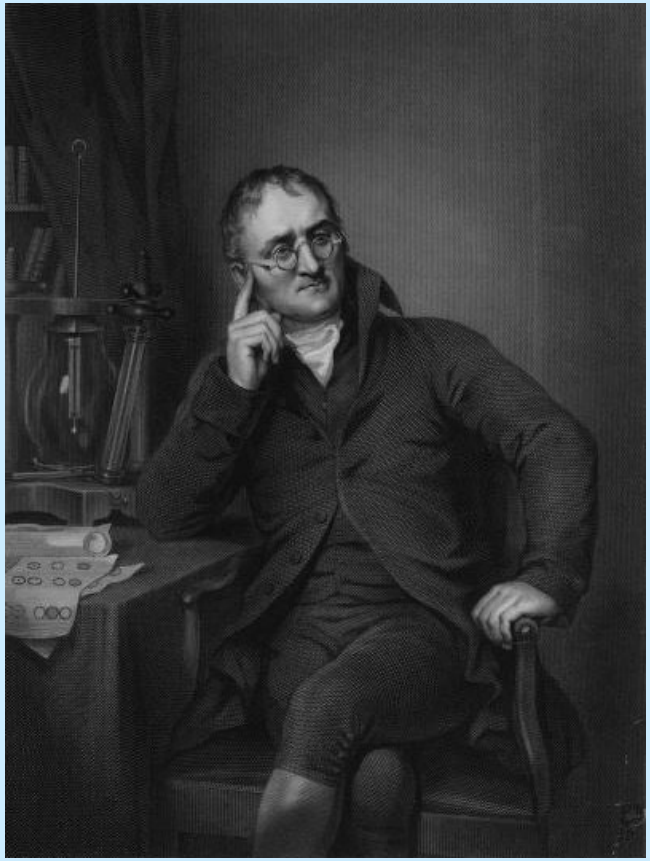
量子力學的量子Quantum指的就是無法再分割的。

一直不斷分割下去，我們的確發現了一層又一層的物質組成結構



現代物理中，不可再分割的物質組成成分就稱為基本粒子。

基本粒子不止一種，而且很可能後來才發現它也可以再分割。



John Dalton 1766-1844

現代的原子論從化學反應的觀察開始：

兩個元素可依不同比例，組成不同的化合物時，

化學家發現，當固定其中一個元素的重量時，

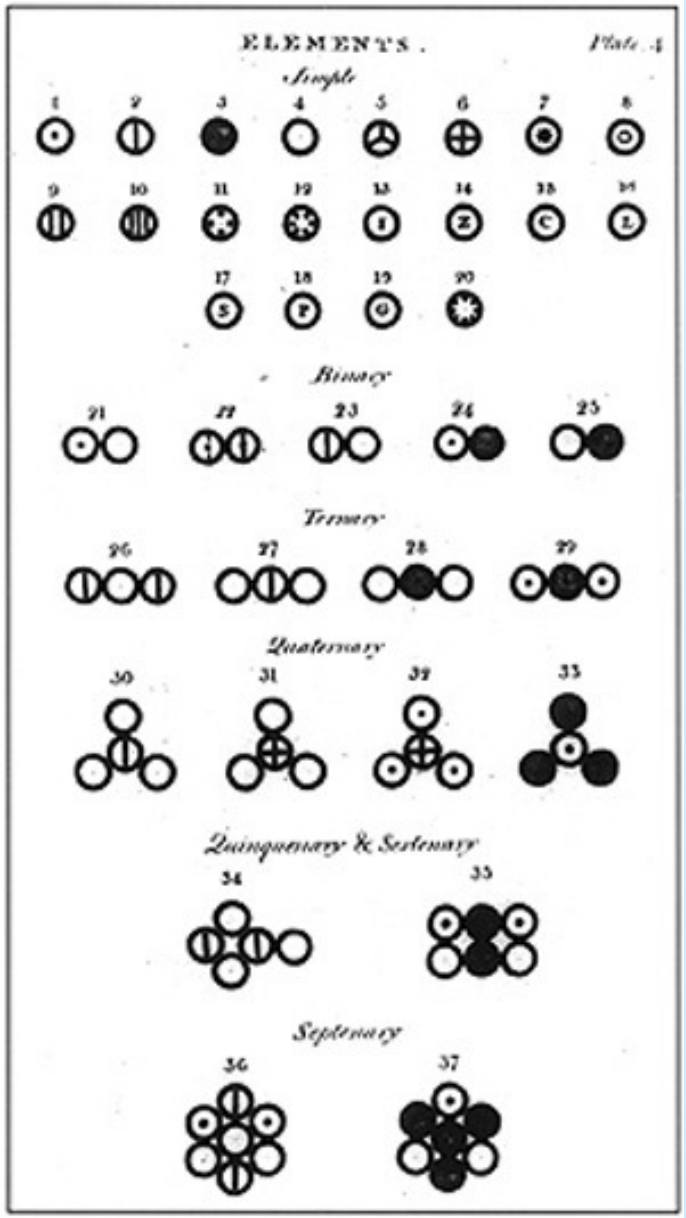
另一個元素的重量比例一定是整數比。

例如：CO or CO_2, but not $CO_{1.3}$

比較油彩調色，我們可以任意連續調整比例，

可見元素不像油彩是連續的，

元素有不可分割的單位！



原子論的基本假設：

元素是由微小的不可分割的原子所組成。

日常中的元素包含極大數量的原子！

$N \sim 10^{23}$ 因此原子極其微小。 $a \sim 10^{-10}$ m

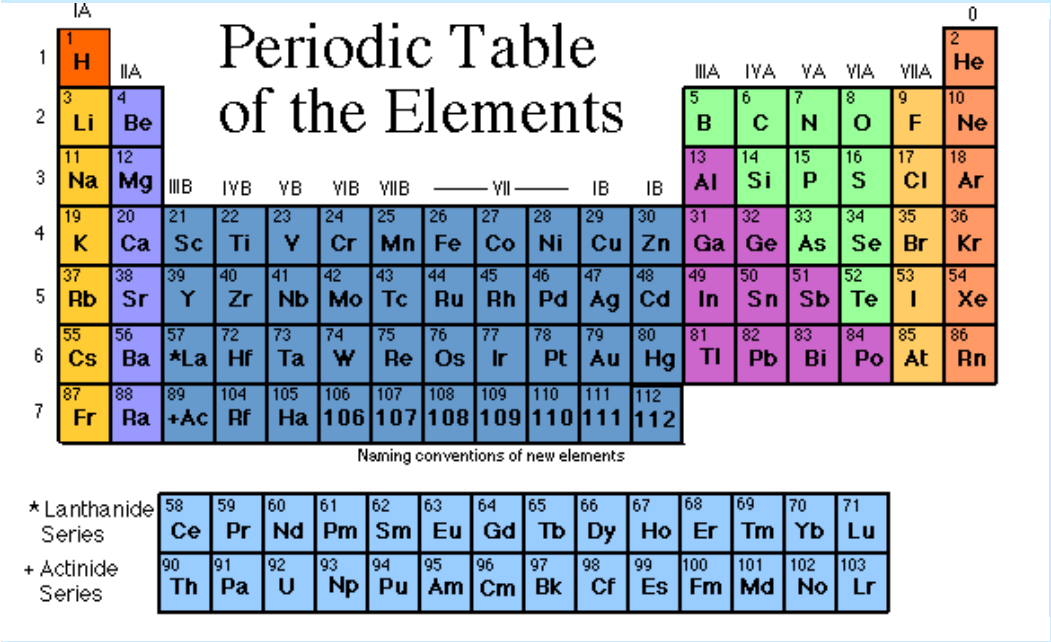
同一元素的組成原子完全相同。

不同的元素之原子不同，且具有不同的質量。

這一點與希臘的原子論不同。

John Dalton (1808) *A New System of Chemical Philosophy.*

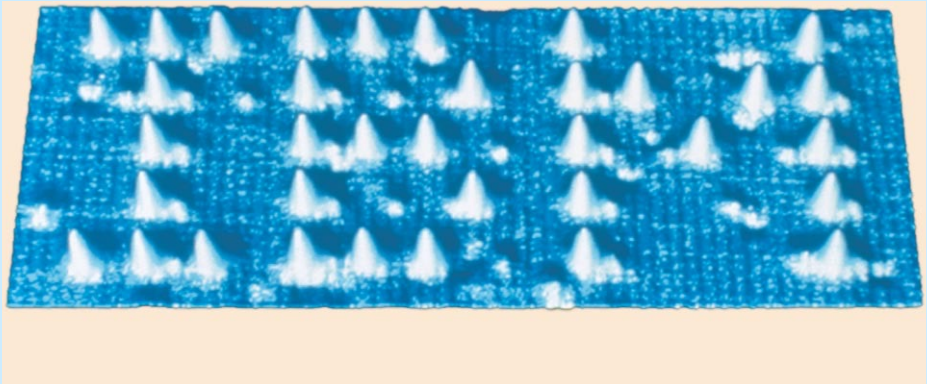
宇宙中的物質由許多種極其微小的原子組成。



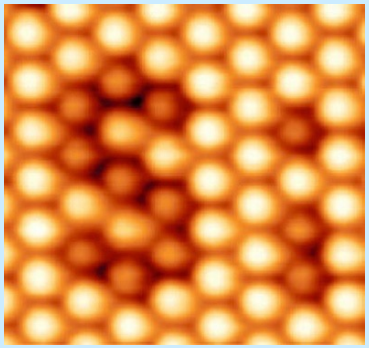


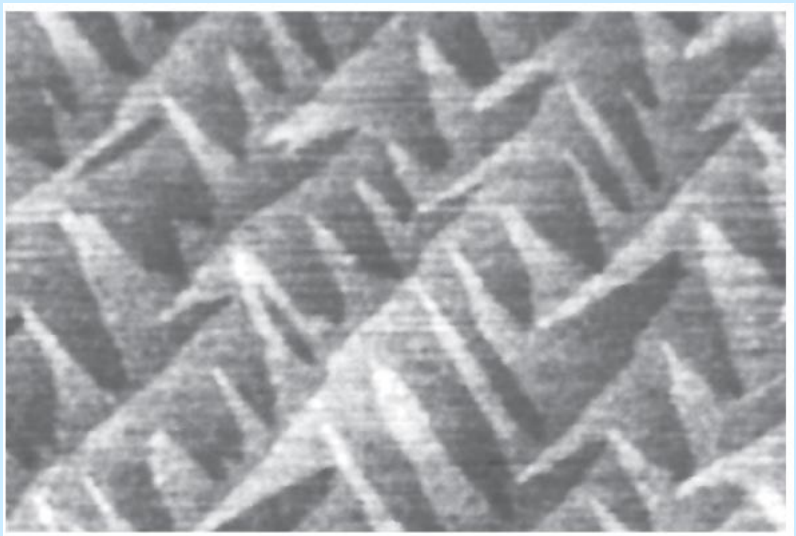
$N \sim 10^{23}$ $a \sim 10^{-10}$ m

原子的世界相對於日常巨觀Macroscopic的世界，是一個極其微觀Microscopic的世界。

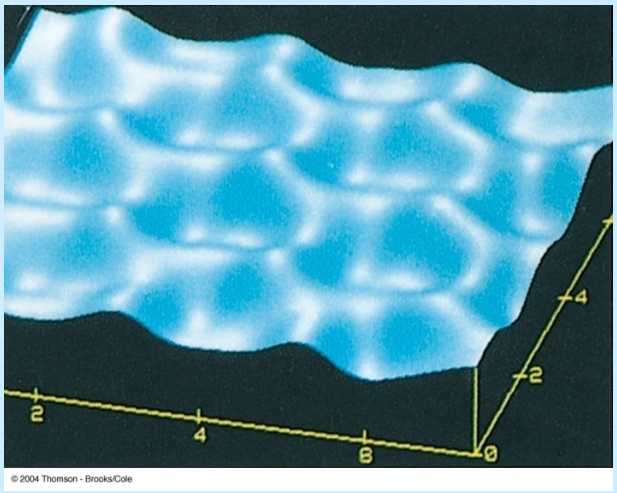


Xenon Atoms





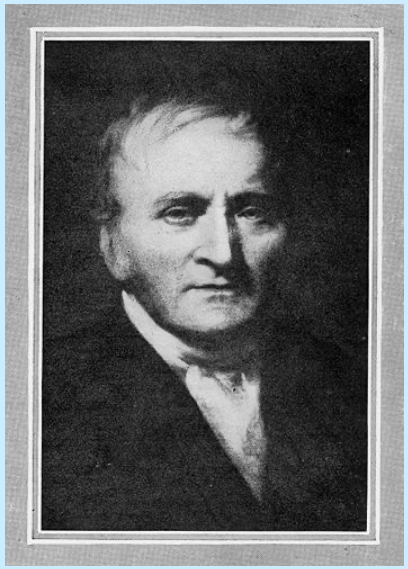
The surface of silicon



© 2004 Thomson - Brooks/Cole

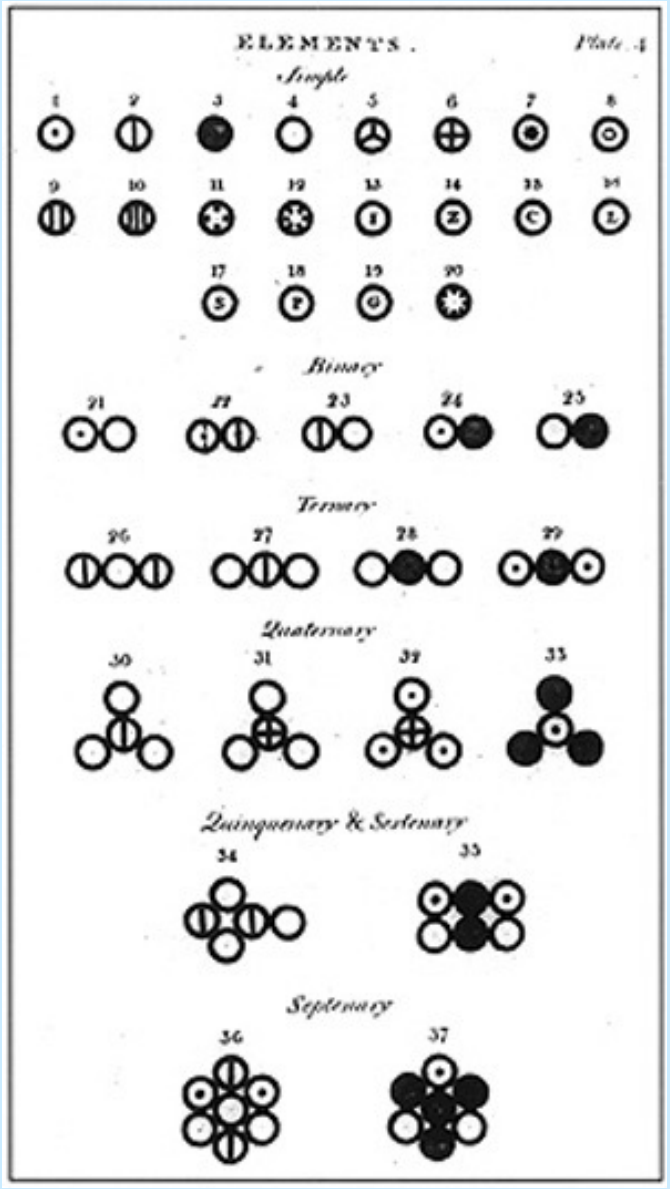
石墨表面的碳原子

元素是由微小的不可分割的粒子:原子構成。





John Dalton (1808)

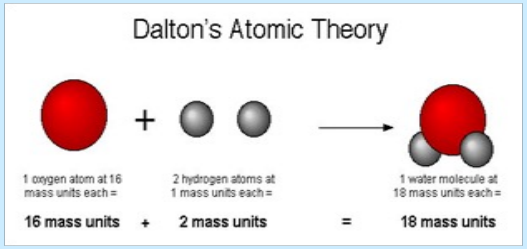


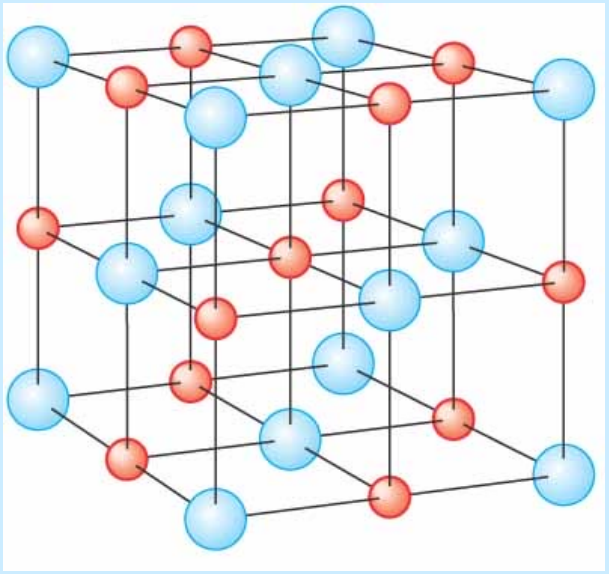
化學家的原子論證實了探索組成成分是真的有用的！

組成化合物的機制不一定只能用在小規模的組合！

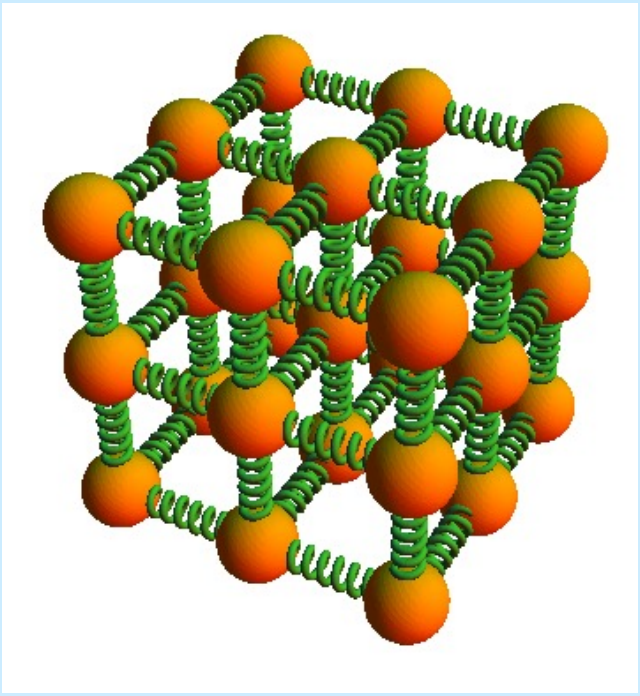
這些原子也可以組成有大規模秩序的晶格，

晶格的大小與形狀無法改變，這就是固態！



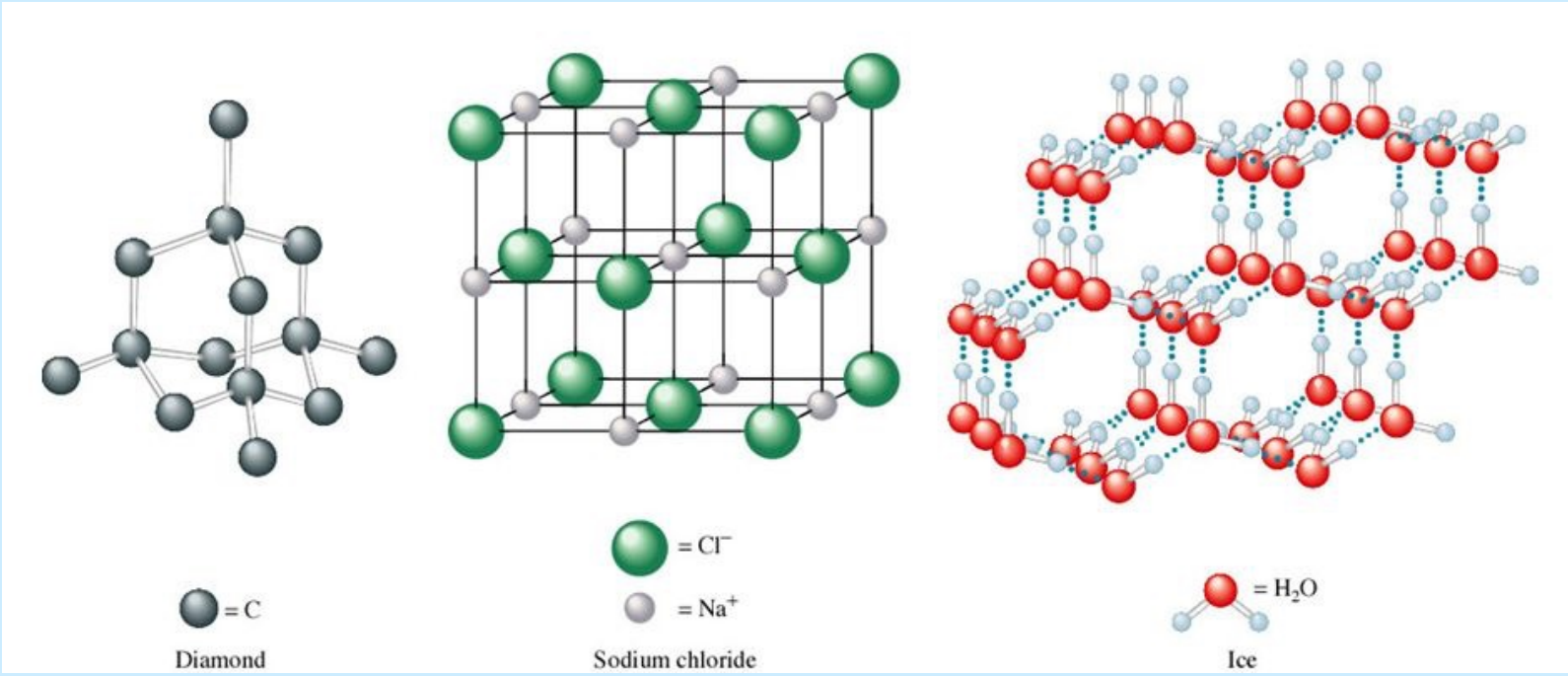






固體中原子彼此之間，如同以彈簧連接。

這是固體的彈性的來源。

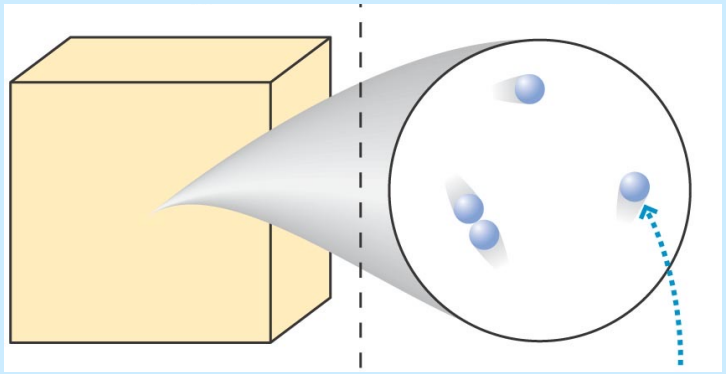


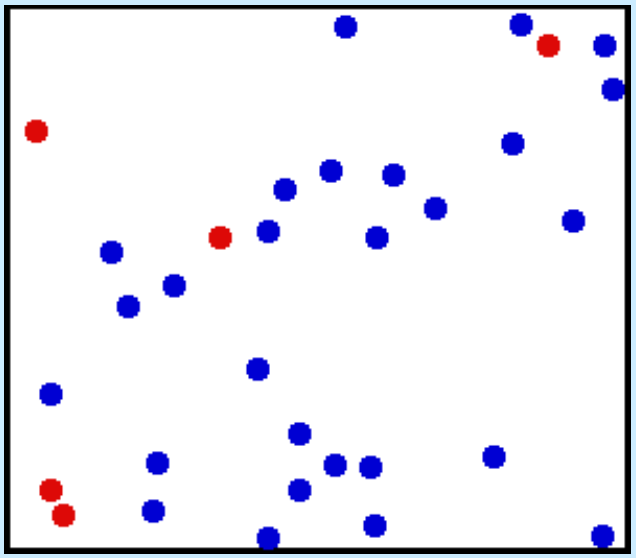
原子對物質的物態有非常自然的解釋

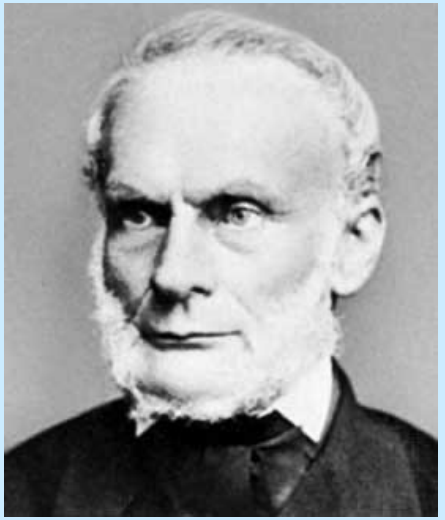


固體有固定大小及形狀　液體有固定大小，無固定形狀　氣體無固定大小及形狀

Kinetic Theory of Gas 氣體動力論







Clausius

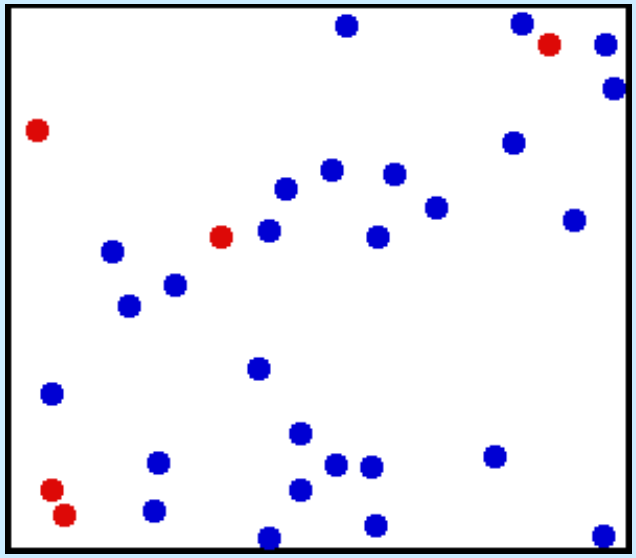
原子就是牛頓力學中的粒子！

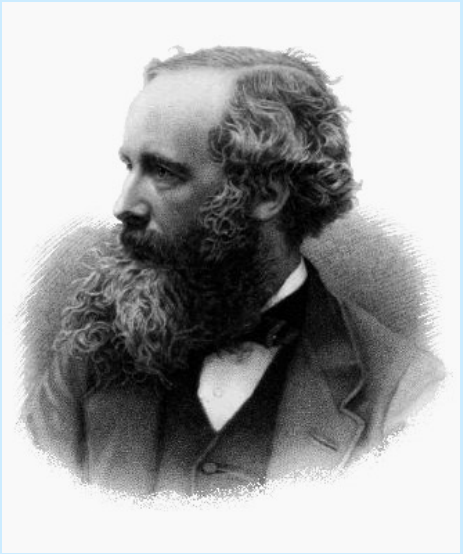
氣體就是一羣不斷運動碰撞的牛頓粒子！

理想氣體的性質為何如此簡單而普遍？

$$T = \frac{PV}{nR}$$

$$E_{\text{int}} = nc_V T$$





Maxwell

因為所有稀薄氣體都是一羣不斷運動碰撞的自由牛頓粒子！

以上兩個公式可以推導出來！

這裡就是古典物理與近代物理的邊界！



Physics is all about particles (and fields).



In Classical Physics, no two particles are the same.

在古典物理中，沒有任何兩個粒子是一模一樣的。

You can not step into the same river twice. Heraclitus 一期一會

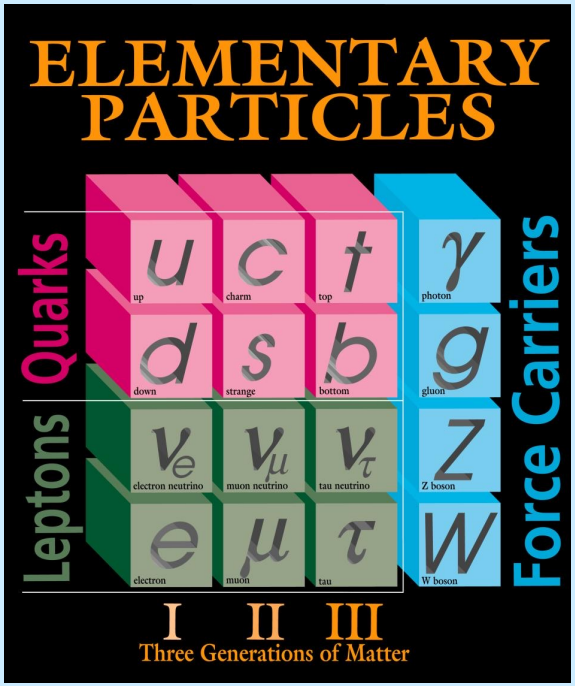
在量子物理中，沒有任何兩個帶電-1.6×10^{-19}C的基本粒子是不一樣的。

In Modern Physics, no two negatively charged particles are any different.

fundamental in nature

Story of electron. Every electron is identical.

電子 **Electron** e^-

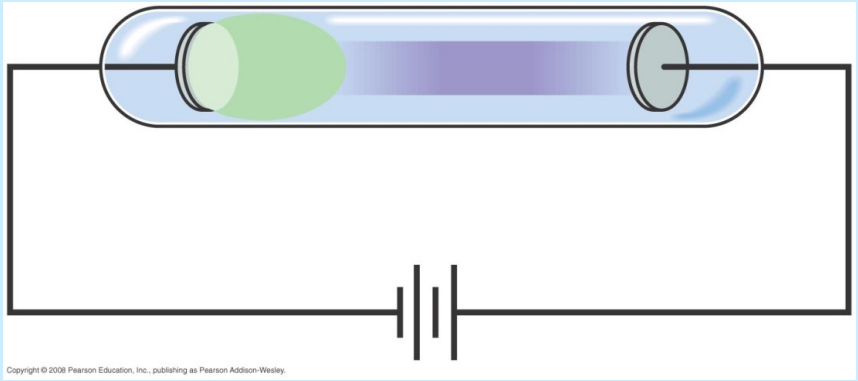






無論你在何時何地找到電子，都與你現在手中的電子完全沒有差異。

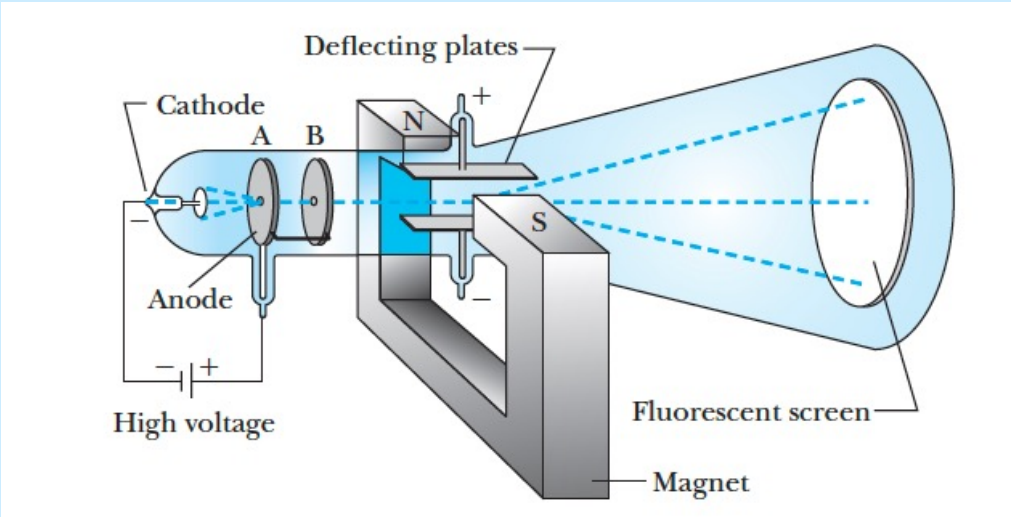
組成物質的基本成分原子是電中性，那電荷從哪來？



陰極射線 Cathode ray

In the 1890s scientists and engineers were familiar with the “cathode rays” that were generated from one of the metal plates in an evacuated tube across which a large electric potential had been established.

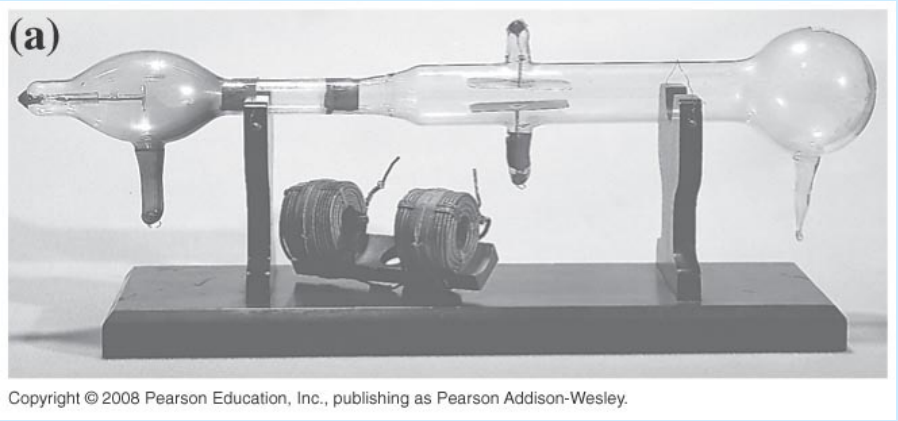
電壓夠大氣體會游離放電，氣體壓力減至極小後，放電消失但瓶子會發出螢光。



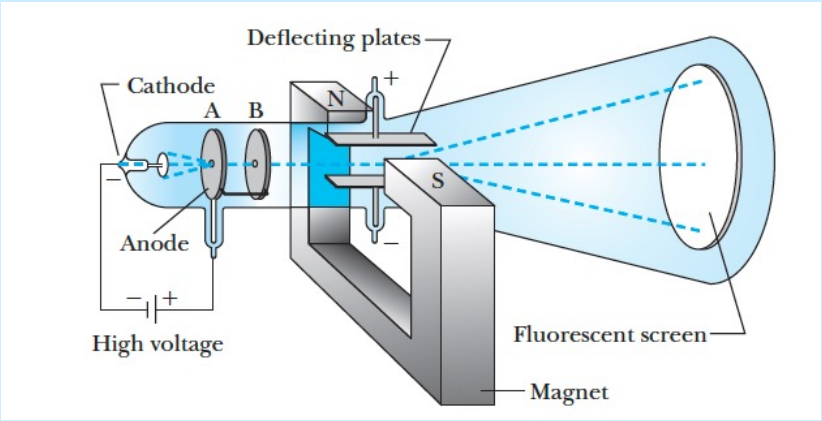


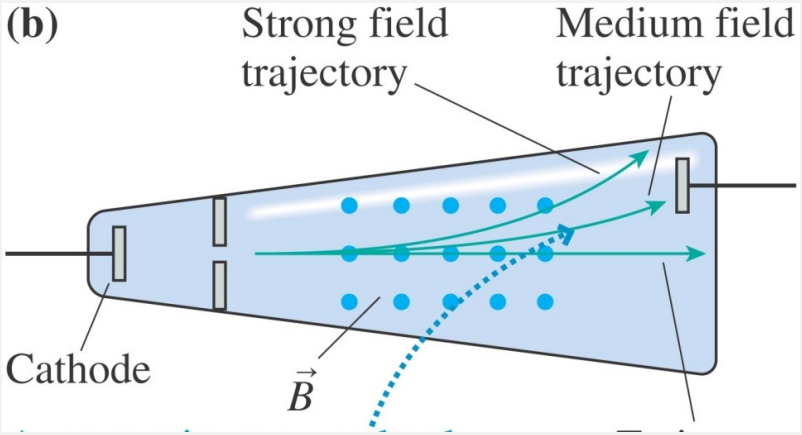


J. J. Thomson 英國



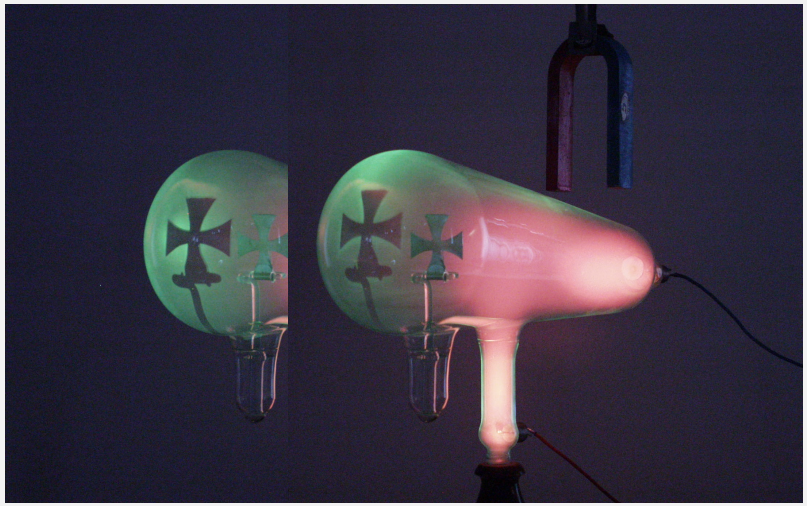
Copyright © 2008 Pearson Education, Inc., publishing as Pearson Addison-Wesley.

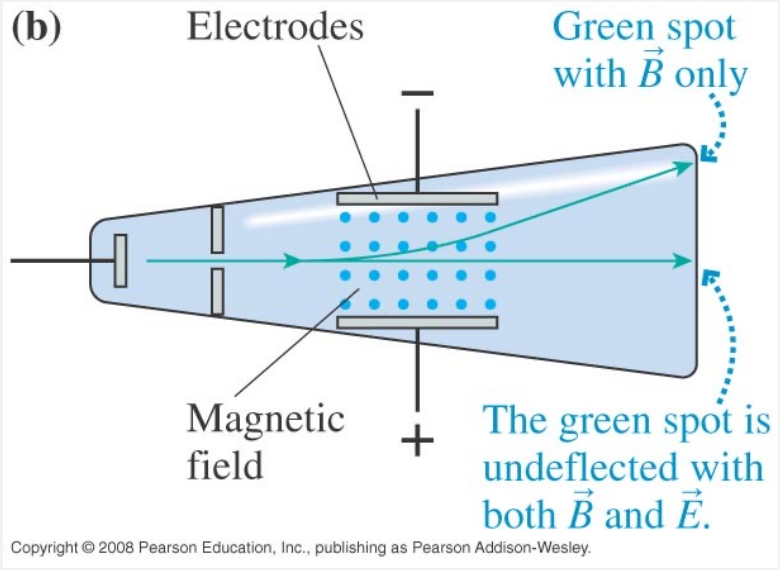


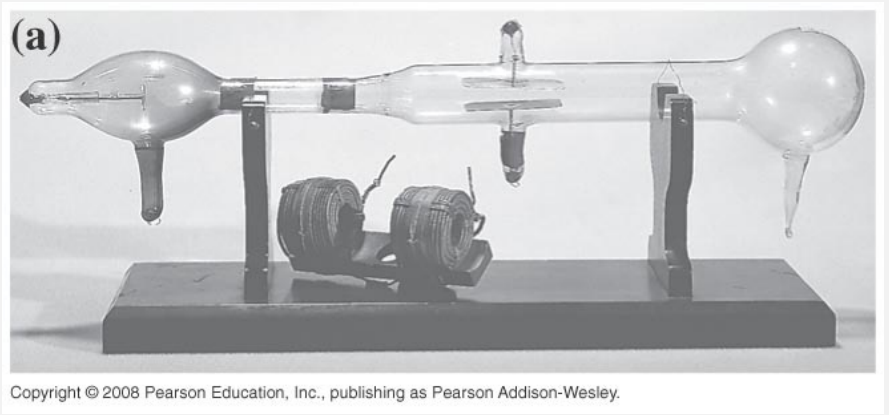


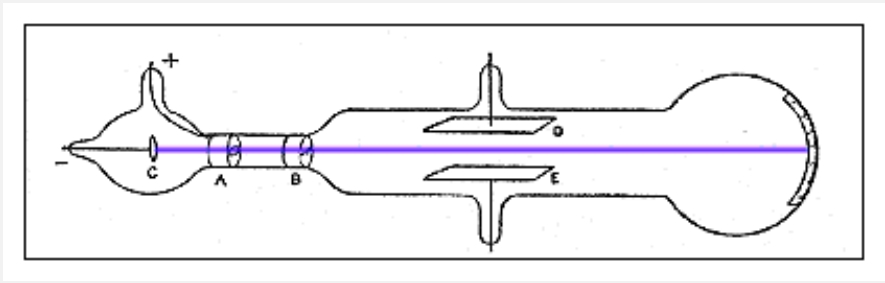
陰極射線加上磁場會發生偏折，

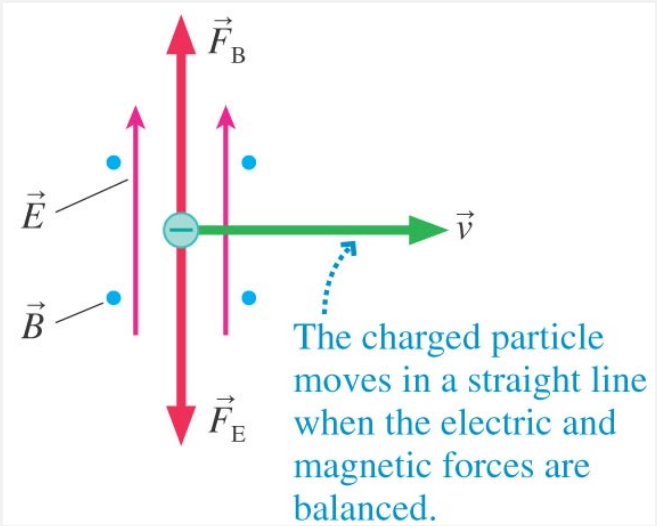
由偏折方向可知陰極射線帶負電，





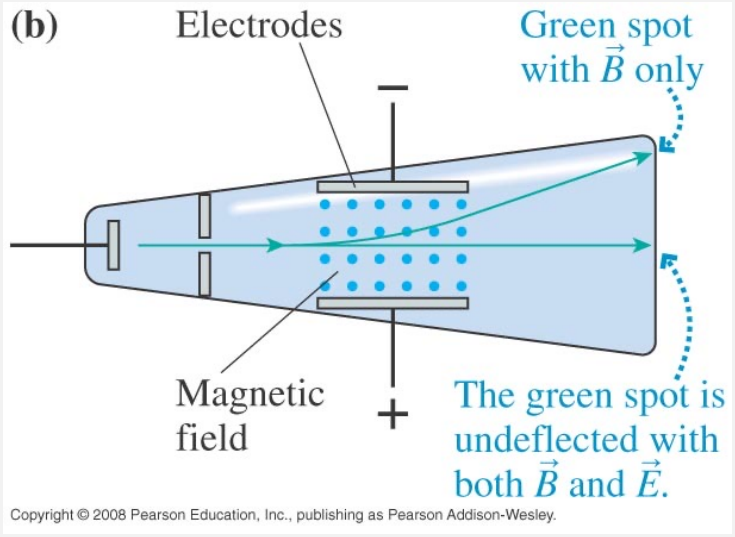






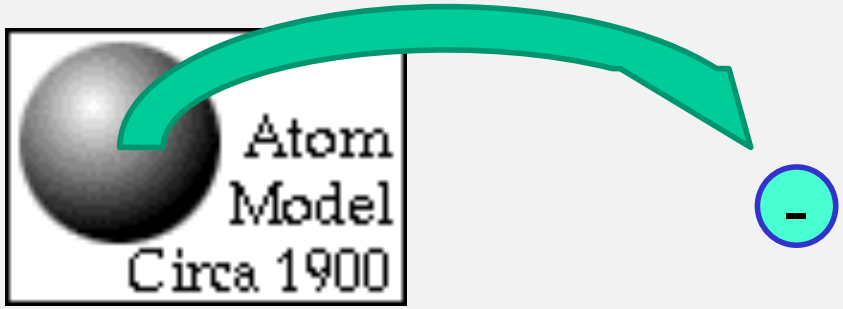
再加上垂直電場可將其折回。

由電場及磁場的值，可得出質量及電荷的比，





英國 J. J. Thomson 1899 測量垂直的電磁場以決定陰極射線的電荷與質量的比。



陰極射線的荷質比遠大於氫離子，所以陰極射線是一束構成原子的組成成分。

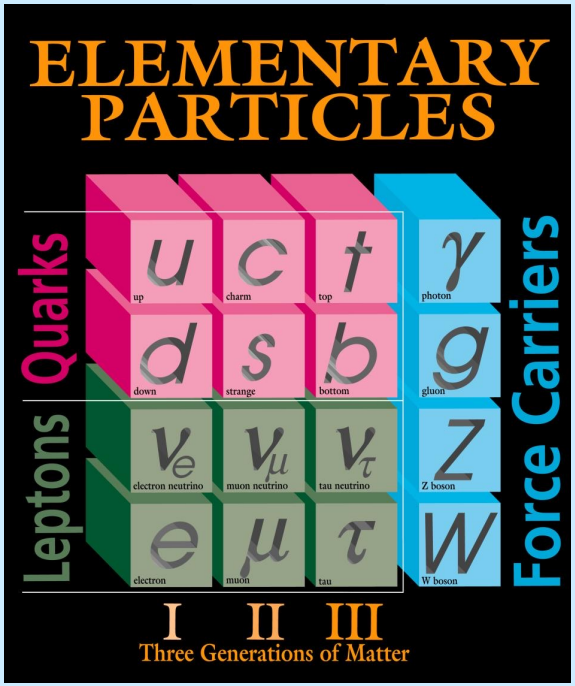
此荷質比對任何材質所發出的陰極射線都相同，可見這個組成成分是共同的。

Thomson found that atoms can be further divided into a smaller particle, electron.

There is only one kind of electron. Every electron is identical.

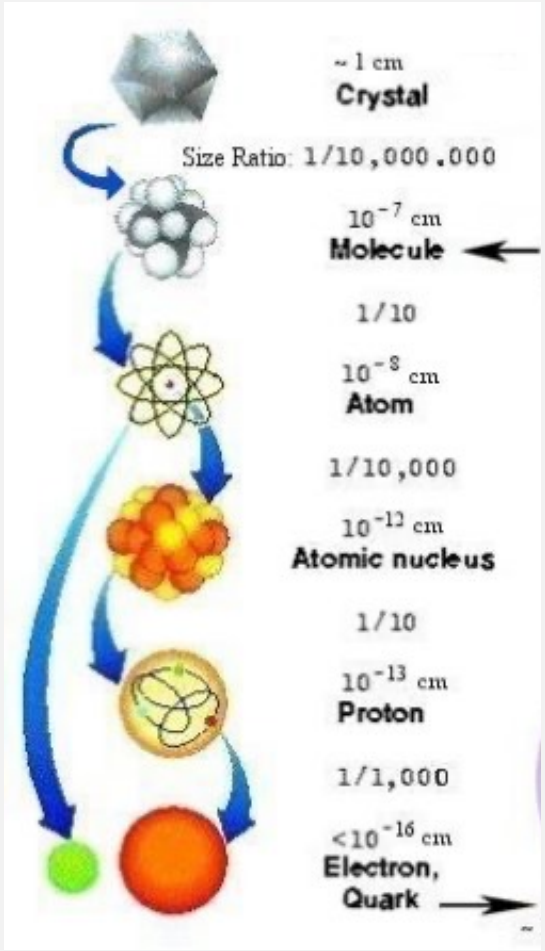
電子 Electron e^-







無論你在何時何地找到電子，都與你現在手中的電子完全沒有差異。



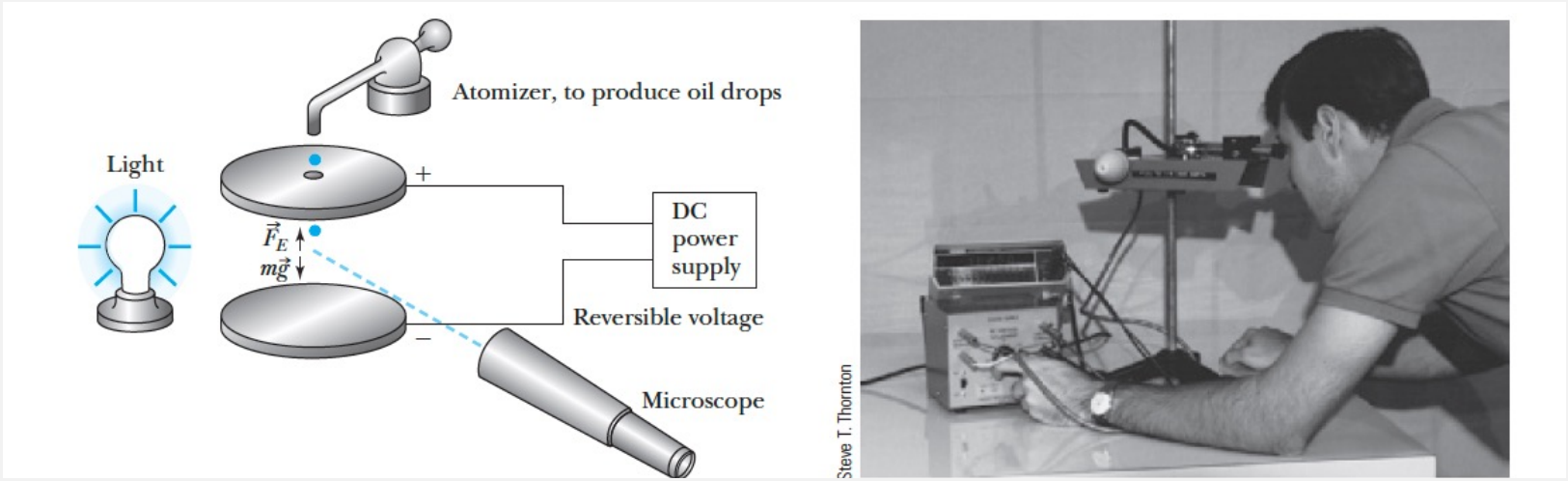
物質組成成分的粒子是否能分割要看分割的能量用得多大。

在某些化學反應的層次，原子的確是不可分割的。

但能量更大如陰極射線時，電子就從原子中被分割出來了！

確定電子的粒子性的另一個實驗：

Millikan’s Oil drop experiment 1909





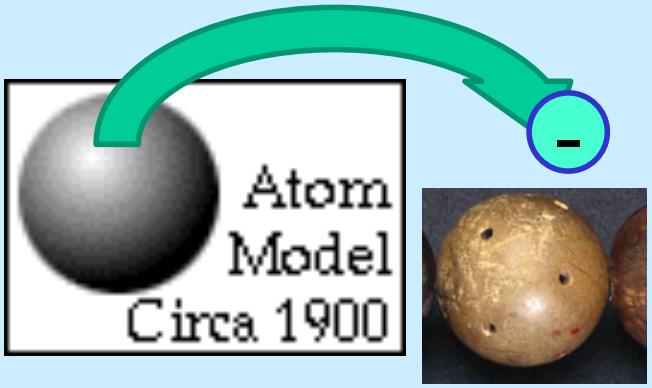


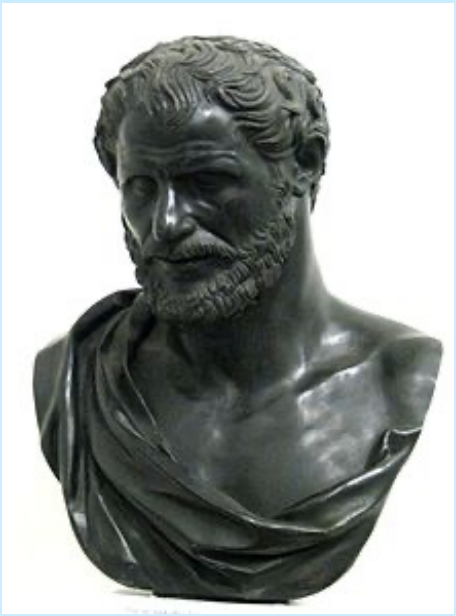
所有油滴所帶電荷都是$1.602176487(40)\times10^{-19}$ C的整數倍

Electrons are indivisible even now.

It seem we finally found the basic constituents of nature as Greeks understand it.







What kind of particle is the electron?

Naturally, we expect it to be a Newton Particle.

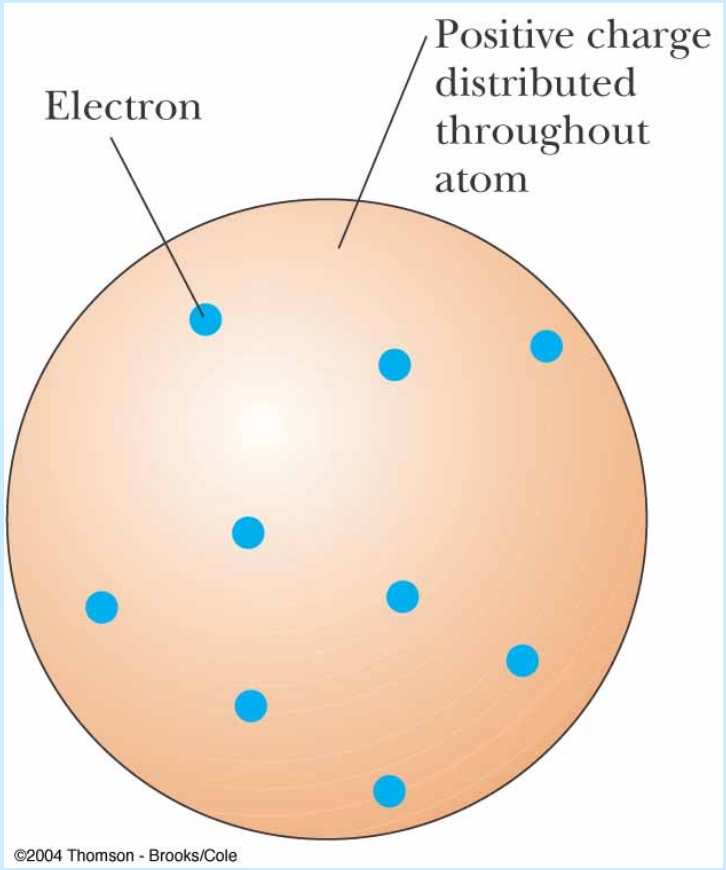
To see if it is true, we ask how is it located inside an atom.

原本物理學家期待微觀世界的物理規則與牛頓的物理沒有差別。
電子就是十分微小的、更像粒子的月亮、地球而已。
但二十世紀初的實驗竟然發現，量子力學與牛頓力學截然不同，十分詭異。



Electrons are negative charged and light.

The rest of the atom must be positive charged and heavy.



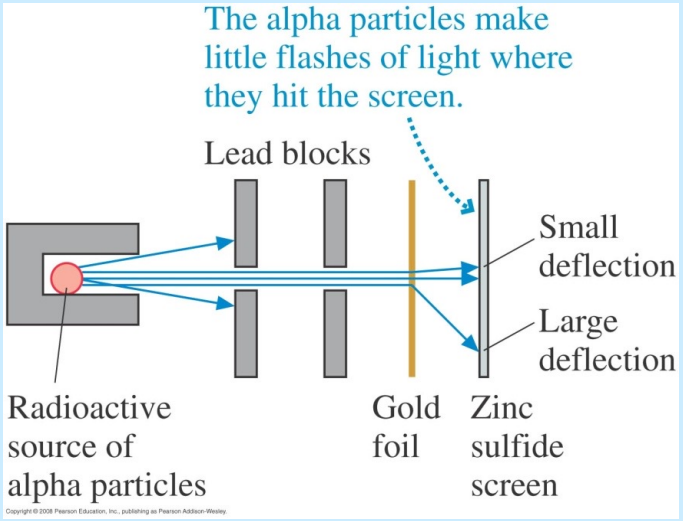


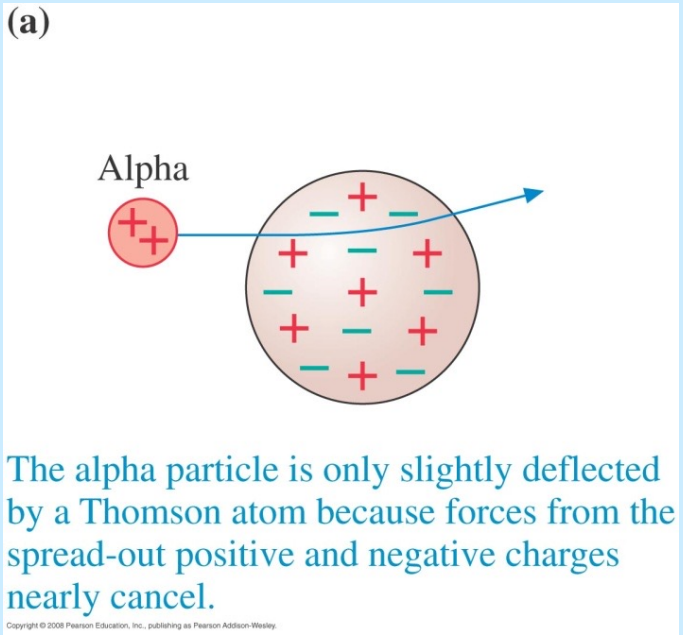
Positive charges and negative charges attract each other.

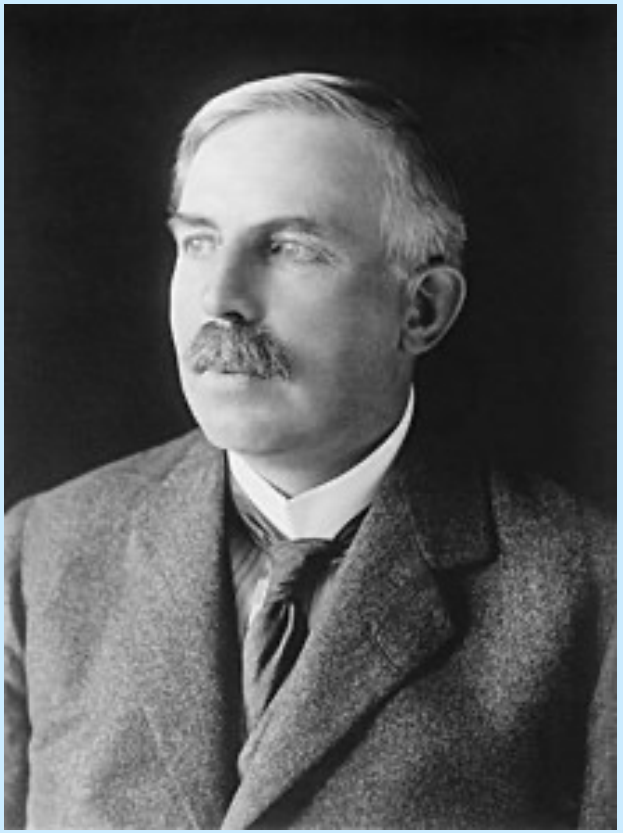
Are positive charges distributed evenly in an atom to keep it stable?

但事不從人願！

If we hit a atom with a very fast, very small α particle,

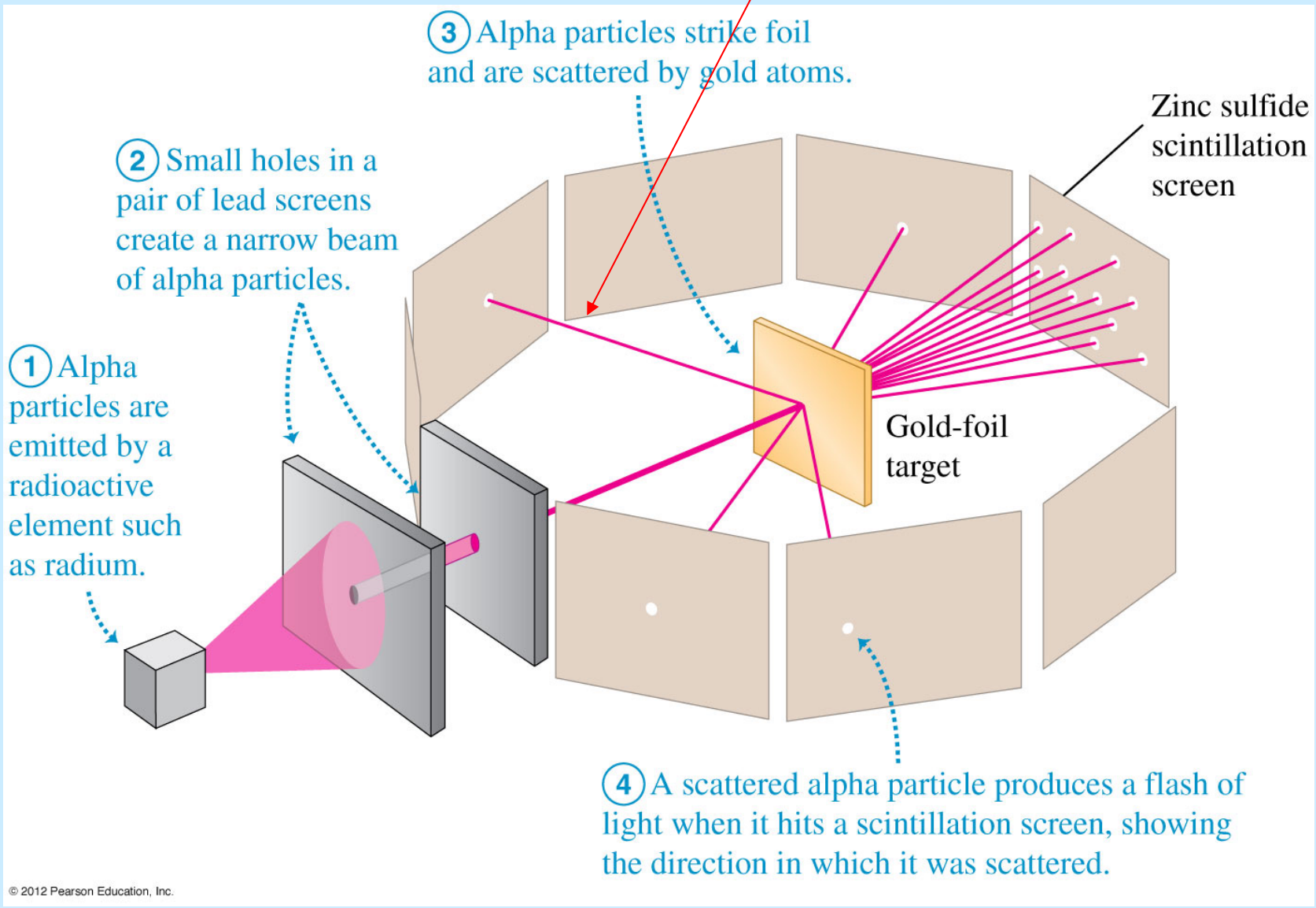




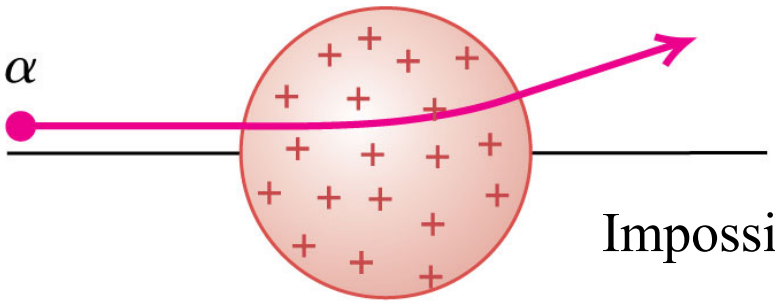


Rutherford 1910

A lot of α particles are reflected in large angle scattering 散射.

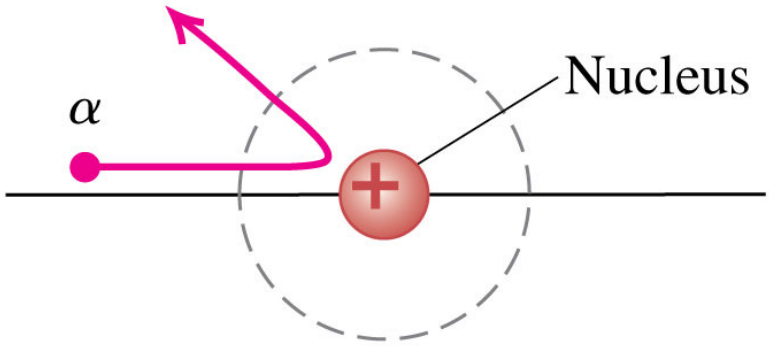


(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.

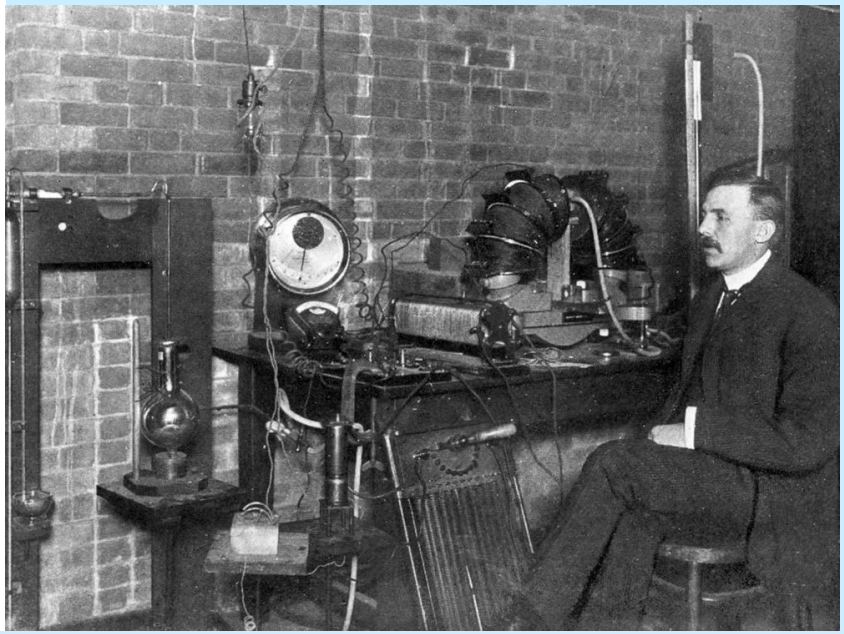


Impossible for uniformly distributed atomic materials.

(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



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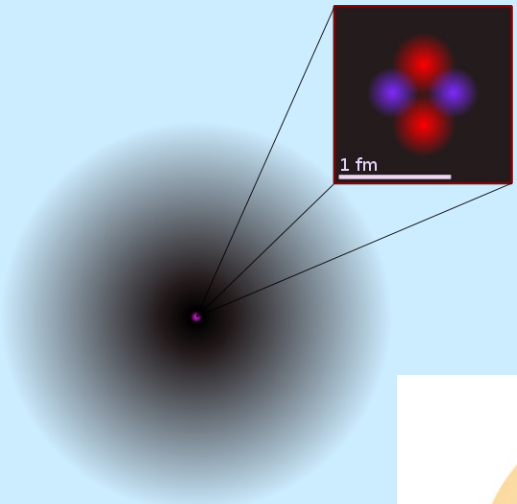


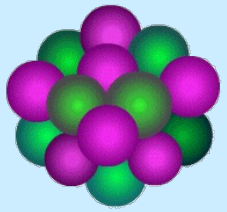
Heavy positive charged material must concentrate in a very small nucleus.

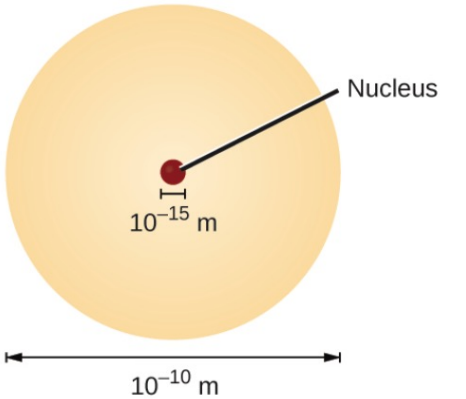


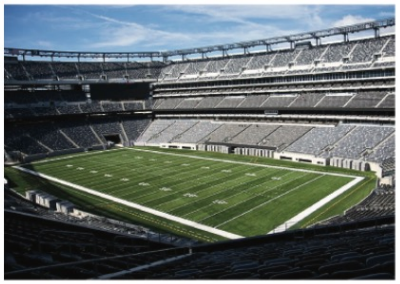

ERNEST RUTHERFORD
(1871–1937)
NOBEL LAUREATE
LED THIS LABORATORY 1907–1919
HEREIN DISCOVERED THE NUCLEAR
ATOM, SPLIT THE ATOM, AND
INITIATED THE FIELD OF
NUCLEAR PHYSICS

Positive charged and heavy material of atom is concentrated in a very small Nucleus.









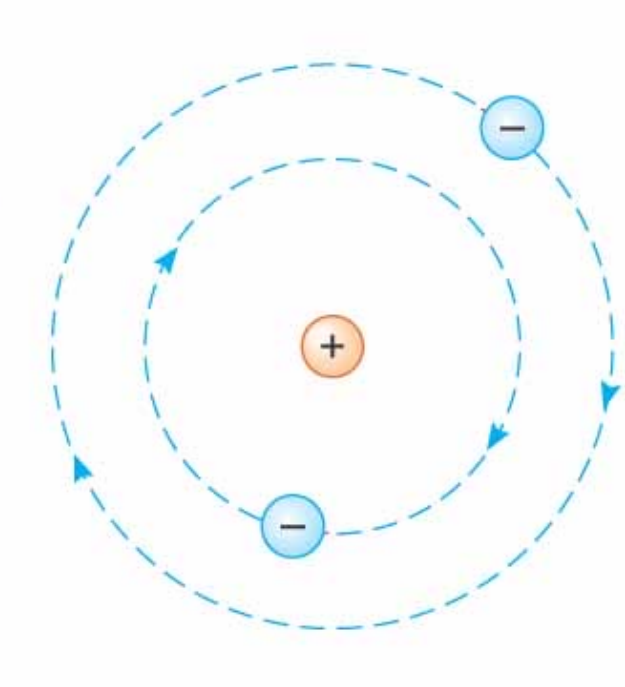


If an atom could be expanded to the size of a football stadium, the nucleus would be the size of a single blueberry. (credit middle: modification of work by "babyknight"/Wikimedia Commons; credit right: modification of work by Paxson Woelber)

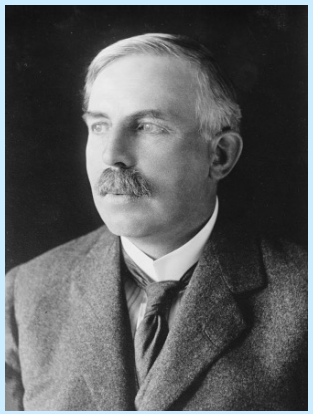


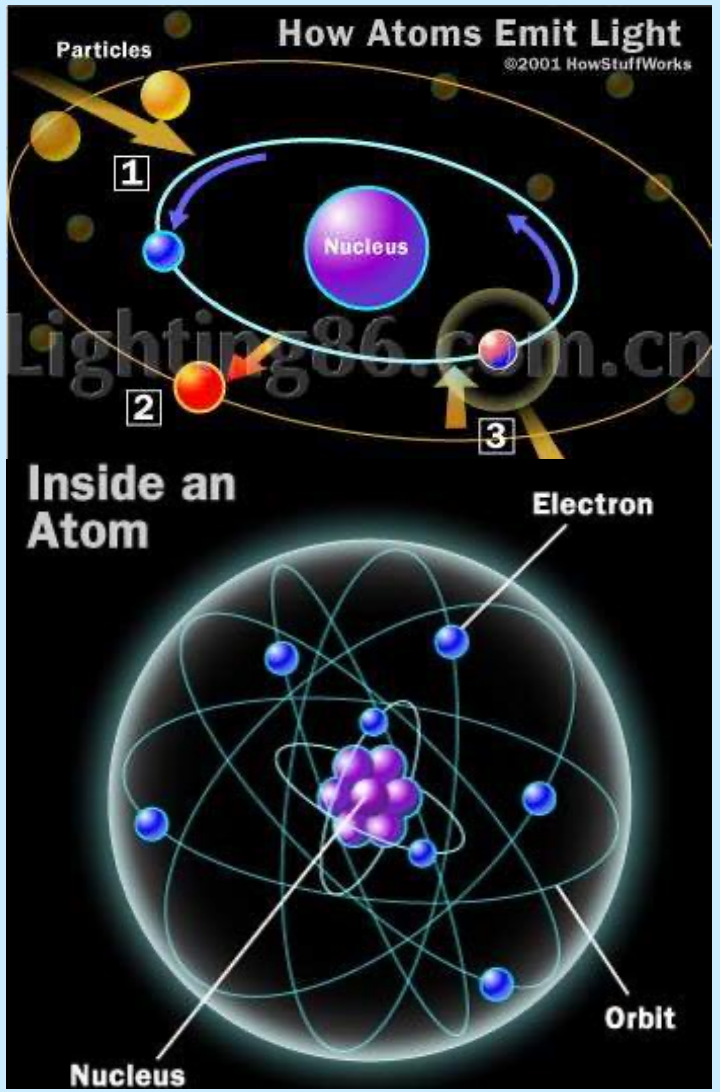
How are electrons located inside atom?

Electrons can rotate around Nucleus like solar system.



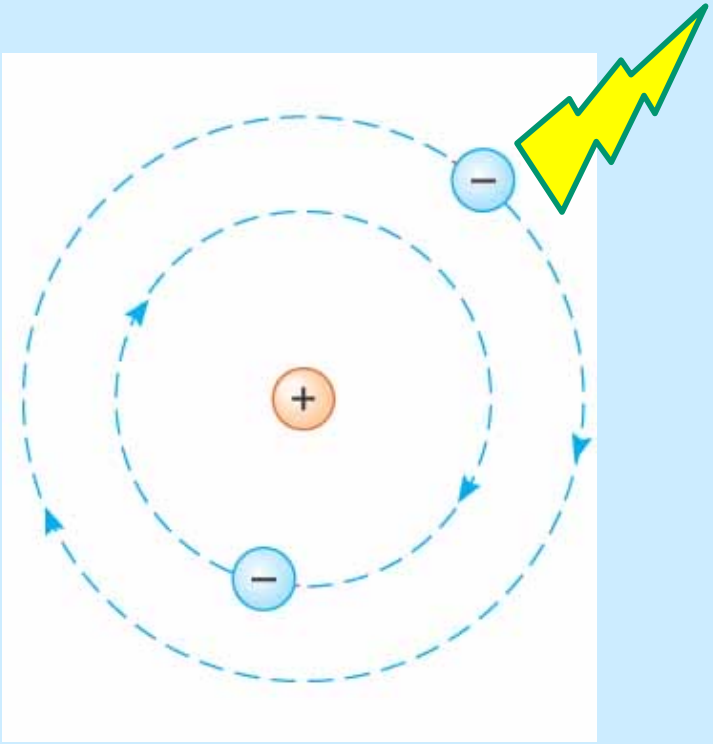
E. Rutherford



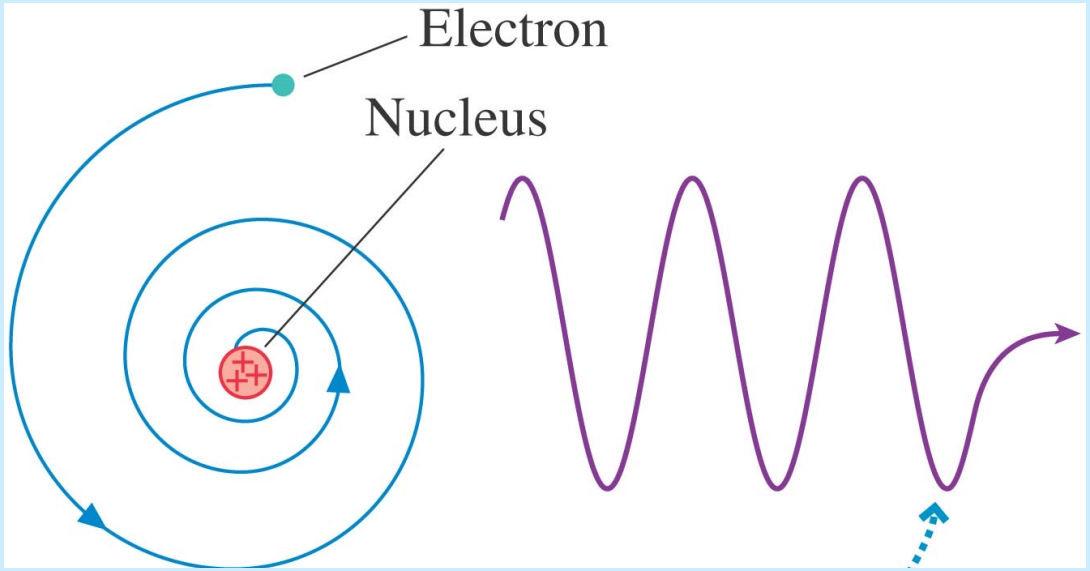


But, this is not right!

Rotating charge emits electromagnetic wave and it caries away energy.



Rotating electrons, losing energy, fall into Nucleus very soon.



Solar system picture of atom does not work!

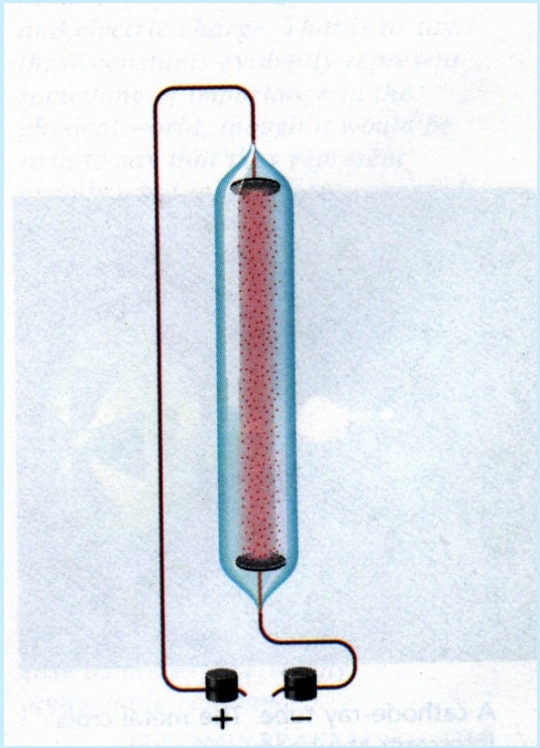
What is to be revealed in emission spectrum is even more shocking!

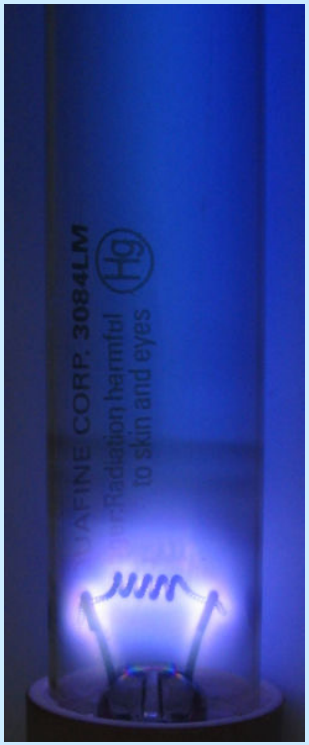
Atomic emission spectrum

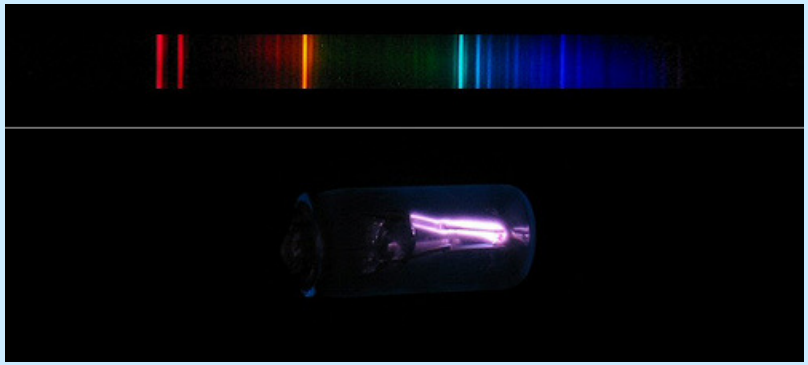
Gas-discharge lamp: Cathode tube with dilute gas inside.

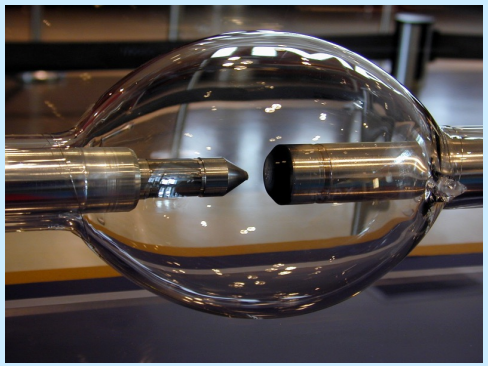
Gas atoms are ionized under high voltage.

Atoms start to emit EM wave as electrons pass through gas and excite atoms.



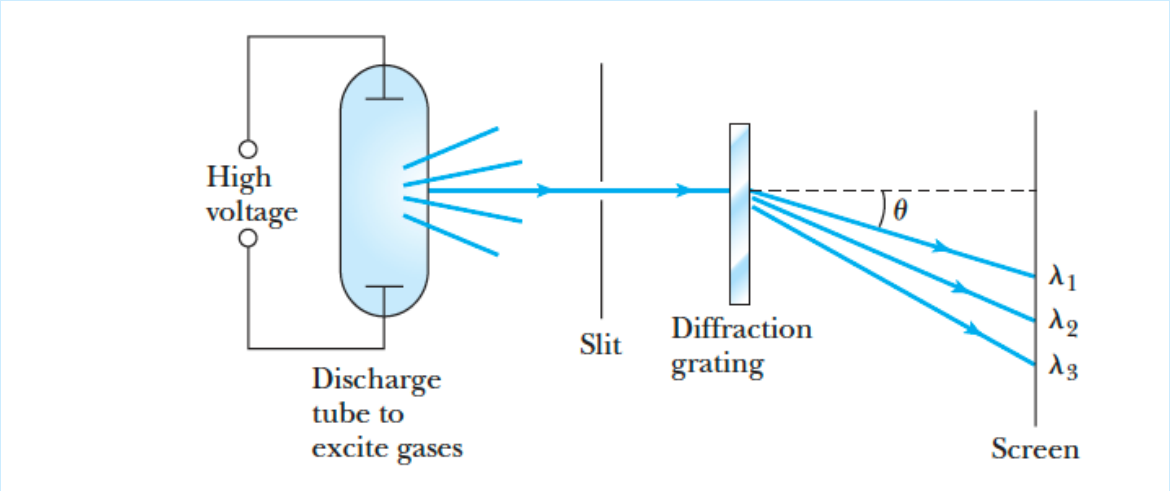


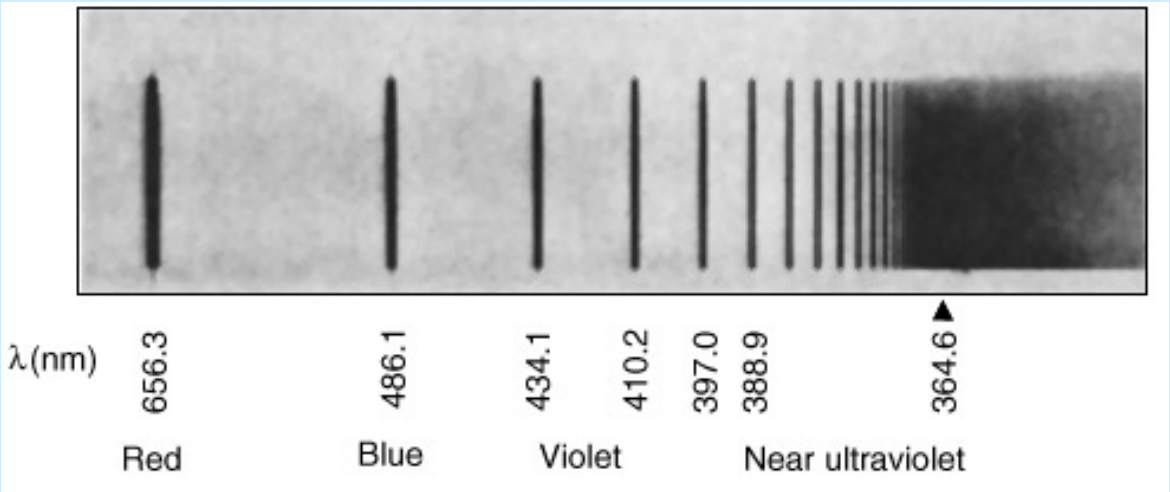




此實驗中氣體並不被加熱，所放出的光非黑體輻射！

The emission EM wave is analyzed by wavelength: 原子光譜 Spectrum。



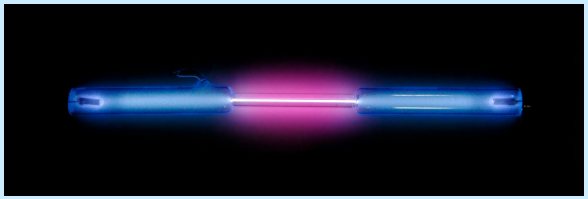


Shockingly, the emission distribution is not continuous, but discreet！

Spectrum of Hydrogen 紅光是它的特徵。

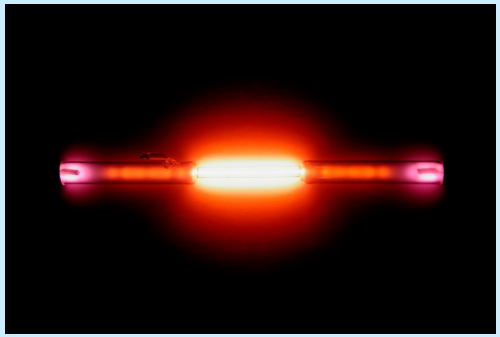


原子放出的光的頻率是離散的。



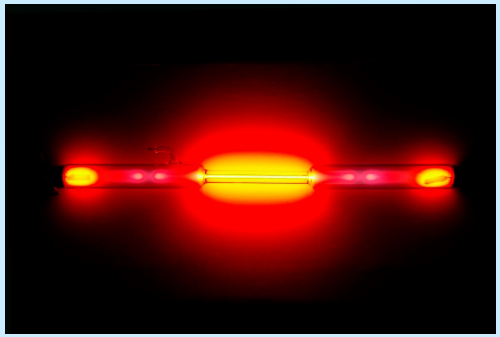
He





Ne

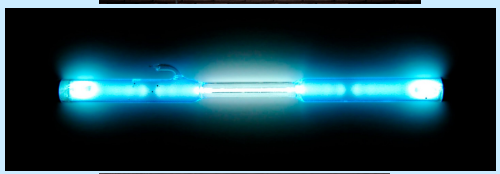




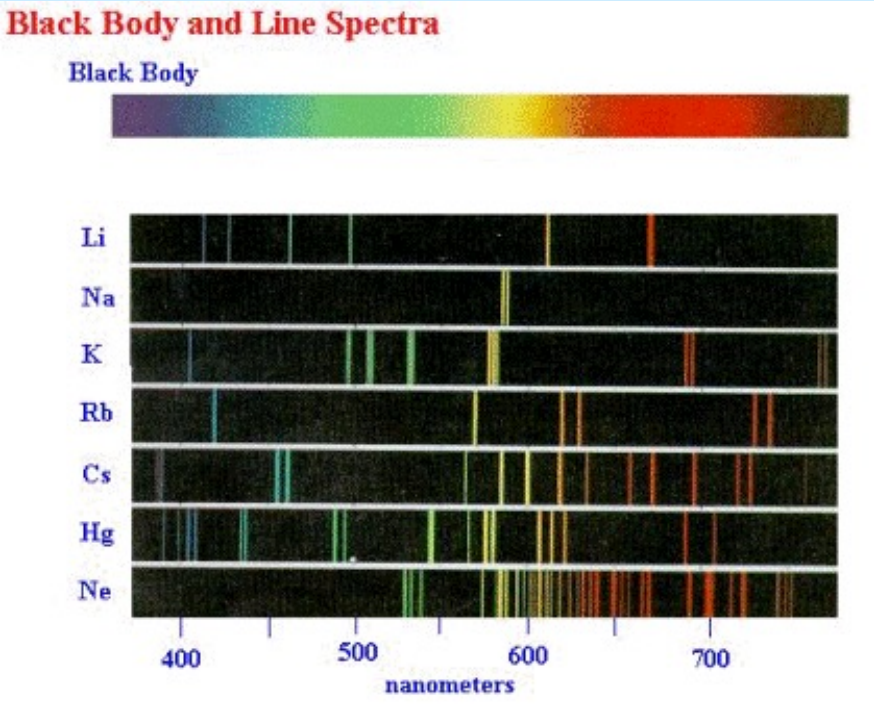


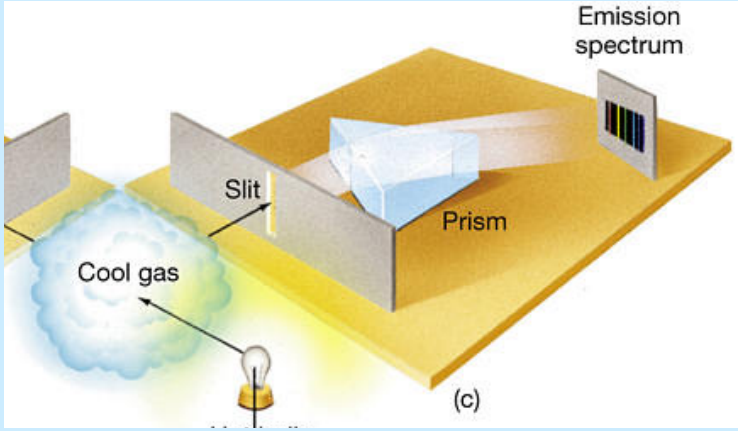
Mercury











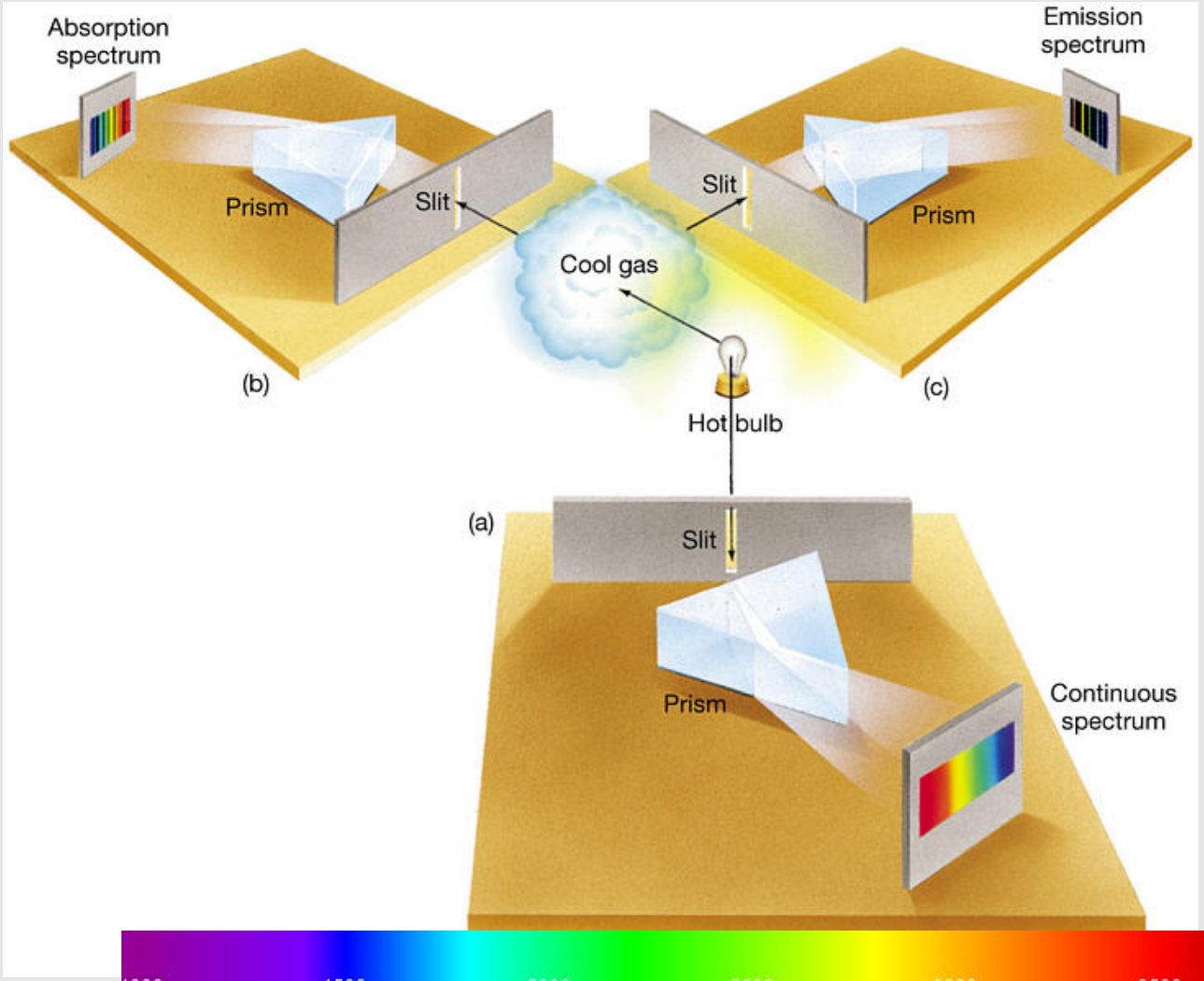
In contrast, Thermal Blackbody radiation spectrum is continuous.

Atomic emission has shorter wavelength than blackbody radiation.

We believe it comes from mechanism inside atoms.

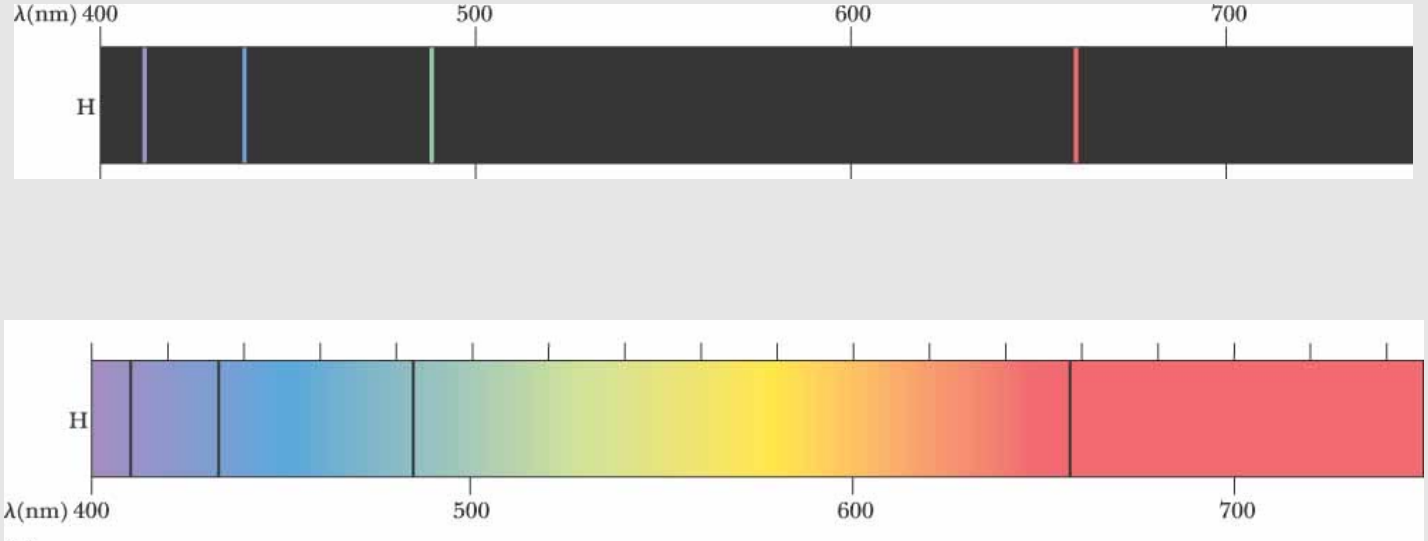
除了放射光譜，還可以有吸收光譜



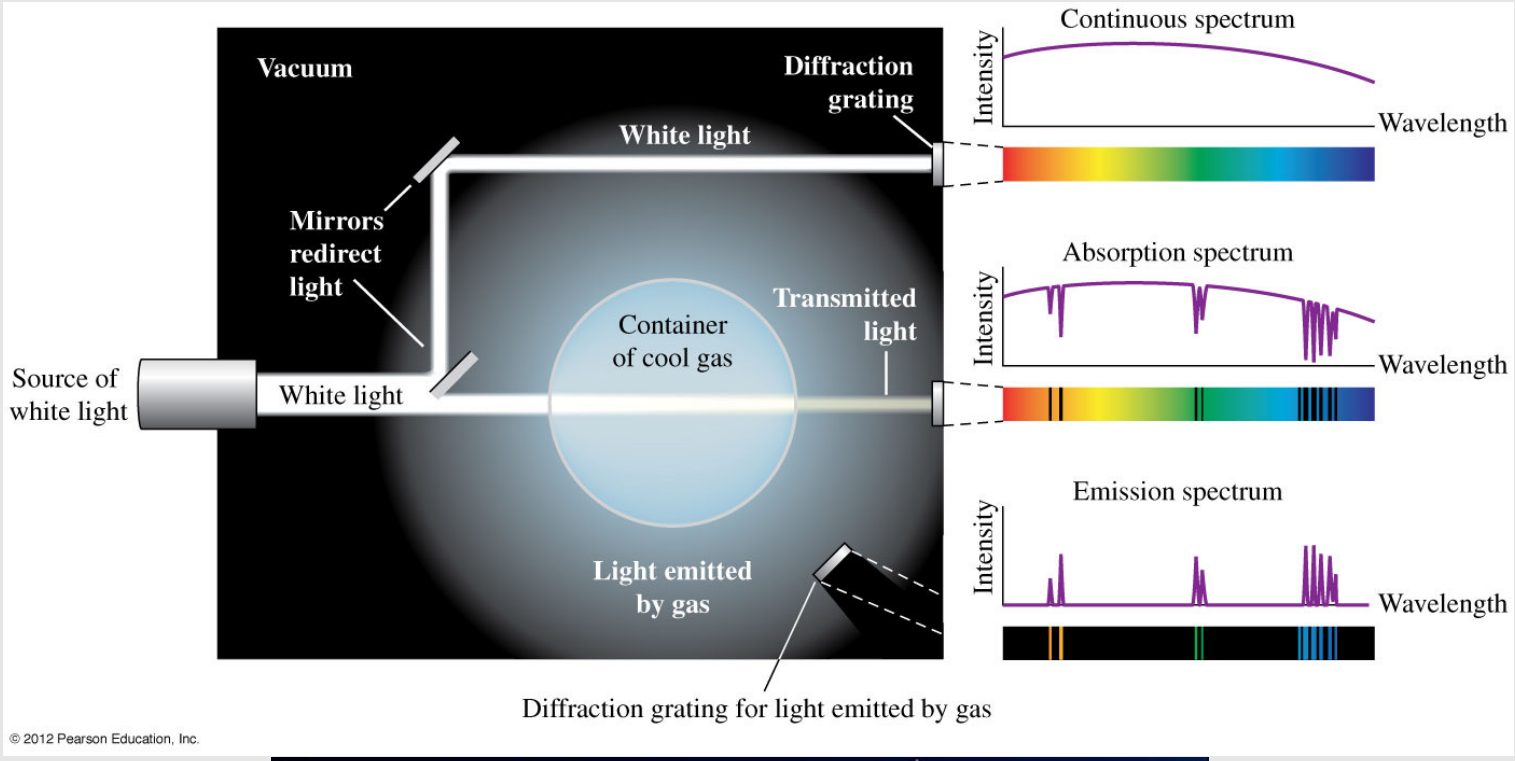




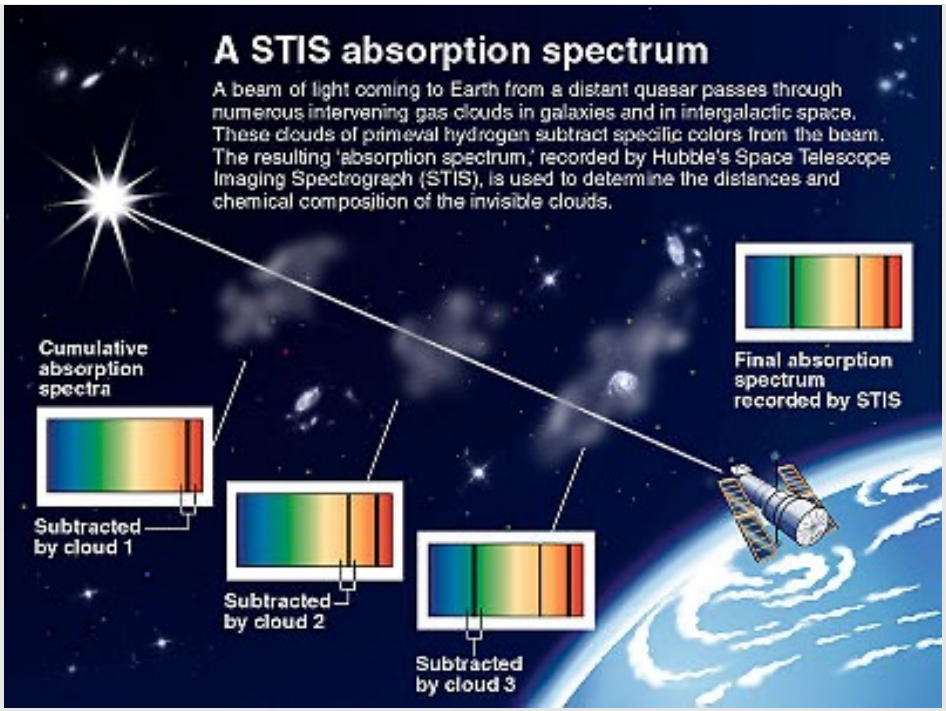
吸收光譜反映了黑體輻射所通過的氣體的特徵

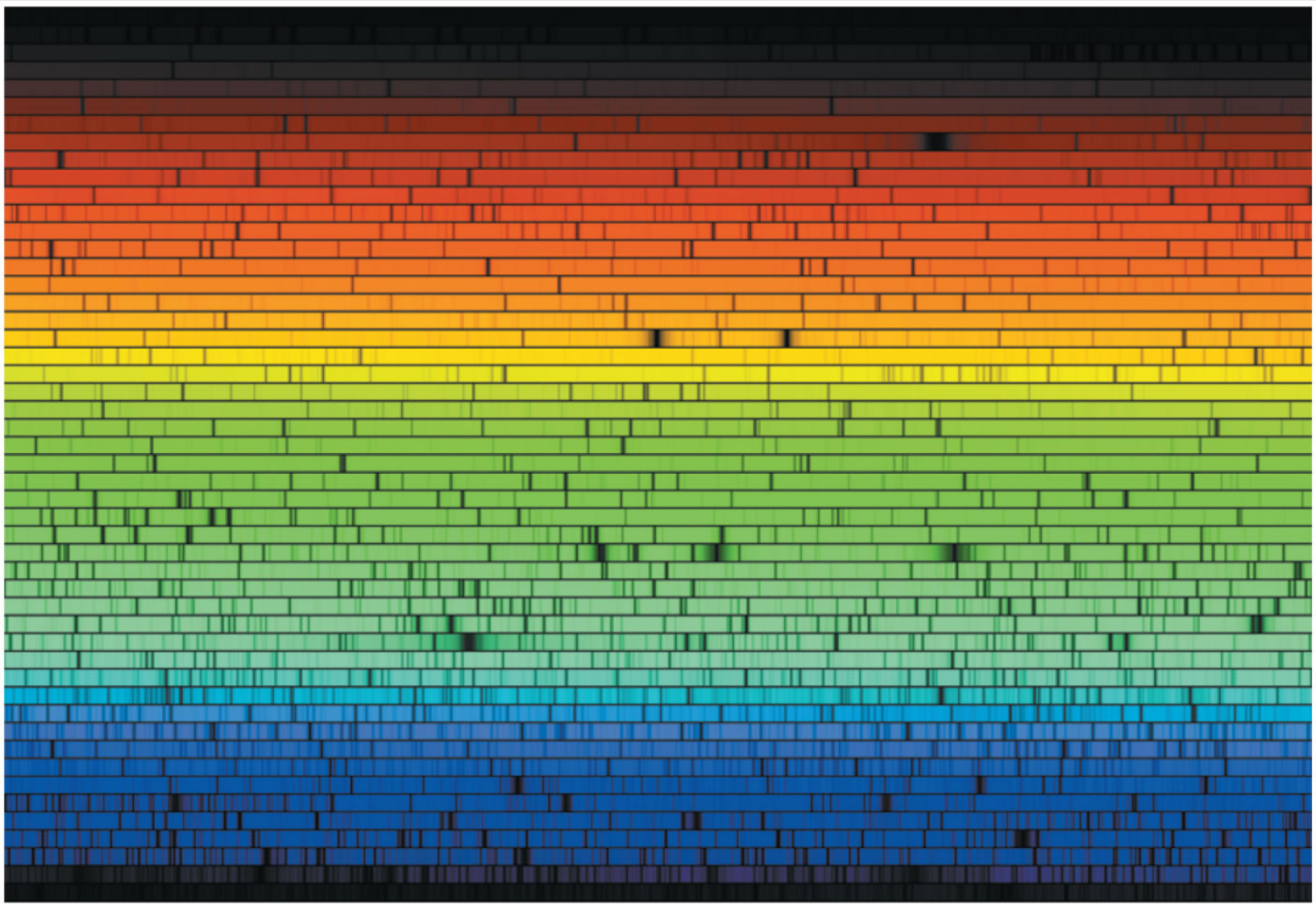


吸收光譜與放射光譜是對應的



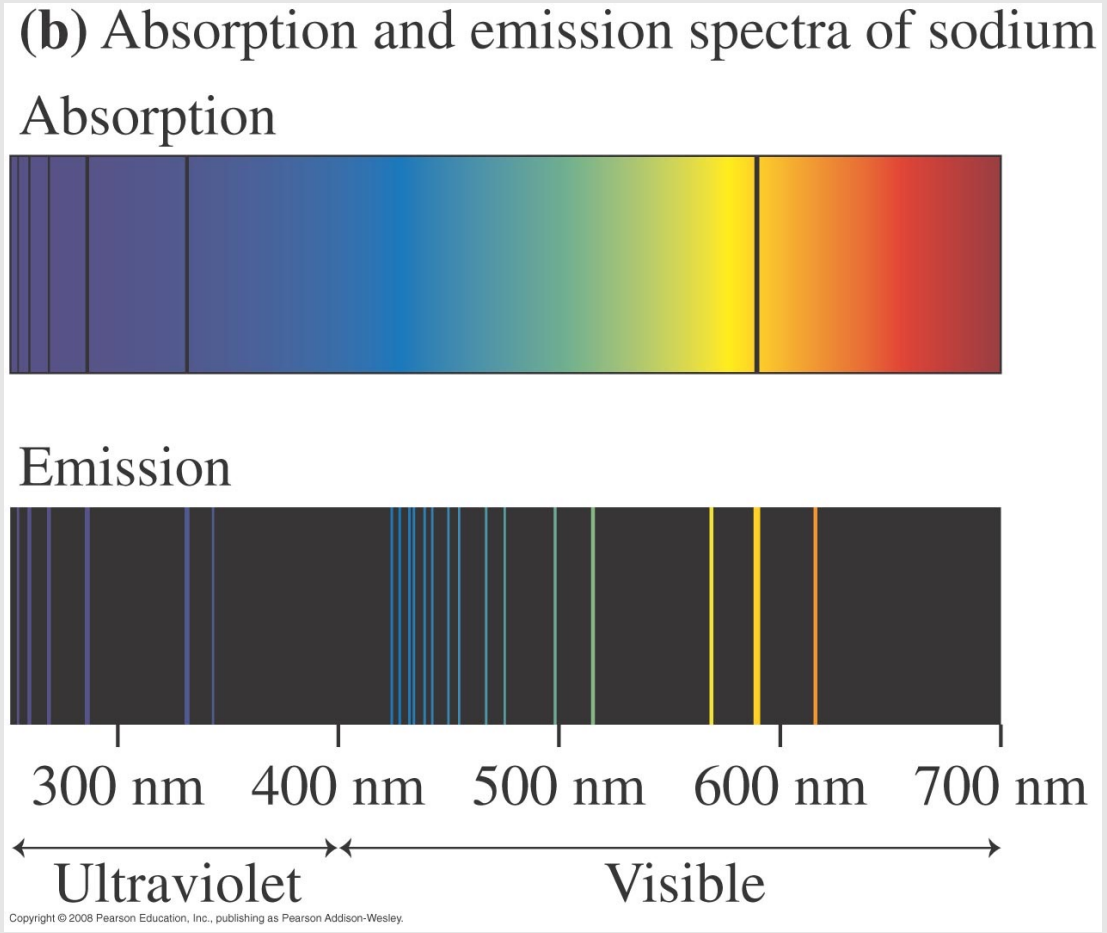
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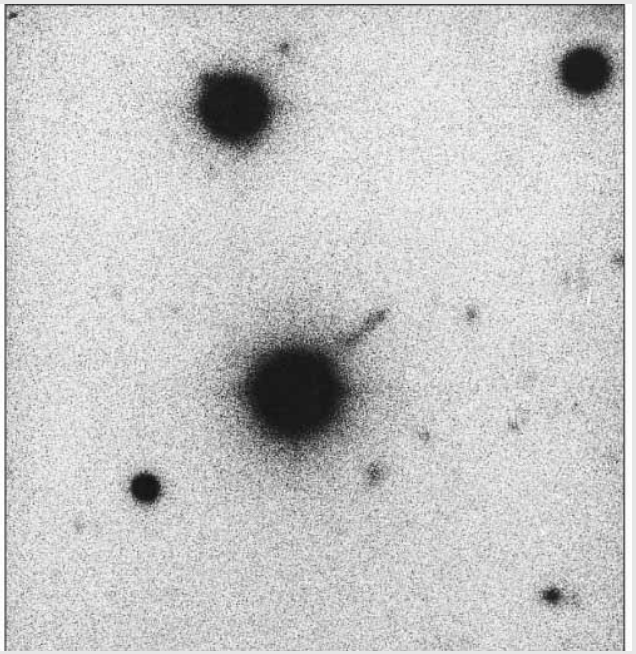
太陽的吸收光譜

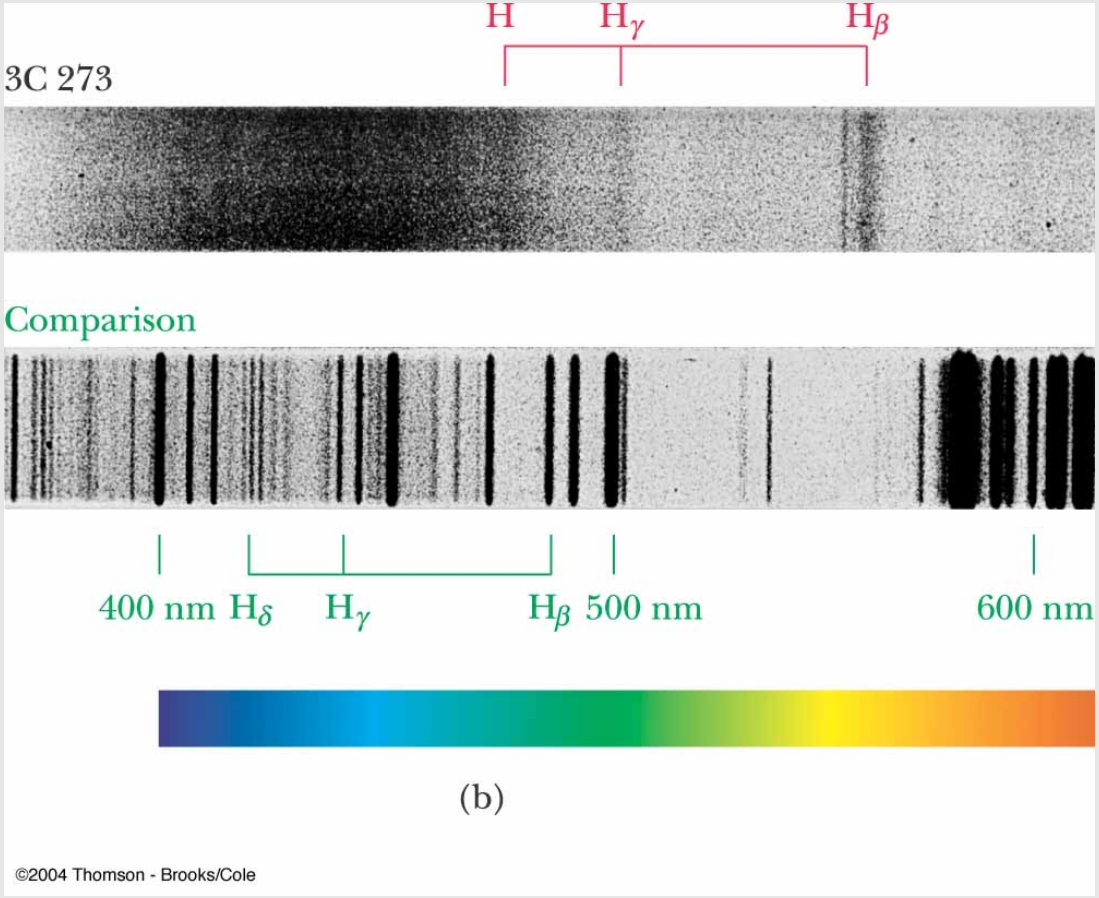


吸收光譜一定會出現在放射光譜上，但反之並不一定。

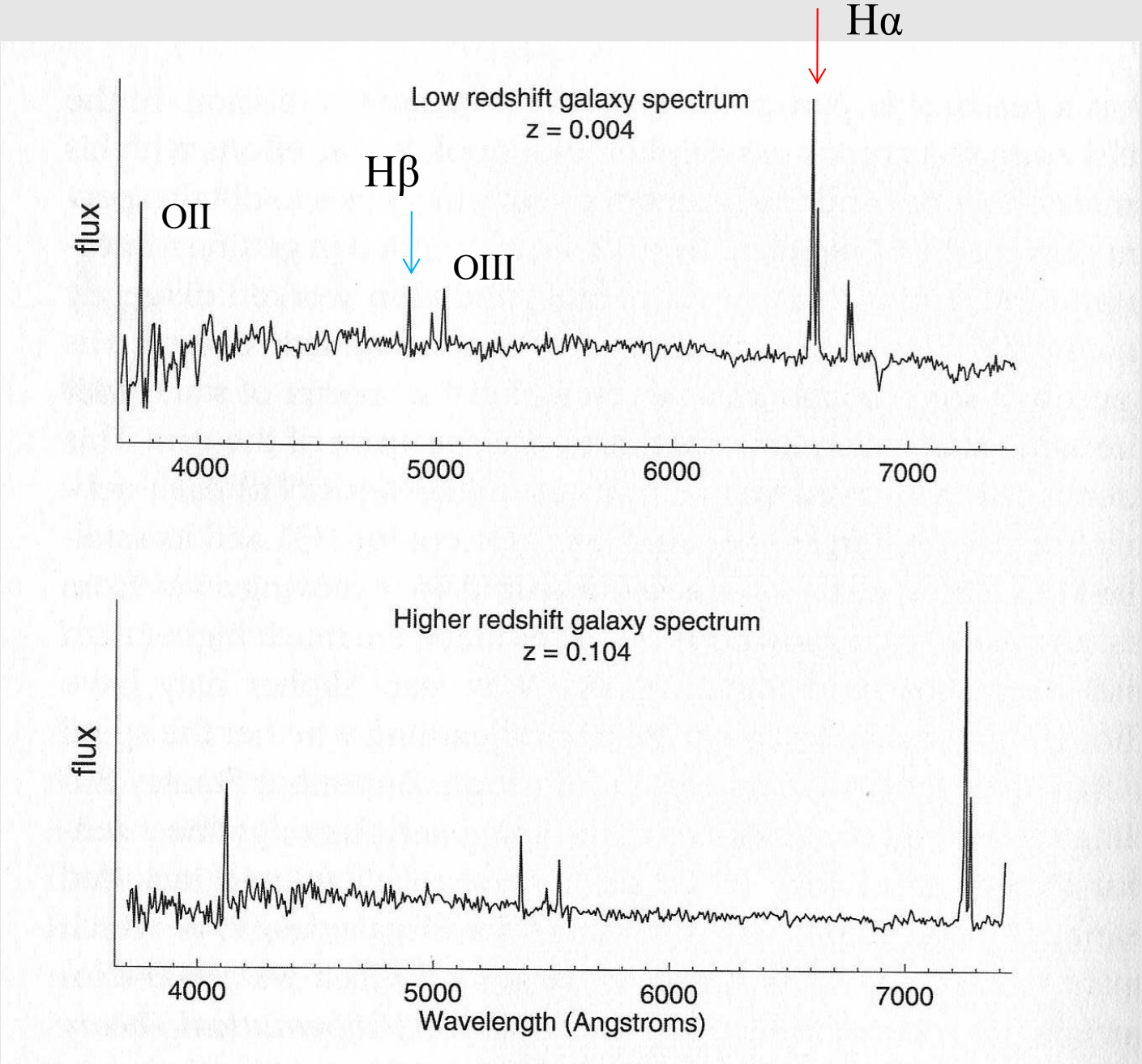
光譜對天文研究非常重要。

遠處星體的紅位移。



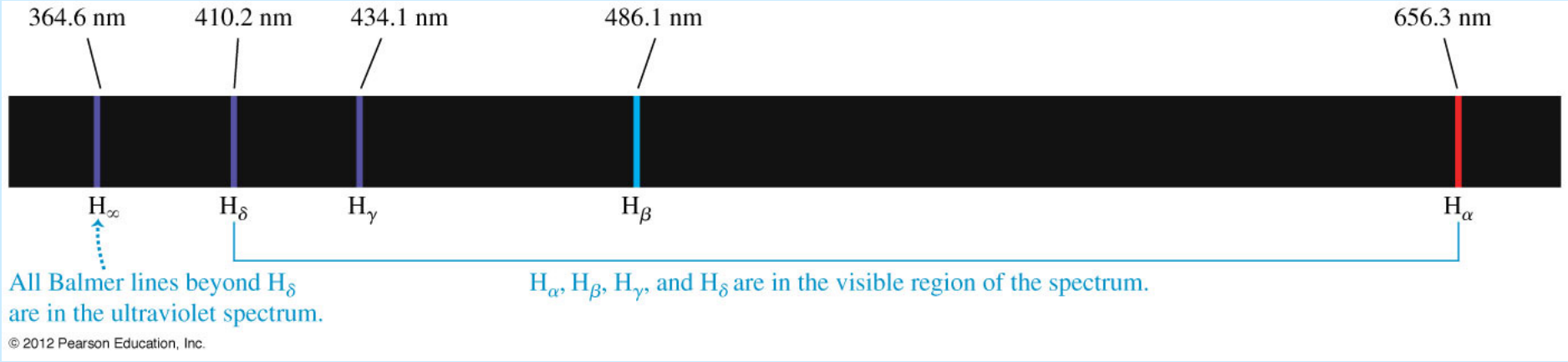


The universe is expanding!!!!!!


Hα
Low redshift galaxy spectrum
z = 0.004
Hβ
OII
OIII
flux
4000
5000
6000
7000
Higher redshift galaxy spectrum
z = 0.104
flux
4000
5000
6000
7000
Wavelength (Angstroms)

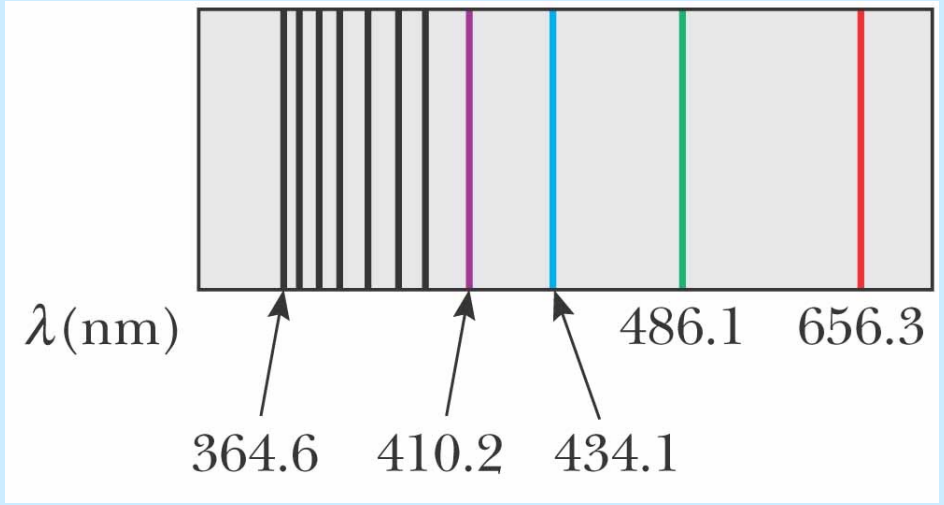
氫原子光譜

Visible light Balmer series of Hydrogen atom



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∞





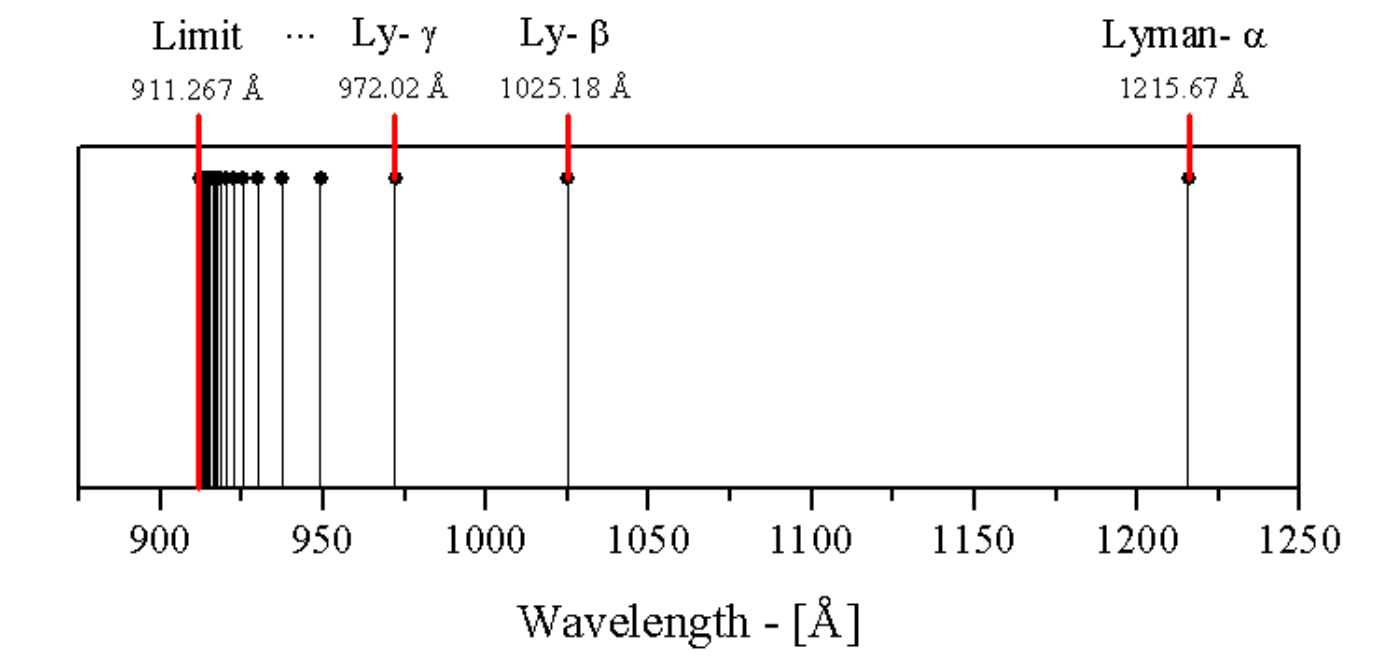
女子高校一位數學教師 Balmer 在1885 找到了光譜線波長的規則：

J. Balmer

$$\frac{1}{\lambda} = R_H \cdot \left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

R_H Rydberg Constant

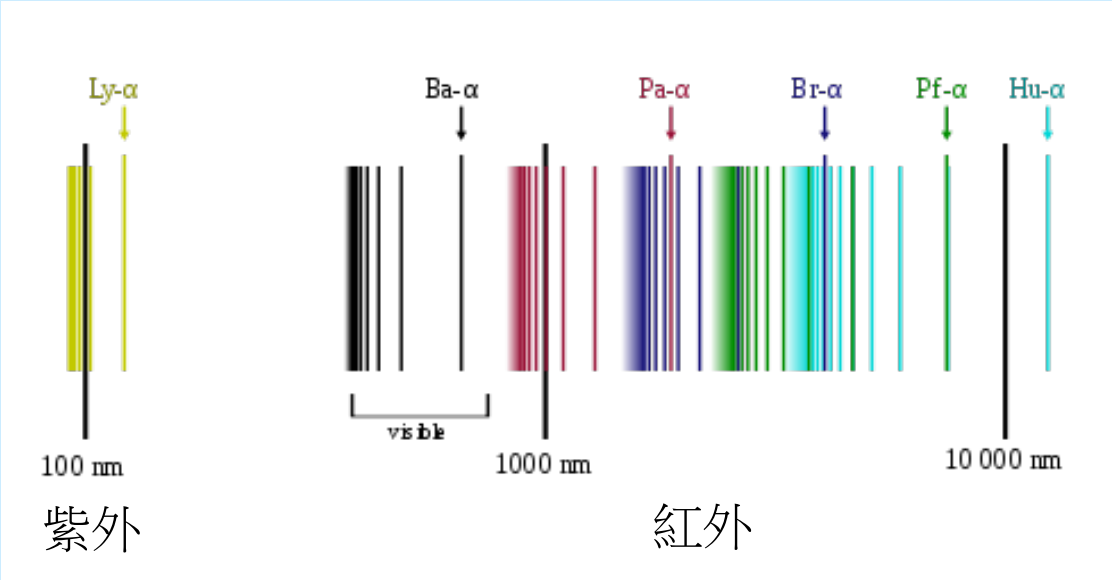
Ultraviolet Lyman Series系列



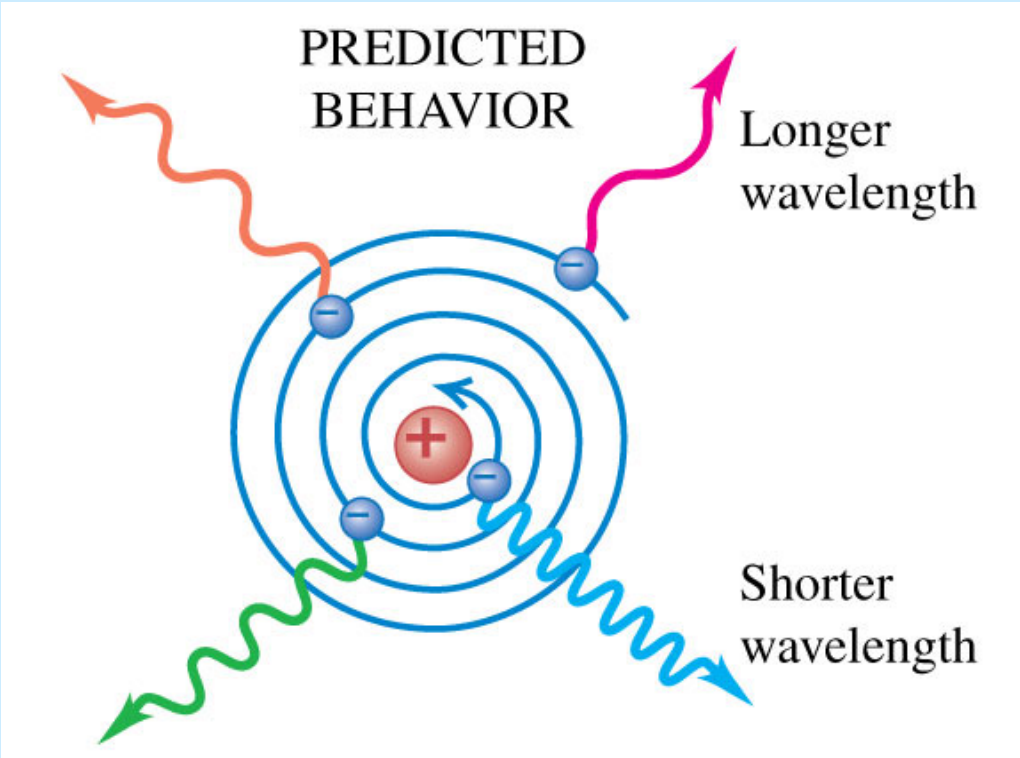
$$\frac{1}{\lambda} = R_H \cdot \left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \rightarrow \quad \frac{1}{\lambda} = R_H \cdot \left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

氫原子光譜全系列 Spectrum of Hydrogen

Lyman series **Balmer series** **Paschen series** **Brackett series** **Pfund series** **Humphreys series**



$$\frac{1}{\lambda} = R_H \cdot \left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad \Rightarrow \quad \frac{1}{\lambda} = R_H \cdot \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$



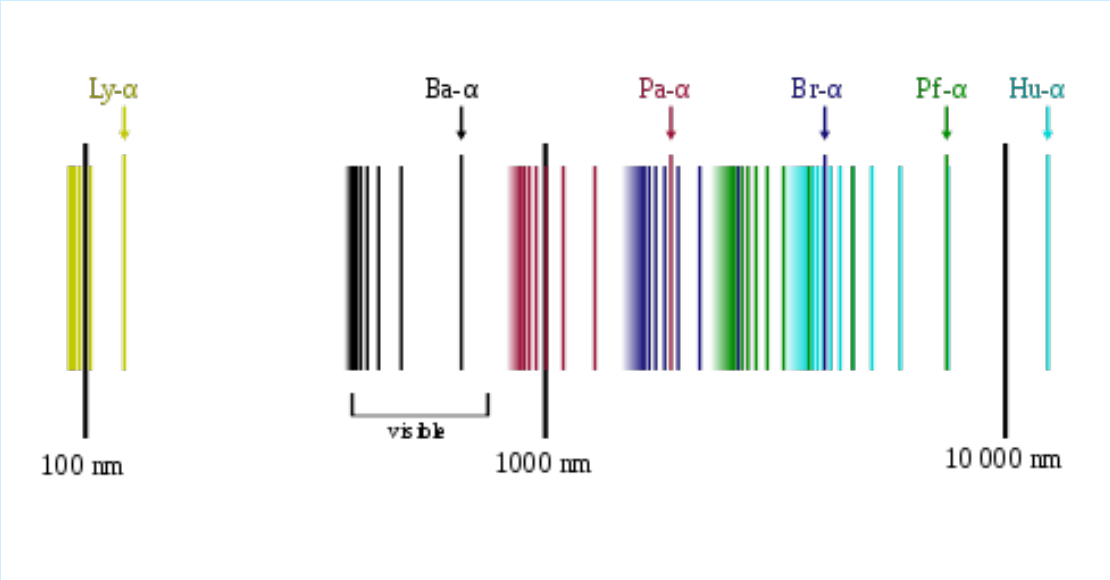
Atomic emission comes from mechanism inside atoms, such as electrons.

Rotating charges emit EM wave with corresponding frequencies.

Electrons in a solar system like atom could rotate at arbitrary frequencies.

We would expect continuous spectrum.

Lyman series **Balmer series** **Paschen series** **Brackett series** **Pfund series** **Humphreys series**



$$\frac{1}{\lambda} = R_H \cdot \left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

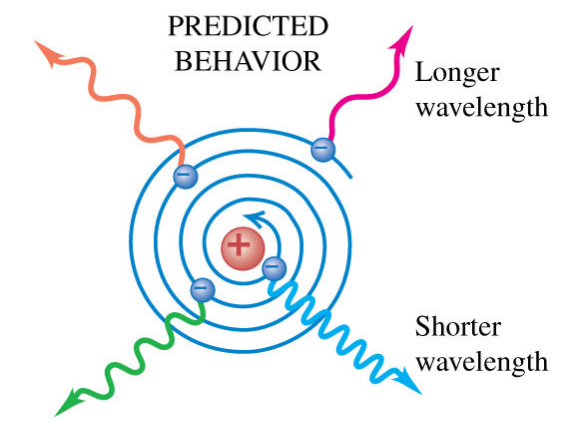
But apparently the spectrum is discreet!

氫原子的光譜顯然是太陽系般的原子模型所無法解釋的。

如此清晰的規則，似乎指點了正確的原子模型最重要的特徵！

Let's back up a bit to explain the discovery that light is made of particles.





原子的光譜與原子內電子的狀態直接相關！

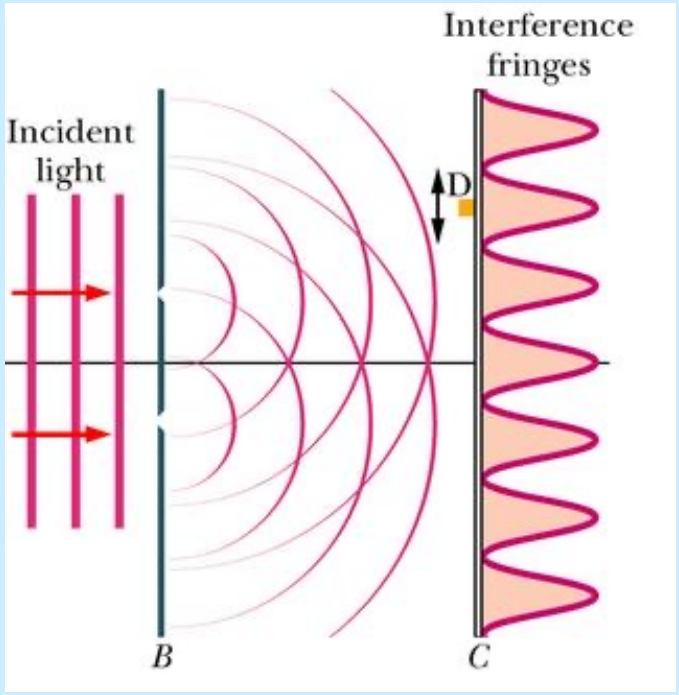
了解光譜的規則，就瞭解了原子結構，也就瞭解了原子中的電子！

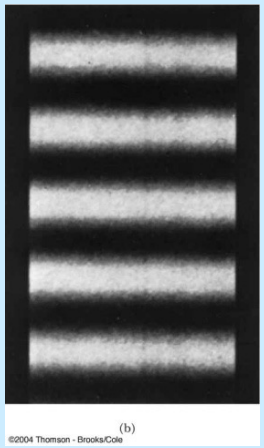
但我們要倒回去介紹兩個革命性的概念：量子以及光子。

Let’s back up a bit to explain light is made of particles.

Light is wave!

達文西，惠更斯：光傳播如此之快，不可能是粒子。







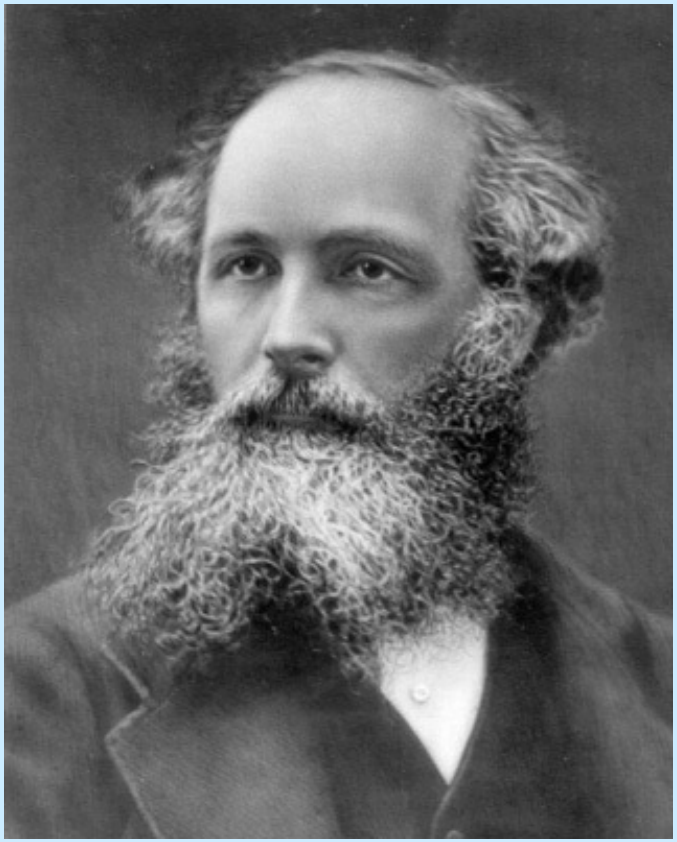
Thomas Young
1773-1829

因為光有干涉現象，光是波！　　楊　1804

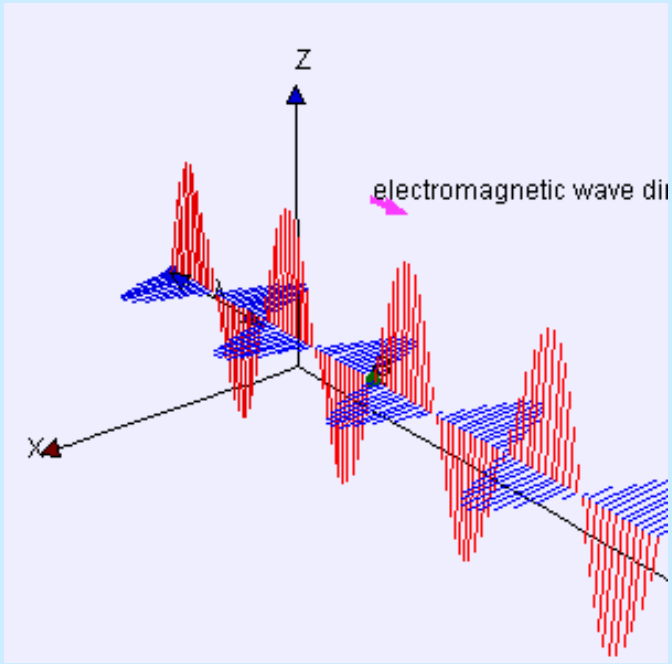
馬克斯威爾推導出光是電磁場的波，將電磁場的擾動傳播到遠方的波。

傳播的獨立的電磁場就是一種波！

電磁波：電與磁的自動化



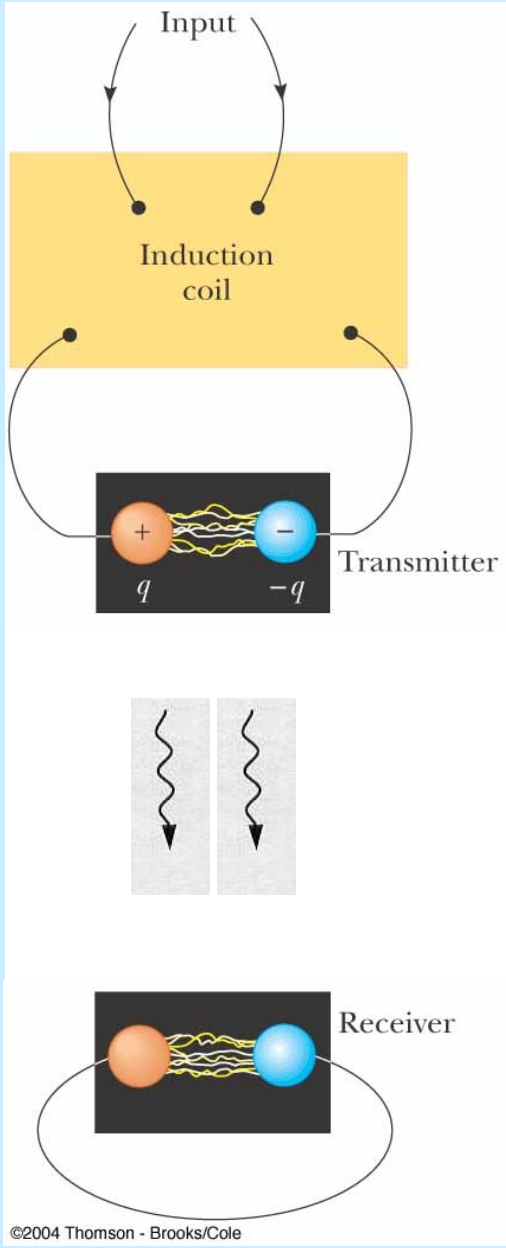
James Clerk Maxwell (1831–1879)



many thanks to Prof. Fu-Kwun Hwang

赫茲在實驗室中第一次人為地製造出電磁波。





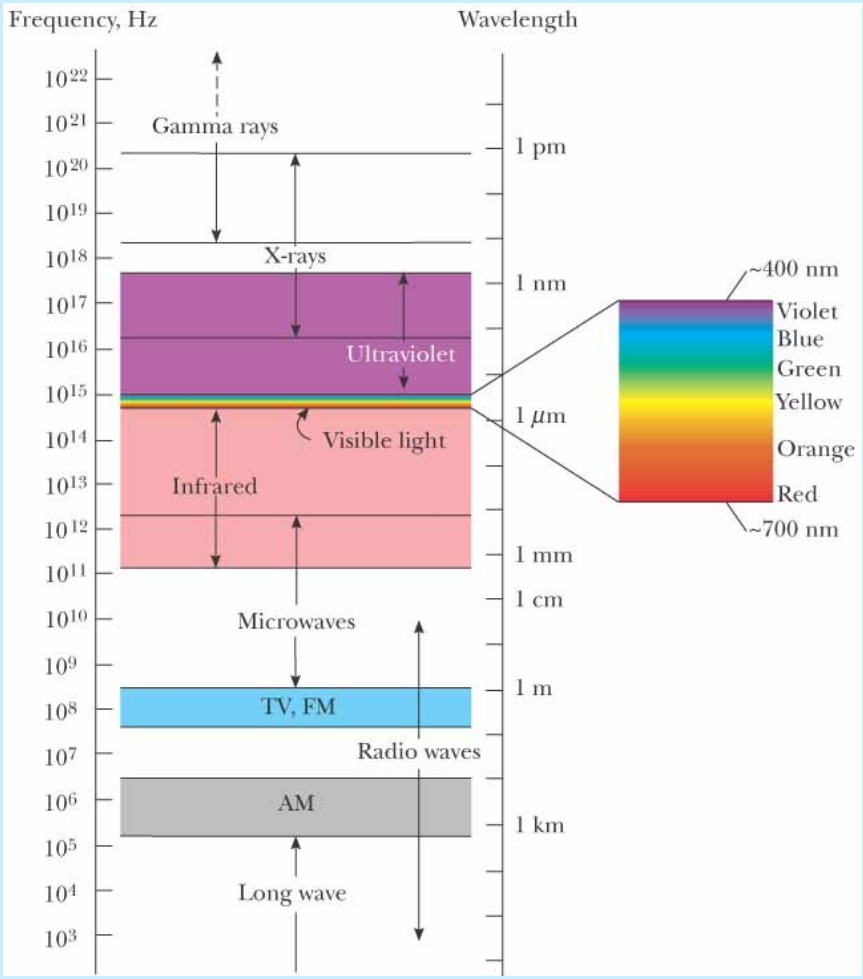
要產生電磁波，就要製造變化的電場或磁場。

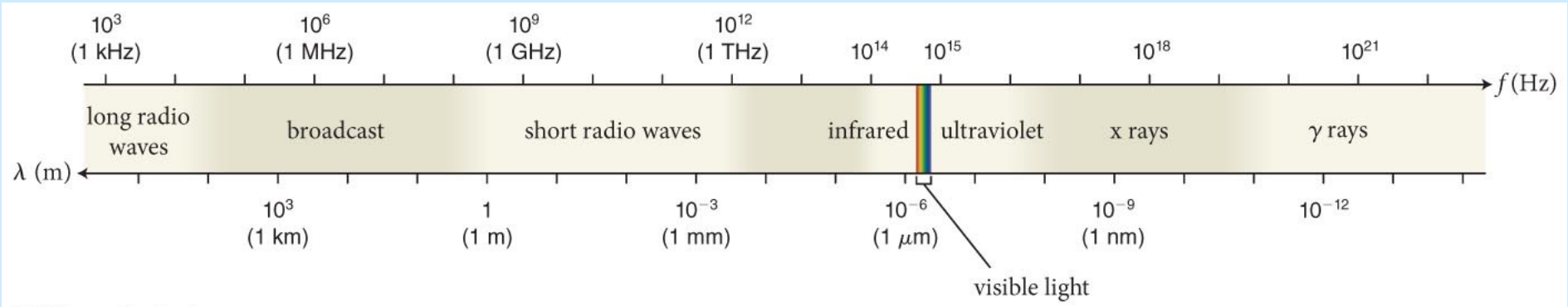
最簡單的方法就是製造變化的電流。LC電路。

電磁波以頻率或波長為特徵

$\lambda f = c$

天線的大小大致與波長相當。











波浪的能量由振幅決定，連續地調整振幅，即可連續地調整、分割能量。

Note that the energy of wave can be adjusted continuously.

3 On a Heuristic Point of View about the Creation and Conversion of Light†

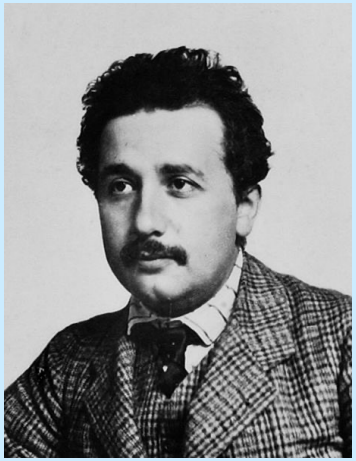
A. EINSTEIN

1905這一年Einstein發表了四篇撼動物理學基礎的論文！
這一篇讓他拿到1921年諾貝爾獎！

THERE exists an essential formal difference between the theoretical pictures physicists have drawn of gases and other ponderable bodies and Maxwell's theory of electromagnetic processes in so-called empty space. Whereas we assume the state of a body to be completely determined by the positions and velocities of an, albeit very large, still finite number of atoms and electrons, we use for the determination of the electromagnetic state in space continuous spatial functions, so that a finite number of variables cannot be considered to be sufficient to fix completely the electromagnetic state in space. According to Maxwell's theory, the energy must be considered to be a continuous function in space for all purely electromagnetic phenomena, thus also for light, while according to the present-day ideas of physicists the energy of a ponderable body can be written as a sum over the atoms and electrons. The energy of a ponderable body cannot be split into arbitrarily many, arbitrarily small parts, while the energy of a light ray, emitted by a point source of light is according to Maxwell's theory (or in general according to any wave theory) of light distributed continuously over an ever increasing volume.

The wave theory of light which operates with continuous functions in space has been excellently justified for the representation of purely optical phenomena and it is unlikely ever to be replaced by another theory. One should, however, bear in mind that optical observations refer to time averages and not to

† *Ann. Physik* **17**, 132 (1905).







In fact, it seems to me that the observations on "black-body radiation", photoluminescence, the production of cathode rays by ultraviolet light and other phenomena involving the emission or conversion of light can be better understood on the assumption that the energy of light is distributed discontinuously in space. According to the assumption considered here, when a light ray starting from a point is propagated, the energy is not continuously distributed over an ever increasing volume, but it consists of a finite number of energy quanta, localised in space, which move without being divided and which can be absorbed or emitted only as a whole.

電磁輻射的粒子是不可分割的！

8. On the Production of Cathode Rays by Illumination of Solids

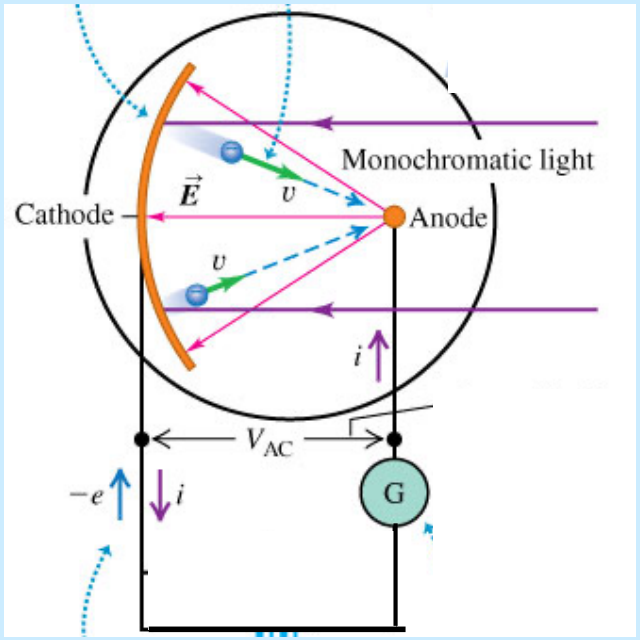
The usual idea that the energy of light is continuously distributed over the space through which it travels meets with especially great difficulties when one tries to explain photo-electric phenomena, as was shown in the pioneering paper by Mr. Lenard.[3]

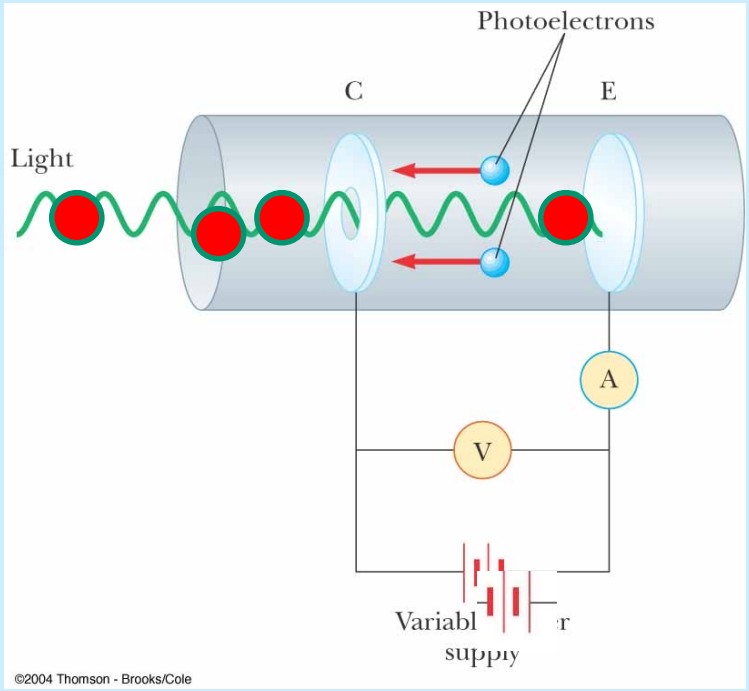
According to the idea that the incident light consists of energy quanta with an energy $R\beta\nu/N$, one can picture the production of cathode rays by light as follows. Energy quanta penetrate into a surface layer of the body, and their energy is at least partly

光電效應 Photoelectric Effect是光粒子性更直接的證據！

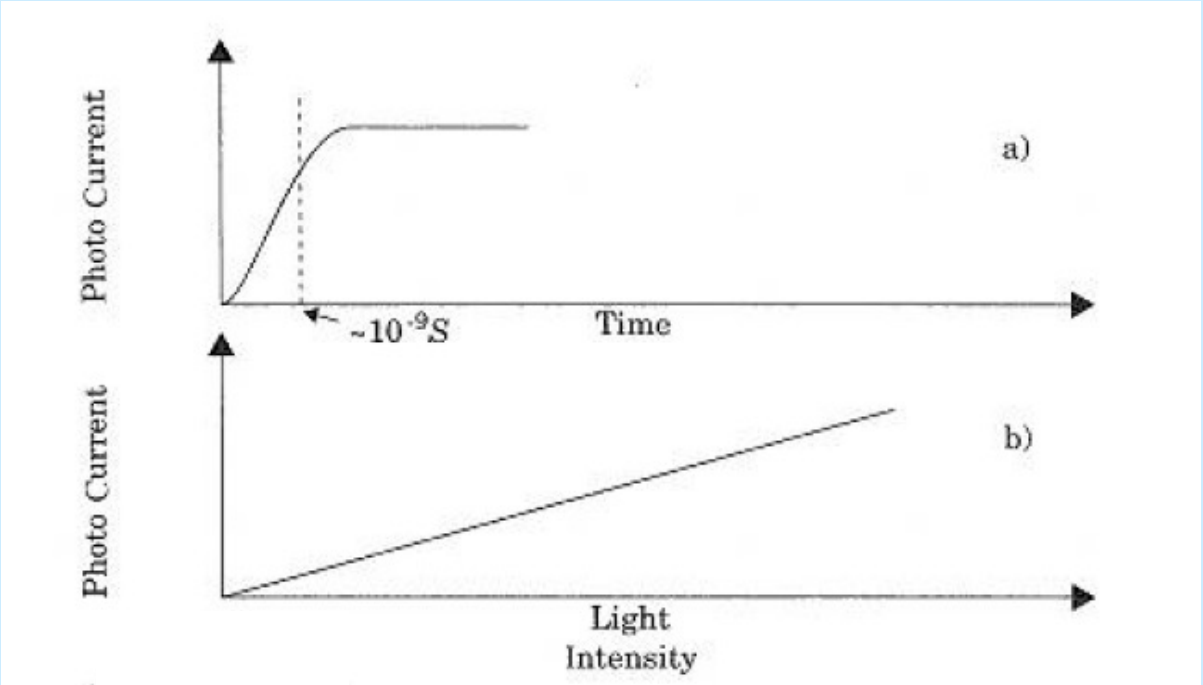


光電效應 Photoelectric Effect



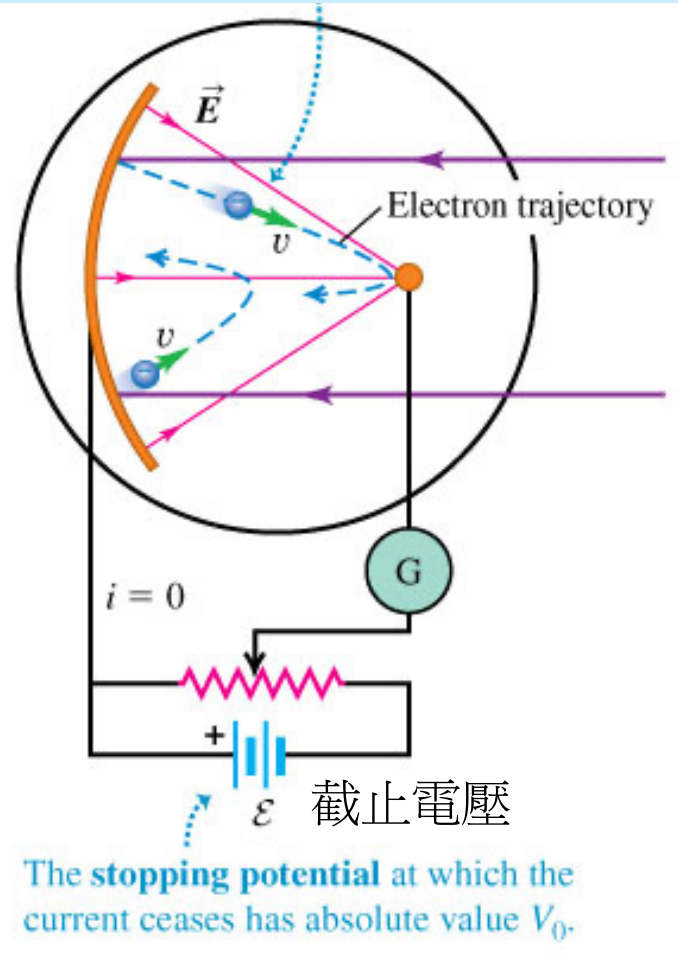


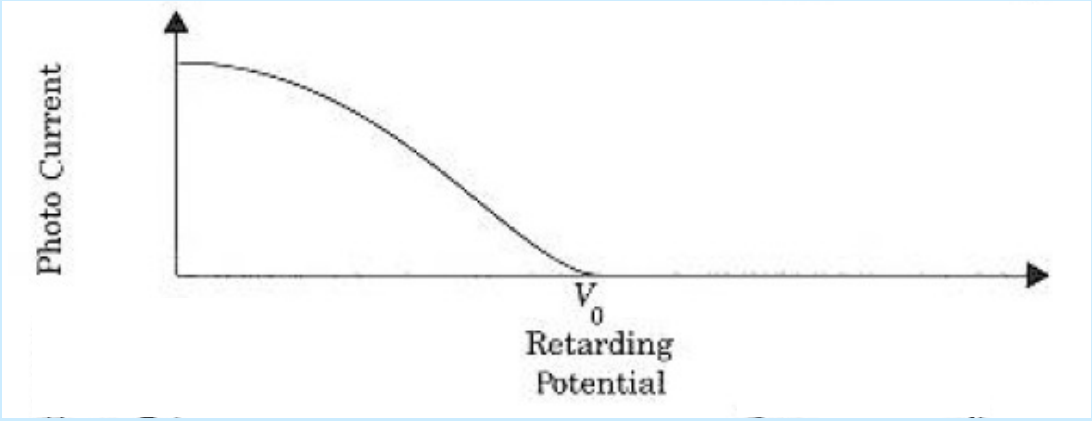
Light hitting metal cathodes could liberate electrons from metal and form current.



Experiments measure that photocurrent occur immediately

Photocurrent is proportional to light intensity, confirming it is indeed photo.





Electrons fly out of cathode with a kinetic energy.

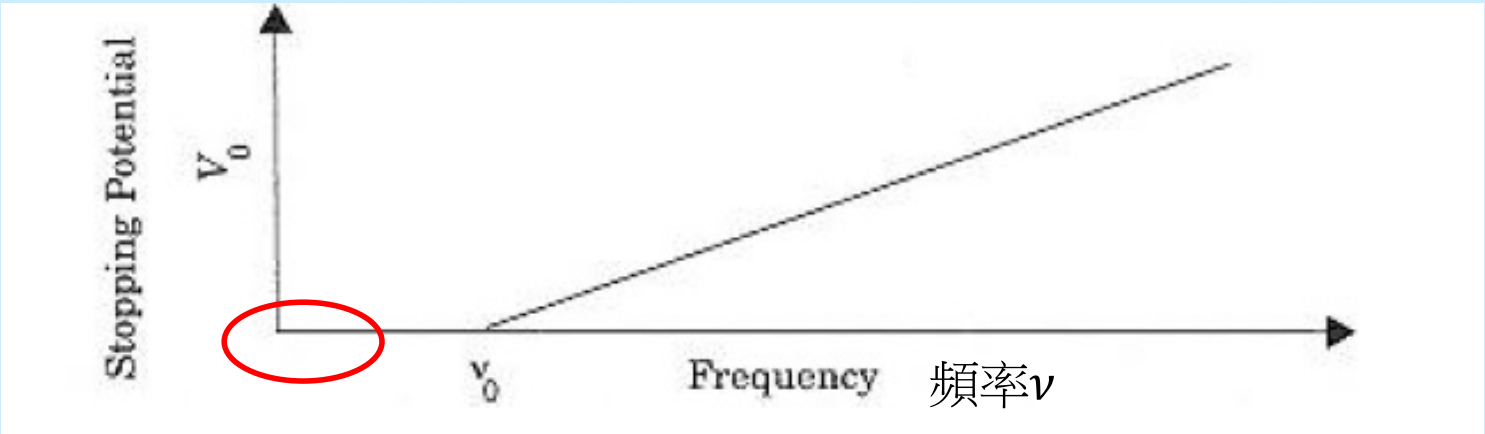
We can add a reverse voltage to decelerate electrons.

The voltage that just stop current is called stopping potential.

Stopping voltage times *e* equals kinetic energy of electrons.



Surprise! Stopping voltage is totally determined by the frequency of light, independent of its intensity.



Note that $\nu < \nu_0, V_0 = 0$.

If frequency is smaller than a certain ν_0, there would be no photocurrent at all, even without reverse voltage



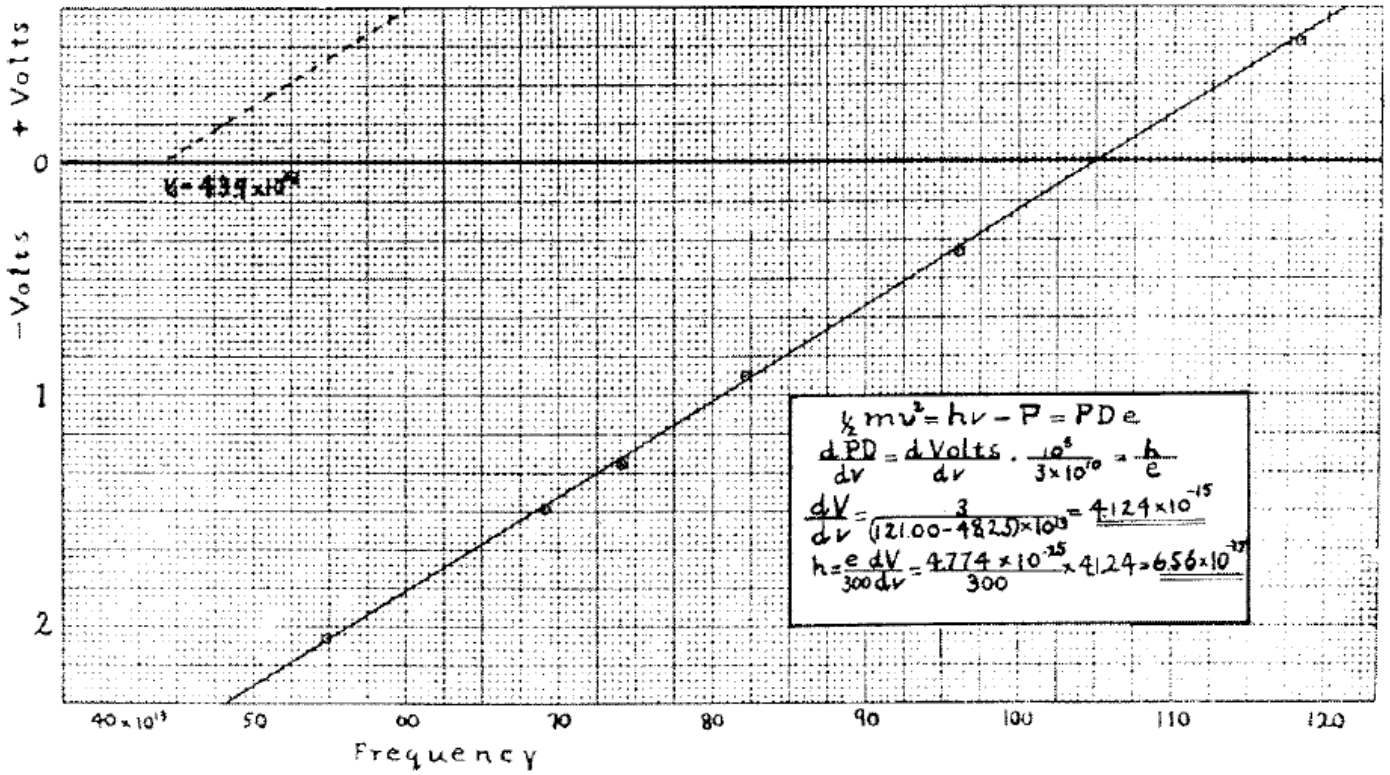


Fig. 3.4 Millikan's results on the photoelectric effect compared with the predictions of Einstein's quantum theory (Millikan, 1916).

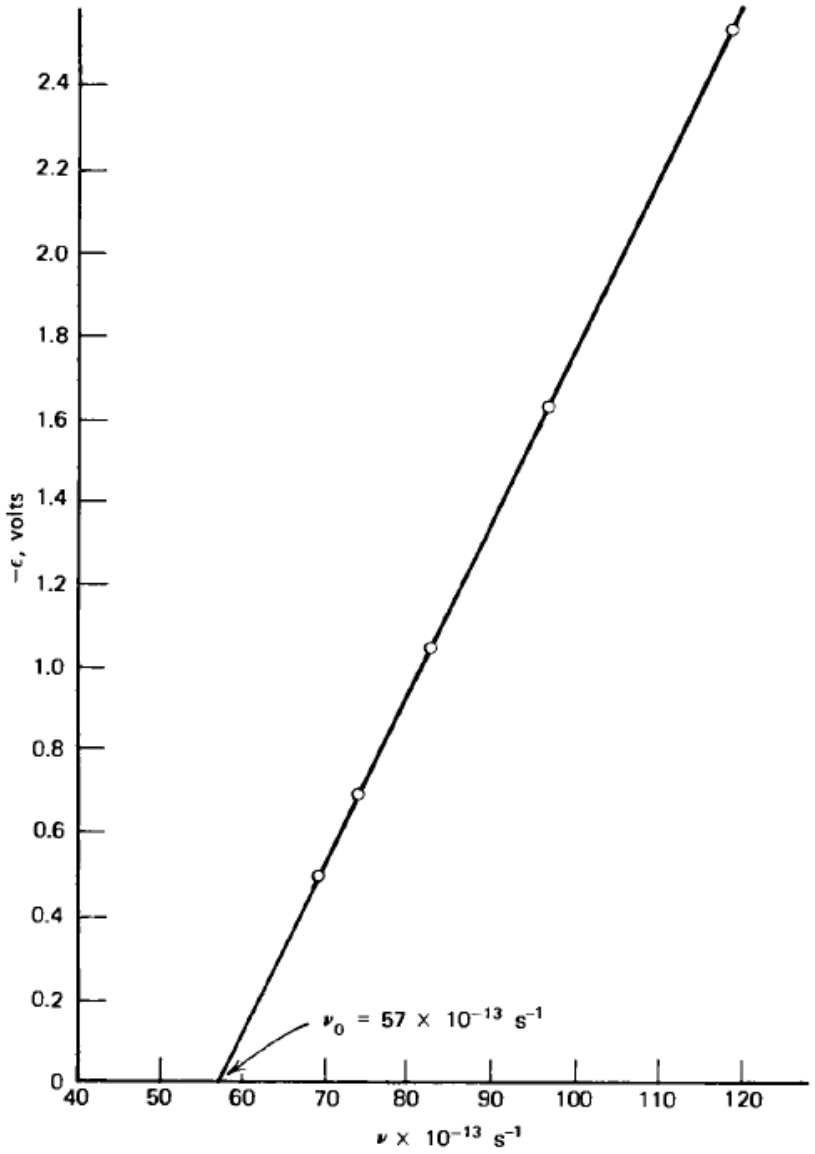
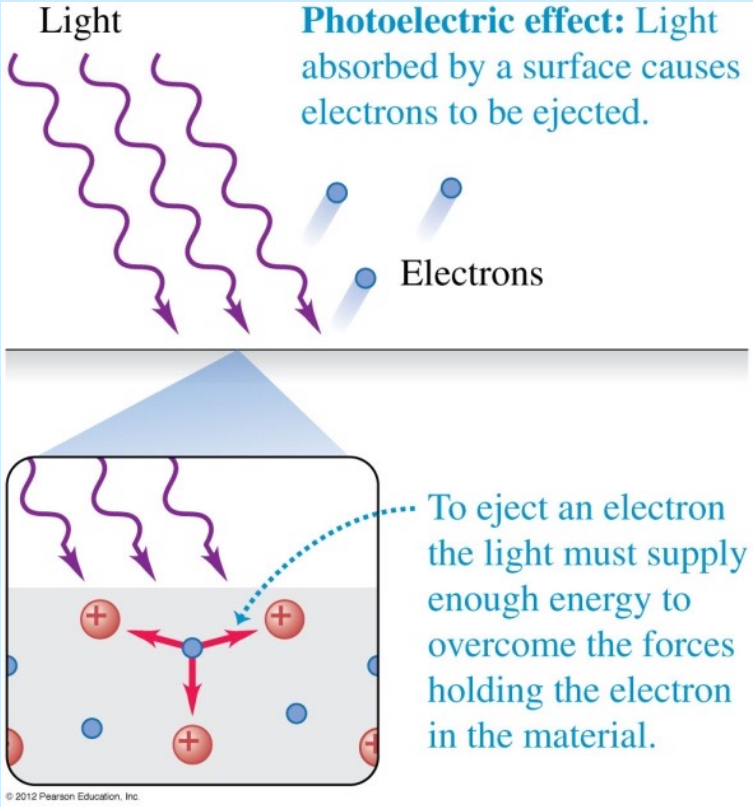


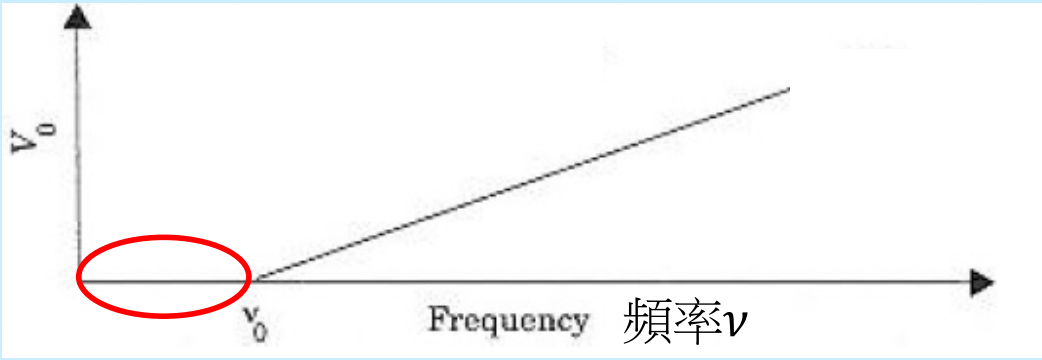
Figure 1-3 Photoelectric effect data showing a plot of retarding potential necessary to stop electron flow from a metal (lithium), or equivalently, electron kinetic energy, as a function of frequency of the incident light. The slope of the line is h/e.



Einstein realized this result could not be explained by light as wave.





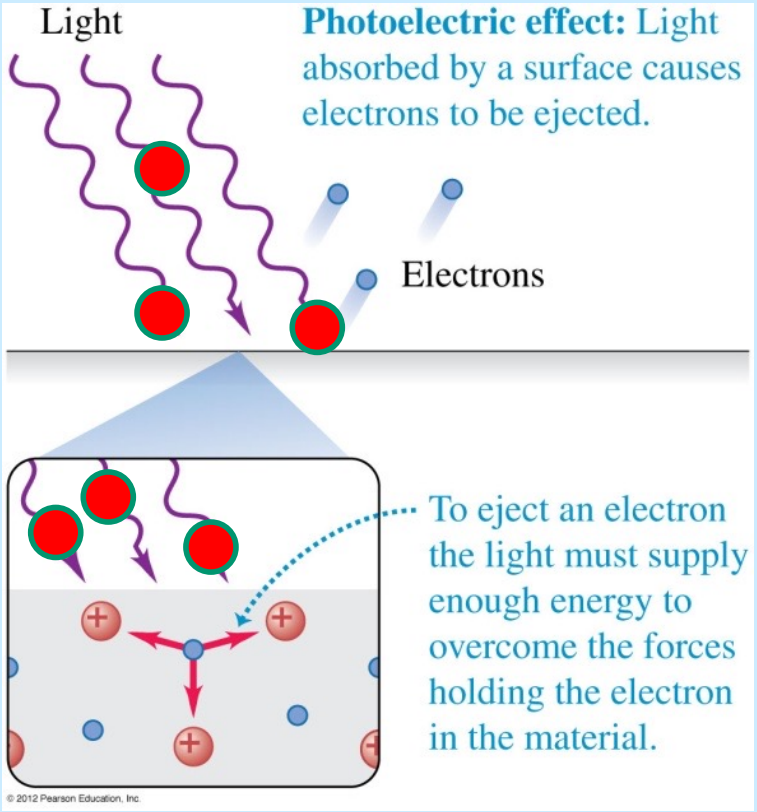


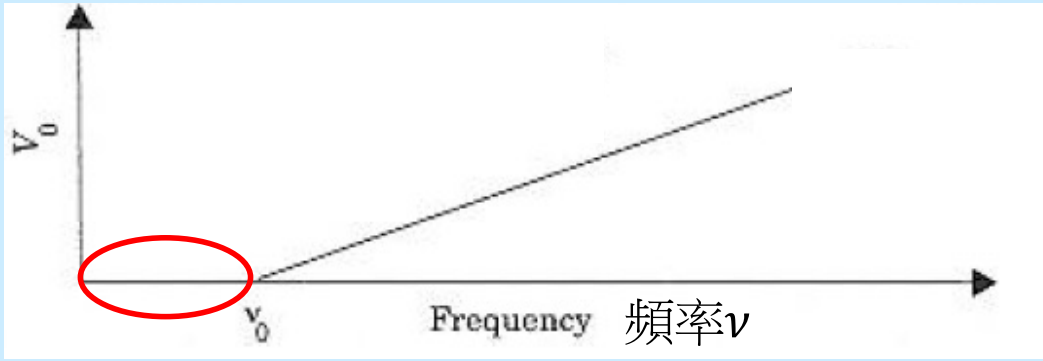
Electrons in metal absorb light energy to overcome bound energy and escape.

For $\nu < \nu_0$, no electrons can absorb enough energy no matter how strong the light.

The energy of wave is continuous and can be accumulated as large as possible.







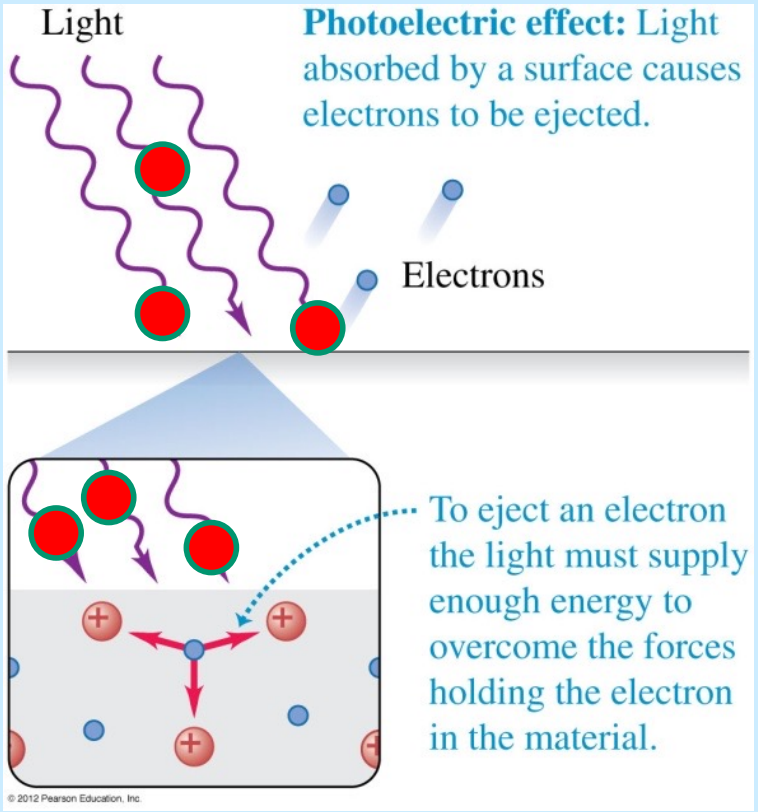
Einstein found that data would be reasonable if light is mode of particles.

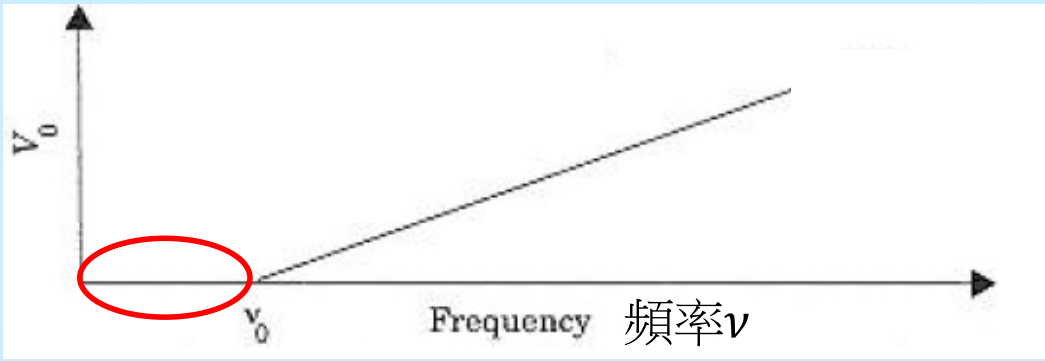
Electrons absorb one and only one light particle to escape or nothing at all.

The key assumption is photon is indivisible. There is no fraction of one photon.

If a light particle has fixed energy solely determined by frequency,

it is natural that frequencies solely determine whether there is photocurrent.



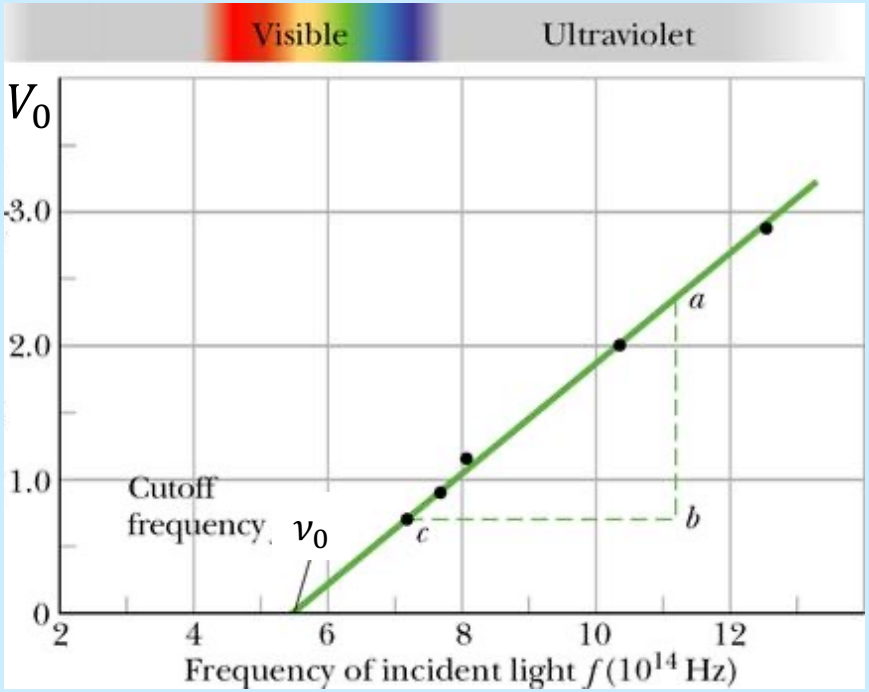


The stopping voltages measure photoelectron kinetic energy.

Photoelectron kinetic energy equals light particle energy minus bound energy.

Linear relation between V_0 and ν indicates light particle energy is proportional to ν.

$$E = h\nu$$



$$eV_0 = K_e = h\nu - W$$

$$V_0 = \frac{h}{e}\nu - \frac{W}{e}$$

$$E = h\nu$$

$$W = h\nu_0$$

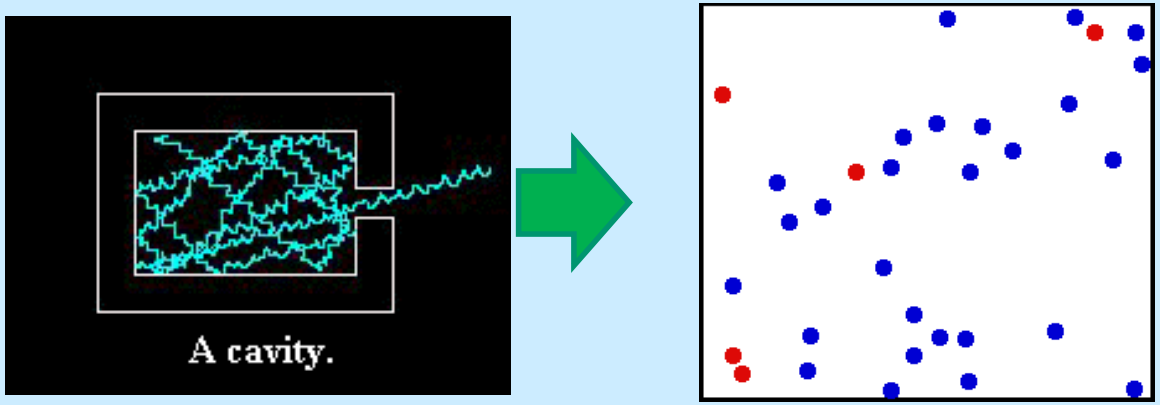
W是光電子離開電極所需克服的束縛能：Work Function：

光量子能量$h\nu$低於W，$\nu < \nu_0$，即無法打出電子。

直線的斜率即是Planck常數h/e。這是最容易測蒲朗克常數的辦法。

電磁波及光都帶有能量也帶有動量！

$$p = \frac{E}{c}$$



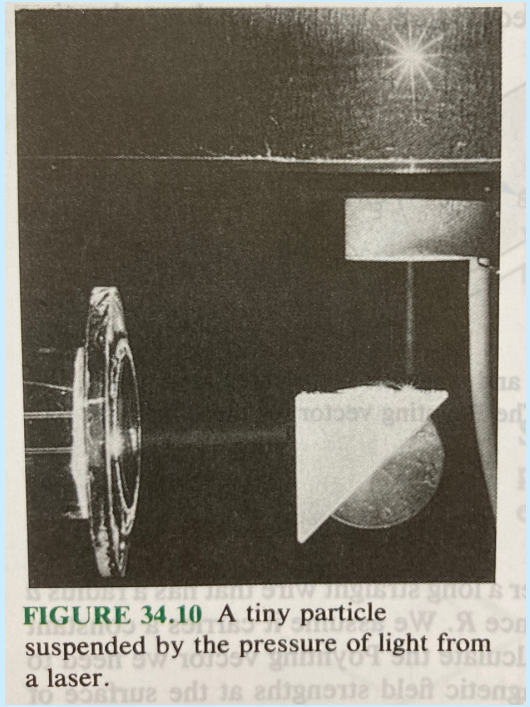


FIGURE 34.10 A tiny particle suspended by the pressure of light from a laser.

光量子如果像一個粒子，那麼就具有動量。

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k \qquad \hbar \equiv \frac{h}{2\pi}$$

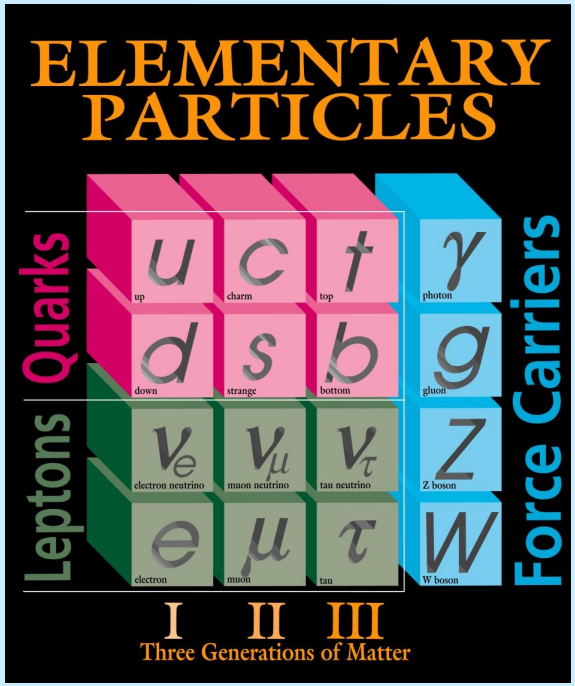
光子動量與波長成反比。

$$E = h\nu = \frac{h}{2\pi} 2\pi\nu = \hbar\omega$$



光子 **Photon** γ





$E = h\nu$ $E = \hbar\omega$

$p = \frac{h}{\lambda}$ $p = \hbar k$

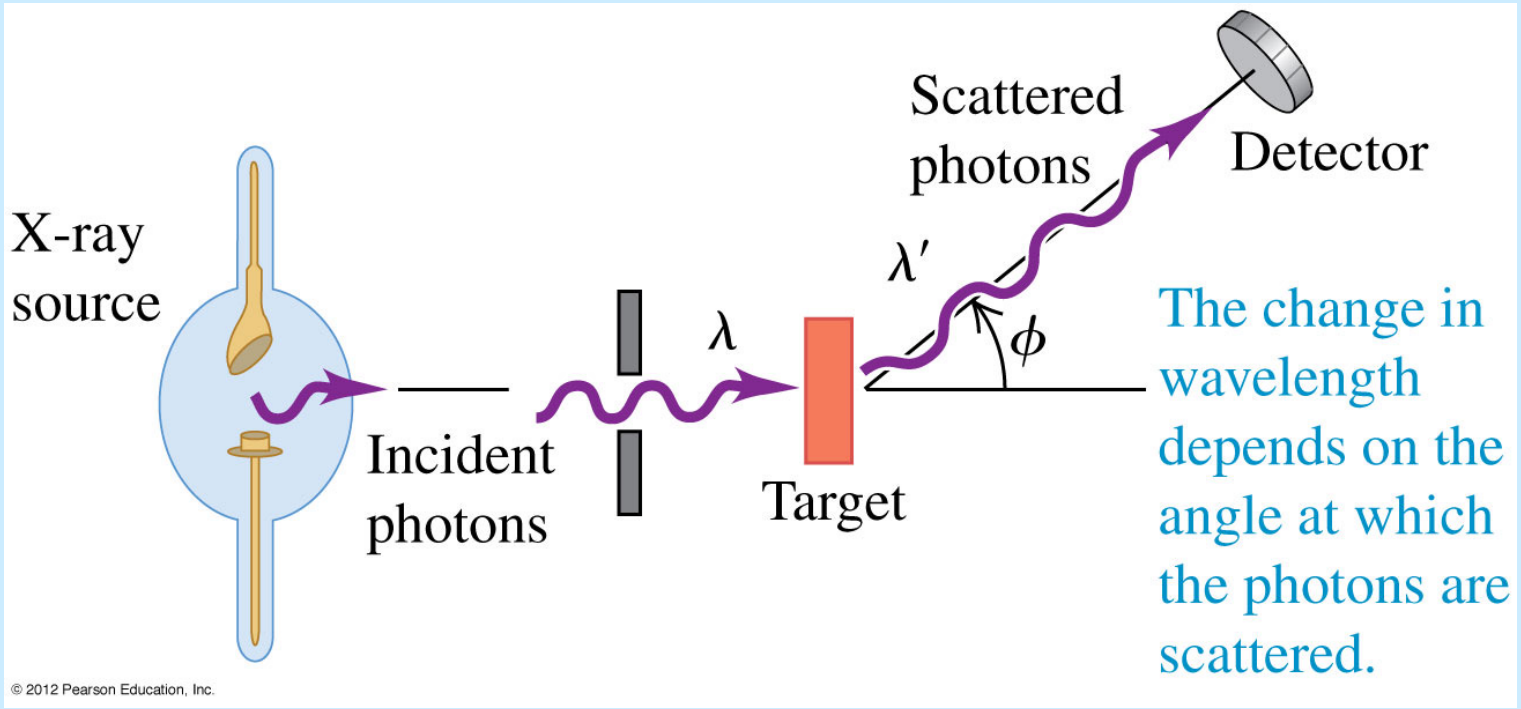
$h = 6.626\times10^{-34}\ \mathrm{J\cdot s} = 4.136\times10^{-15}\ \mathrm{eV\cdot s}$

The key assumption is photon is indivisible. There is no fraction of one photon.

如果光是粒子應該可以與其他粒子、例如電子碰撞。

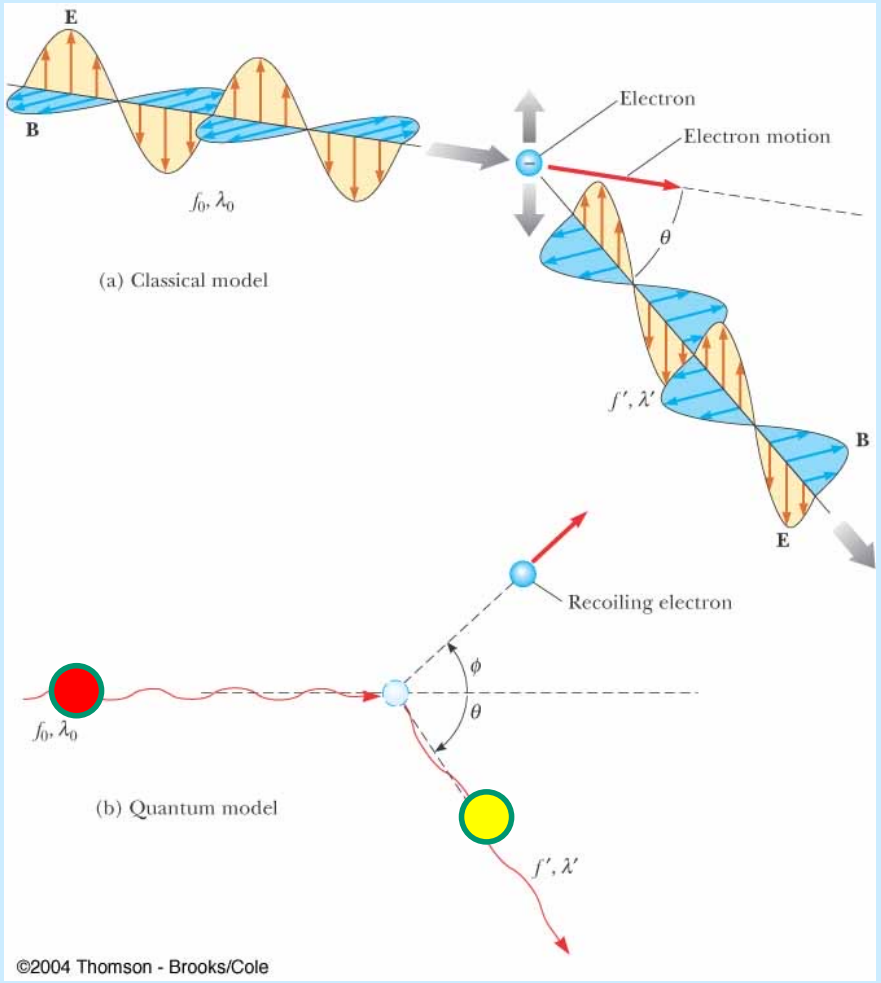
碰撞後，動量會改變。於是光子波長也會改變。

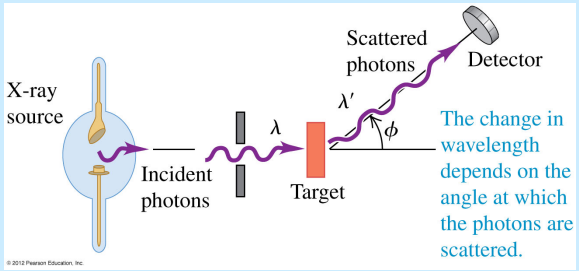
$$p = \frac{h}{\lambda}$$





若光是波，與粒子碰撞一般稱為散射，散射後頻率不會變





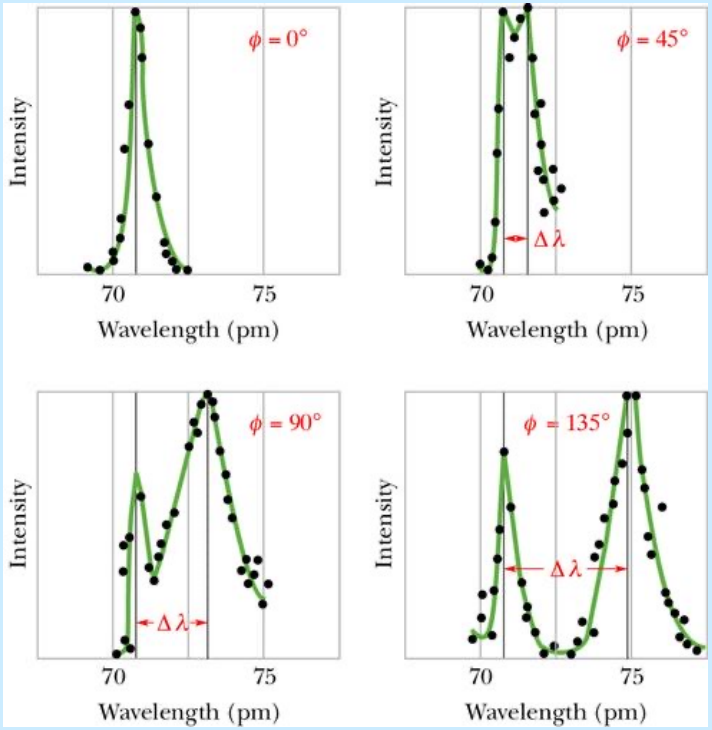
但若是光子，與電子碰撞後，光子動量會改變。因此波長會改變。

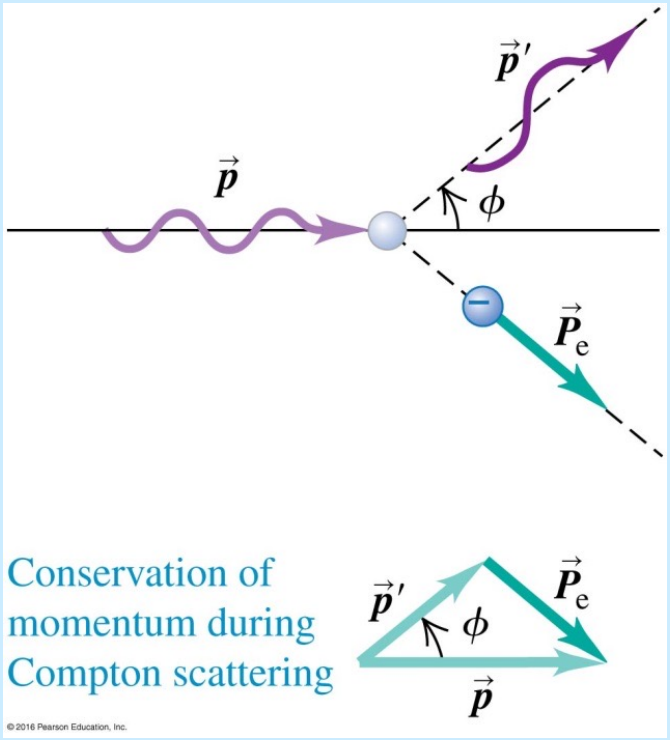


康普頓效應

動量的改變與散射角ϕ有關，

因此波長的變化也與ϕ有關。





$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

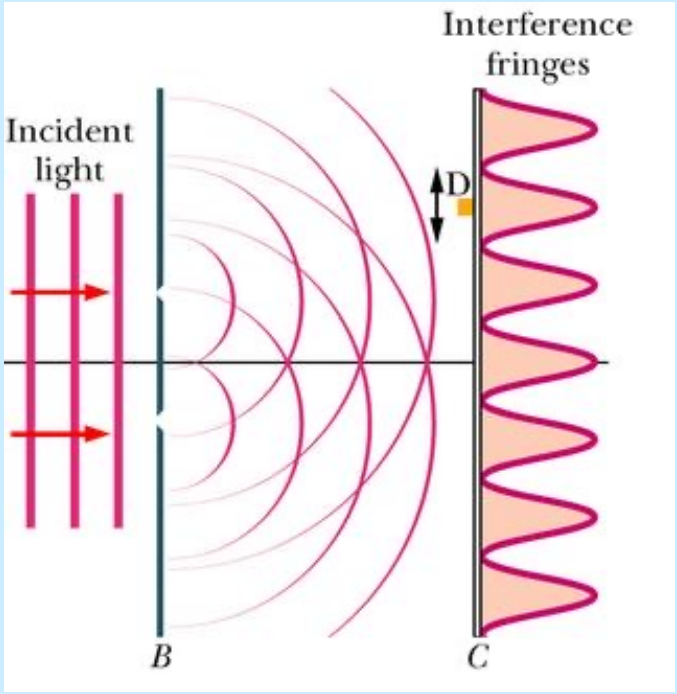
Despite Einstein's advocacy and Millikan's dramatic verification of Einstein's expression for the relation between the frequency of the incident radiation and the stopping voltage of the photoelectrons in the photoelectric effect (Sect. 3.7), the concept of light quanta was not taken seriously by the majority of physicists. As expressed by Mehra and Rechenberg,[1]

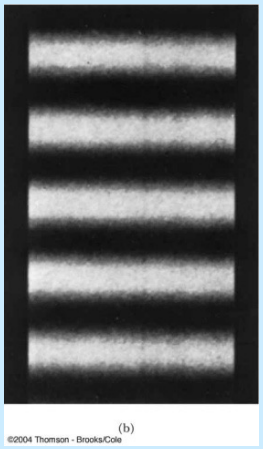
> '. . . the large majority of physicists working on quantum theory . . . subscribed to what Planck, Nernst, Rubens and Warburg wrote in 1913 about Einstein: "that he may have sometimes missed the target in his speculations as, for example, in his theory of light-quanta." And during the following decade hardly anyone took light-quanta seriously until there appeared a paper to settle this question completely: this was the paper presented by Arthur Holly Compton in December, 1922.' (Compton, 1923)

Compton demonstrated that the law of scattering of energetic X-rays by electrons in atoms follows precisely from the assumption that the quanta of light have energy $\varepsilon = h\nu$ and momentum $\boldsymbol{p} = (h\nu/c)\,\boldsymbol{i}_k$. This was irrefutable evidence for the *wave–particle duality* which was to play a central role in the development of quantum theory.

康普頓效應說服了物理界光是由光子組成！







Light is made of particles. But light is also wave.

粒子與波的二重性

PHOTONS: LIGHT WAVES BEHAVING AS PARTICLES 38







Light is an example that there could be something of both particle and wave natures.

波爾的原子模型



Niels Bohr 波爾 1985-1962





Scanned at the American
Institute of Physics

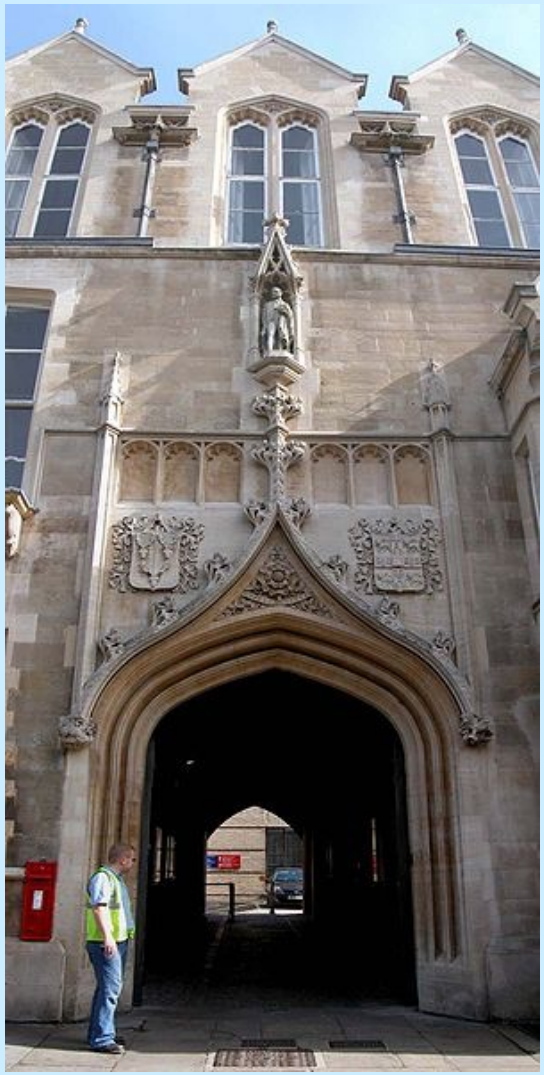




Bohr's engagement photo, taken shortly before his departure for England.
(AIP Emilio Segrè Visual Archives, Margrethe Bohr Collection)

Bohr went to work with JJ Thomson at Cavendish Lab of Cambridge U







The whole thing was very interesting in Cambridge, but it was absolutely useless.

失望之餘，波爾往北到Manchester與Rutherford共事，就此改變了歷史。



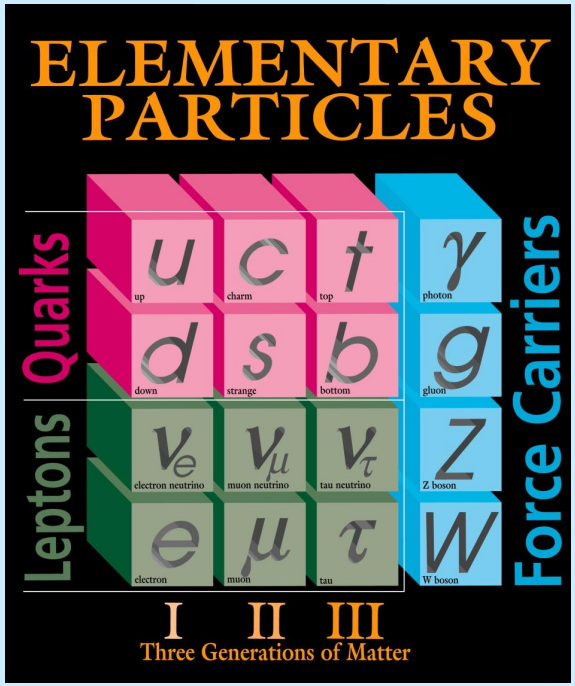




光子 **Photon** γ







$$E = h\nu$$

$$p = \frac{h}{\lambda}$$

The young Bohr is easier to accept new ideas such as photon!

Bohr look at spectrum emission as photons: 1913.

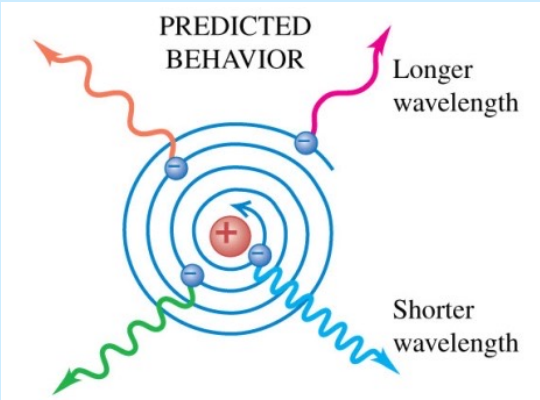


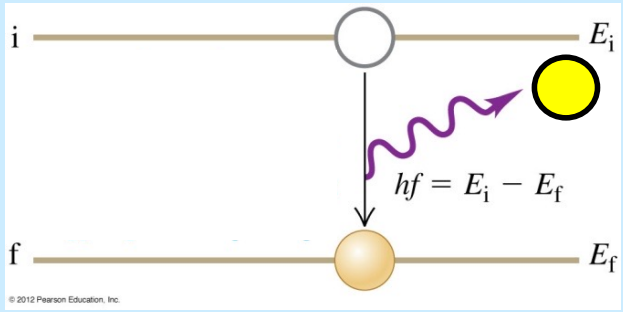
The energy of a photon emitted by Hydrogen atom equals

$$h\nu \propto \frac{1}{\lambda} = \frac{R_H}{m^2} - \frac{R_H}{n^2}$$

It looks like the difference of similar formulae.

The energy of a photon can be understood as energy difference of two states.





Atomic Emission is not generated as electrons rotate, but as electrons jump.

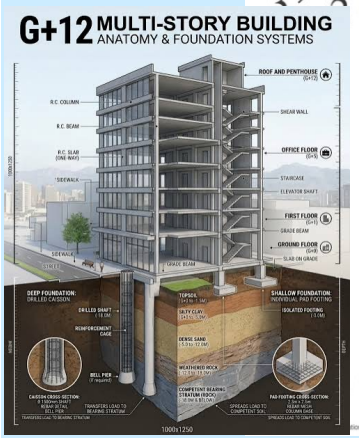
Jump from one state to the other.

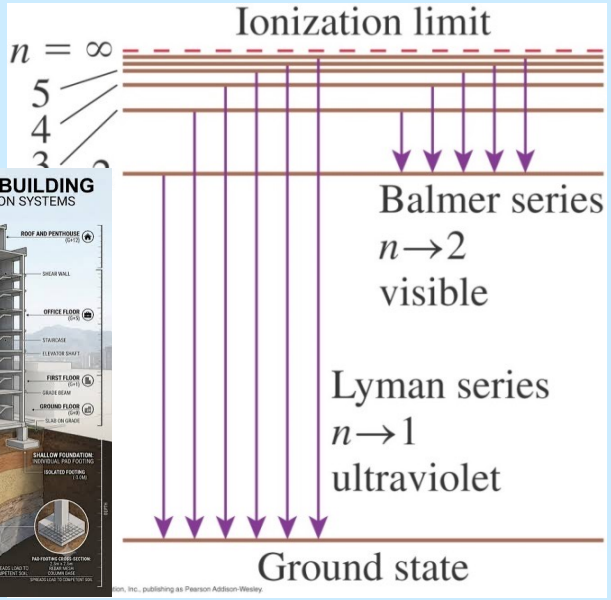
光不是在電子在繞的時候發出，而是在跳的時候發出！

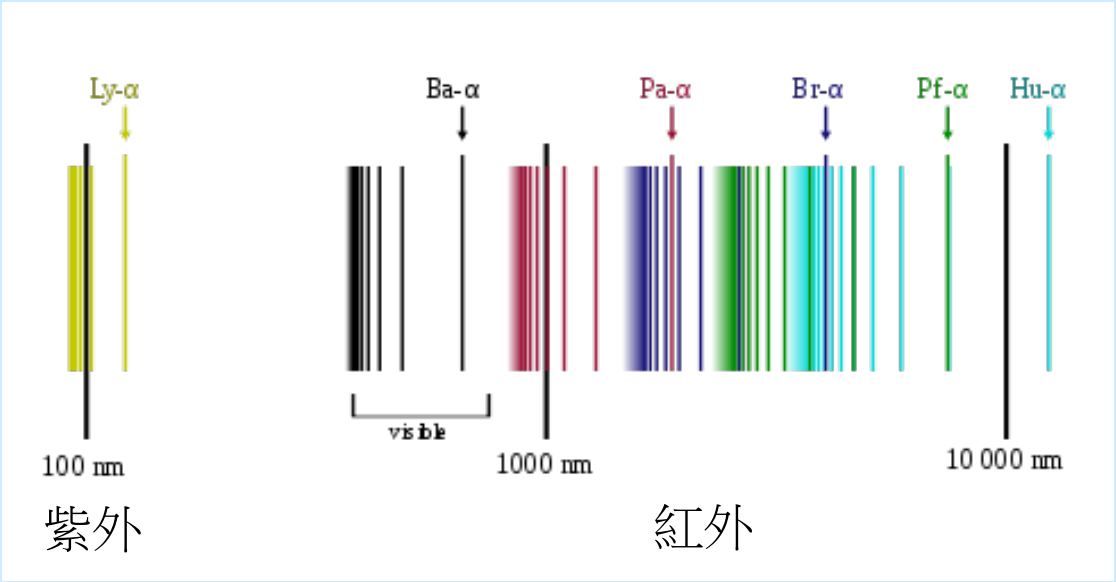
$$h\nu \propto \frac{1}{\lambda} = \frac{R_H}{m^2} - \frac{R_H}{n^2}$$ is energy difference of two states.

Energy of an electron state equals $\frac{1}{n^2}$, identified by a natural number n.

Quantum states are not continuously distributed, but discretely.







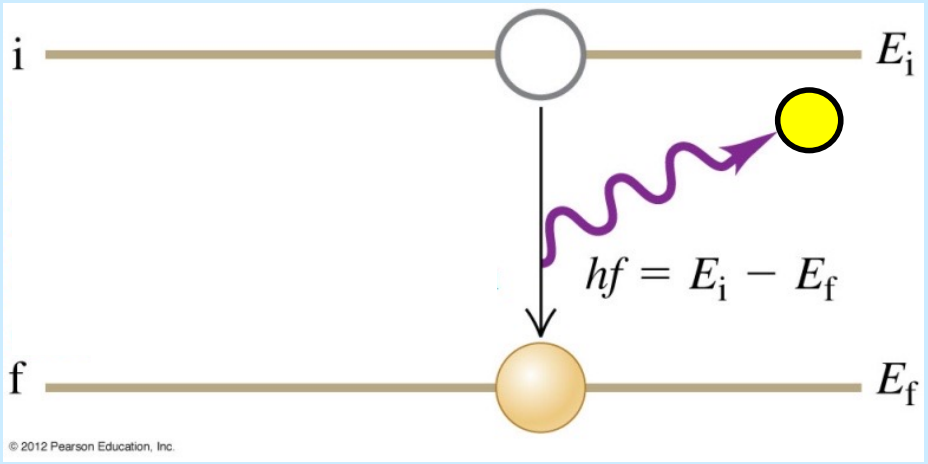
It is like a multi-storied building without stairs.

Discretely distributed states are why spectrum is discreet.

如一棟沒有樓梯的大樓！這就是光譜線是離散的原因！

離散discrete與連續相對，是指分散開來的、不存在中間值的量。

波爾原子模型的基本假設 Basic Assumptions of Bohr Atomic Model



$$E_i \propto -\frac{R_H}{n^2}$$

$$h\nu \propto \frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$E_f \propto -\frac{R_H}{m^2}$$

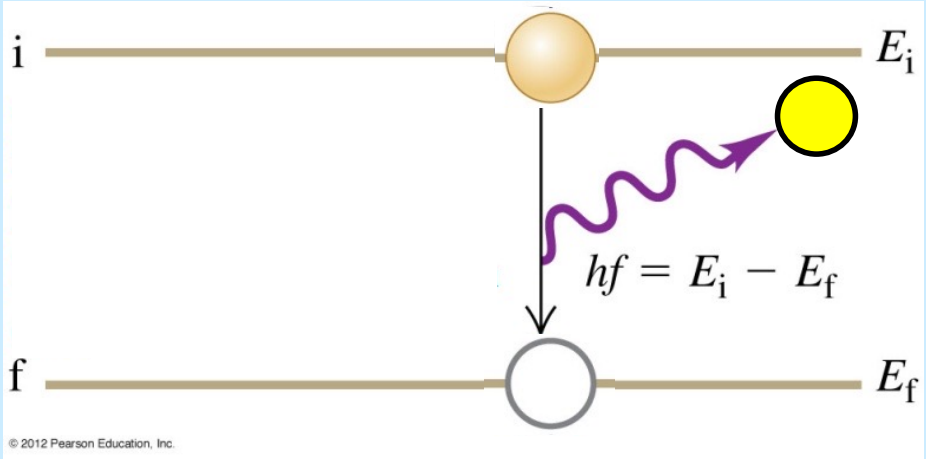
Electron Quantum states inside atom are not continuously distributed, but discretely.

We'll say: Energy is quantized. Quantization of energy is a key feature of QM.

Atomic Emission is not generated as electrons rotate, but as electrons jump.

Energy difference between two states are released by emitting one (just one) photon.

量子躍遷！ Quantum Jump



It's "jump" since electrons don't move through space in between two states over time.

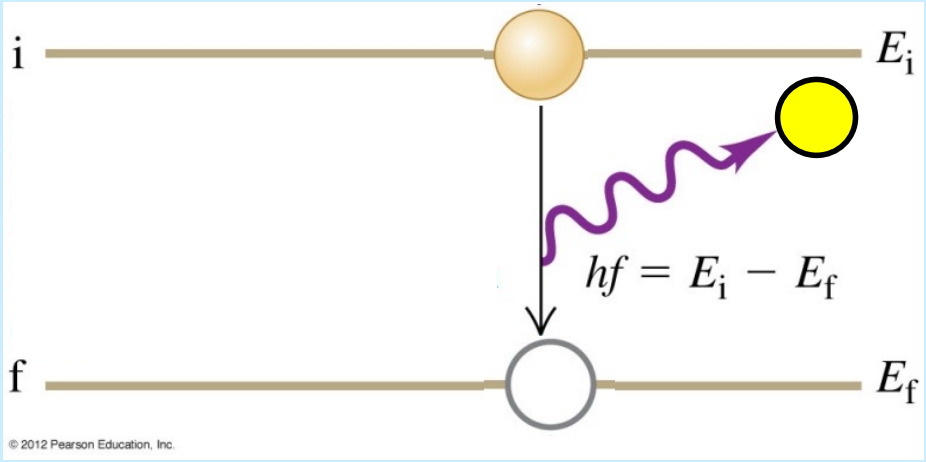
If it is like moving, you can ask what fraction of a photon is emitted in the process.

But photon is indivisible! Therefore, electrons are not moving. They are jumping!

光子是不能分割的！因此，躍遷的時候不經過中間連續的變化！

否則你就可以問跳到一半時，放出了多少光子。

量子躍遷！ Quantum Jump

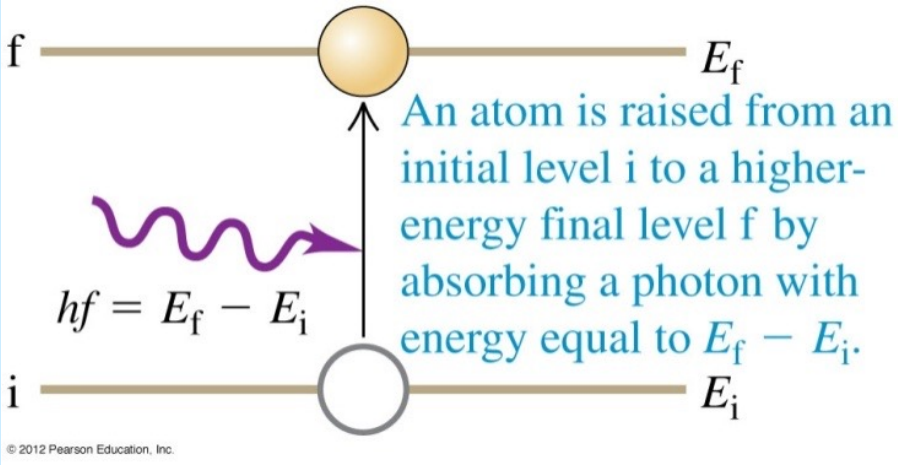


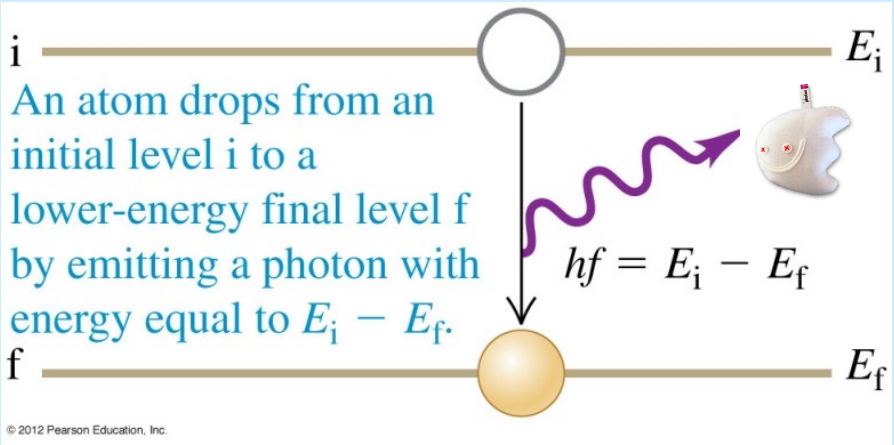
“Jumping” consist of only starting state and final state. That is it.

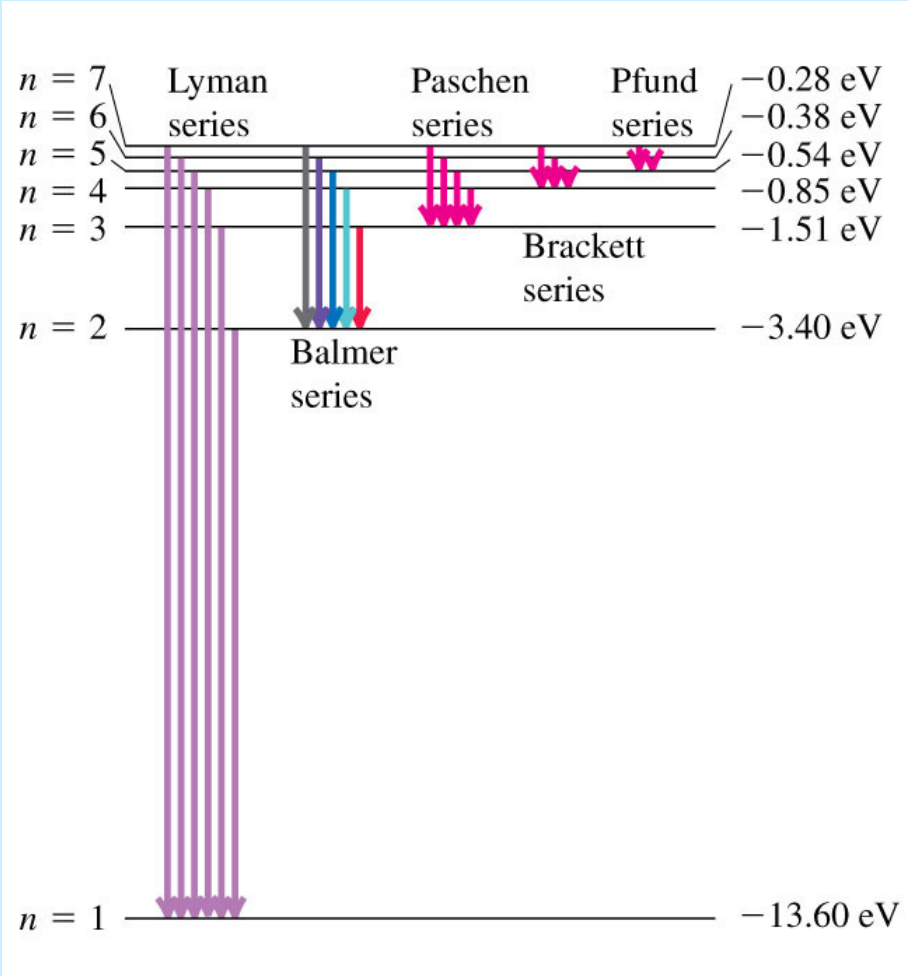
There is not a continuous transition process.

This is the essence of a quantum process, in contrast to classical process.

也可以吸收能量往上跳！


f
E_f
An atom is raised from an initial level i to a higher-energy final level f by absorbing a photon with energy equal to $E_f - E_i$.
$hf = E_f - E_i$
i
E_i
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i
E_i
An atom drops from an initial level i to a lower-energy final level f by emitting a photon with energy equal to $E_i - E_f$.
$hf = E_i - E_f$
f
E_f
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Emission photon energy:

$$h\nu = \Delta E = hcR_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$
$$= (-13.6\text{eV})\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

n, m 分別標定跳躍前後的穩定態，

Energy level inside atom 能階

$$E_n = (-13.6\text{eV})\left(\frac{1}{n^2}\right)$$

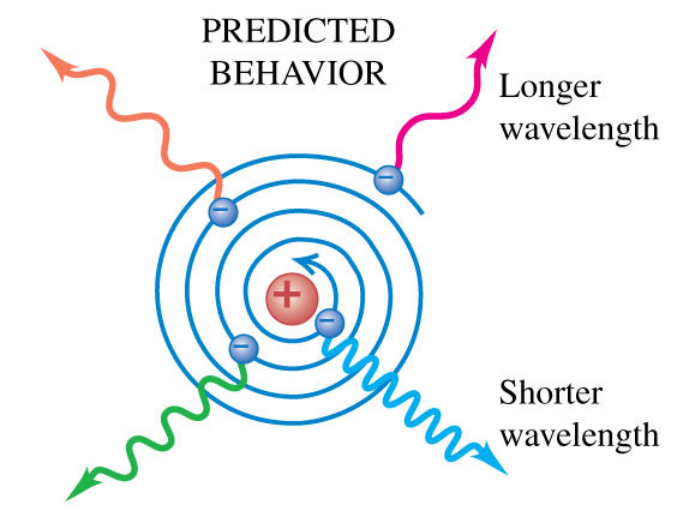
There is one state with the lowest energy $n = 1$, called ground state.

Other states $n > 1$ with higher energies are called excited states

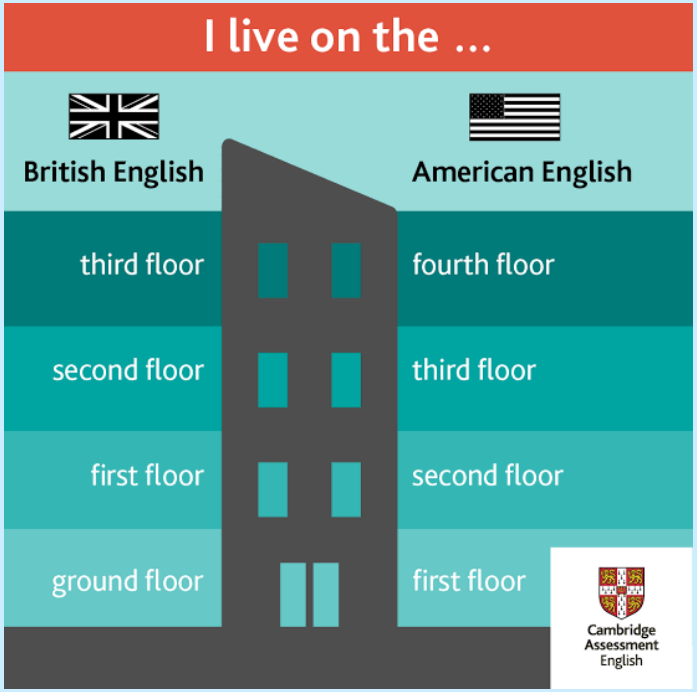
Electrons in ground state is stable since there is no lower state to go.

This is not the case in classical mechanics, and this is why atoms are stable.

This does not happen.



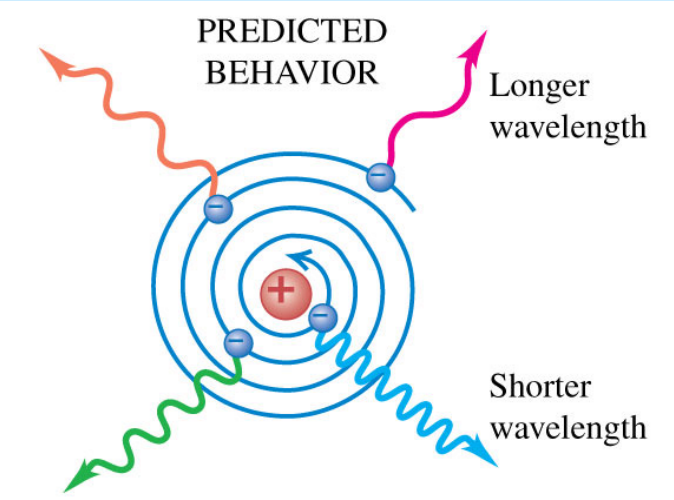
Atom has a ground floor.

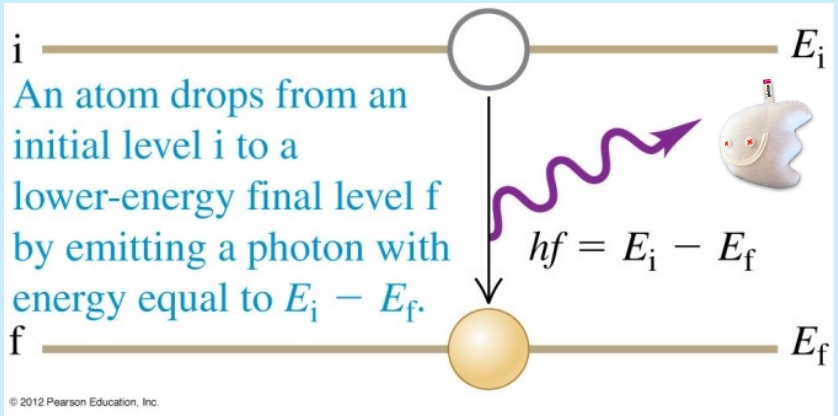


這是一個驚天動地的大洞見！

電子在原子中，只能處在特定、分立而不是連續分布的狀態！

光不是在電子在繞的時候發出，而是在分立的穩定態之間跳的時候發出！





這是連續的！

這是不連續的！

量子躍遷 (Quantum Jump) 的概念正式誕生！

微觀世界的物理過程基本上是不連續的跳！

量子躍遷是一個驚天動地、革命性的大洞見！This is revolutionary!





革命不是一日就可以達到目標！

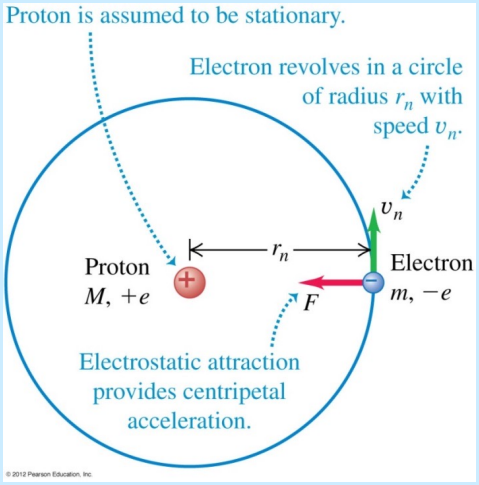
The greatest insight of Bohr is that atomic electrons exist in discrete stable states.

We know their energies. Then, what are the orbits of these states? Bohr asked.

Bohr plug in the energies formula to find the corresponding circular orbits.

This step is anti-revolutionary and it turns out atomic electrons do not have “orbits”.

（後來證實這部分不是正確的，原子中的電子沒有軌道可言）



Cloud is more appropriate.

（這部分證實是部分正確的）

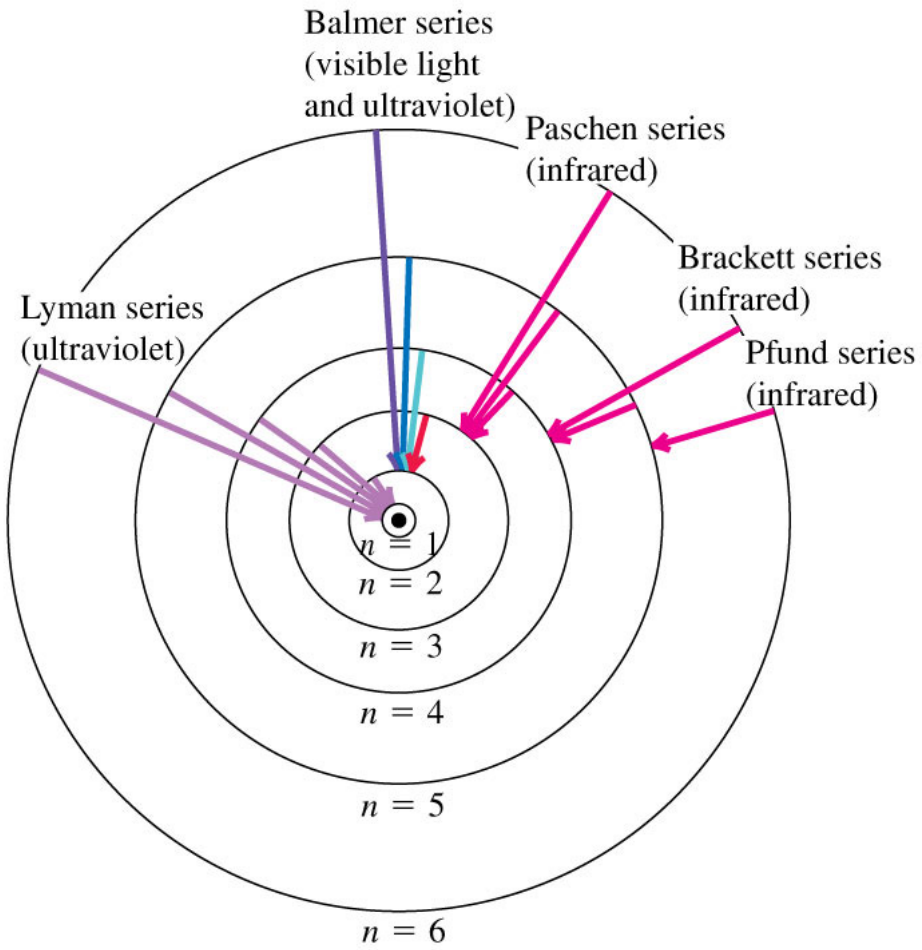
Bohr was surprised to find anugular momentum of these orbit is integer multiple of $\hbar$.

$$L = n\hbar \qquad \hbar = \frac{h}{2\pi}$$

Bohr Quantization condition. He believe energy is quantized since L is quantized.

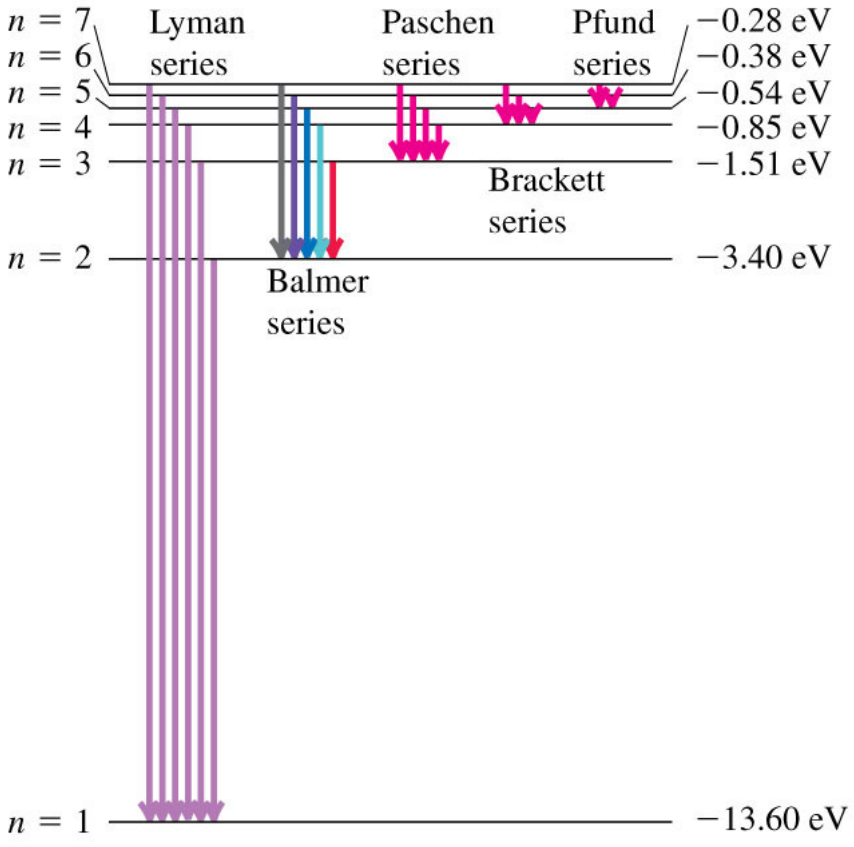
對於波爾，能階是一系列滿足量子化條件的同心軌道。

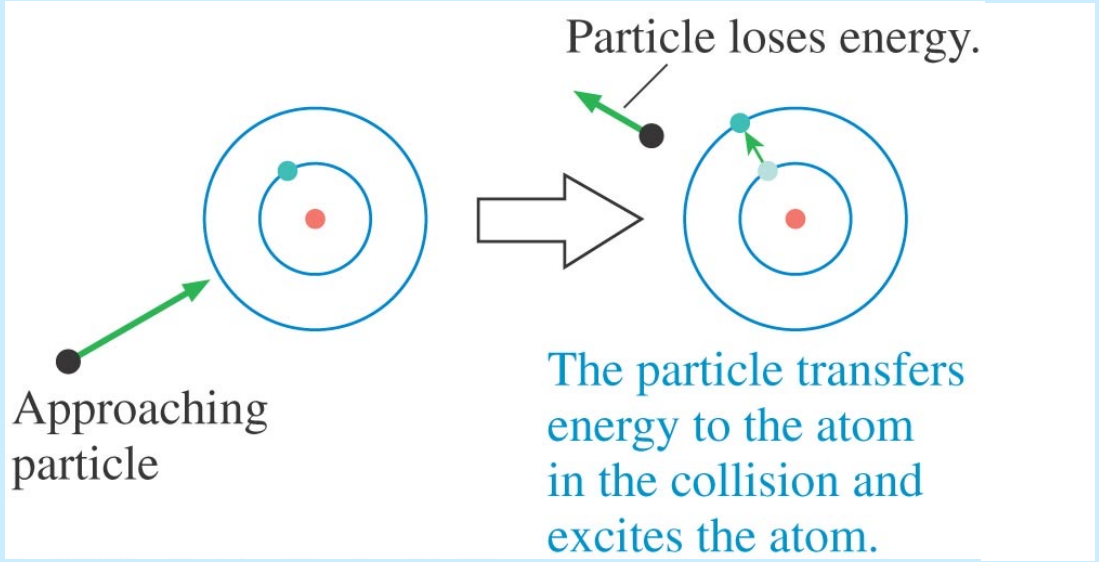
(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.



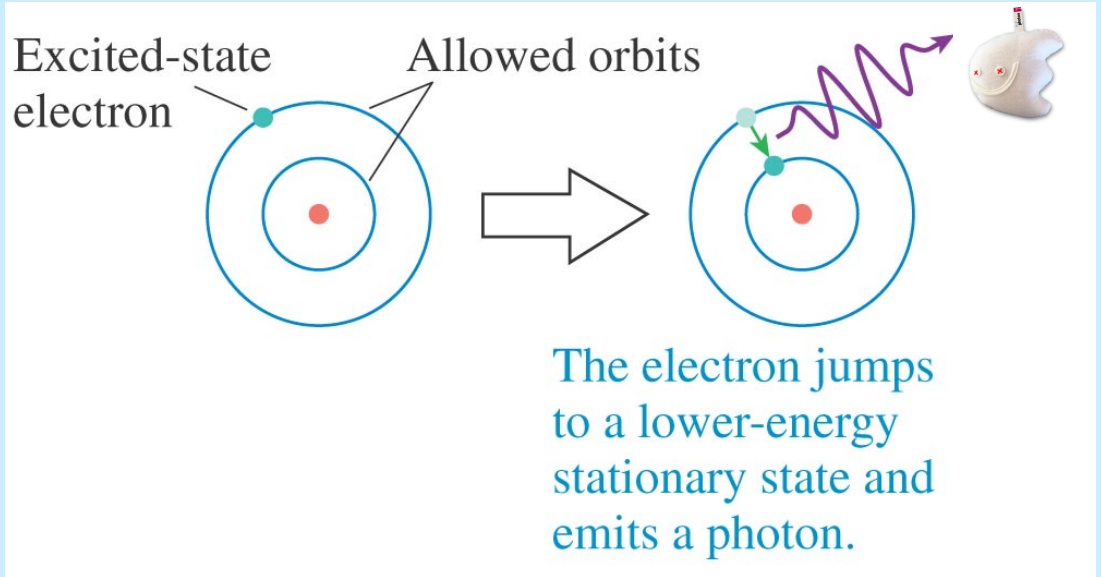
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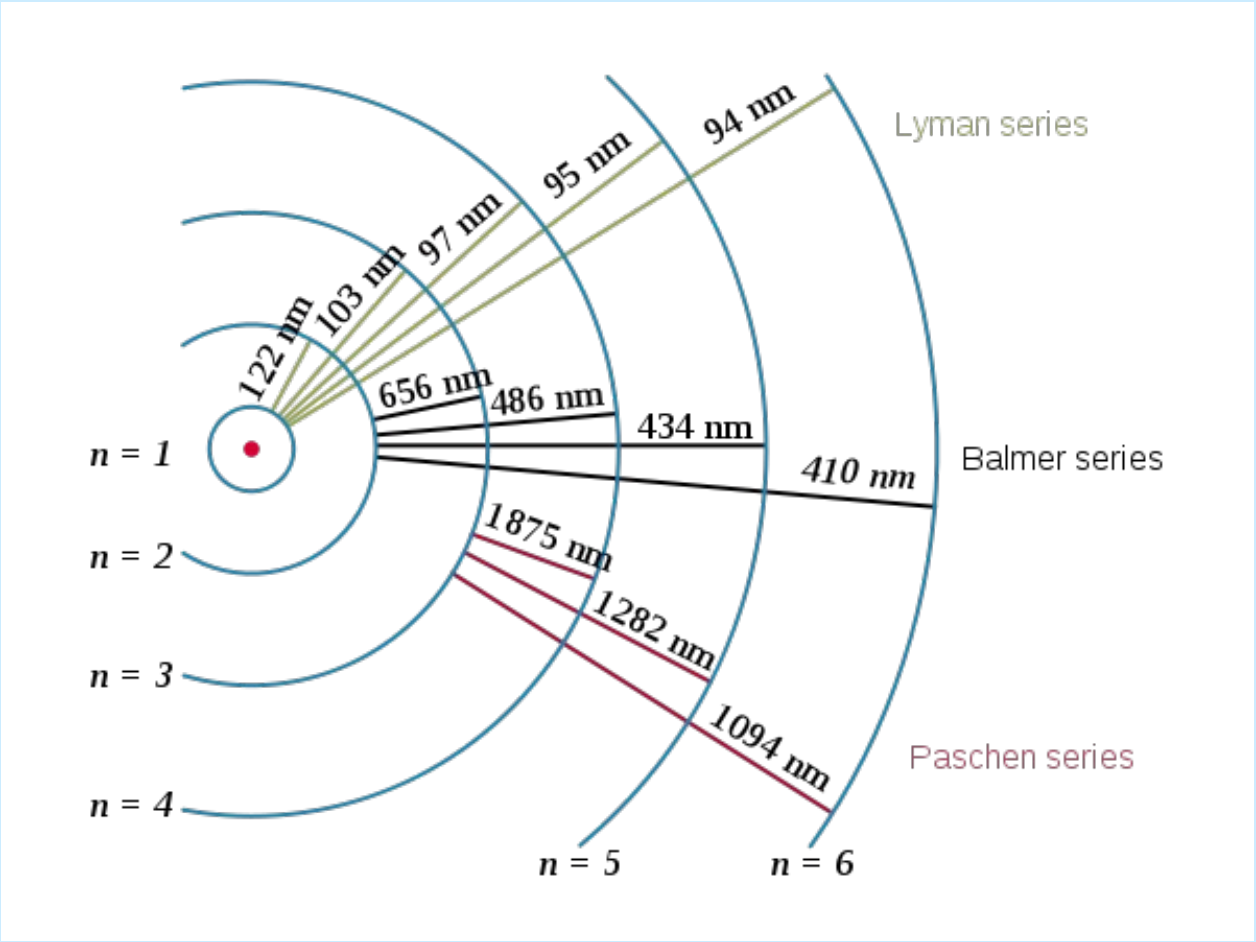
(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series





光是在電子從一個軌道跳到另一個軌道的時候發出！




Lyman series
94 nm
95 nm
97 nm
103 nm
122 nm
Balmer series
656 nm
486 nm
434 nm
410 nm
Paschen series
1875 nm
1282 nm
1094 nm
n = 1
n = 2
n = 3
n = 4
n = 5
n = 6

如此光譜線是這樣解釋！

存在一能量最低、穩定的態， $n = 1$，稱基態。這是氫原子的自然狀態。

$$a_0 = \frac{4\pi\varepsilon_0}{mZe^2}\hbar = 0.53\times10^{-10}\ \text{m}$$ 大致就是原子半徑

(a) The radius of the lowest ($n = 1$) Bohr orbit is

$$a_0 = \frac{137}{Z}\frac{\hbar}{m_e c} = \frac{0.053}{Z}\ \text{nm} \equiv \frac{0.53}{Z}\ \text{Å} \qquad (1\text{-}39)$$

We have introduced here the useful unit of length, the Ångstrom, where $1\ \text{Å} \doteq 10^{-10}\ \text{m} = 0.1\ \text{nm}$

(b) The binding energy of the electron in the lowest Bohr orbit—that is, the energy required to put it in a state with $E = 0$ (corresponding to $n = \infty$)—is

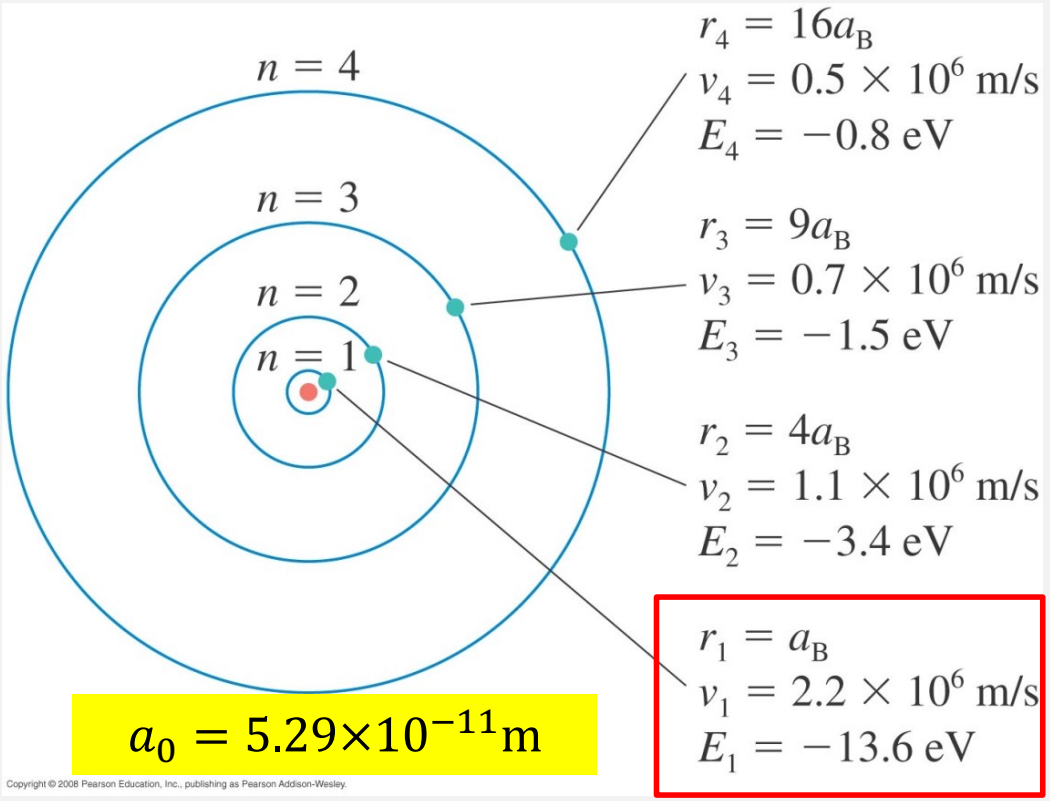
$$E = \tfrac{1}{2}m_e c^2(Z\alpha)^2 = 13.6Z^2\ \text{eV} \qquad (1\text{-}40)$$

$$E = -\frac{m}{2}\left(\frac{Ze^2}{4\pi\varepsilon_0\hbar}\right)^2 = -13.6\text{eV}$$

這兩個重要的關於氫原子的物理觀測量，可以由電子的性質計算出來！

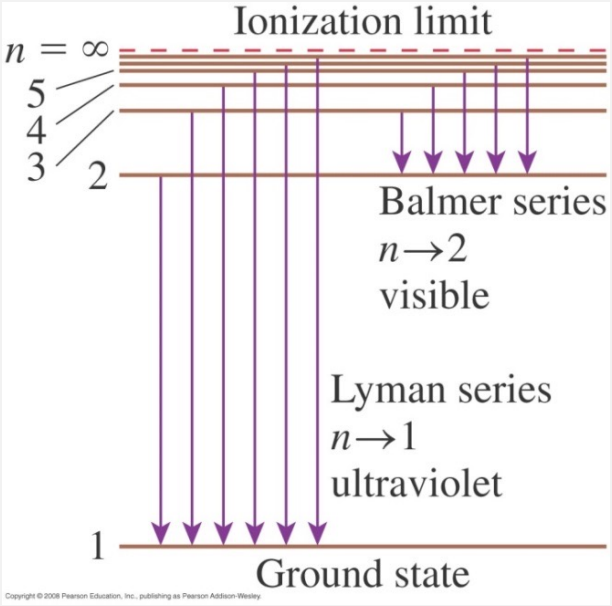
由電子繞行的軌道圖像可以計算出神秘常數R_{H}

$$\frac{1}{\lambda} = \left(\frac{R_H}{m^2} - \frac{R_H}{n^2}\right)$$

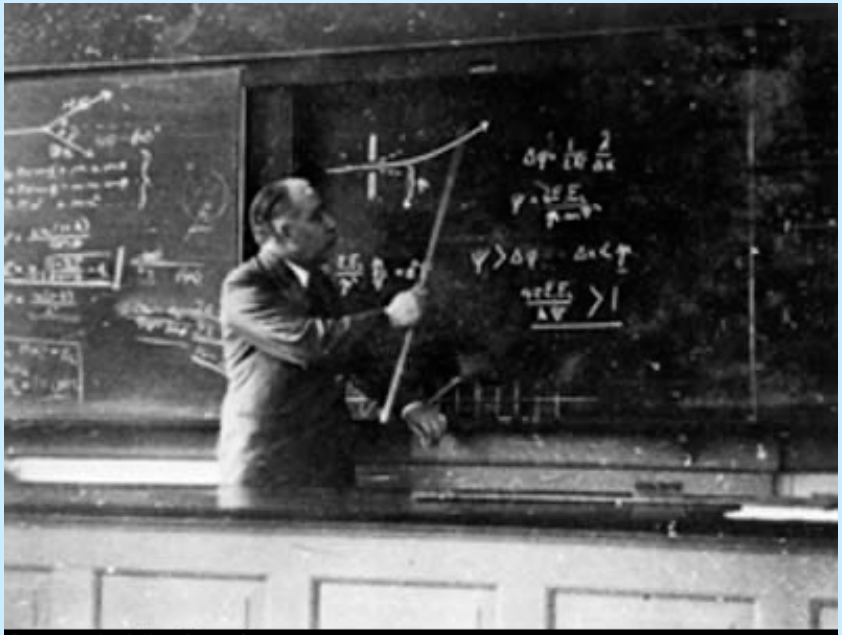


$$a_0 = 5.29\times10^{-11}\mathrm{m}$$

$$R_H = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097\times10^7\,\mathrm{m}^{-1}$$



1922 Nobel Prize



Scanned at the American
Institute of Physics



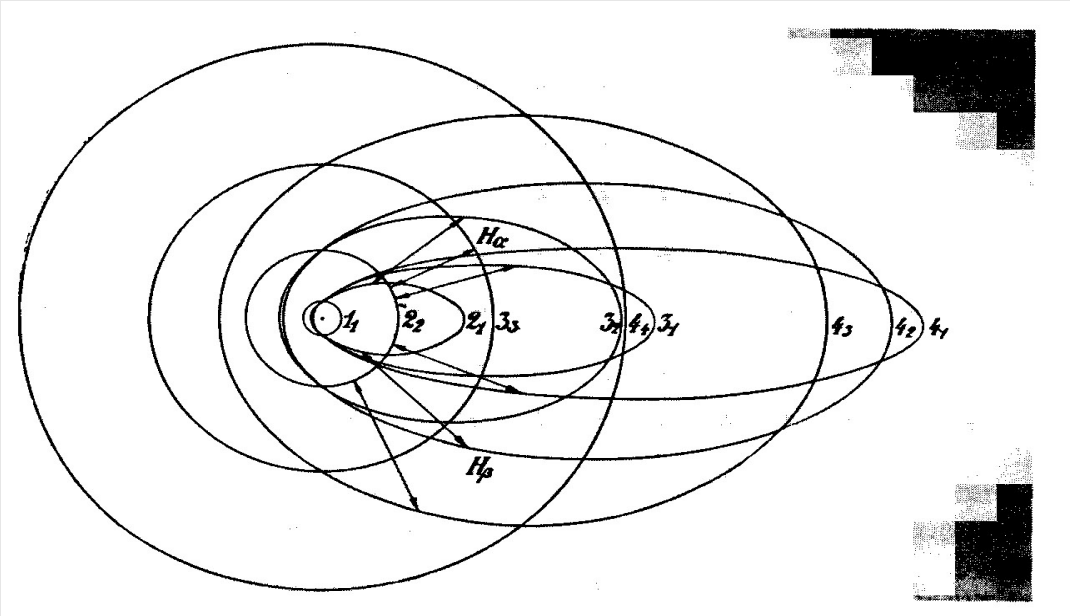
Scanned at the American
Institute of Physics



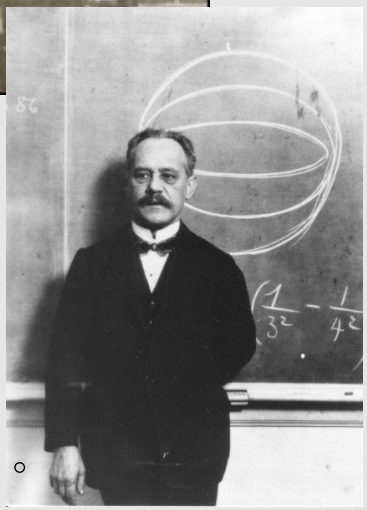


橢圓軌道應該也是可能的！

Sommerfeld 將波爾的想法推廣到橢圓形軌道（這部分是錯的）。







並且將量子化條件，寫成角座標的積分，

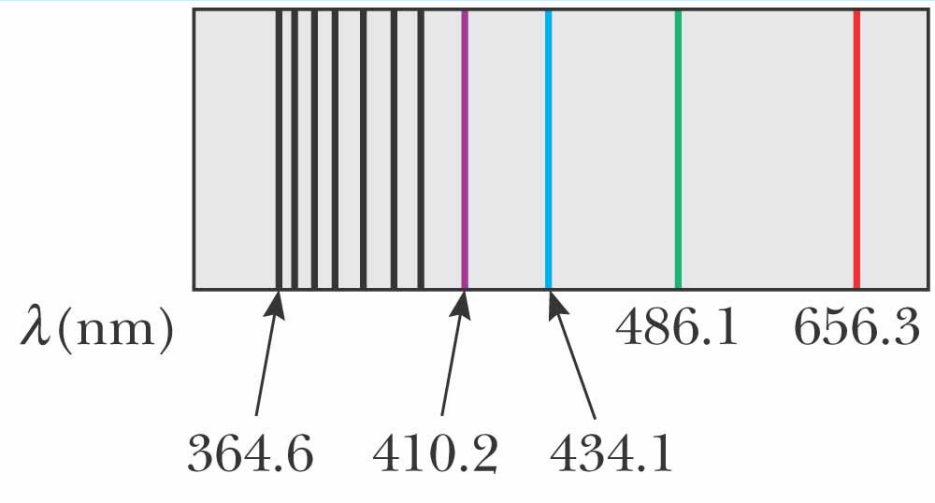
推廣到其他兩個極座標，由新的量子化條件可以得出能階量子化。

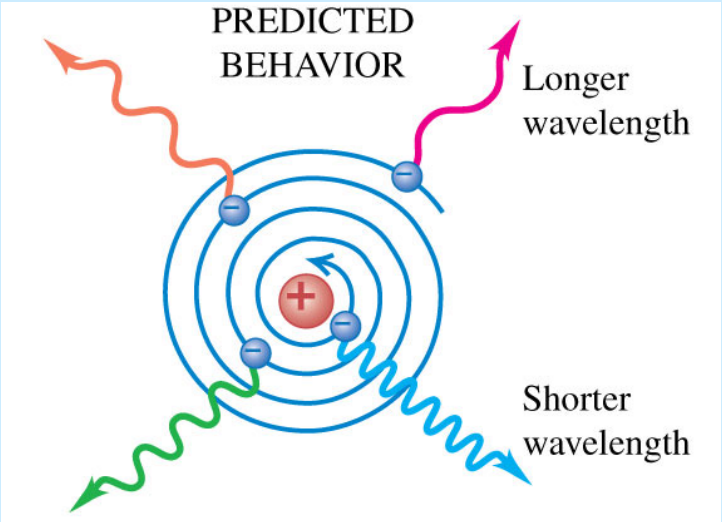
$$L = p_\phi = n\hbar \quad \Rightarrow \quad \oint d\phi \cdot p_\phi = n_\phi h \quad \Rightarrow \quad \oint dr \cdot p_r = n_r h \quad \Rightarrow \quad E \propto \frac{1}{(n_\phi + n_r)^2}$$

猜想對任一廣義座標，量子化條件：$\oint dq \cdot p_q = nh$　這部分猜對了！

波爾的原子模型是一個極為正確又精確但肯定是錯誤的理論！

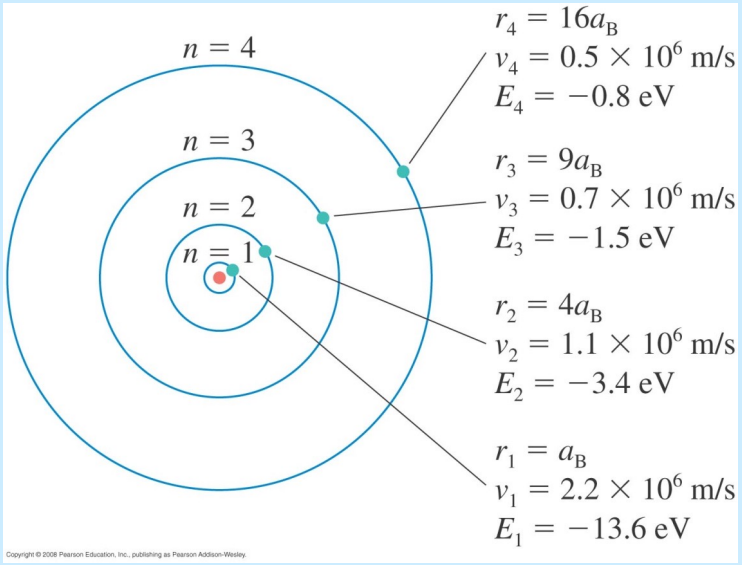
Bohr model is a splendidly accurate but apparently wrong theory.





關鍵的原子內電子的穩定態是一個非常矛盾奧秘的觀念。

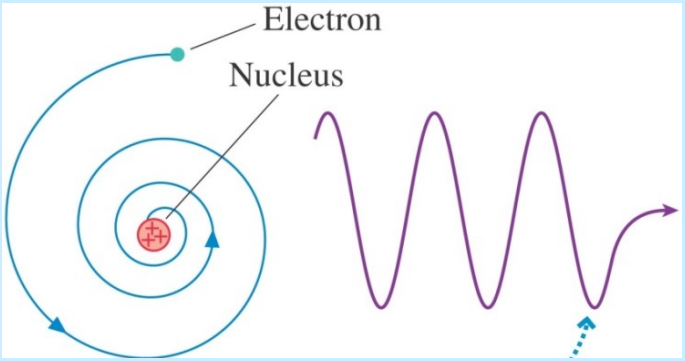
The crucial thing is the mysterious electron stable states inside atoms.



電子只能選擇符合量子化條件的軌道形成穩定態。

為什麼？量子化條件是從何而來？

為什麼基態軌道上的電子會穩定？加速的點電荷不是會放出電磁波的嗎？



畢竟這是巨觀觀察到的事實！

難道微觀世界與巨觀世界不同？

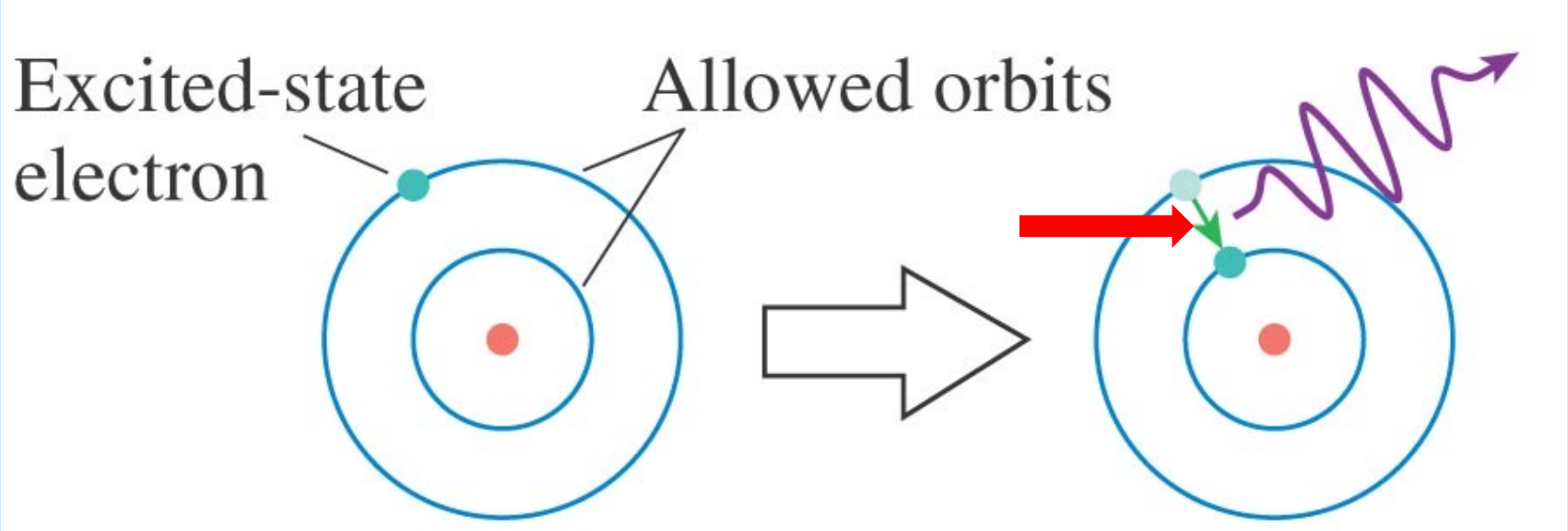
更糟的是，波爾模型對光譜的預測極其成功，

老一輩的物理學家對波爾模型的成功非常不安！

因為波爾模型的潛在後果非常嚴重！



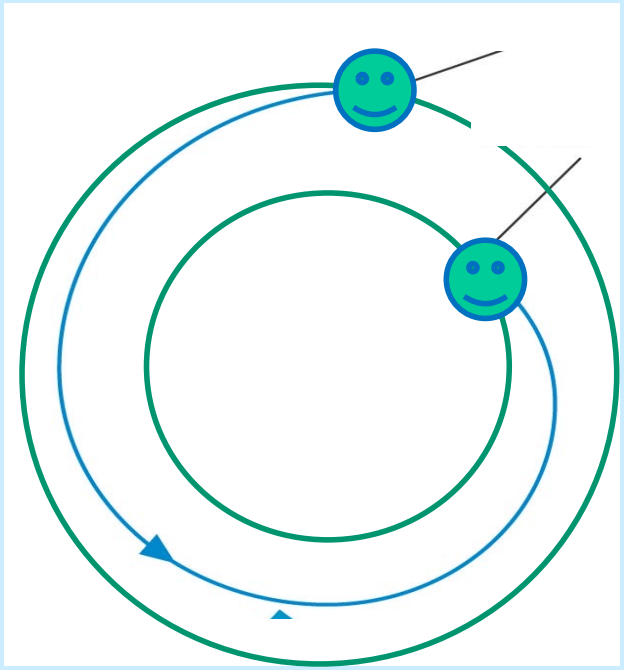
光不是在電子繞的時候發出，而是在跳的時候發出！

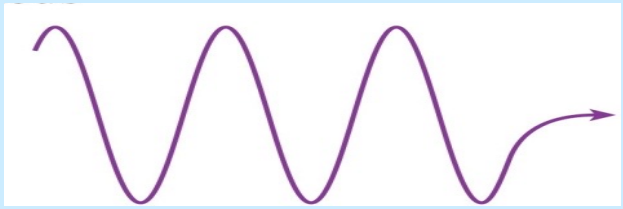


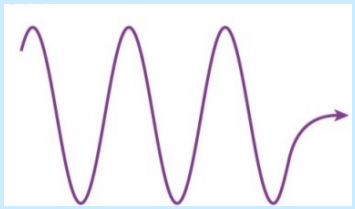
電子在穩定態之間躍遷，釋放的能量以一個光子釋出。

因光子無法分割，所以電子不能慢慢挪移過去！

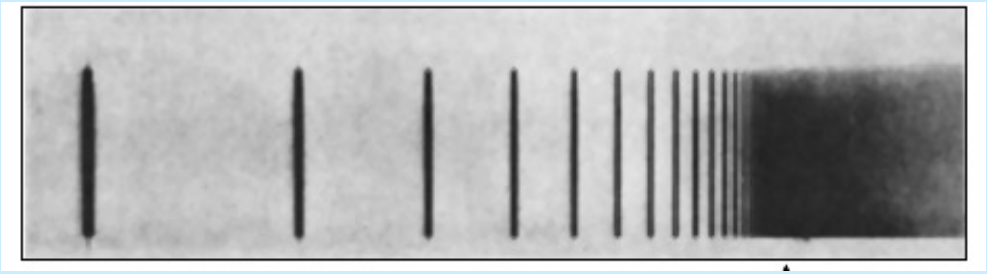
躍遷的過程



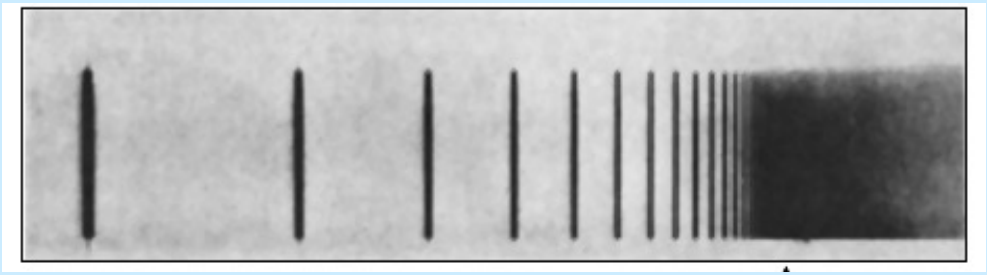




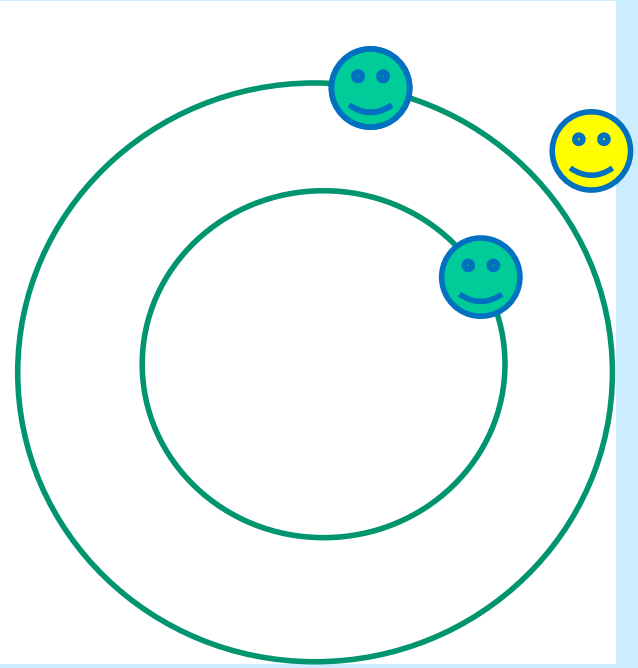
若躍遷是一個連續地、挪移的、過渡的過程，必然放出連續的光譜！



實驗上卻只觀察到極細的譜線。



極細的譜線顯示躍遷是沒有過程的：



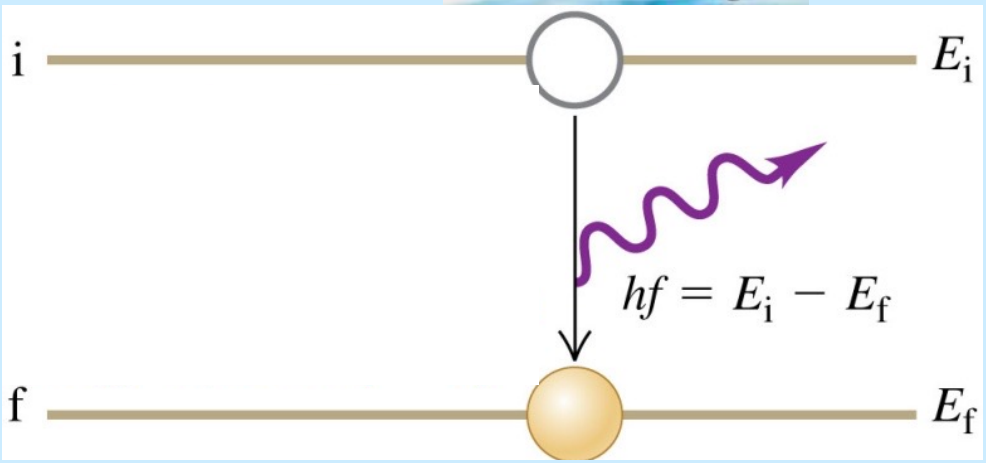
這躍遷是一個量子的，非連續性的，跳躍！

躍遷沒有過程，我們將沒有過程的細節來決定這躍遷要到那兒去。

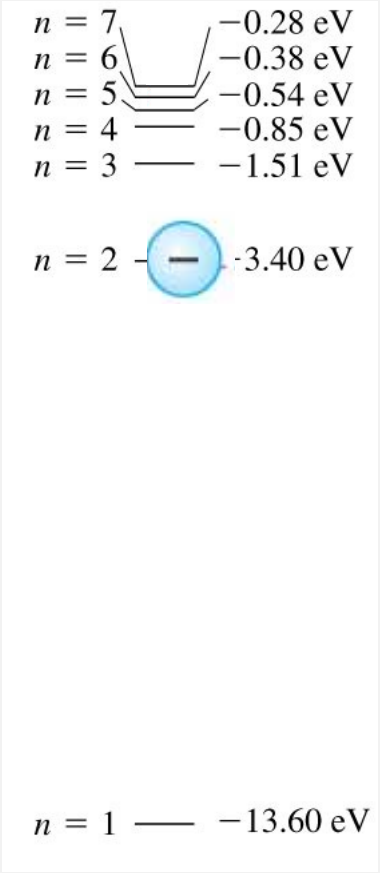
躍遷的終點與何時發生只能靠起始條件決定！

問題是，由相同的激態出發，起始條件是完全相同的！









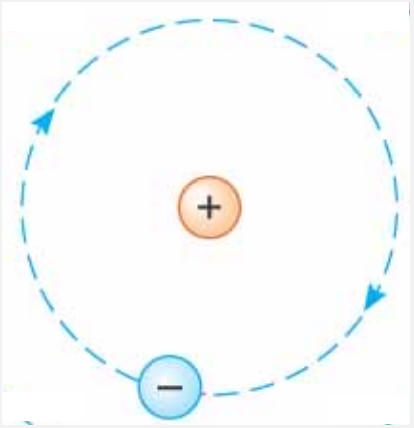


電子在原子中處於離散的能態，

任意兩顆電子，若處於原子內的同樣狀態，將完全相同。

這類似數位的1與0，所有的 1 都是完全相同。

起始條件不容許有細微差異。



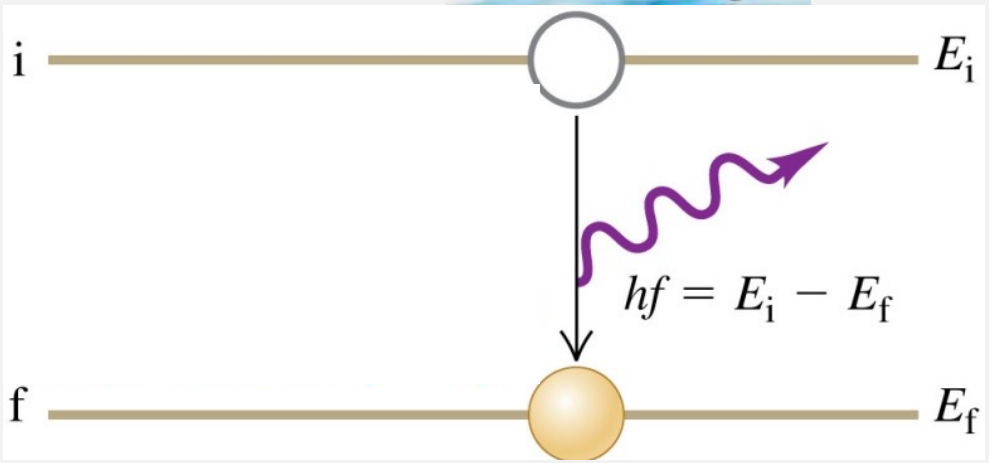
古典的電子在原子中處於連續的軌道，

軌道的大小可以連續的微調，

沒有任何兩顆電子的起始狀態是完全相同的。

起始條件可以容許有細微差異。







處於同樣軌道(上)的電子，起始狀態完全相同，照理說就只能在同時開始躍遷！跳到同樣的另一軌道，同樣的因、同樣的果。

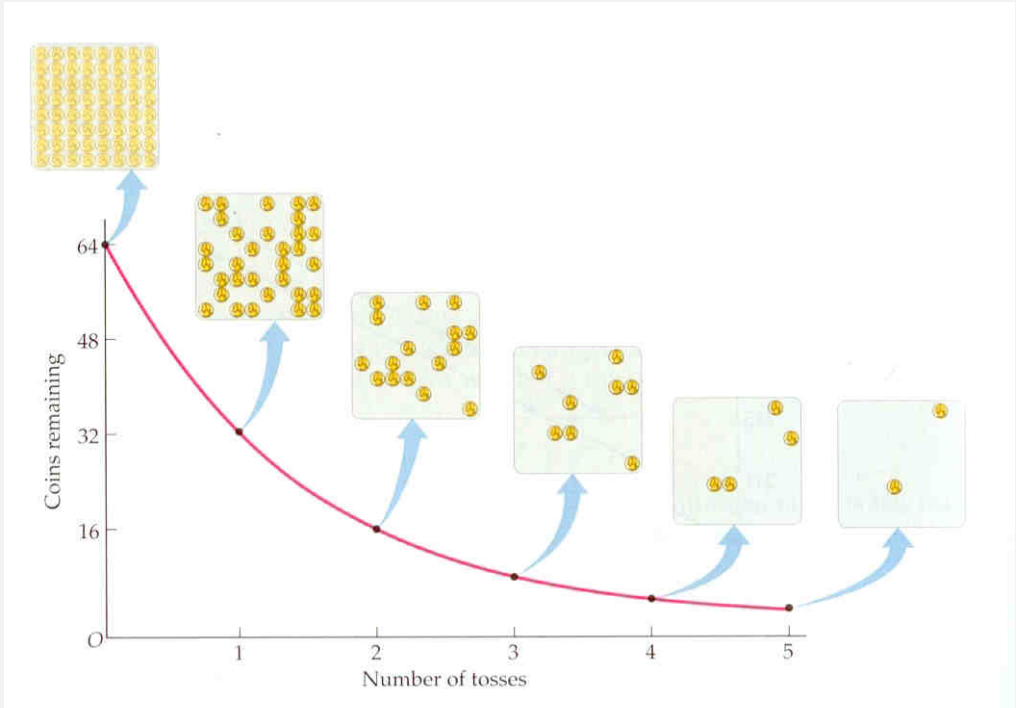
實驗上應該看到一群同樣激發原子的向下躍遷是同時發生、跳到同樣目標。

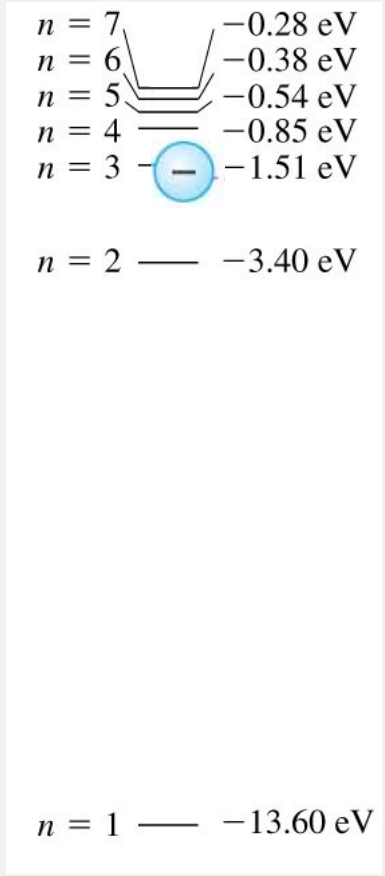
但事實是：觀測一大群已激發的原子：

經過時間t後尚未躍遷的數量是時間t的指數函數：

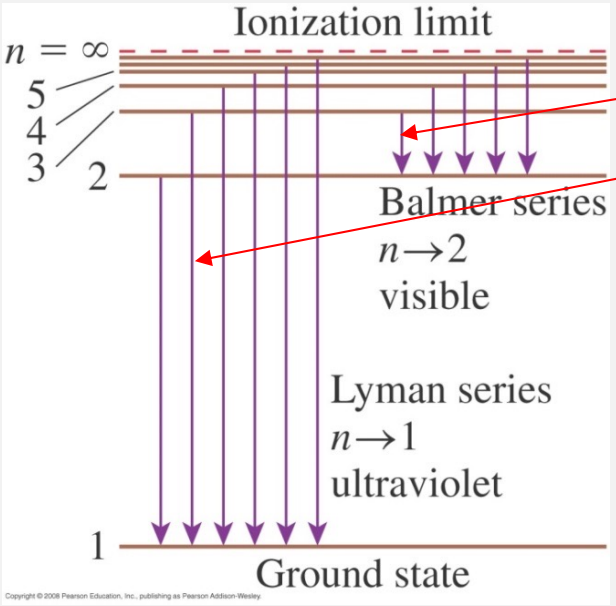
$$N = N_0 e^{-\lambda t}$$

有的躍遷得快，有的躍遷得慢！這是實驗事實。



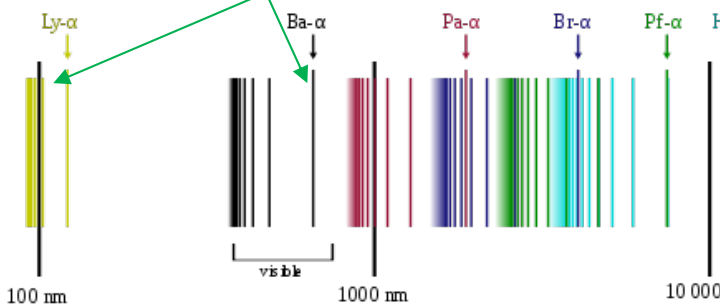


事實是：同樣起點，躍遷後的電子就是會出現在不同的能態上



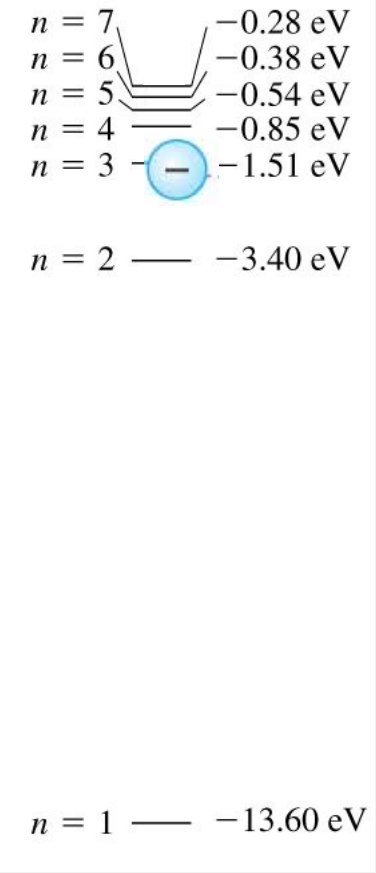
$n = 3 \rightarrow n = 2$

$n = 3 \rightarrow n = 1$



紫外

紅外



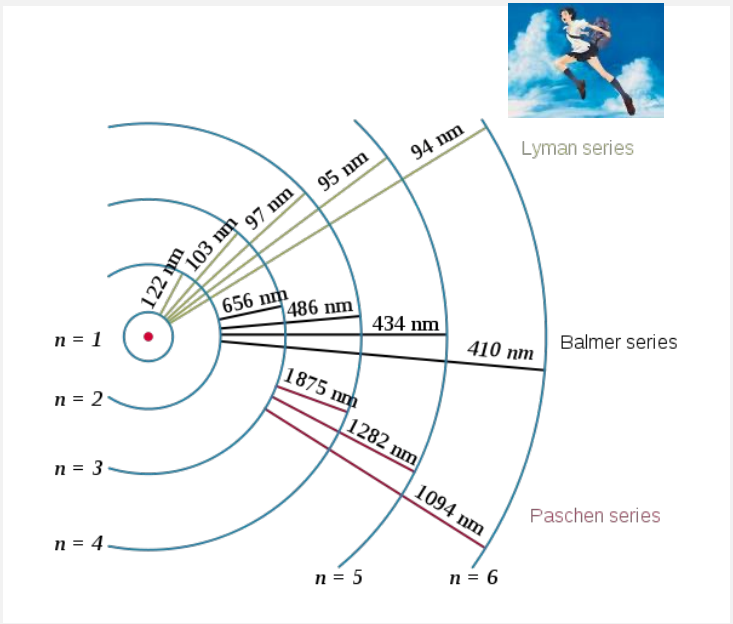
處於$n = 3$的電子完全相同，

沒有過程，就沒有“因”，可以決定為何有一些跳到$n = 2$，有一些跳到$n = 1$的果。

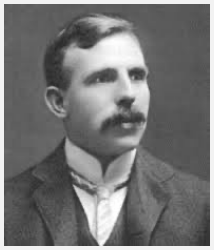
而且，從同樣的穩定態出發，電子如何決定它要躍遷”跳”到那一個狀態？

你必須假設電子事先得知道他要停在哪裡，

從$n = 6$出發的電子，必須事先決定它是跳到 $n = 2$還是 $n = 3$。



How does an electron decide which stationary state to pass to and when to go? You would have to assume that the electron knows beforehand where it is going to stop.



Criticism by Rutherford

電子所停的軌道的果，必定在出發時就必須有一個因！

但處於同樣軌道的電子，狀態完全相同，

在量子躍遷中，同樣的因卻造成不同的果！

量子躍遷的觀念與物理內在的因果律可能是牴觸的！



完全相同的實驗，所得的結果不會一樣。

如此的話，面對單一一顆激發原子，我們是無法確定預測它的躍遷的！



農場的每一株菜都不同：即使一樣品種的菜，還是有些許後天的差異。

而每一個激發原子都是一模一樣的，無從分辨，

電子 Electron  e^-

面對單一一顆激發氫原子內的電子，我們是無法預測它何時向下躍遷的！

科學的預測確定性受到前所未見的威脅！

我們對電子作為一個終極的牛頓粒子寄予厚望

但光譜及原子結構的實驗結果，顯然不支持這樣的期待。

電子不是單純的牛頓力學的粒子！

更糟的是它可能是一隻特洛伊木馬！





量子不確定性的敵人可能藉此入侵！

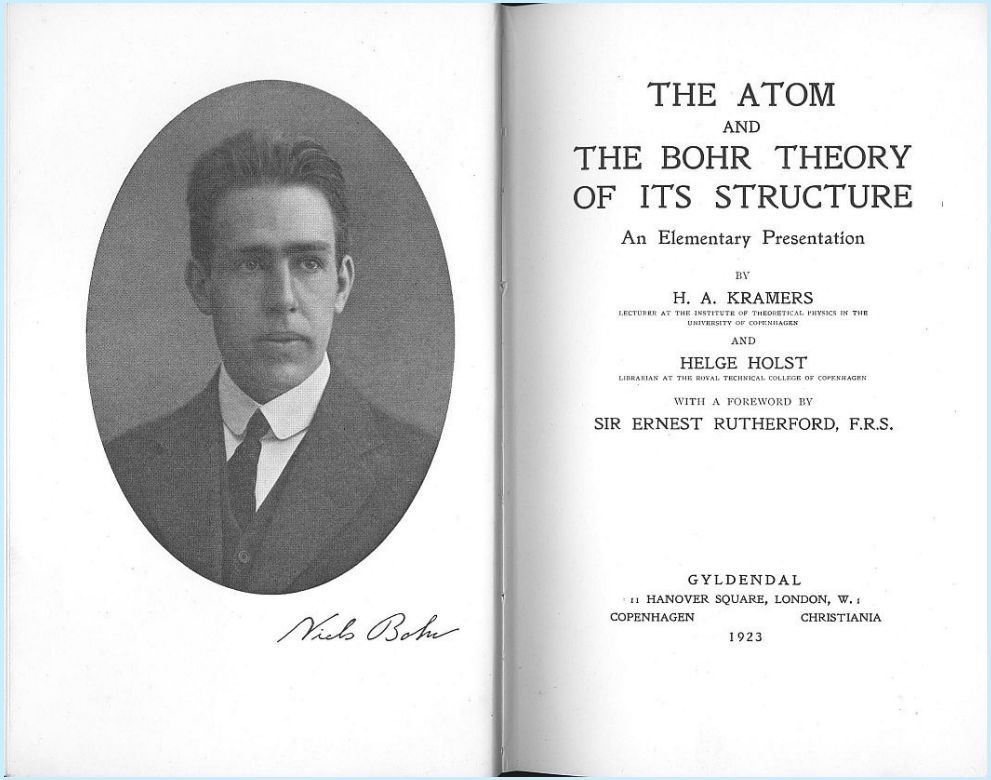
科學的因果原則受到前所未見的威脅！



當時的教科書是這樣描述量子物理：1923

It has been necessary in many respects to grope in the dark, guided in part by the experimental results and in part by various assumptions, often very arbitrary.

量子物理如在黑暗中摸索，
能憑藉的就是實驗結果的導引，偶而參考各式各樣的假設，
這些假設大部分是沒有太大的原因，非常隨意的。



新的量子世界，本來就應該有全新的、不可置信的、impossible的觀念，

Alice laughed.
“There‘s no use trying,” she said: “one can’t believe impossible things.”

“I dare say you haven‘t had much practice,” said the Queen. “When I was your age, I always did it for half-an-hour a day. Why, sometimes I’ve believed as many as six impossible things before breakfast” (Lewis 127).



Quantum Wonderland



1920波爾在哥本哈根創立了Niles Bohr Institute，成為量子物理發展的中心！









一個接著一個謎一般的新理論在這裡誕生！



1929



新思想的堅兵



1930



1933



1936

革命的前鋒！

Heisenberg and Pauli





漸漸有人了解，古典的物理已經到達極限。

新的時代已經來臨，量子現象需要全新的物理與力學來描述。

Max Born 波恩1924 über Quantenmechanik (About Quantum Mechanics)

Über Quantenmechanik.

Von **M. Born** in Göttingen.

(Eingegangen am 13. Juni 1924.)

Die Arbeit enthält einen Versuch, den ersten Schritt zur Quantenmechanik der Kopplung aufzustellen, welcher von den wichtigsten Eigenschaften der Atome (Stabilität, Resonanz für die Sprungfrequenzen, Korrespondenzprinzip) Rechenschaft gibt und in natürlicher Weise aus den klassischen Gesetzen entsteht. Diese Theorie enthält die Dispersionsformel von Kramers und zeigt eine enge Verwandtschaft zu Heisenbergs Formulierung der Regeln des anomalen Zeemaneffekts.

Einleitung. Das Versagen der Quantentheorie in allen Fällen, wo es sich um Bewegungen mehrerer Elektronen handelt (z. B. beim Helium), ist schon mehrfach darauf zurückgeführt worden, daß dabei auf jedes Elektron ein Wechselfeld wirkt, dessen Frequenz von derselben Größenordnung ist wie bei einer Lichtwelle. Da man nun weiß, daß die Atome auf Lichtwellen unter Umständen gänzlich „unmechanisch" reagieren (nämlich zu Quantensprüngen angeregt werden), so wird man auch nicht erwarten dürfen, daß die Wechselwirkung zwischen Elektronen desselben Atoms nach den Gesetzen der klassischen Mechanik erfolgt; damit wird die Anwendung der durch Quantenregeln ergänzten klassischen Störungstheorie zur Berechnung der stationären Bahnen hinfällig. Solange man die Gesetze der Einwirkung des Lichtes auf Atome, also den Zusammenhang der Dispersion mit dem Atombau und den Quantensprüngen, nicht kennt, wird man erst recht über die Gesetze der Wechselwirkung zwischen mehreren Elektronen eines Atoms im Dunkeln sein.

In neuerer Zeit ist nun aber gerade auf diesem Gebiete des Zusammenhanges von Strahlung und Atombau ein wesentlicher Fortschritt durch Bohr, Kramers und Slater[1]) erzielt worden. Dieser besteht meiner Meinung nach vor allem darin, daß die klassische Optik wieder in hohem Maße zu ihrem Rechte kommt. Die Fruchtbarkeit dieser Ideen hat sich auch darin gezeigt, daß es Kramers[2]) gelungen ist, eine Dispersionsformel aufzustellen und zu begründen, die allen Anforderungen der Quantentheorie, vor allem dem Korrespondenzprinzip, genügt.

Bei dieser Sachlage kann man daran denken, ob es nicht möglich ist, die von Kramers so erfolgreich auf die Wechselwirkung zwischen Lichtfeld und Leuchtelektron angewandten Überlegungen sinngemäß auf

[1]) N. Bohr, H. A. Kramers und J. C. Slater, ZS. f. Phys. **24**, 69, 1924.
[2]) H. A. Kramers, Nature **113**, 673, Nr. 2845, 10. Mai 1924.

This paper contains an attempt to make a first step towards a quantum theory…..

這篇文章是嘗試走向量子理論的第一步….

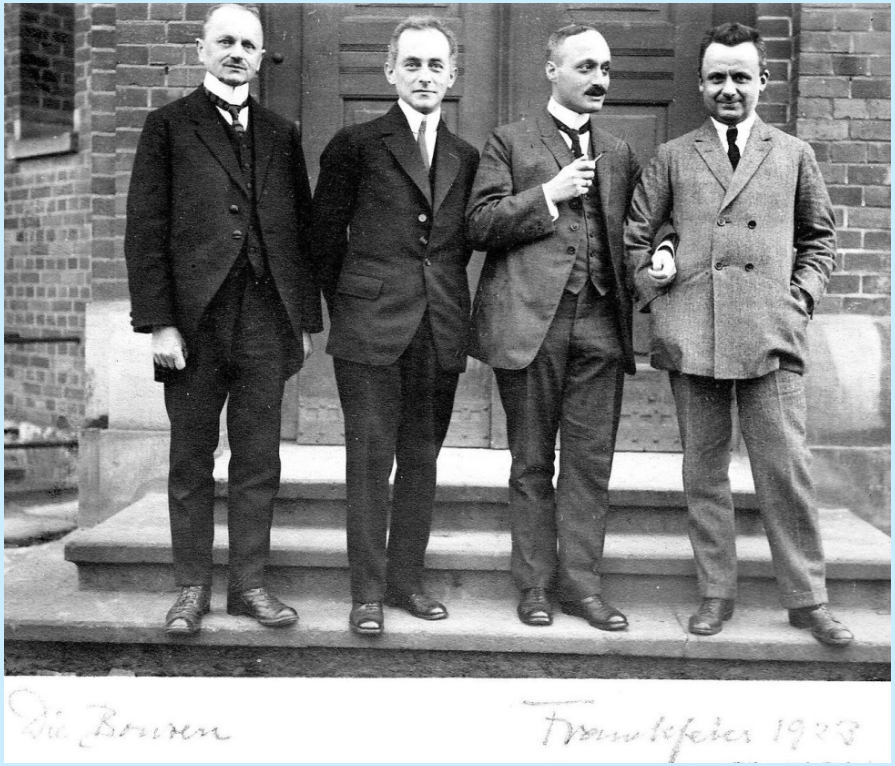
序言

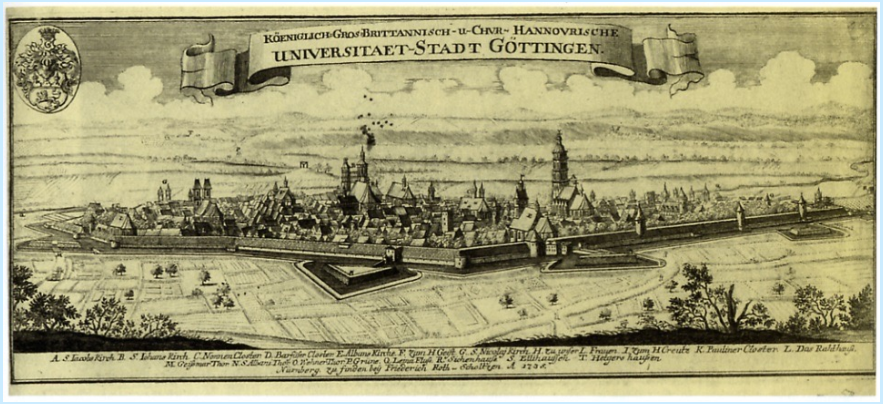


Max Born 波恩1882-1970

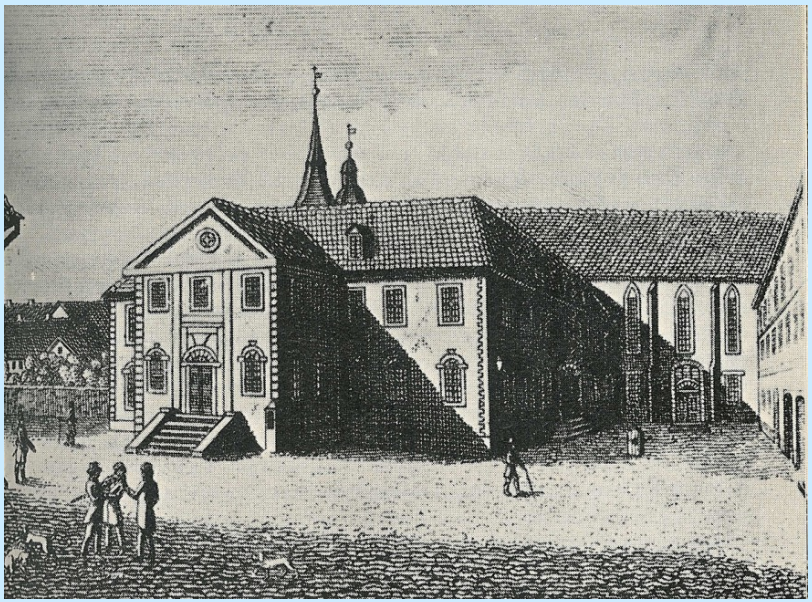
University of Göttingen











但老一輩的物理學家不希望看到古典的物理原則被推翻，

更加厭惡這些革命新理論拼拼湊湊的虛浮與含混！

Bohr’s new theory is “Quite artificial. I would rather be a cobbler or a casino worker than a physicist if this is where physics is headed.

波爾的新理論實在非常人工而做作。如果物理是朝著這個方向發展的話，

我寧可當個修補的鞋匠，或是在輕浮的賭場工作，也不要當物理學家。



Einstein to Born

這是最壞的時代，這是最好的時代。1915-1924

對革命派，這正是大好機會！

舊觀念必須打倒，革命已無可避免！



沒有人能了解為什麼波爾模型會是對的！

但新的神秘世界使人非常興奮！

革命的前鋒！

Heisenberg and Pauli







沒有比他們更不同的了

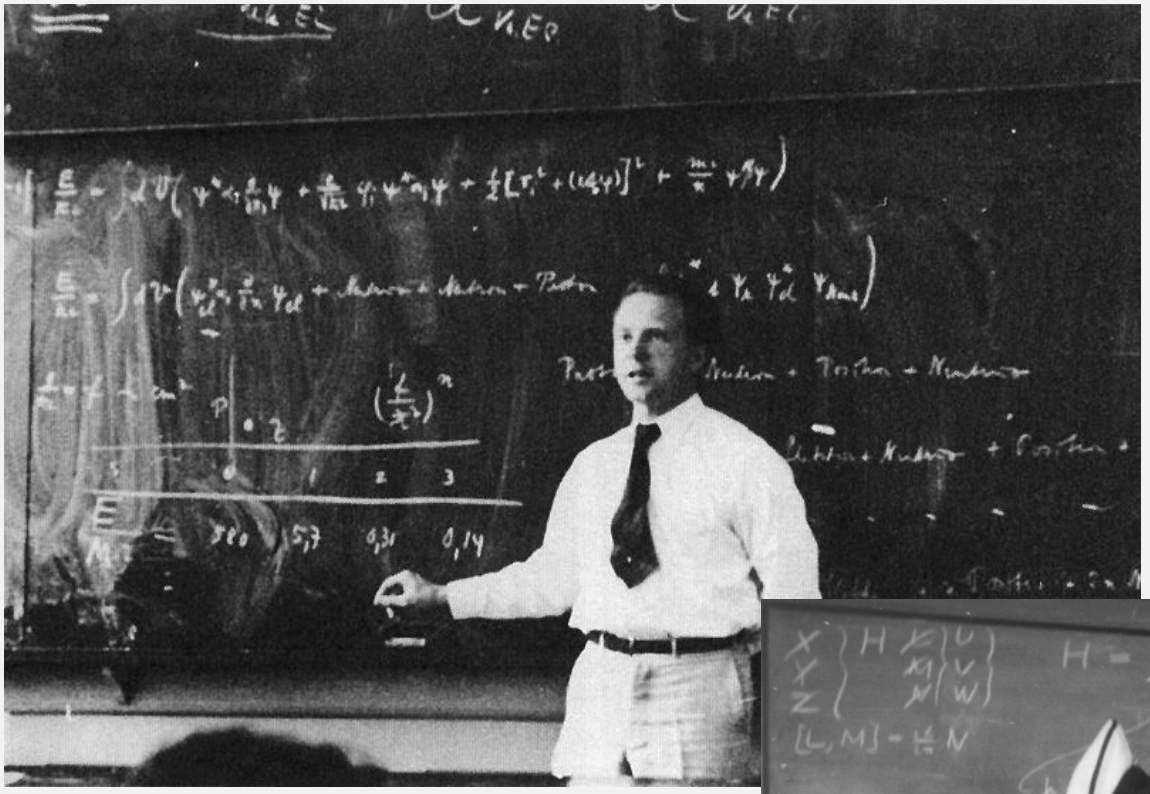


Werner Heisenberg 1901-1976 海森堡

Wolfgang Pauli 1900-1958 包立










Scanned at the American
Institute of Physics

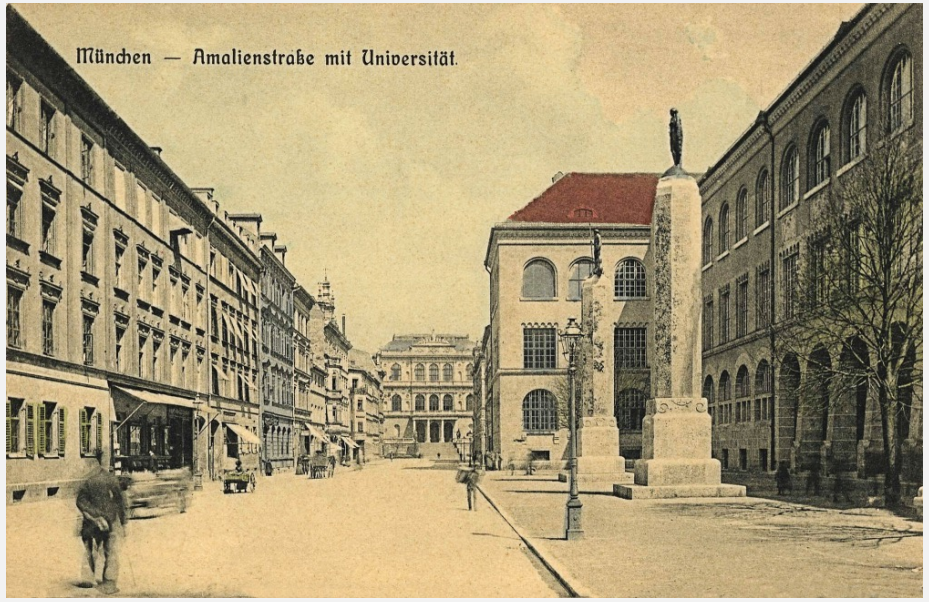

© Archiv Max-Planck-Gesellschaft

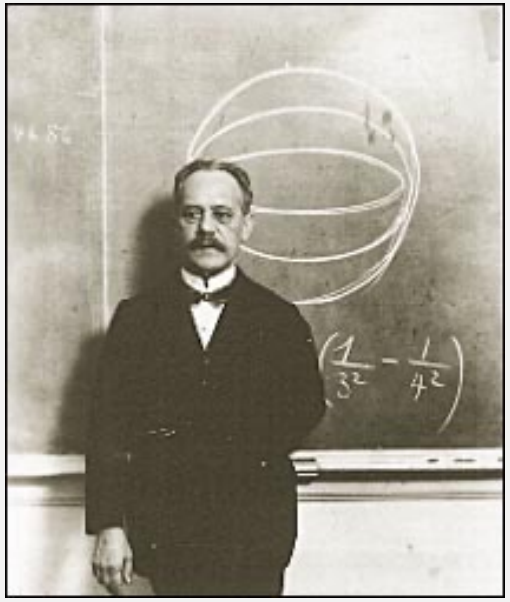
Heisenberg與Pauli一樣來自一個學者家庭。













Heisenberg與Pauli一起來到 U of Munich上大學。

這時還是大學生的Pauli已經是為深奧的廣義相對論寫回顧review的成熟的天才。

海森堡卻極度生嫩，但也因此毫無包袱、毫無顧忌。





It is much easier if one does not know a lot and is not too familiar with classical physics. You have a decided advantage there, but then lack of knowledge is no guarantee of success.

Pauli to Heisenberg,

有時懂得少一點，做起學問來反而容易，在這一點上，你就佔了優勢！

當然啦，反過頭來講，無知也不保證成功。

二十歲的年輕人成為進步的主要動力！



Knabenphysik (**boy-physics**).



Physics is now very confused again, anyway it's too difficult for me and I wish I am a movie comedian and had never heard of physics. I only hope that Bohr will save us with some new idea. Pauli

Now everything is in Heisenberg's hands, to find a way out of the difficulties. Bohr

Heisenberg 嘗試用革命性的新想法來解決原子物理的問題！

革命的基本原則就是懷疑揚棄所有舊觀念！



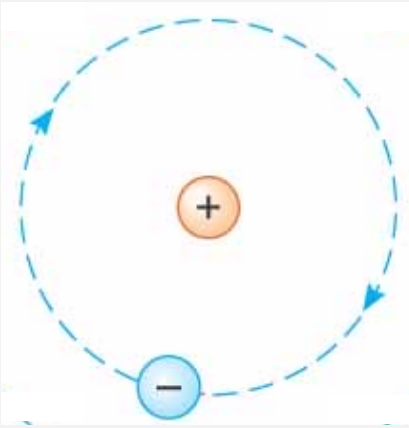
1925

如何革命? 如何揚棄舊觀念？

古典物理認定電子是一個牛頓粒子，

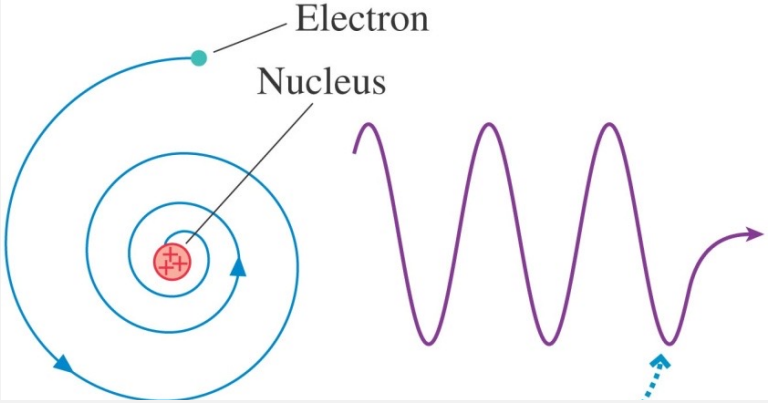
因此電子一直有位置這個性質，以這性質可以畫出電子軌跡：

這似乎是先天自明的。



可是原子中的電子的軌道從未被觀察到過!

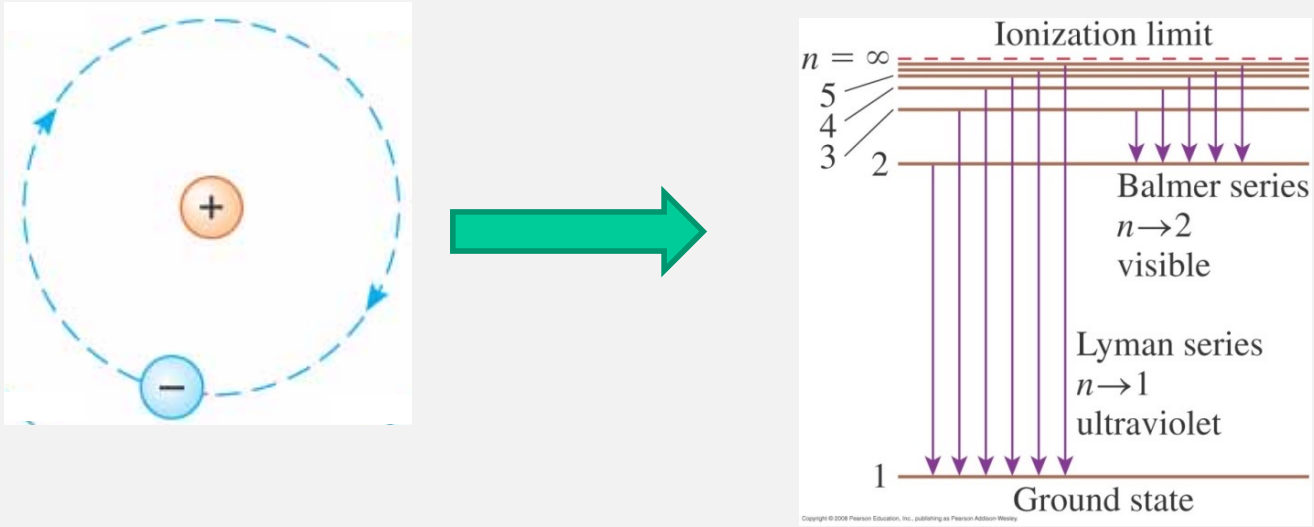
只有可觀察的才算數！所以首先要放棄電子軌跡的概念！



如此原子中的電子並沒有所謂軌跡！

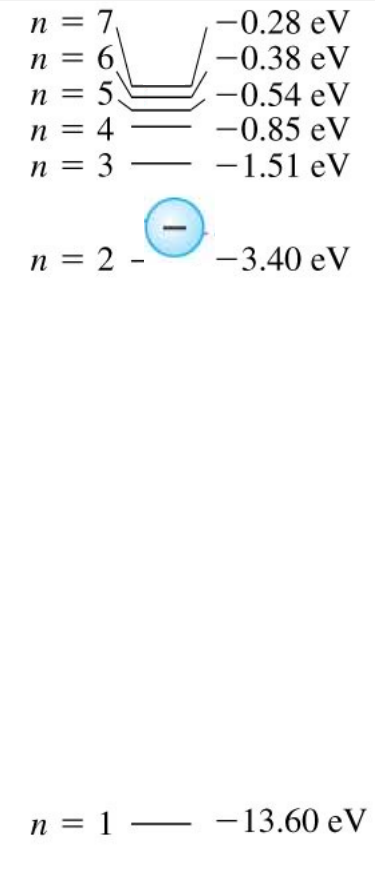
自然也就沒有釋放電磁波的問題！

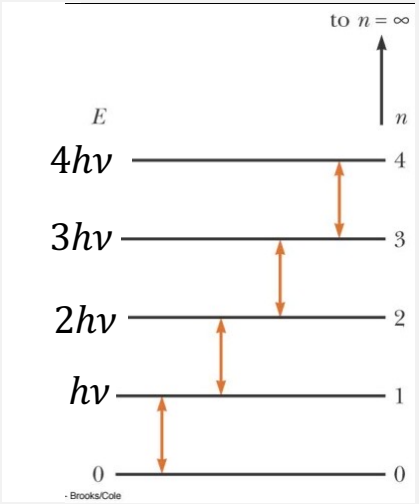
原子中的電子唯一可觀察的是能階之間的躍遷！



從原子中的電子出發，來了解電子，先暫時忘記自由電子，

暫時捨棄無法觀察的位置。重新以可觀察的能階來定義"原子中的電子"！

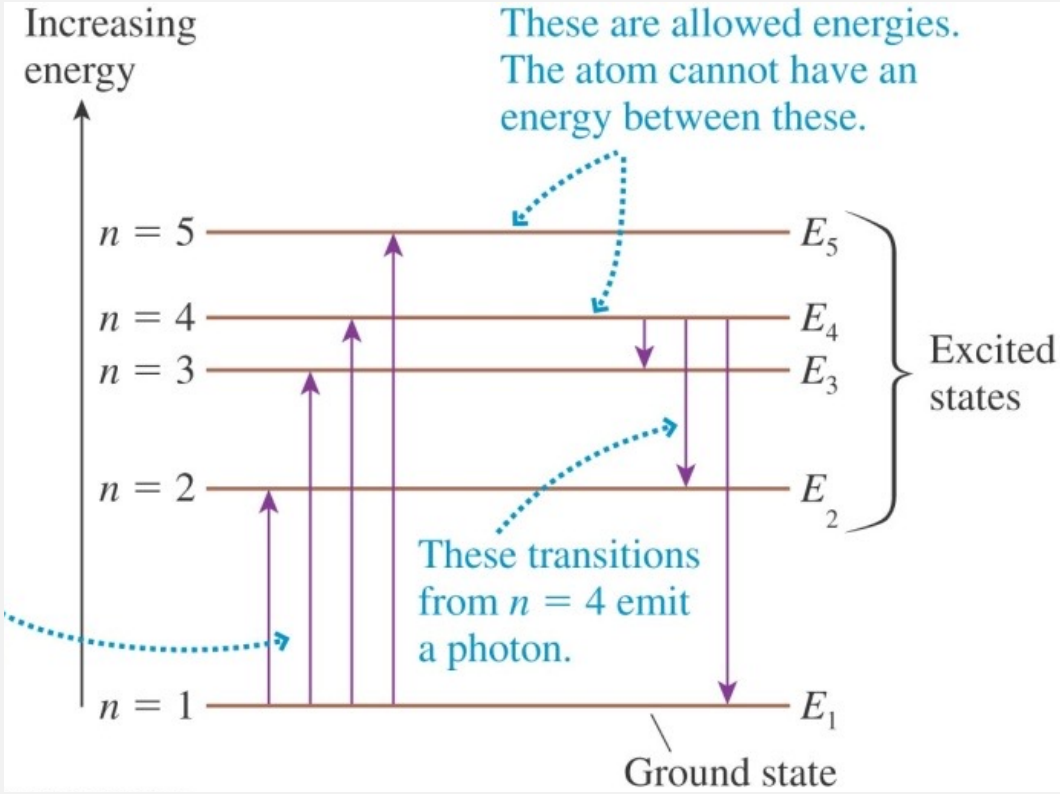




對海森堡來說，一系列的能階就是討論原子中電子的唯一出發點！

無人理會時（無作用時），就留在穩定的能階上！

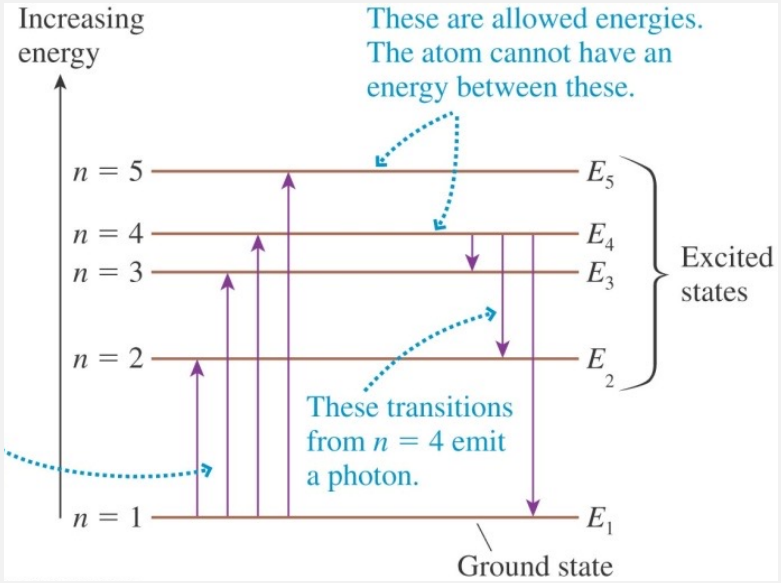
有人打擾時(觀測時)，就會發生躍遷！就可以被觀測到！



如果由可觀察量出發，躍遷就成為量子物理的觀測量的核心！

那我們如何描述躍遷、以及躍遷時的觀測量呢？

描述躍遷，就如同描述一個火車旅程，有起點有終點。







而描述或討論火車旅程時，就需要如價格表或里程表這樣的表格：



商務車廂　標準車廂

車站	台北	板橋	桃園	新竹	台中	嘉義	台南	左營
台北	-	$ 260	$ 440	$ 640	$ 1,250	$ 1,820	$ 2,230	$ 2,440
板橋	$ 40	-	$ 400	$ 590	$ 1,210	$ 1,780	$ 2,180	$ 2,390
桃園	$ 160	$ 130	-	$ 400	$ 1,010	$ 1,580	$ 1,990	$ 2,200
新竹	$ 290	$ 260	$ 130	-	$ 820	$ 1,390	$ 1,790	$ 2,000
台中	$ 700	$ 670	$ 540	$ 410	-	$ 770	$ 1,180	$ 1,390
嘉義	$ 1,080	$ 1,050	$ 920	$ 790	$ 380	-	$ 620	$ 820
台南	$ 1,350	$ 1,320	$ 1,190	$ 1,060	$ 650	$ 280	-	$ 410
左營	$ 1,490	$ 1,460	$ 1,330	$ 1,200	$ 790	$ 410	$ 140	-



								土場
							濁水	4.5 20
						牛鬪	3.5 16	8.0 32
					清水	1.5 5	5.0 20	9.5 40
				天送埤	2.1 12	3.6 16	7.1 32	11.6 48
			三星	2.2 12	4.3 20	5.8 24	9.3 40	13.8 56
		二萬五	2.6 12	4.8 20	6.9 28	8.4 36	11.9 48	16.4 68
	大洲	2.7 12	5.3 24	7.5 32	9.6 40	11.1 48	14.6 60	19.1 80
歪歪	1.8 8	4.5 20	7.1 32	9.3 40	2.4 48	12.9 52	16.4 68	20.9 84
竹林	1.9 8	3.7 16	6.4 28	9.0 36	11.2 48	13.3 56	14.8 60	18.3 76	22.8 92

羅東森林鐵道各站里程暨票價表〈數字上方為公里數；下方為票價〉

縣道186號線

維新0K+000(省道17號線201K+900)

1.961	本洲1K+961								
5.621	3.660	岡山5K+621(台19甲線68K+100)							
6.491	4.530	0.870	劉厝6K+491(介壽西路交叉口)						
7.040	5.079	1.419	0.549	岡山7K+040(舊省道交叉路口)					
10.000	8.039	4.379	3.509	2.960	瓊林10K+850(瓊興路交叉口)				
14.250	12.289	8.629	7.759	7.210	4.250	燕巢14K+250(中興路交叉口)			
18.343	16.382	12.722	11.852	11.303	8.343	4.093	鳳山厝18K+343		
19.450	17.489	13.829	12.959	12.410	9.450	5.200	1.107	大社19K+450	
23.450	21.489	17.829	16.959	16.410	13.450	9.200	5.107	4.000	仁武23K+450
34.074	32.113	28.453	27.583	27.034	24.074	19.824	15.731	14.624	10.624	大樹34K+074(台21線291K+223)

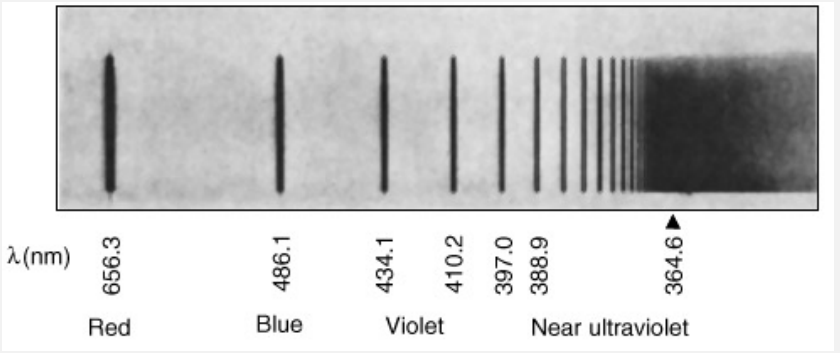
縣道186號甲線

大社0K+000

12.634	姑婆寮12k+634

商務車廂　標準車廂

車站	台北	板橋	桃園	新竹	台中	嘉義	台南	左營
台北	-	$ 260	$ 440	$ 640	$ 1,250	$ 1,820	$ 2,230	$ 2,440
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嘉義	$ 1,080	$ 1,050	$ 920	$ 790	$ 380	-	$ 620	$ 820
台南	$ 1,350	$ 1,320	$ 1,190	$ 1,060	$ 650	$ 280	-	$ 410
左營	$ 1,490	$ 1,460	$ 1,330	$ 1,200	$ 790	$ 410	$ 140	-



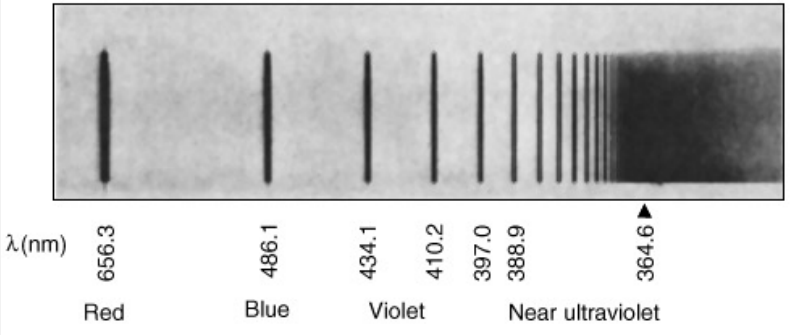
收集電子躍遷時所放出的光的頻率，就形成一個表格：

$$\begin{pmatrix} \nu_{11} & \nu_{12} & \nu_{13} & \cdots \\ \nu_{21} & \nu_{22} & \nu_{23} & \cdots \\ \nu_{31} & \nu_{32} & \nu_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

ν_{nm}

這就是一個可觀察的量。就是光譜線的頻率！

光譜線的光的強度也可以測量！



光譜線的光強度也應該可以寫成一個表格：

$$a = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

a_{nm} n態到m態的躍遷強度

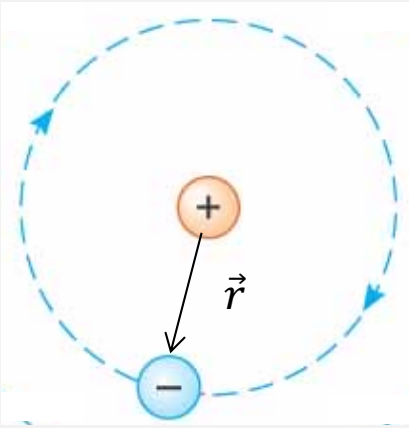
這就十分有趣了！

因為在電磁學中，一個電荷發光的強度，正比其電偶極矩(的時間兩次微分)：

$a \propto p$　　$\vec{p} = e \cdot \vec{r}$

而電偶極矩又正比於位置，因此光譜線光強度就正比於電子的位置：

$a \propto r$　這樣定義的位置是可量的，說不定這才是電子位置的真實意義！





測量躍遷的強度a，就等同測量電子的位置！

如果a是一個表格，或許電子位置r不是完全沒有意義，只是非單純的數。

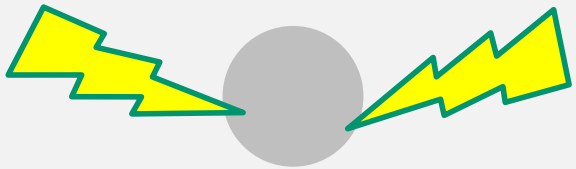
而是：位置是一個表格！

對原子中電子唯一的觀察，就是以電磁波的吸收與放射來探測電子

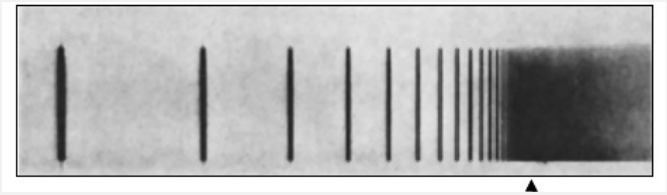
光譜線光強度就正比於電子的位置：$a \propto r$

$$a = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

E, B

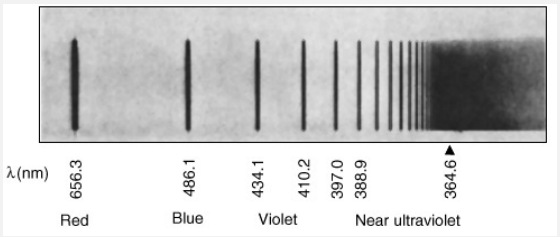


光譜線的強度是一個表格：



電子位置自然也是一個表格！

$$x = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

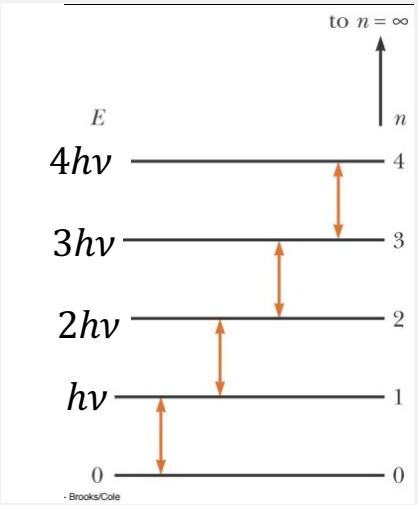


於是，我們可以由一個可觀測量，重新定義電子的位置，它是表格。

海森堡大膽假設：

$$x = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

位置是一個表格！



$$p = \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ p_{31} & p_{32} & p_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

動量也是一個表格！



以能階的量子數為排列的足標，所有電子性質的物理量都是表格！

電子位置，動量都是表格！　這的確很酷！

表格如何計算？如何從電子的位置與動量表格算出電子的能量呢？

海森堡從最簡單的力學問題出發：簡諧振盪器、量子彈簧！

$E = \frac{p \cdot p}{2m} + \frac{1}{2} kx \cdot x$　表格怎麼乘？

在迷惑中，又發作嚴重的花粉熱，1925夏天Heisenberg逃到Helgoland島！



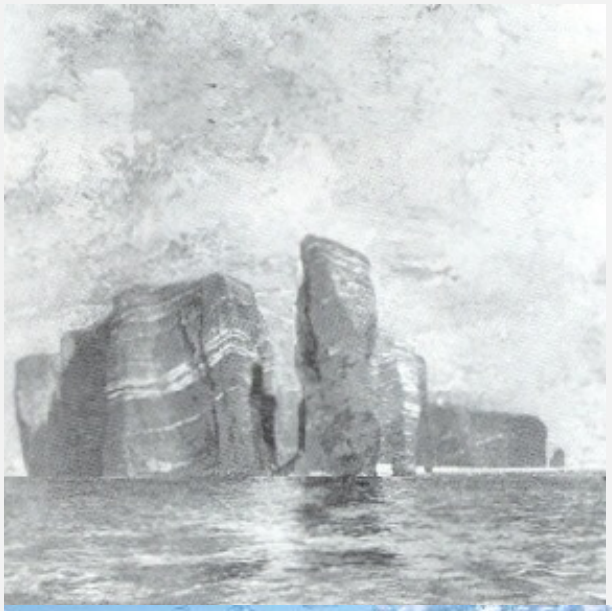












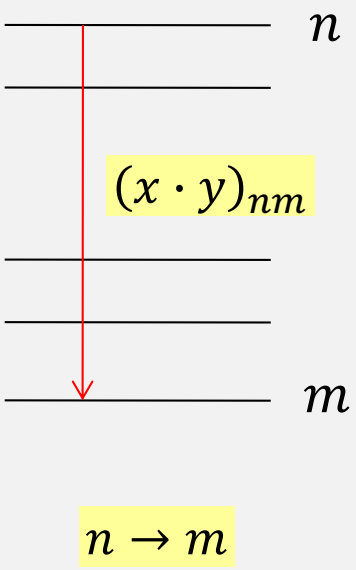


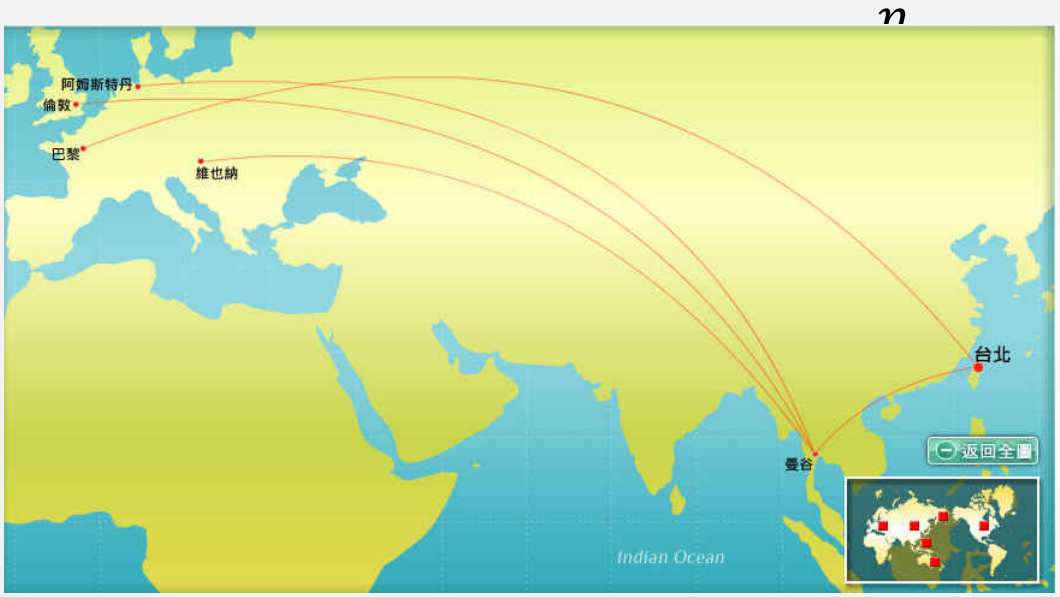
也許是自然的海風，也許是文學的催化。

海森堡漸漸猜到了一個可能的表格的乘法！

考慮兩表格x、 y的乘積$x \cdot y$表格的一個元素：

$(x \cdot y)_{nm}$　這個 $x \cdot y$表格的元素對應n態到m態的躍遷。



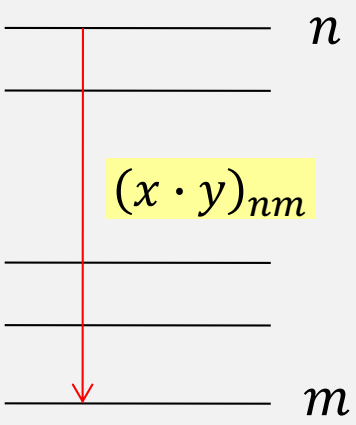


若要將它拆成兩個表格x、 y的元素的乘積，最自然的方法：

就是轉乘：找一個中繼站，分兩段來完成躍遷。

考慮兩表格x、 y的乘積$x \cdot y$表格的一個元素：

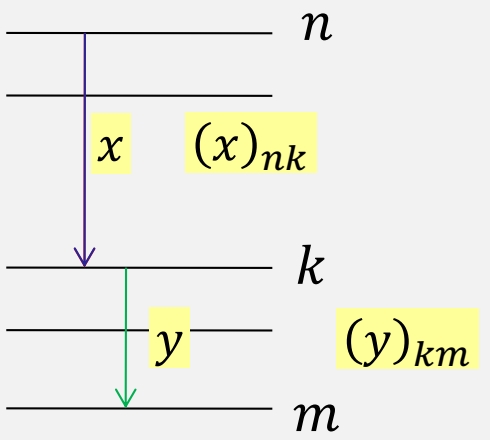
$(x \cdot y)_{nm}$ 若要將它拆成兩個表格x、 y的元素的乘積，最自然的方法：



$n \to m$

$(x \cdot y)_{nm} = (x)_{nk} \cdot (y)_{km}$

$\sum_k$
任意 k

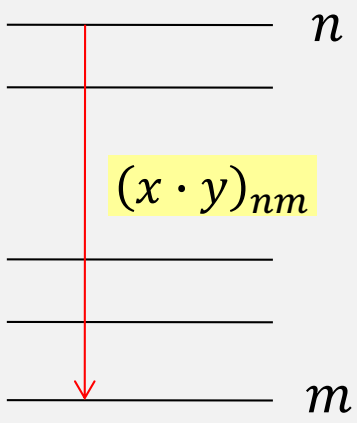


$\sum_k$ $n \to k, k \to m$

中繼站可以任意，必須加總所有可能！

$$(x \cdot y)_{nm} = \sum_k (x)_{nk} \cdot (y)_{km}$$

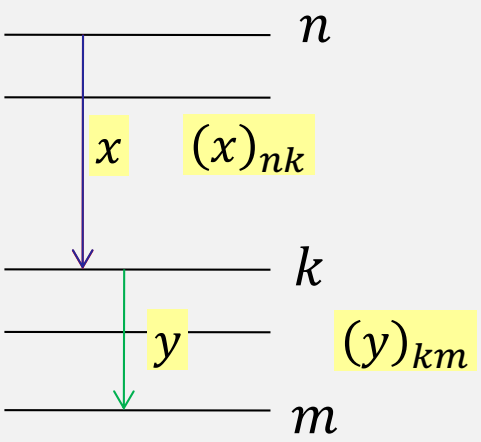
兩個表格的乘法，正是兩個矩陣的乘法，當然年輕的海森堡根本不知道矩陣是什麼：



$n \to m$

$$\sum_k$$

任意 k



$$\sum_k$$

$n \to k, k \to m$

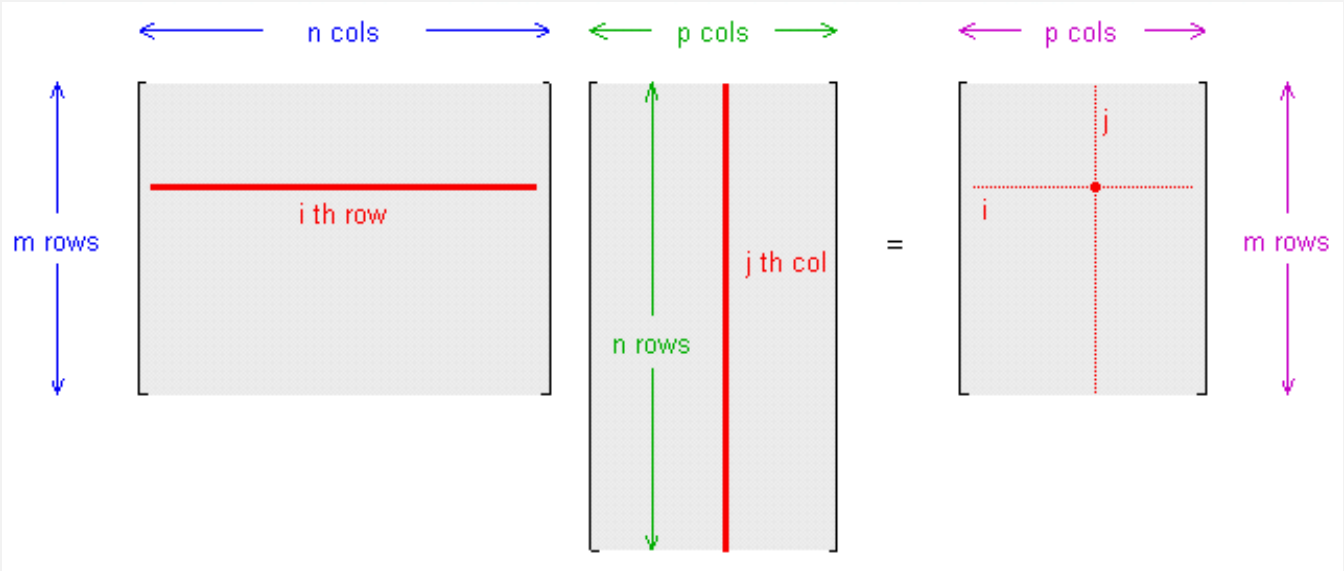
$$(x \cdot y)_{nm} = \sum_k (x)_{nk} \cdot (y)_{km}$$

$$(A \cdot B)_{nm} = \sum_k (A)_{nk} \cdot (B)_{km}$$

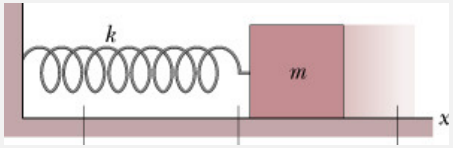
$$(A \times B)_{32} = a_{31} \cdot b_{12} + a_{32} \cdot b_{22} + a_{33} \cdot b_{32} + \cdots = \begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & b_{13} & \cdots \\ b_{21} & b_{22} & b_{23} & \cdots \\ b_{31} & b_{32} & b_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\underset{3\times 4 \text{ matrix}}{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & 2 & 3 & 4 \end{bmatrix}} \underset{4\times 5 \text{ matrix}}{\begin{bmatrix} \cdot & \cdot & \cdot & a & \cdot \\ \cdot & \cdot & \cdot & b & \cdot \\ \cdot & \cdot & \cdot & c & \cdot \\ \cdot & \cdot & \cdot & d & \cdot \end{bmatrix}} = \underset{3\times 5 \text{ matrix}}{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & x_{3,4} & \cdot \end{bmatrix}}$$

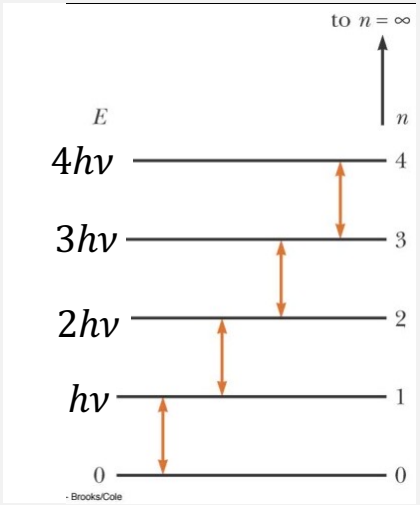
$$x_{3,4} = (1,2,3,4) \cdot (a,b,c,d) = 1 \times a + 2 \times b + 3 \times c + 4 \times d.$$



將表格的乘法，運用於量子彈簧的能量計算！



$$E = \frac{p \cdot p}{2m} + \frac{1}{2} kx \cdot x$$



以能階的量子數為排列的足標，

$$x = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

位置是一個表格！

$$p = \begin{pmatrix} p_{11} & p_{12} & p_{13} & \cdots \\ p_{21} & p_{22} & p_{23} & \cdots \\ p_{31} & p_{32} & p_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

動量也是一個表格！



$$(A \cdot B)_{nm} = \sum_k (A)_{nk} \cdot (B)_{km}$$

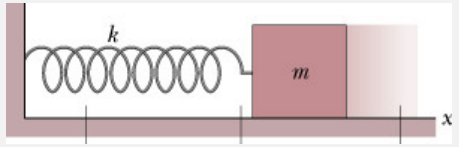
再於結果代入量子化條件：

$$\oint dx \cdot p = nh$$

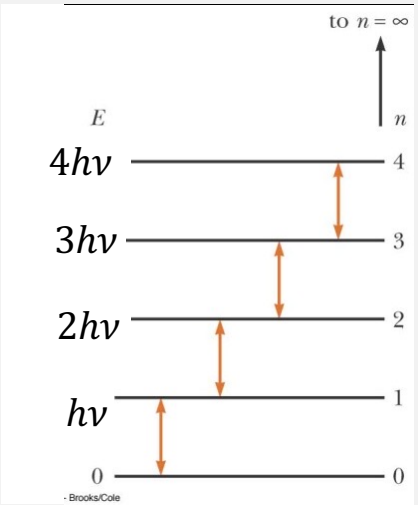
在一天的深夜裡，海森堡用這種方法去計算一個簡諧振盪器的能量，

結果發現能量的表格只有位於對角線的元素不為零，

而且其值就剛好就是Planck所假設的量子化能量值：

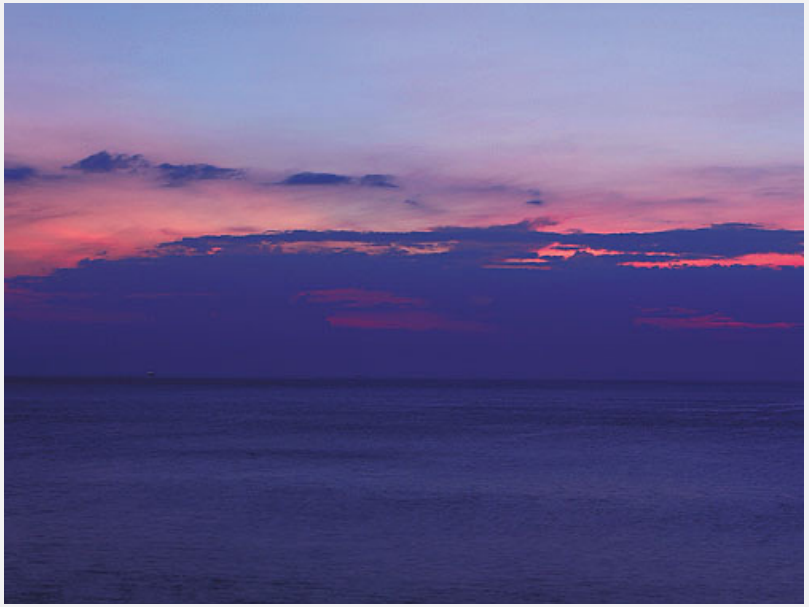


$$E = \frac{p \cdot p}{2m} + \frac{1}{2} kx \cdot x = \begin{pmatrix} h\nu & 0 & 0 & \cdots \\ 0 & 2h\nu & 0 & \cdots \\ 0 & 0 & 3h\nu & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$



電子位置，動量都是表格！➡ 能量是對角化的表格，表格元素是量子化的！

能量量子化，這個展開量子革命的第一個的謎，現在可以被計算推導出來！

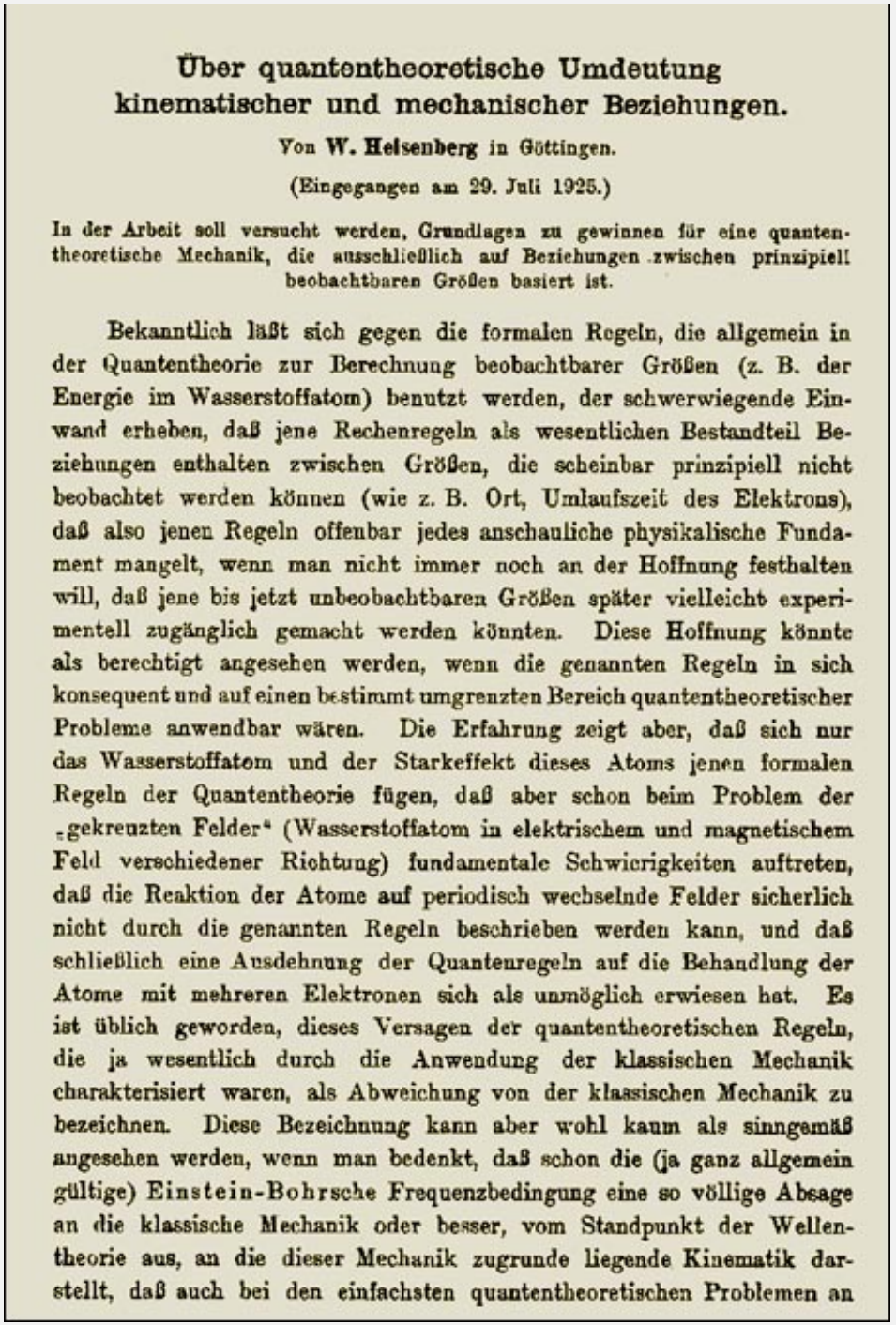




量子力學的革命正式展開！

量子物理的數學第一次被正確的寫下來！





Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.

Von **W. Heisenberg** in Göttingen.

(Eingegangen am 29. Juli 1925.)

In der Arbeit soll versucht werden, Grundlagen zu gewinnen für eine quantentheoretische Mechanik, die ausschließlich auf Beziehungen zwischen prinzipiell beobachtbaren Größen basiert ist.

Bekanntlich läßt sich gegen die formalen Regeln, die allgemein in der Quantentheorie zur Berechnung beobachtbarer Größen (z. B. der Energie im Wasserstoffatom) benutzt werden, der schwerwiegende Einwand erheben, daß jene Rechenregeln als wesentlichen Bestandteil Beziehungen enthalten zwischen Größen, die scheinbar prinzipiell nicht beobachtet werden können (wie z. B. Ort, Umlaufszeit des Elektrons), daß also jenen Regeln offenbar jedes anschauliche physikalische Fundament mangelt, wenn man nicht immer noch an der Hoffnung festhalten will, daß jene bis jetzt unbeobachtbaren Größen später vielleicht experimentell zugänglich gemacht werden könnten. Diese Hoffnung könnte als berechtigt angesehen werden, wenn die genannten Regeln in sich konsequent und auf einen bestimmt umgrenzten Bereich quantentheoretischer Probleme anwendbar wären. Die Erfahrung zeigt aber, daß sich nur das Wasserstoffatom und der Starkeffekt dieses Atoms jenen formalen Regeln der Quantentheorie fügen, daß aber schon beim Problem der „gekreuzten Felder" (Wasserstoffatom in elektrischem und magnetischem Feld verschiedener Richtung) fundamentale Schwierigkeiten auftreten, daß die Reaktion der Atome auf periodisch wechselnde Felder sicherlich nicht durch die genannten Regeln beschrieben werden kann, und daß schließlich eine Ausdehnung der Quantenregeln auf die Behandlung der Atome mit mehreren Elektronen sich als unmöglich erwiesen hat. Es ist üblich geworden, dieses Versagen der quantentheoretischen Regeln, die ja wesentlich durch die Anwendung der klassischen Mechanik charakterisiert waren, als Abweichung von der klassischen Mechanik zu bezeichnen. Diese Bezeichnung kann aber wohl kaum als sinngemäß angesehen werden, wenn man bedenkt, daß schon die (ja ganz allgemein gültige) Einstein-Bohrsche Frequenzbedingung eine so völlige Absage an die klassische Mechanik oder besser, vom Standpunkt der Wellentheorie aus, an die dieser Mechanik zugrunde liegende Kinematik darstellt, daß auch bei den einfachsten quantentheoretischen Problemen an

"Everything is still vague and unclear to me, but it seems as if the electrons will no more move on orbits."

一切都還不清楚，

但至少原子中電子已經不再有軌道了。

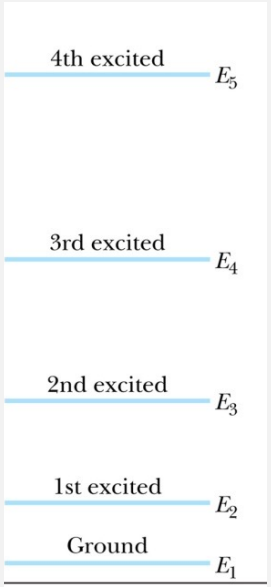
The present paper seeks to establish a basis for theoretical quantum mechanics founded exclusively upon relationships between quantities which in principle are observable.

本文嘗試為量子力學建立一個基礎，

這個基礎完全植基於可觀測的物理量彼此的關係！







October 1925 Pauli 用表格法算出了氫原子能階。

Heisenberg's form completely avoids a mechanical-kinematic visualization of the motion of electrons! Pauli

海森堡的理論完全避免使用電子運動軌跡這一類視覺化的概念。

M. Born, Z. Phys. **34,** 858 **1925**

On Quantum Mechanics

M. Born

Received 1925

— — ◇◇◇ — —

Translation into English: *Sources of Quantum Mechanics,*
Ed. by B. L. van der Waerden, North Holland, Amsterdam (1967) 277.

— — ◇◇◇ — —

The recently published theoretical approach of Heisenberg is here developed into a systematic theory of quantum mechanics (in the first place for systems having one degree of freedom) with the aid of mathematical matrix methods. After a brief survey of the latter, the mechanical equations of motion are derived from a variational principle and it is shown that using Heisenberg's quantum condition, the principle of energy conservation and Bohr's frequency condition follow from the mechanical equations. Using the anharmonic oscillator as example, the question of uniqueness of the solution and of the significance of the phases of the partial vibrations is raised. The paper concludes with an attempt to incorporate electromagnetic field laws into the new theory.

但表格……..實在聽起來非常的業餘！

經驗與資歷有時還是有用的。



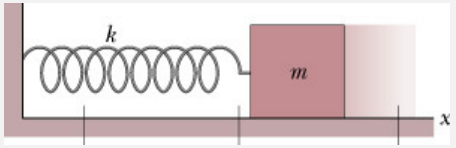
波恩指出表格就是矩陣！

海森堡發明的表格乘法就是矩陣乘法。

電子位置、動量及所有物理量都是矩陣！

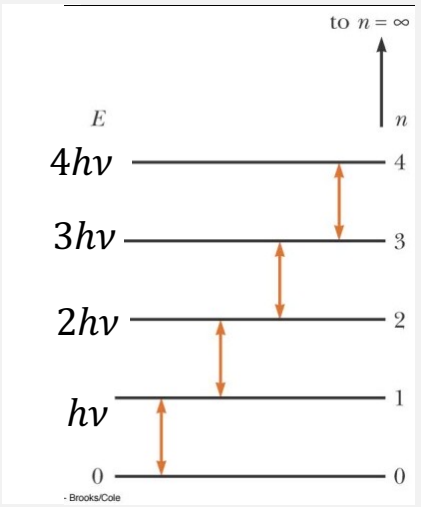
矩陣力學 Matrix Mechanics

運用現代的符號：一樣以能階量子數n為足標。



量子簡諧振盪器的位置矩陣：

$$x=\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix}0&1&0&0&0&\cdots\\1&0&\sqrt{2}&0&0&\cdots\\0&\sqrt{2}&0&\sqrt{3}&0&\cdots\\0&0&\sqrt{3}&0&0&\cdots\\0&0&0&0&0&\cdots\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots\end{pmatrix}$$



動量矩陣：

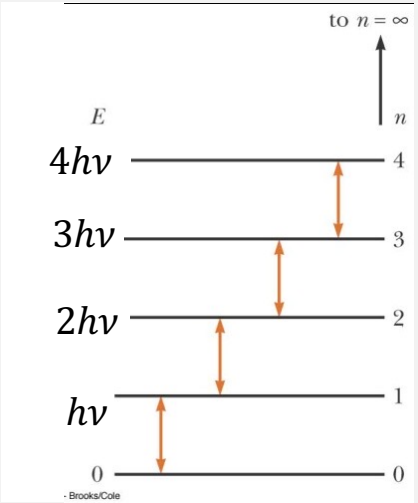
$$p=\sqrt{\frac{m\omega\hbar}{2}}\begin{pmatrix}0&i&0&0&0&\cdots\\-i&0&i\sqrt{2}&0&0&\cdots\\0&-i\sqrt{2}&0&i\sqrt{3}&0&\cdots\\0&0&-i\sqrt{3}&0&i\sqrt{4}&\cdots\\0&0&0&-i\sqrt{4}&0&\cdots\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots\end{pmatrix}$$

注意它們都是Hermitian Matrices.

在何種意義上，這些物理量是矩陣呢？

1. 能量矩陣與位置矩陣及動量矩陣保持古典物理中的代數關係！

$$E = \frac{p \cdot p}{2m} + \frac{1}{2} kx \cdot x = \begin{pmatrix} h\nu & 0 & 0 & \cdots \\ 0 & 2h\nu & 0 & \cdots \\ 0 & 0 & 3h\nu & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

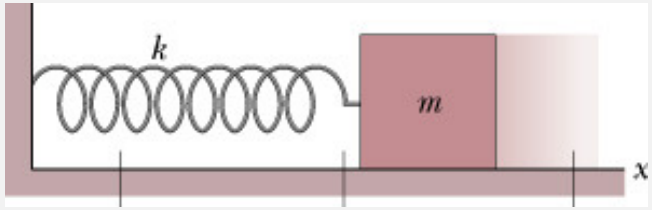


能量表格在以能階量子數為足標時是對角化的！

基底是能量矩陣的本徵向量Eigenvectors。

海：一個電子的點狀的位置沒有意義，

但我們還是可以測量它的點狀的位置，



海：或許測量結果沒有確定值！

我們只能就特定狀態下，電子點狀的位置作測量，將結果取平均值！

只有平均值有意義！

2. 測量物理量的平均值等於將物理量矩陣夾在兩個代表狀態的向量間：

以基態$n=1$為例：

$$\langle x\rangle=(1,0,0,0,0\cdots)\sqrt{\frac{\hbar}{2m\omega}}\begin{pmatrix}0&1&0&0&0&\cdots\\1&0&\sqrt{2}&0&0&\cdots\\0&\sqrt{2}&0&\sqrt{3}&0&\cdots\\0&0&\sqrt{3}&0&0&\cdots\\0&0&0&0&0&\cdots\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\\0\\\vdots\end{pmatrix}=0$$

在基態$n=1$測量位置的平均值，就是位置矩陣的11矩陣元：x_{11}!

這適用於任何物理量！

$$\langle p\rangle=(1,0,0,0,0\cdots)\sqrt{\frac{m\omega\hbar}{2}}\begin{pmatrix}0&i&0&0&0&\cdots\\-i&0&i\sqrt{2}&0&0&\cdots\\0&-i\sqrt{2}&0&i\sqrt{3}&0&\cdots\\0&0&-i\sqrt{3}&0&i\sqrt{4}&\cdots\\0&0&0&-i\sqrt{4}&0&\cdots\\\vdots&\vdots&\vdots&\vdots&\vdots&\ddots\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\\0\\\vdots\end{pmatrix}=0$$

位置與動量的平均值為零不奇怪。

在基態$n=1$測量物理量的平均值，就是該物理量矩陣的11矩陣元：x_{11}^2!

$$\langle x^2\rangle = \left(\frac{\hbar}{2m\omega}\right)(1,0,0,0,0\cdots)\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ 1 & 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 & \cdots \\ 0 & 0 & \sqrt{3} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ 1 & 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 & \cdots \\ 0 & 0 & \sqrt{3} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

$$= \frac{\hbar}{2m\omega}$$

可以計算位置測量的不準度。

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{\hbar}{2m\omega}$$

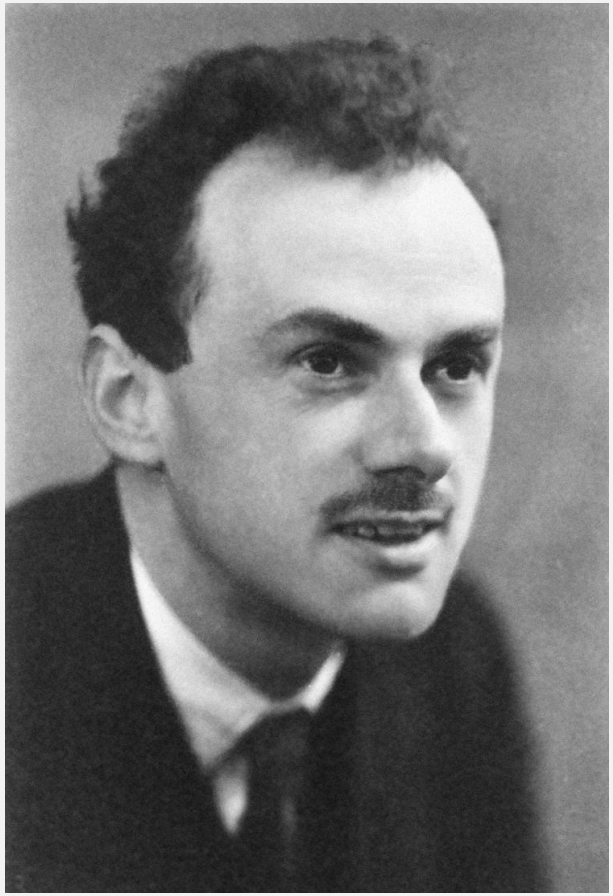
$$\langle p^2 \rangle = \left(\frac{m\omega\hbar}{2}\right)$$

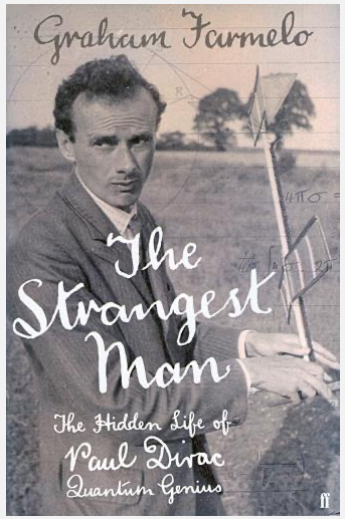
$$(1,0,\cdots)\begin{pmatrix} 0 & i & 0 & 0 & 0 & \cdots \\ -i & 0 & i\sqrt{2} & 0 & 0 & \cdots \\ 0 & -i\sqrt{2} & 0 & i\sqrt{3} & 0 & \cdots \\ 0 & 0 & -i\sqrt{3} & 0 & i\sqrt{4} & \cdots \\ 0 & 0 & 0 & -i\sqrt{4} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix} 0 & i & 0 & 0 & 0 & \cdots \\ -i & 0 & i\sqrt{2} & 0 & 0 & \cdots \\ 0 & -i\sqrt{2} & 0 & i\sqrt{3} & 0 & \cdots \\ 0 & 0 & -i\sqrt{3} & 0 & i\sqrt{4} & \cdots \\ 0 & 0 & 0 & -i\sqrt{4} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

$$= \frac{m\omega\hbar}{2}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \frac{m\omega\hbar}{2}$$

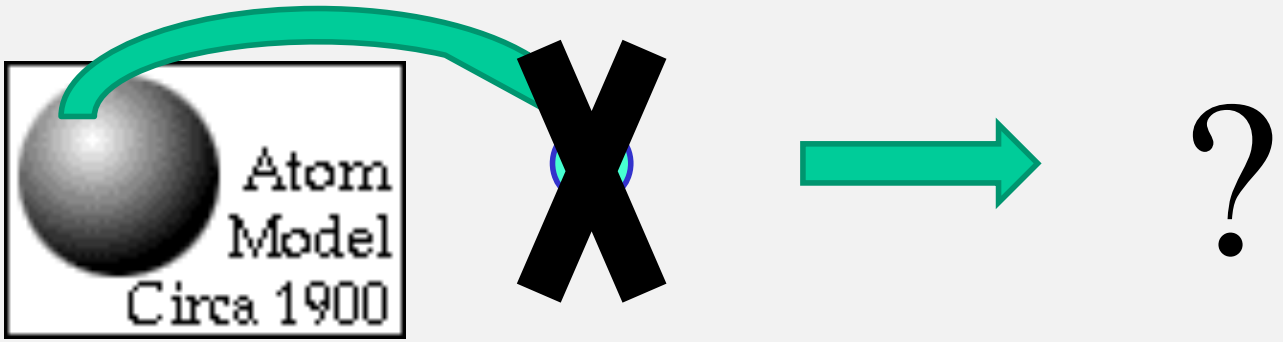
$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{\hbar}{2m\omega}$$





此時，英國的Dirac也將海森堡的想法推廣為一套新的力學。

好不容易發現的基本粒子



把原子內的電子看成一個牛頓粒子顯然是不對的！

如何對待它呢？

電子是一個無法分割的粒子，它的位置是矩陣！

$x(t)$ → $x = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots \\ x_{21} & x_{22} & x_{23} & \cdots \\ x_{31} & x_{32} & x_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

 → ?

如果電子的位置連一個數都不是，它還算一個粒子嗎？

更糟的是：波恩注意到，矩陣一般是不對易的。

where in the right-hand expression those $q(nm)$, $p(nm)$ which take on a negative index are to be set equal to zero. In this way we obtain the quantization condition corresponding to (36) as

$$\sum_k (p(nk)q(kn) - q(nk)p(kn)) = \frac{h}{2\pi i}. \tag{37}$$

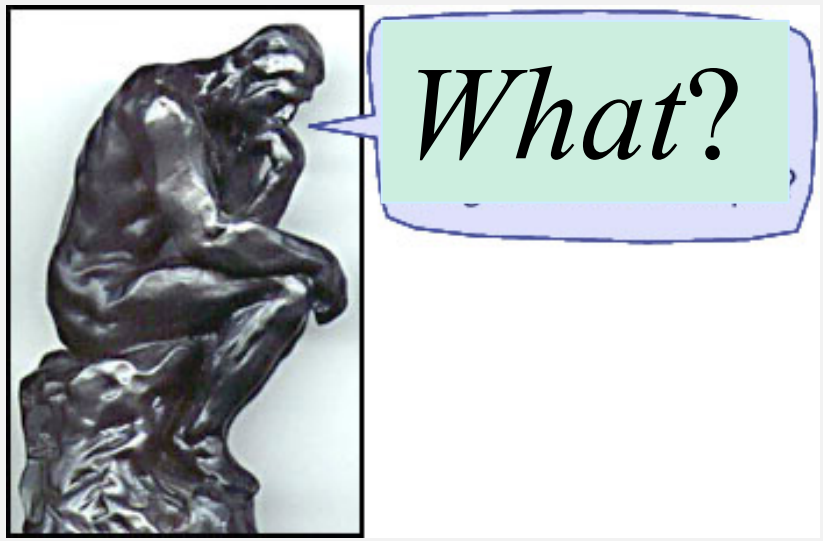
This is a system of infinitely many equations, namely one for each value of n.

In particular, for $\mathbf{p} = m\dot{\mathbf{q}}$ this yields

But...Oops!

$pq \neq qp$

這是怎麼一回事？



negative index are to be set equal to zero. In this way we obtain the quantization condition corresponding to (36) as

$$\sum_k (p(nk)q(kn) - q(nk)p(kn)) = \frac{h}{2\pi i}. \qquad (37)$$

若動量及座標p, q都是矩陣

Sommerfeld量子化條件 $\oint dq \cdot p_q = nh$ ➡ $qp - pq = i\hbar$



最後的量子化條件。

對當時的物理學家，甚至波恩本人，並不知道這個結果的重要性，只覺得疑惑。

現在我們已知道，假設動量及座標p, q都是矩陣，並規定它們滿足量子化條件：

$$qp - pq = i\hbar$$

整個電子的量子力學就可以從這兩個假設中推導出來！



????





I felt discouraged, not to say repelled, by the Heisenberg methods of transcendental algebra.

Schrodinger 1926

我對海森堡的這些超越經驗超越感覺的代數實在覺得十分沮喪！