老師：張嘉泓　　　　　物理系

Office:   A213     Phone #: 7749-6038

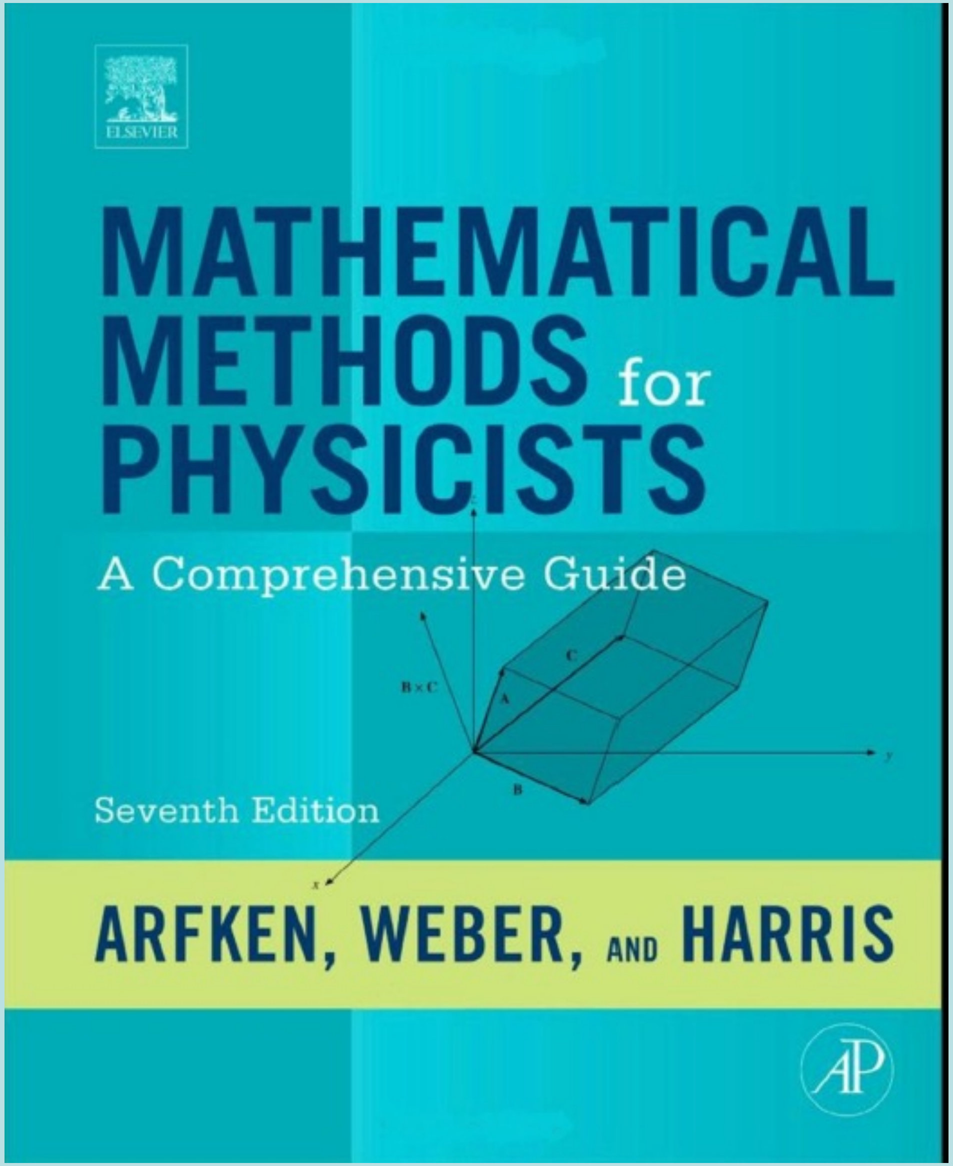
手機：0922-492150

E-mail:   chvchang@hotmail.com

FB:   https://www.facebook.com/CHVincentChang

個人網址：http://phy.ntnu.edu.tw/~chchang

課程網址：https://phy.ntnu.edu.tw/~chchang/PMI.htm



ELSEVIER

# MATHEMATICAL METHODS for PHYSICISTS

A Comprehensive Guide

Seventh Edition

**ARFKEN, WEBER, AND HARRIS**

Make Mathematical Physics really physics and really mathematics.

Not just solution manual.

Mathematical methods for application to problems in physics.

數學物理Mathematical Physics典型的三大數學主題：

偏微分方程式 Partial Differential Equation 粒子與場的運動方程式

線性代數 Linear Algebra 向量空間的代數

複變函數 Functions of complex variables

但我們不會堅持完整的介紹這幾個數學科目。

We will not insist on covering these mathematical topics comprehensively.

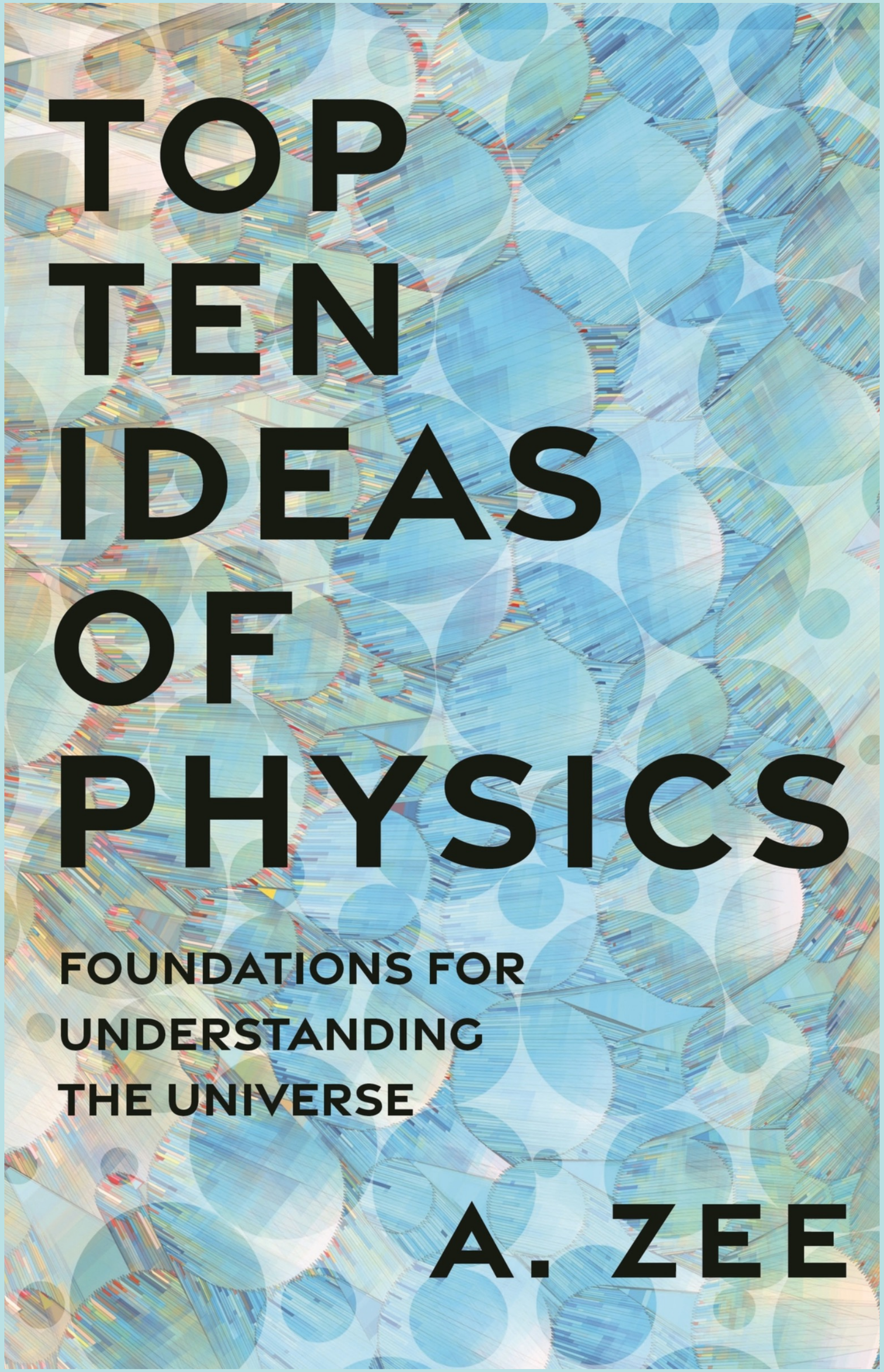
我們反而會堅持完整的介紹相關的物理問題，以及它們的推理。

But we will insist on covering their physics as comprehensively as allowed.

以物理問題為本位！看這些數學能如何幫助我們！

This is a physics centric course and is to show how mathematics can help us.

物理與數學真的密不可分！They are inseperable.

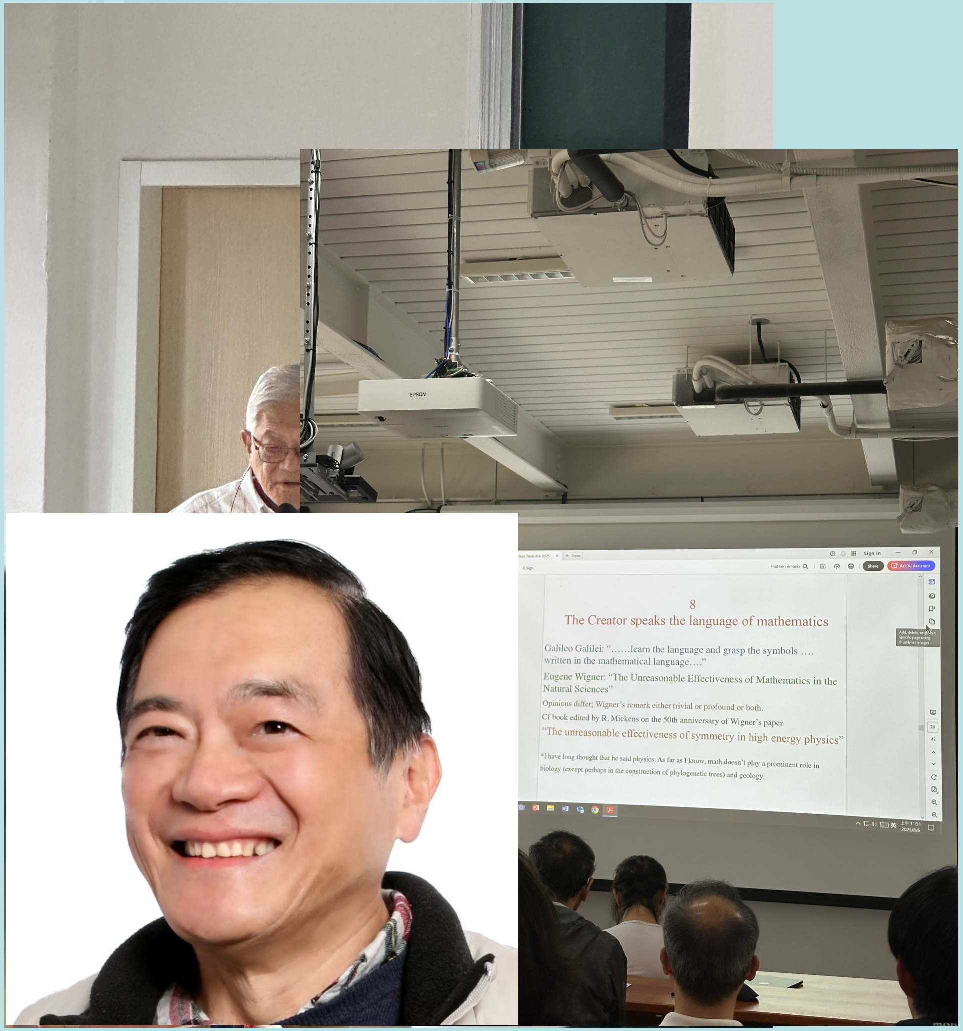


TOP
TEN
IDEAS
OF
PHYSICS

FOUNDATIONS FOR
UNDERSTANDING
THE UNIVERSE

A. ZEE

徐一鴻 Tony Zee



8
The Creator speaks the language of mathematics

Galileo Galilei: "……learn the language and grasp the symbols …. written in the mathematical language…."

Eugene Wigner: "The Unreasonable Effectiveness of Mathematics in the Natural Sciences"

Opinions differ; Wigner's remark either trivial or profound or both.

Cf book edited by R. Mickens on the 50th anniversary of Wigner's paper

"The unreasonable effectiveness of symmetry in high energy physics"

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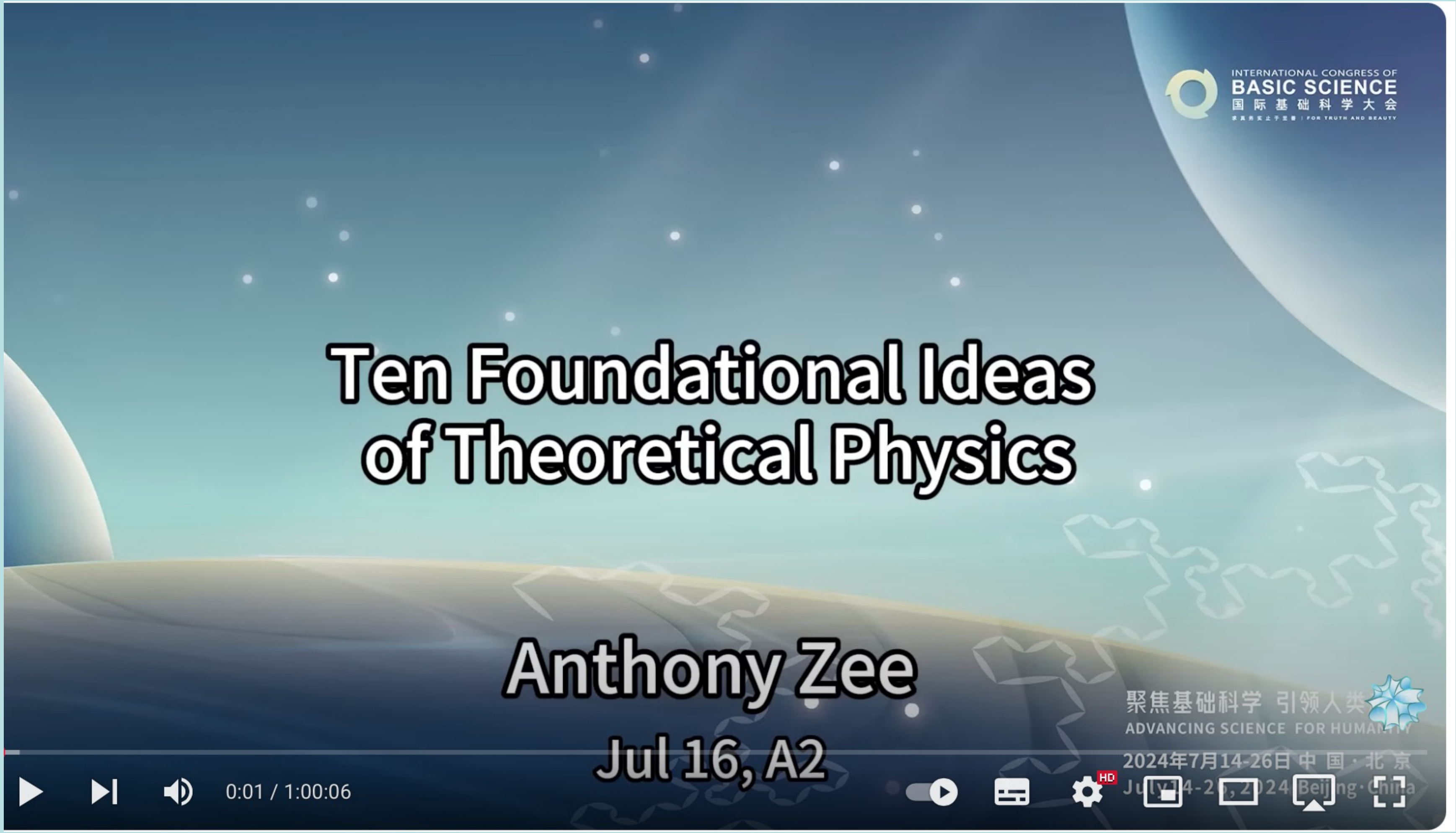
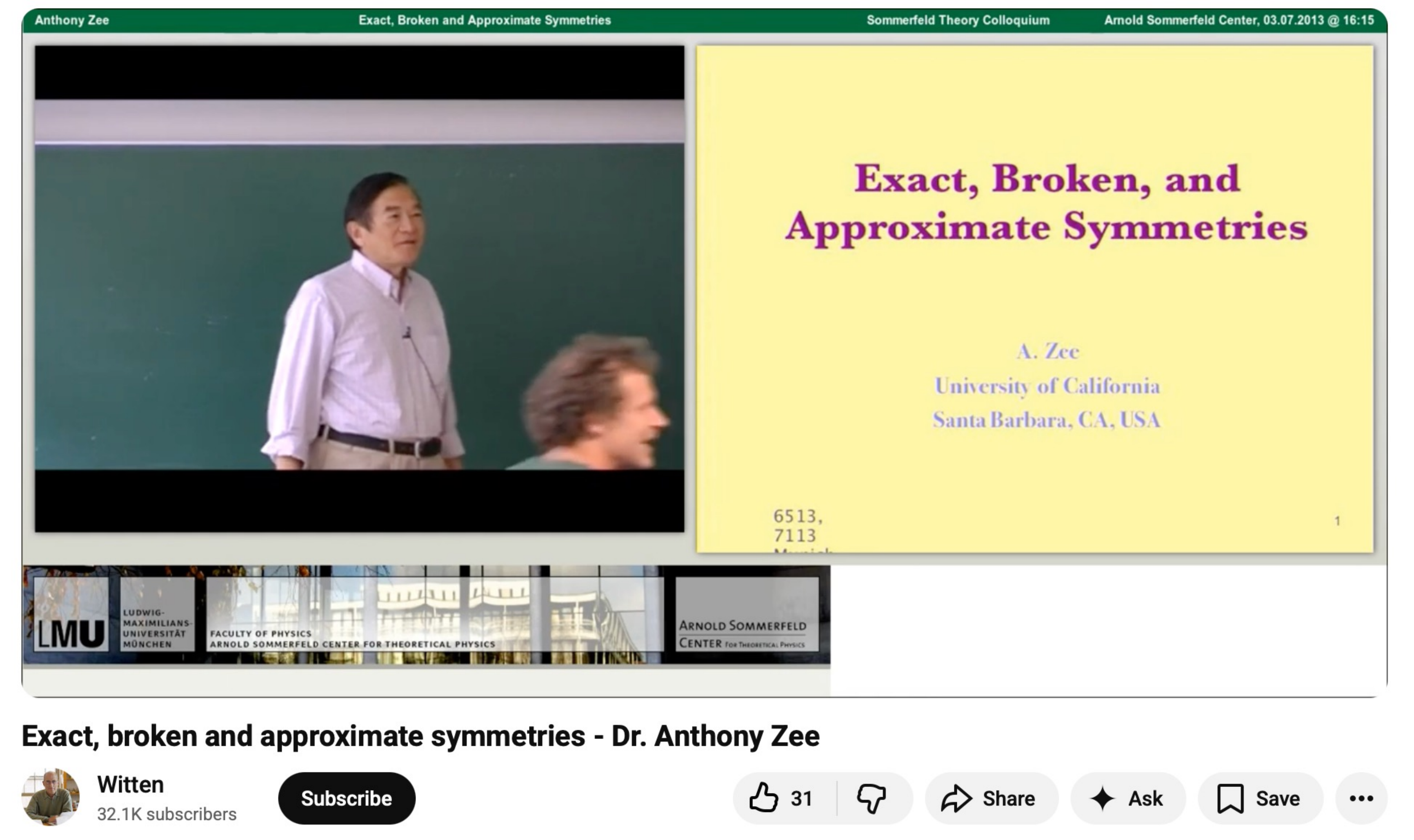


# Ten Foundational Ideas of Theoretical Physics

## Anthony Zee

Jul 16, A2



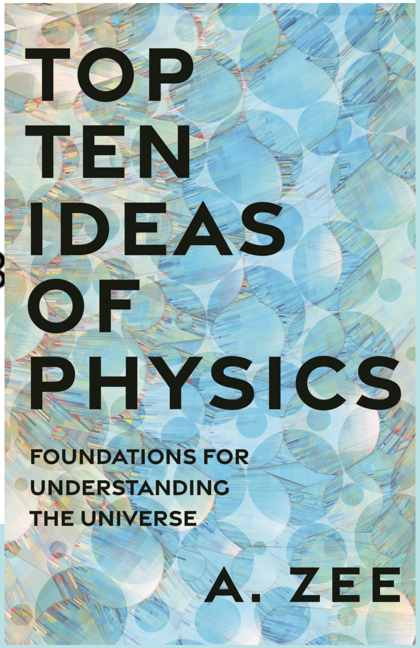
# Exact, Broken, and Approximate Symmetries

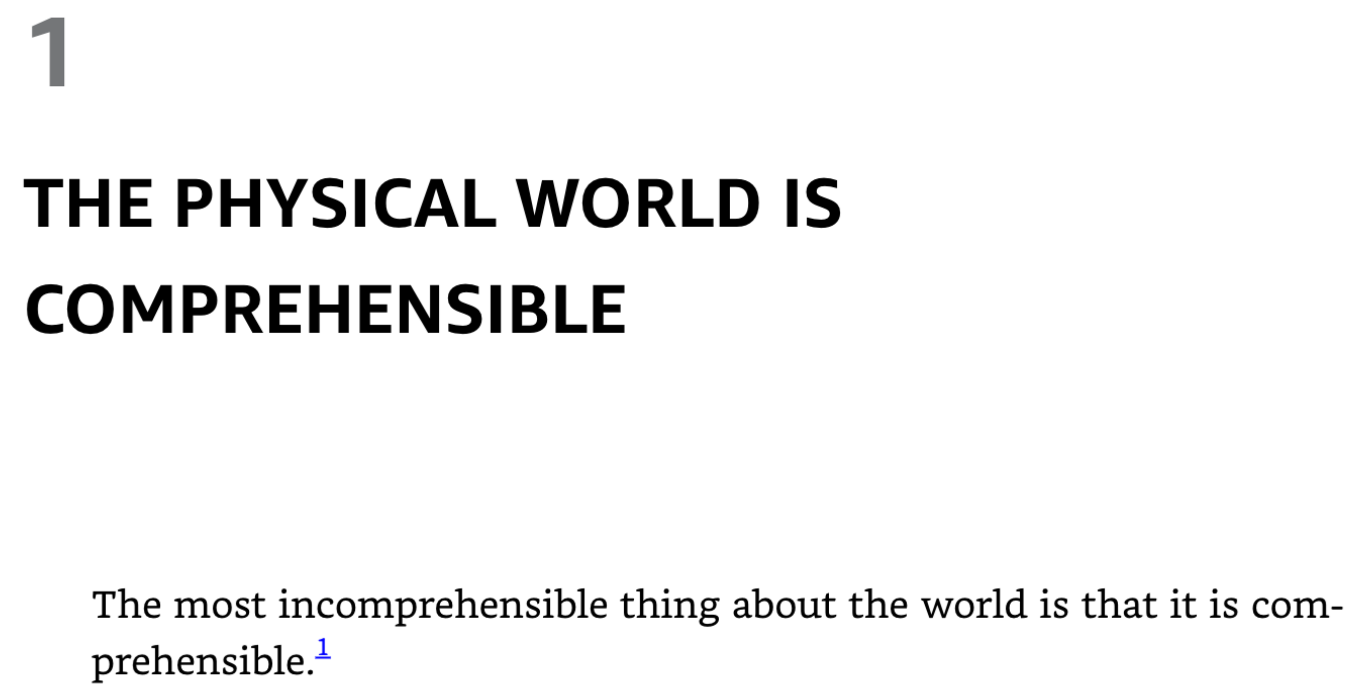
A. Zee
University of California
Santa Barbara, CA, USA

6513,
7113

1

**Exact, broken and approximate symmetries - Dr. Anthony Zee**


TOP TEN IDEAS OF PHYSICS
FOUNDATIONS FOR UNDERSTANDING THE UNIVERSE
A. ZEE



1

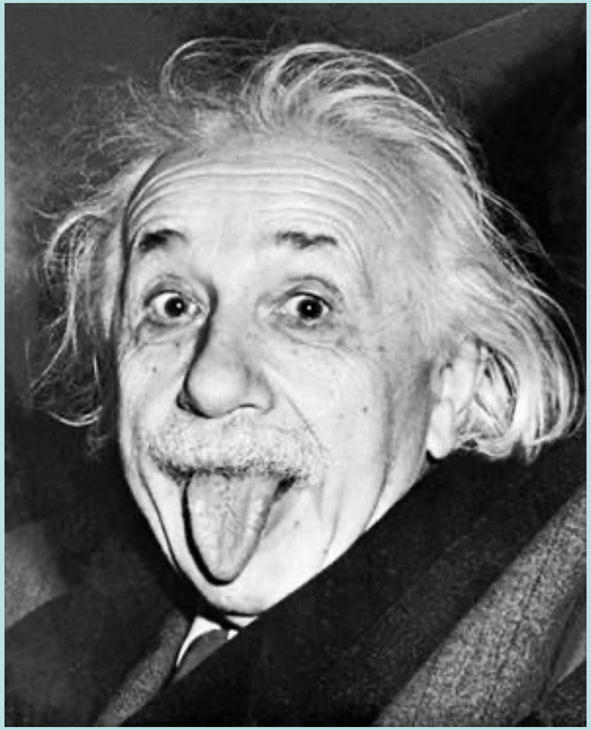
# THE PHYSICAL WORLD IS COMPREHENSIBLE

> The most incomprehensible thing about the world is that it is comprehensible.[1]

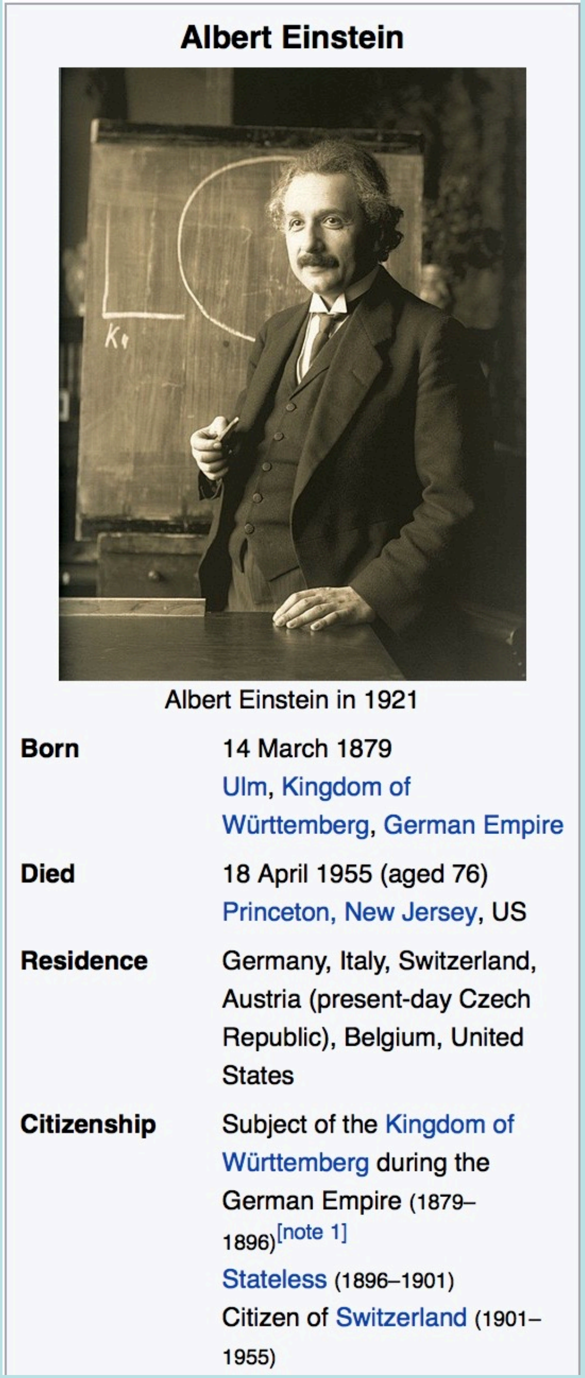
The most incomprehensible thing about the universe is that it is comprehensible.

這個宇宙，最難以理解的，就是它竟然可以理解的。

A. Einstein



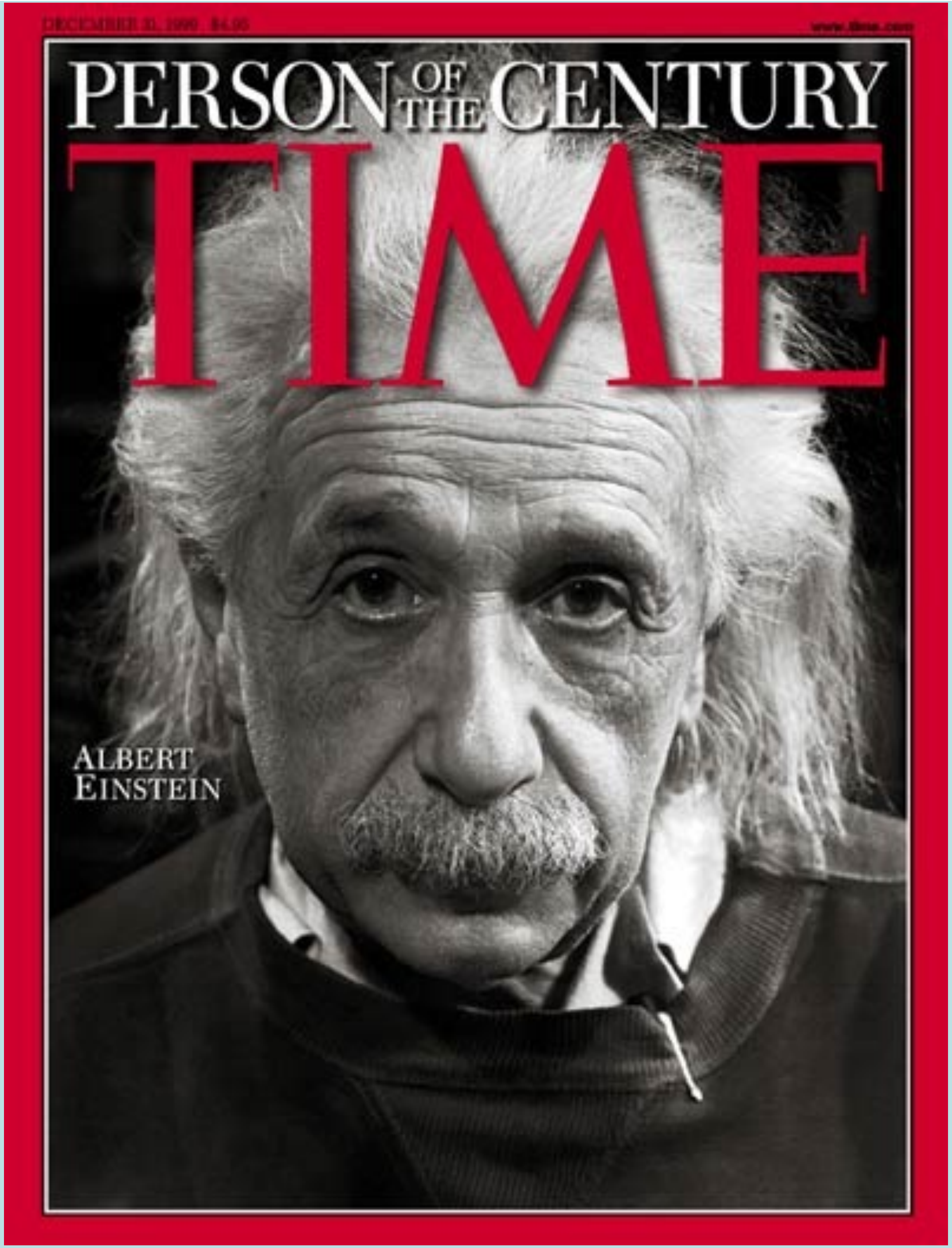
史上最佳代言人



**Albert Einstein**

Albert Einstein in 1921

| | |
|---|---|
| **Born** | 14 March 1879<br>Ulm, Kingdom of Württemberg, German Empire |
| **Died** | 18 April 1955 (aged 76)<br>Princeton, New Jersey, US |
| **Residence** | Germany, Italy, Switzerland, Austria (present-day Czech Republic), Belgium, United States |
| **Citizenship** | Subject of the Kingdom of Württemberg during the German Empire (1879–1896)[note 1]<br>Stateless (1896–1901)<br>Citizen of Switzerland (1901–1955) |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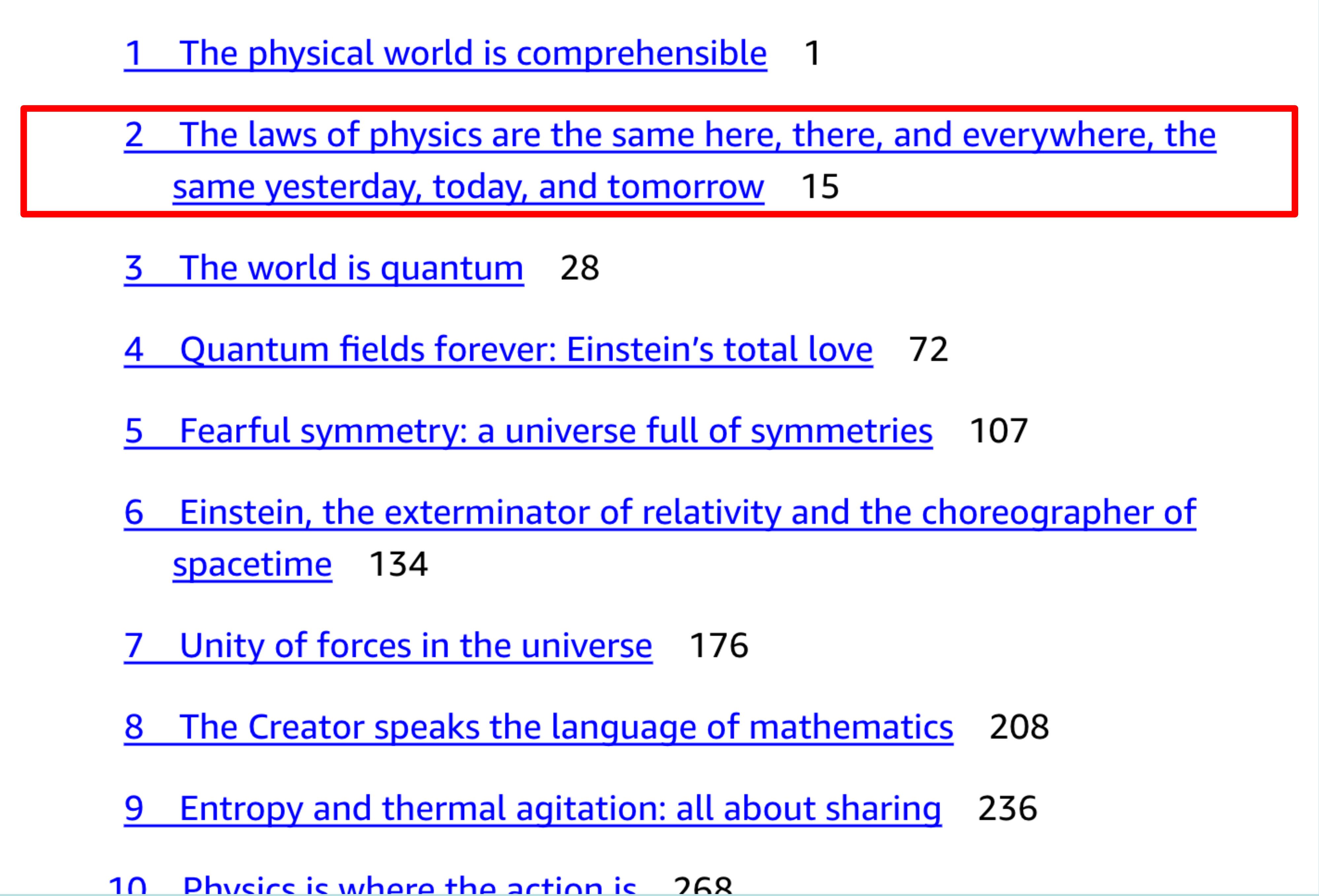
www.time.com
PERSON OF THE CENTURY
TIME
ALBERT
EINSTEIN

物理之光！



# 2

# THE LAWS OF PHYSICS ARE THE SAME HERE, THERE, AND EVERYWHERE, THE SAME YESTERDAY, TODAY, AND TOMORROW

## The laws of physics are the same in heaven and on earth

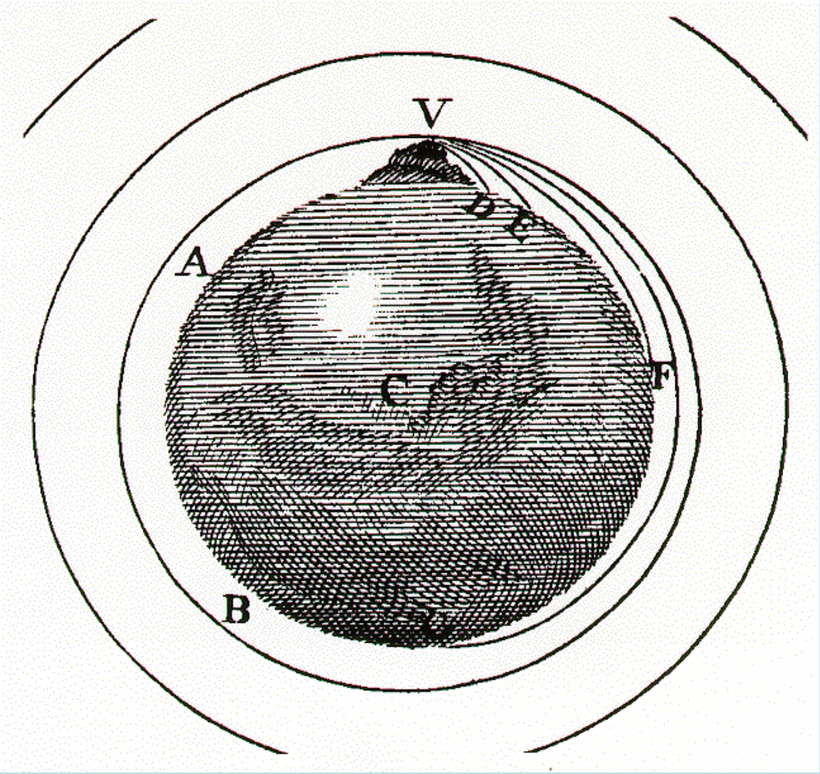
From day one, the entire human race had looked up at the ethereal moon floating there, all the while waxing and waning like the tides. Newton alone among the multitudes realized that the moon was falling.

Yes, falling, just like the apple!

On a moonlit night, the poet inside me trumps the physicist and I can barely believe that the same law applies to that celestial apparition as to a common rock. The full moon appears so still and so round, serenely looking down at us. You are not alone if you find it difficult to visualize the truth, that the moon is constantly falling, pulled in relentlessly by the earth in the physics nerd's version of a fatal attraction. The unceasing and inexorable pull of gravity!

For interminable ages, the savants yakked about celestial mechanics and terrestrial mechanics, but after young Isaac had his blinding insight under the apple tree, there was only one mechan-

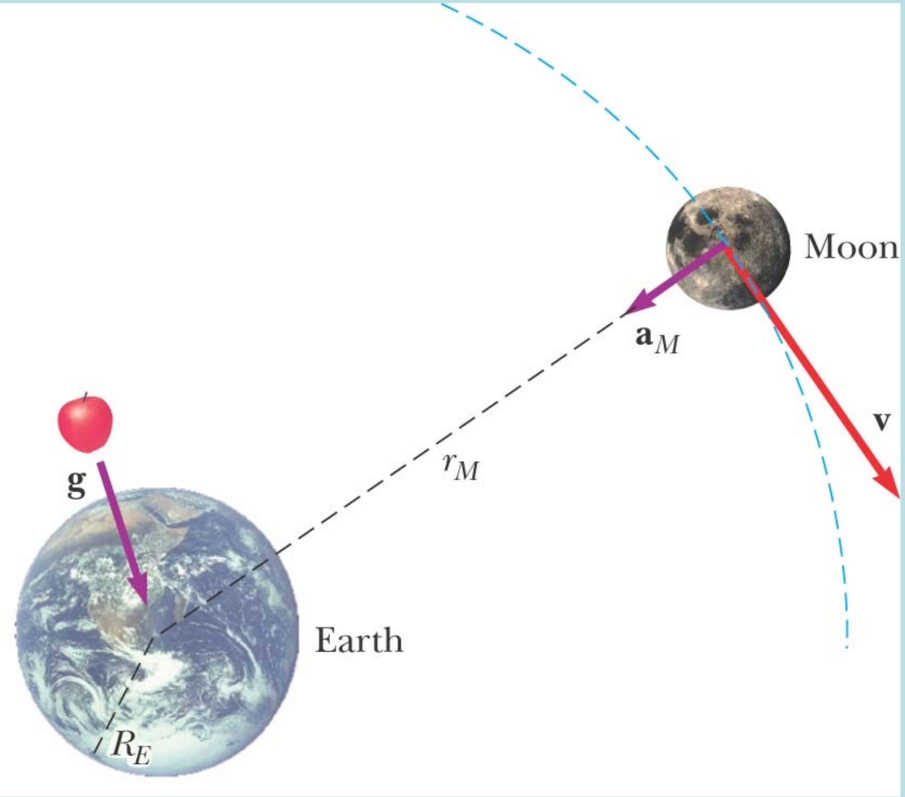
＝

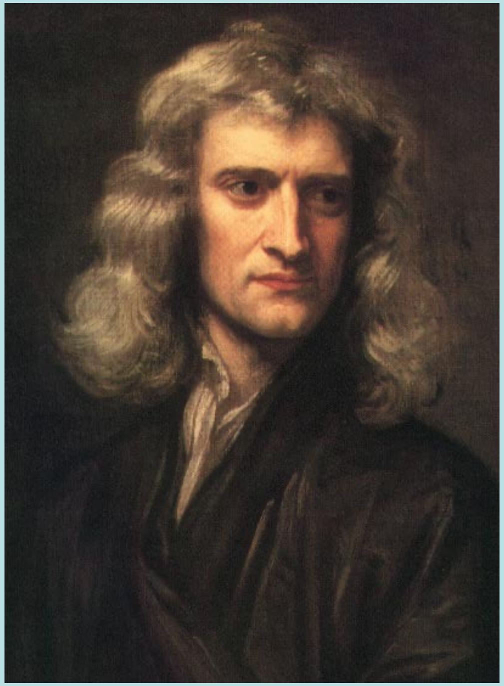


塵世的物體可以成為神聖的天體！

天體與地面物體本質上是平等的！

# 天體與蘋果服從同樣的物理定律

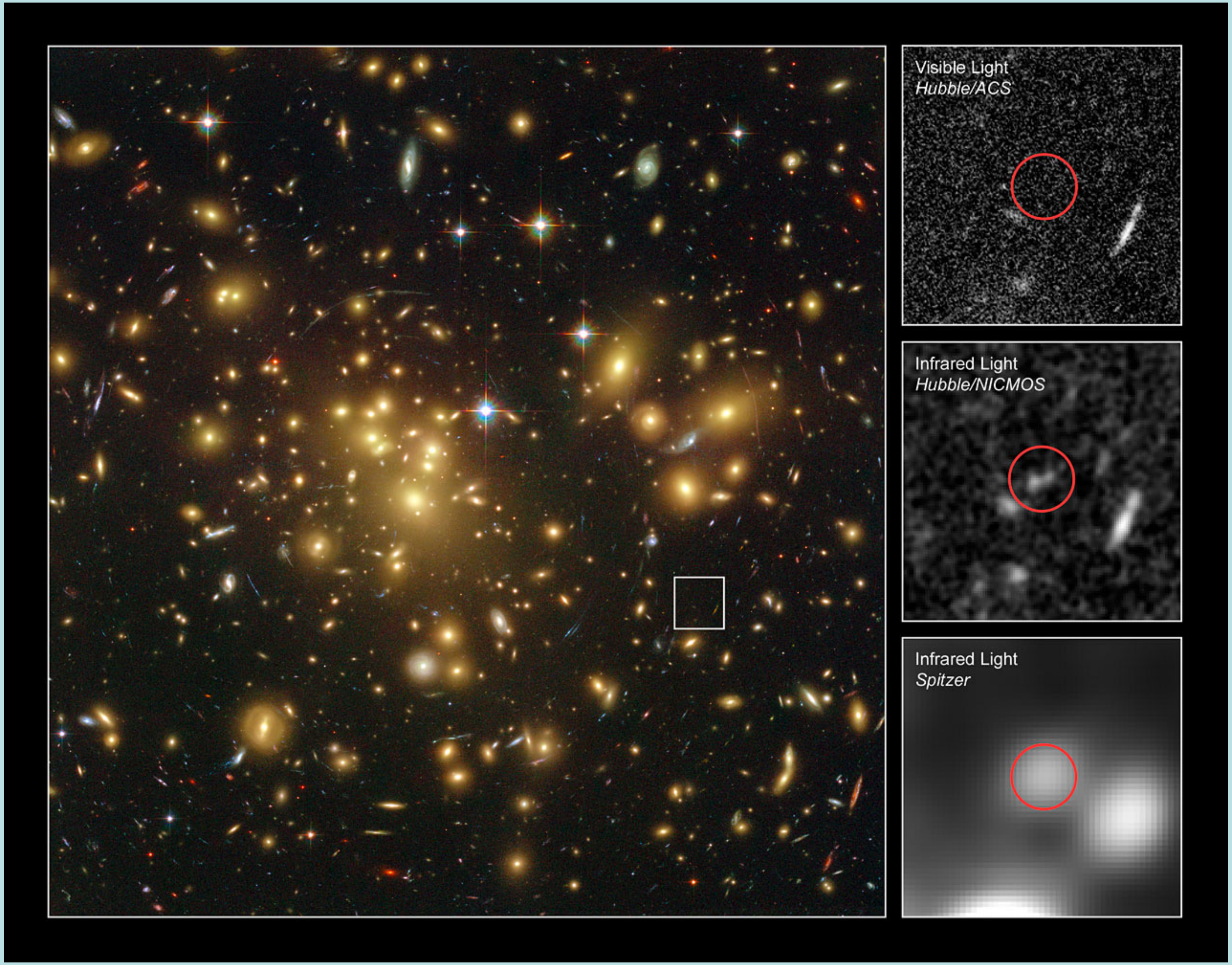




Isaac Newton

(1642-1727)

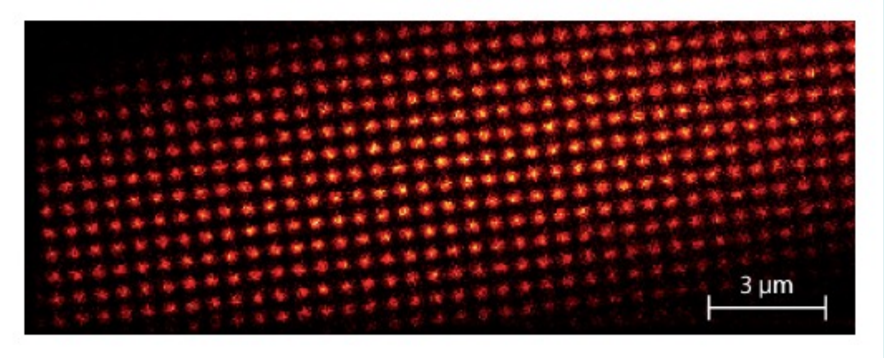
物理定律是普遍的 (universal)

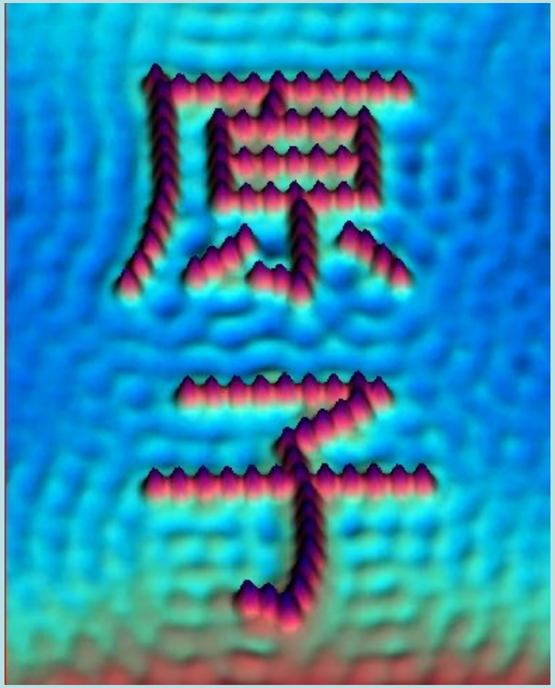


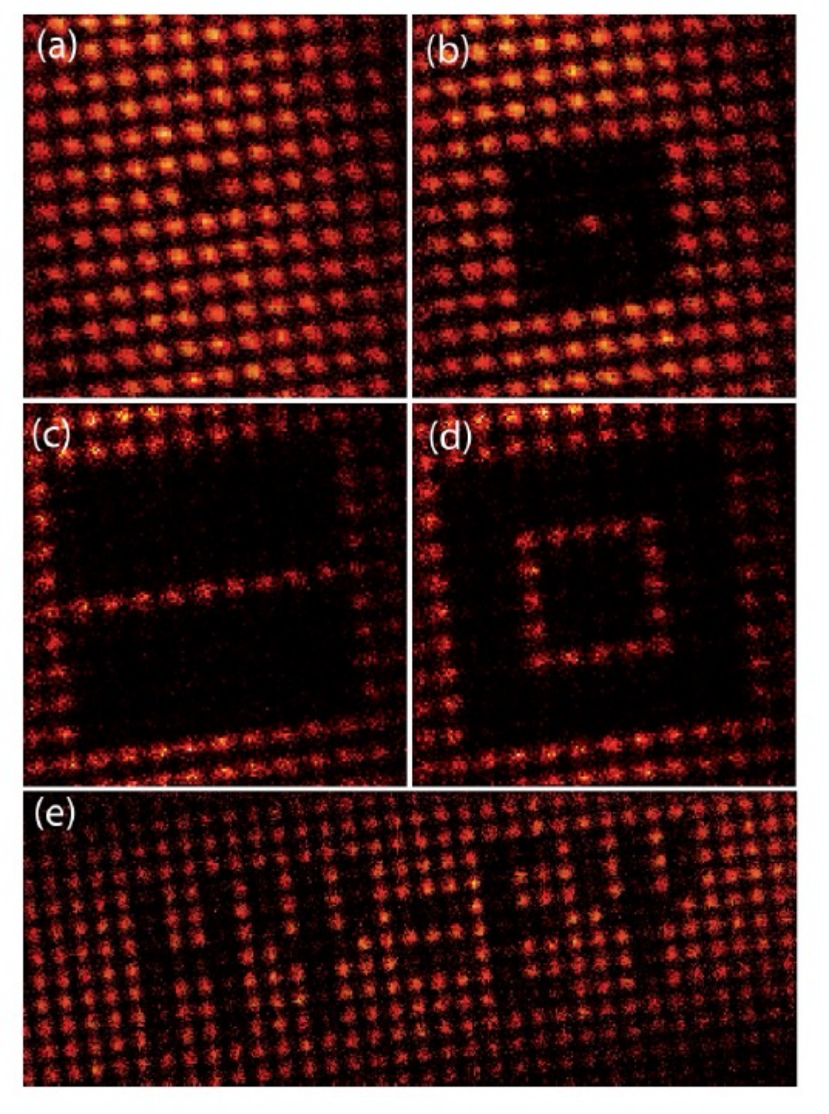
A1689-zD1銀河距離地球130億光年，所發出的光是在130億年前發出！

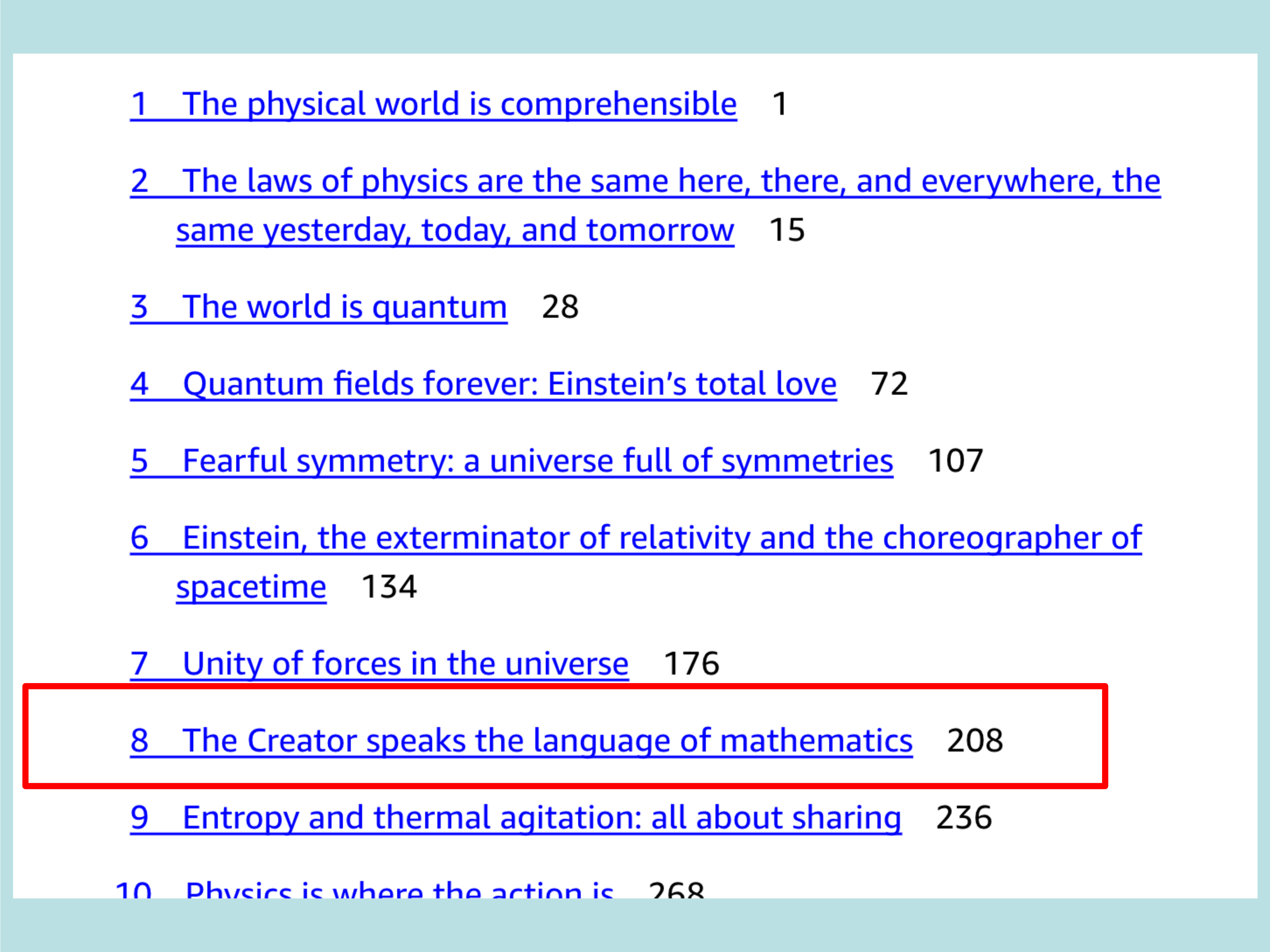
我們預期物理定律在那裏在那時依舊成立！

我們預期物理定律在極微小的範圍依舊成立！



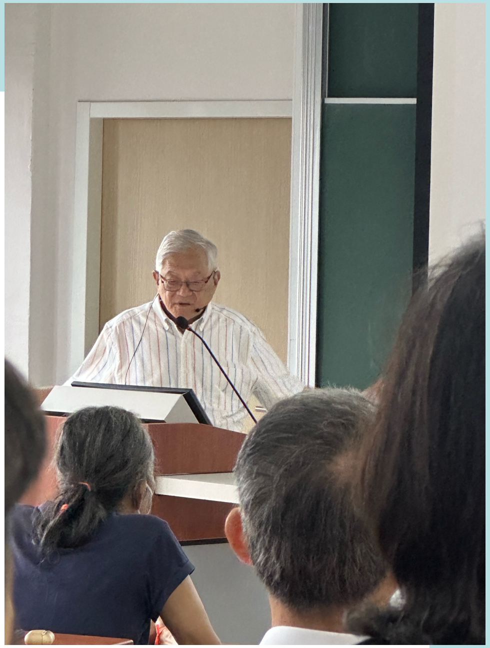






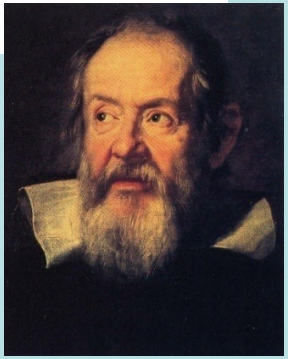
# 8

# THE CREATOR SPEAKS THE LANGUAGE OF MATHEMATICS



> [The universe] … that great book which ever lies before our eyes … but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written. This book is written in the mathematical language, … without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.[1]

要了解宇宙這本書就得了解上帝的語言，而上帝的語言是數學



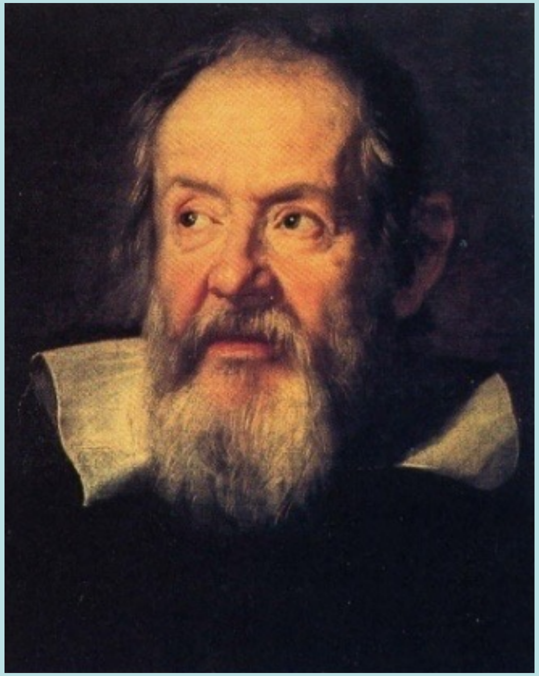
物理定律要如何尋找？

科學是描述自然現象的！

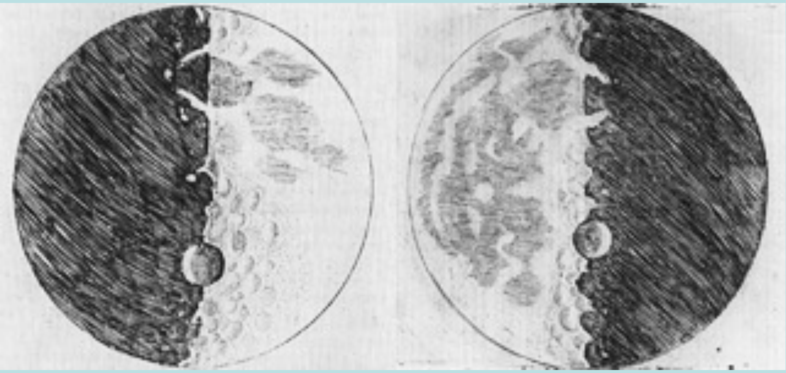
觀察現象是科學最基本的方法

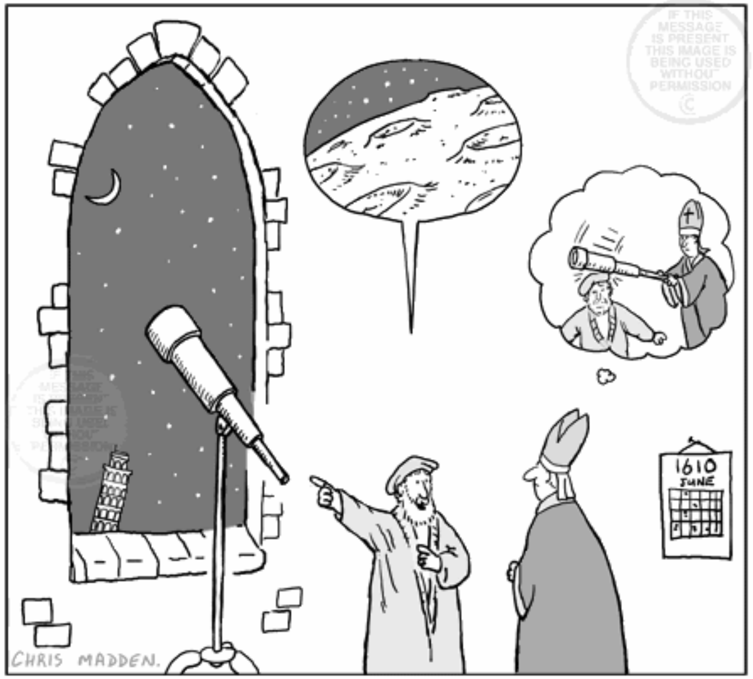
Galileo Galilei (1564-1642)

Galileo Galilej

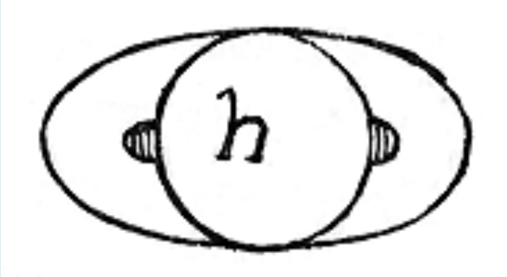


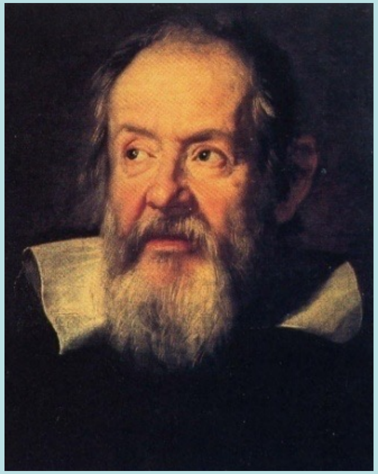
# Galileo Galilei (1564-1642)



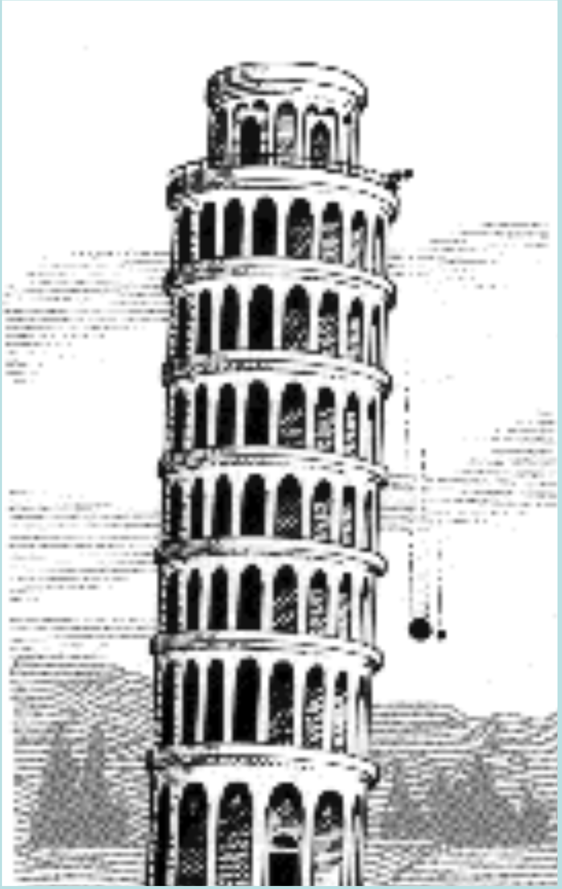


GALILEO DESCRIBES HIS DISCOVERIES TO THE CHURCH





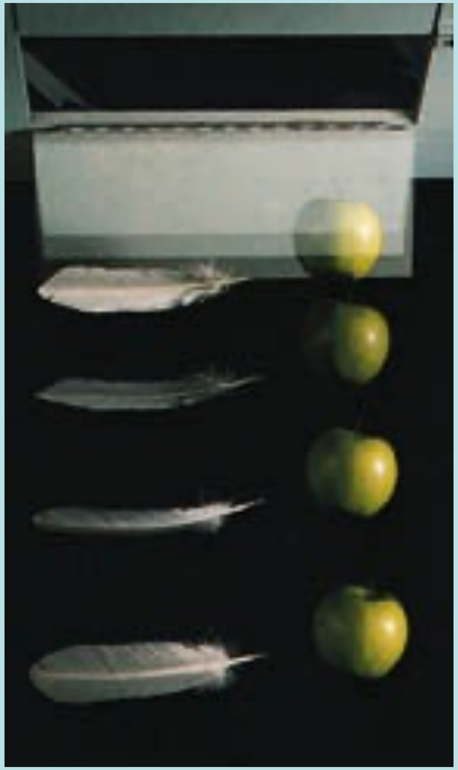
伽利略更進一步主張：所有的知識都要通過實驗來檢驗！



哲學通過辯論來檢驗！

自由落體運動

沒有介質阻力下，所有物體將以同樣的方法下落

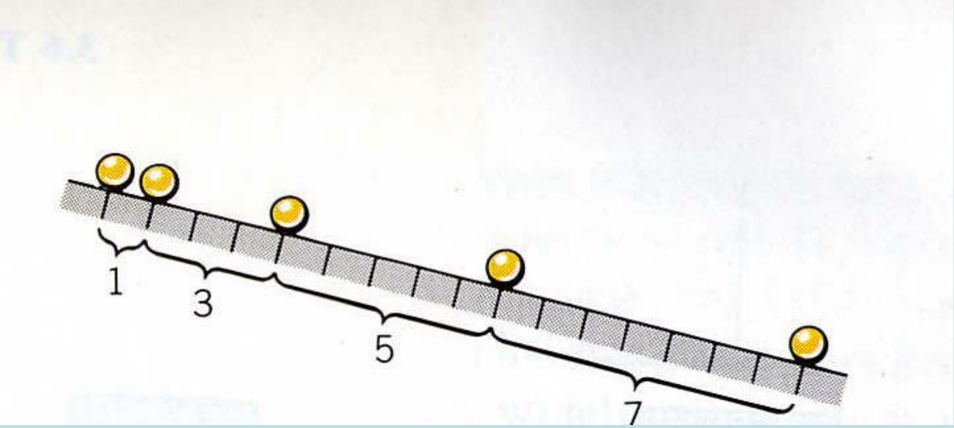


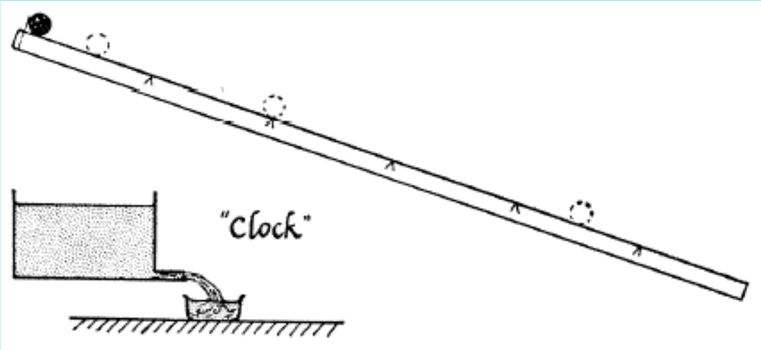
地表上物體的下落運動是普遍的，因此非常特別。

地表上物體的下落運動是普遍的，因此非常特別。

觀察這個特別的現象就很重要！如何觀察？測量！

測量並記錄物體下落的距離，這些是數！找出這些數與時間的關係。





物體的下落是等加速度運動！ 加速度是定值！

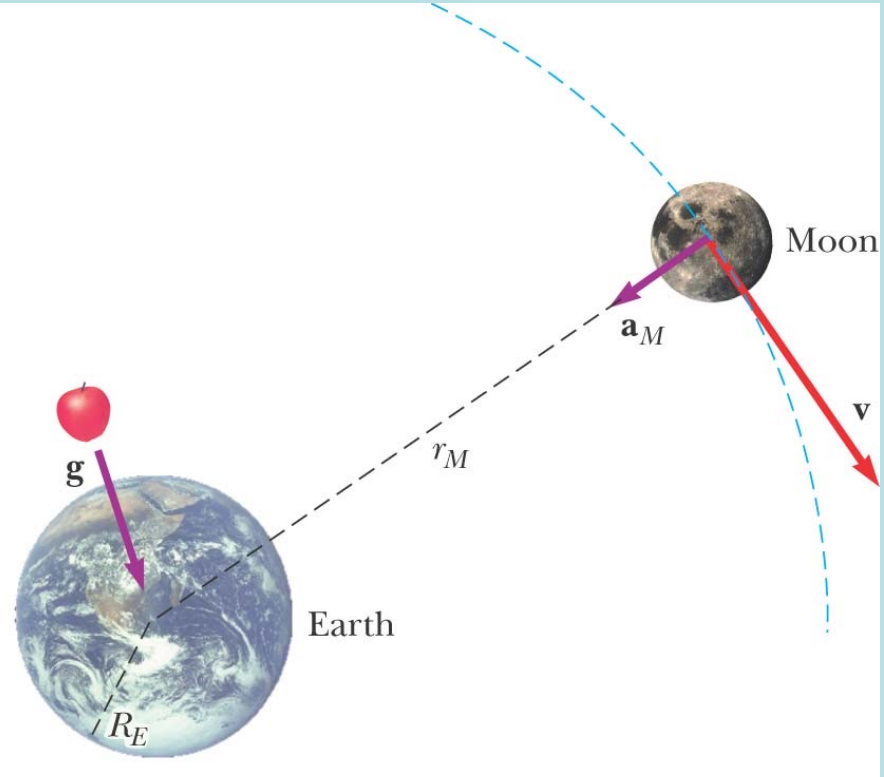
這個非常特別的運動，它的加速度是定值！

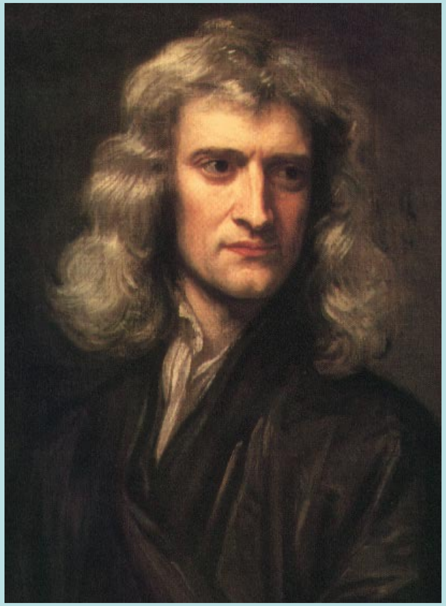
對物體運動的研究應該由速度轉移到加速度！

牛頓問：月球繞地運動的加速度是多少？

圓周運動雖然速率不變，但方向改變

因此是有加速度的！





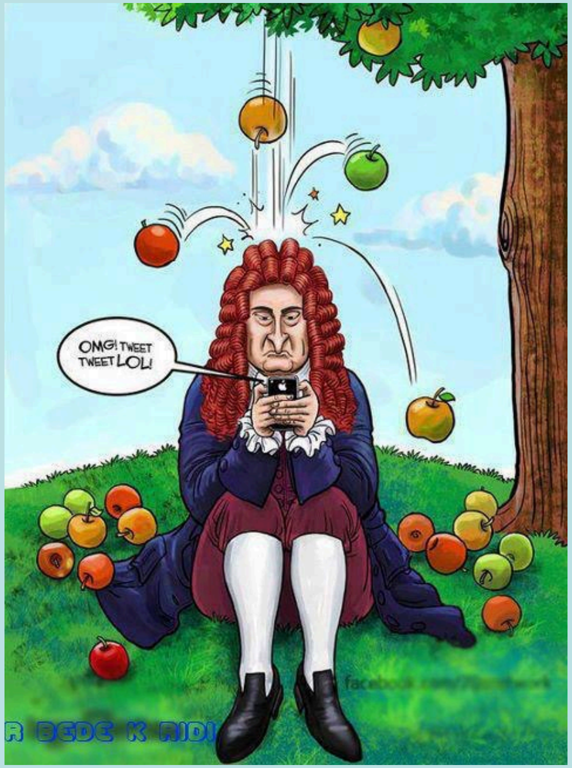
月球的運動也是等加速度運動！

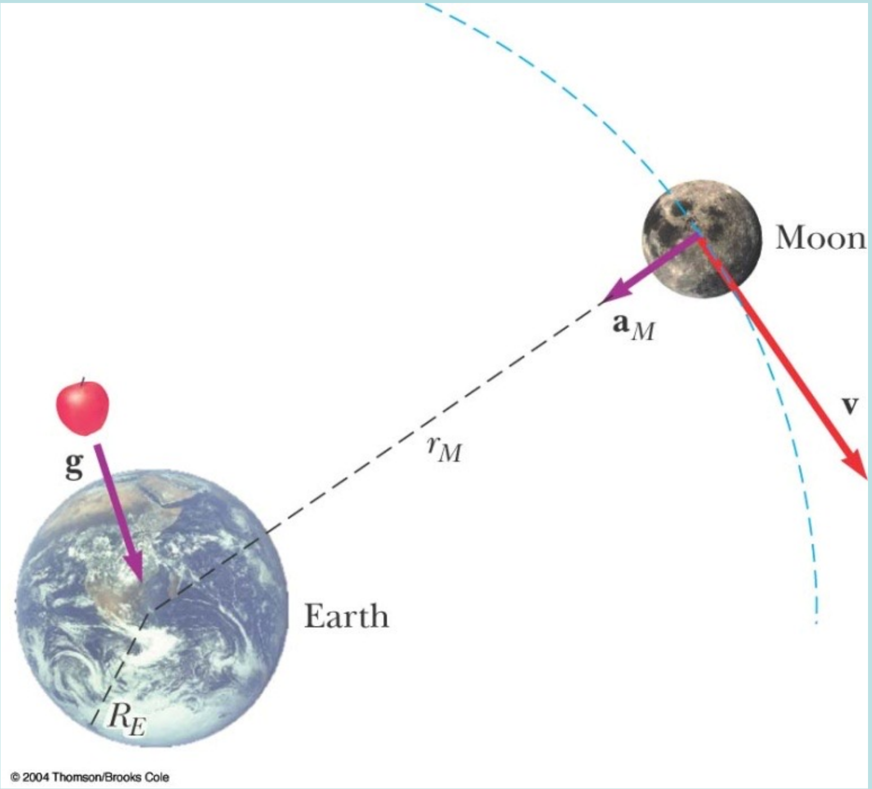
加速度的方向也是指向地球！

Isaac Newton

(1642-1727)

Is. Newton

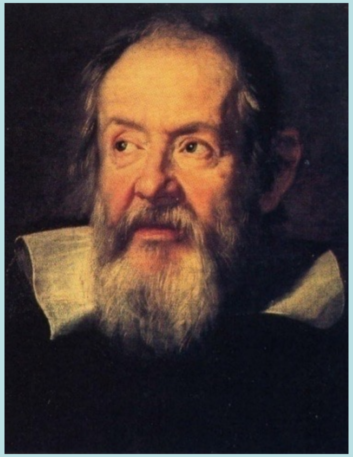




天上的月球與地上的物體，運動速度（特別是方向）雖然完全不同，但運動的加速度卻都指向地心！

如果以加速度來研究，月球與蘋果的運動在本質上是一樣的。

以上都是數學！

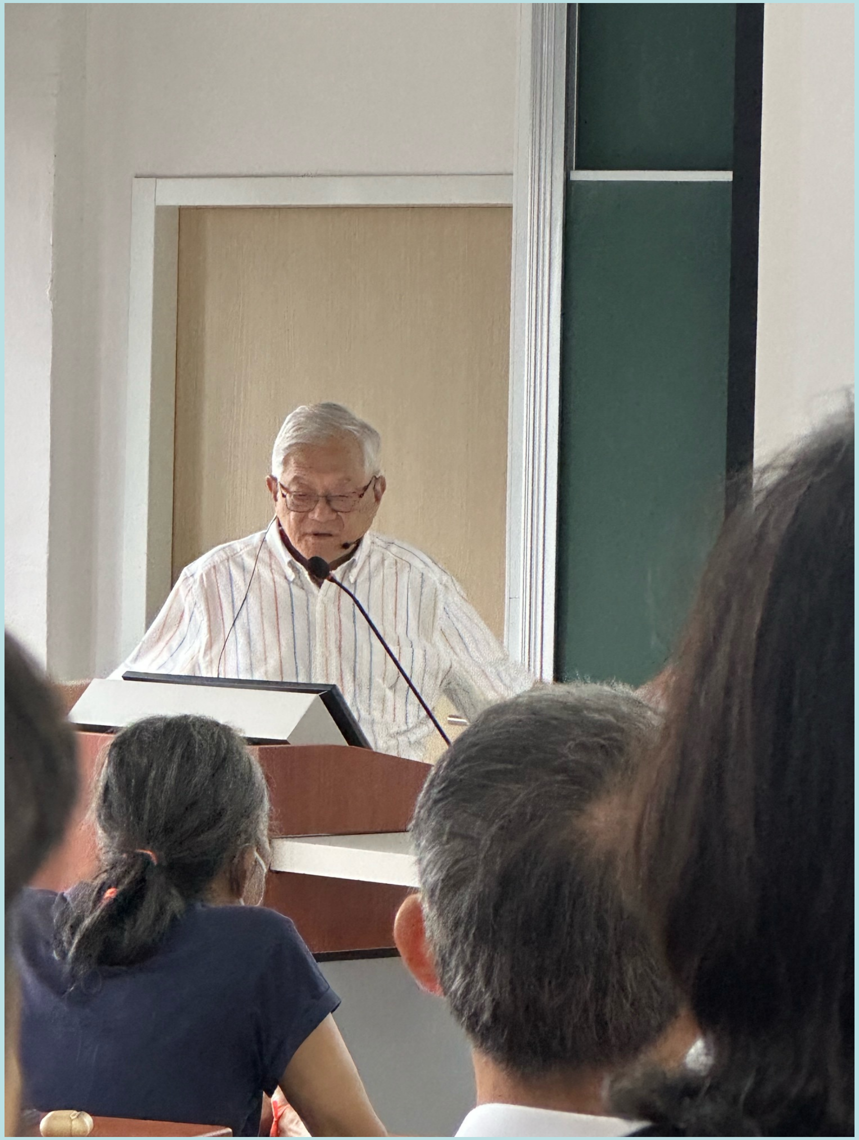


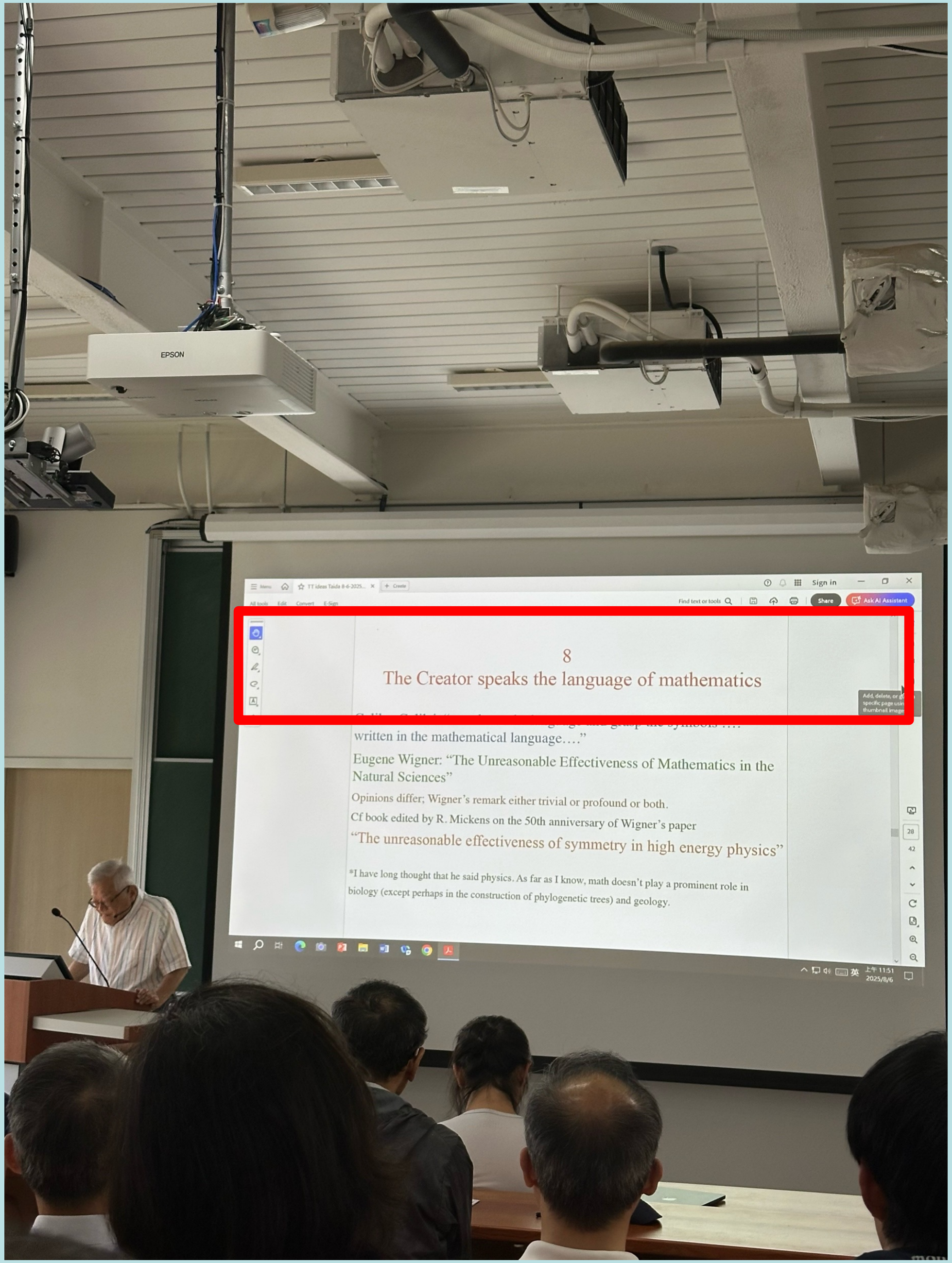
一門廣博精深的新科學已經誕生，我的工作只是一個開端，其他更聰明的心靈將可以利用其中的方法與手段，來探索新科學以致最遙遠的角落

物理定律是以數學描述的，因此是抽象的（mathematical and abstract）

要了解宇宙就得了解上帝的語言，而上帝的語言是數學

The Creator speaks the language of mathematics.





8

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written in the mathematical language….”

Eugene Wigner: “The Unreasonable Effectiveness of Mathematics in the Natural Sciences”

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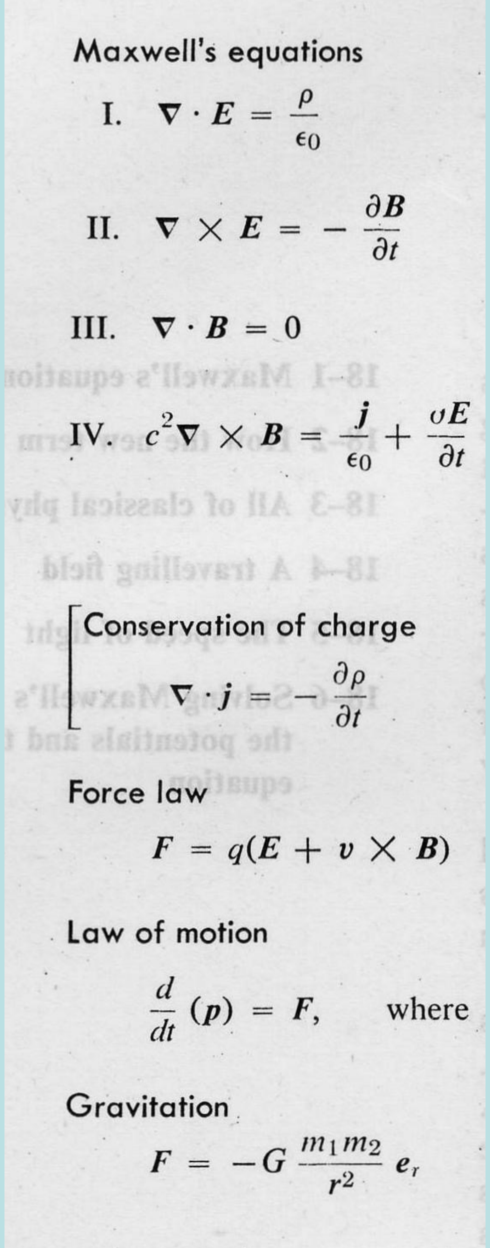
Cf book edited by R. Mickens on the 50th anniversary of Wigner’s paper

“The unreasonable effectiveness of symmetry in high energy physics”

*I have long thought that he said physics. As far as I know, math doesn’t play a prominent role in biology (except perhaps in the construction of phylogenetic trees) and geology.

# 古典物理定律全部由數學寫成



Maxwell's equations

I. $\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$

II. $\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$

III. $\nabla \cdot \boldsymbol{B} = 0$

IV. $c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$

Conservation of charge

$$\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t}$$

Force law

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Law of motion

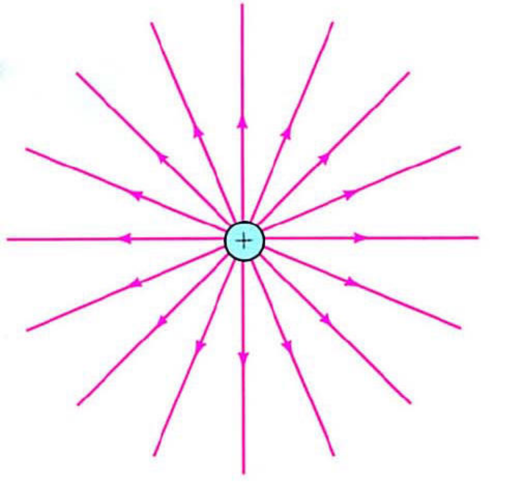
$$\frac{d}{dt}(\boldsymbol{p}) = \boldsymbol{F}, \qquad \text{where}$$

Gravitation

$$\boldsymbol{F} = -G\frac{m_1 m_2}{r^2}\boldsymbol{e}_r$$

場$\vec{E}$就是一個非常抽象的概念！

物理討論的對象就是抽象的、數學的場！



但別忘了空間中空無一物。

場$\vec{E}$不是個東西，是個概念！

數學與物理是密不可分的！

如果不是物理重實驗，以數學概念為對象的物理應該是數學的一支。

數學物理就是介紹物理用的到的數學方法！

偏微分方程式 Partial Differential Equation

線性代數 Linear Algebra

複變函數 Functions of complex variables

我們將先從古典普通物理中已經介紹過、需要較進階數學方法的問題開始暖身：

We will start with classical physics.

Ordinary Differential Equation 常微分方程式

1. Decay Equation, First order Linear Ordinary Differential Equation.
2. SHO, damped SHO and Forced Oscillation equation. Second order Linear ODE.

3. Coupled SHO. Systems of second order ODE's.
4. Vectors, Matrices, linear Equation and Eigenvector Eigenvalue problem.

第二周　First order ODE, Decay Equation

第三周　Integrating Factor  Second order linear ODE, SHM

第四周　Complex Number Solution of Linear ODE,

第五周　Inhomogeneous Solutions, Coupled SHM, Systems of ODE’s.

第六周　Matrix and Eigenvalue

第十周　Positive definite matrices　System of ODE’s, Coupled oscillation for large

第十一周 Continuous limit and Wave equation

第十二周 Wave Solution, Vector Calculus

第十三周 Vector Calculus

第十四周 Electromagnetic Wave Equation, Wave Equation

第十五周 Seperation of Variables

第十六周 期末考

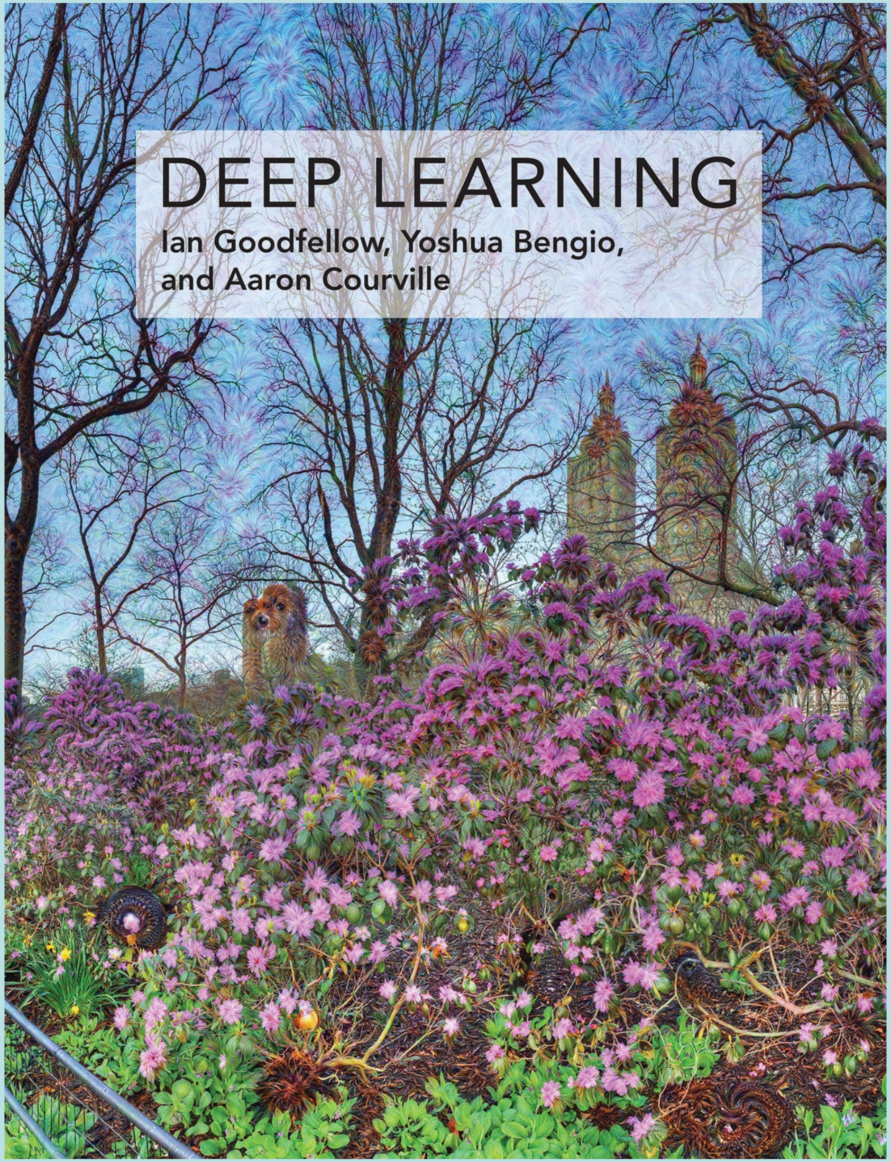
## Partial Differential Equation.

1. Continuous limit of Systems of ODE and Wave equation
2. 3D Vector Calculus: Gradient, Divergence, Curl. Gauss’ Stokes Theorem.
3. Maxwell Equation by 3D vector calculus notations. Feynman Lectures V. 2 Ch. 1,2,3.
4. Electromagnetic Wave Equation. Partial Differential Equation.
5. Separation of variables. Fourier Series and Eigenvalue problems, similar with matrix.

Eigenvalue problems 這個字重複出現！

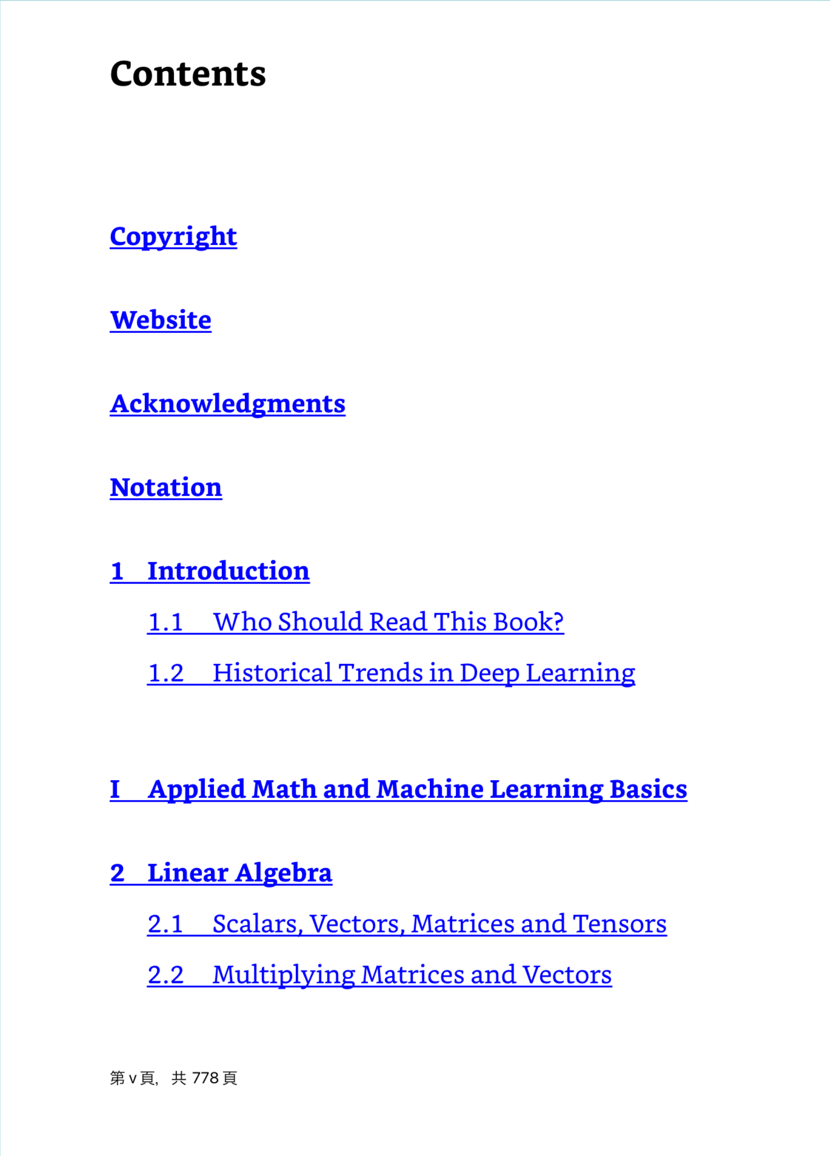
第二個數學主題：Linear Algebra 線性代數

# 線性代數竟然是AI深層學習的數學基礎



DEEP LEARNING

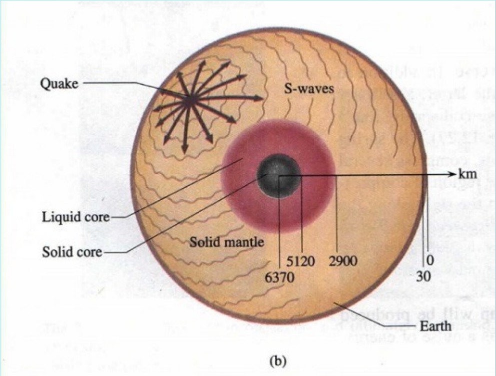
Ian Goodfellow, Yoshua Bengio,
and Aaron Courville

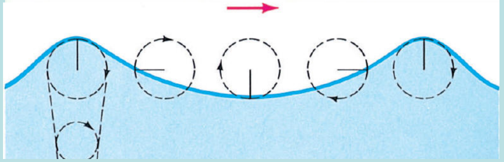


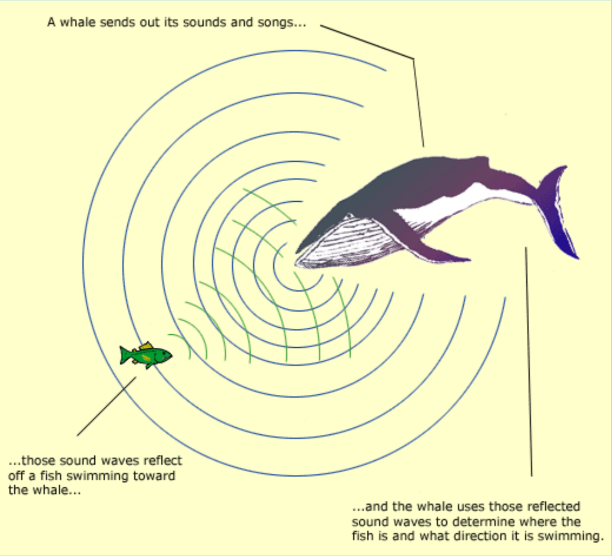
## Contents

# 波方程式 Wave Equation will be the organizing topics of the first semester.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

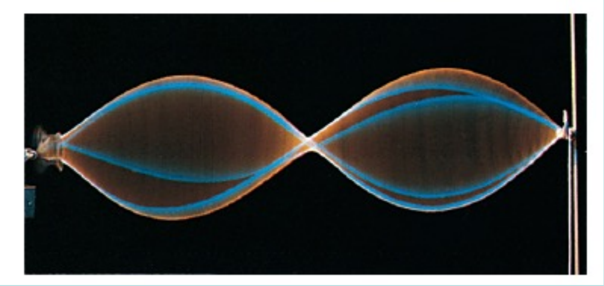





A whale sends out its sounds and songs...

...those sound waves reflect off a fish swimming toward the whale...

...and the whale uses those reflected sound waves to determine where the fish is and what direction it is swimming.



所有波動現象滿足的運動方程式。

This is the identical equation of motion of all wave phenomena.

Identical equation leads to identical solution. All wave systems have identical solutions.

## 不考慮邊界時的波方程式

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

解： $f(x - vt) + g(x + vt)$

在這些波函數的解之中，時間與空間並不獨立，而是連鎖在一起 $x'$，對空間的偏微分與時間的偏微分基本上都是函數 $f$ 對 $x'$的常微分因此對空間的兩次偏微分與對時間的兩次偏微分成正比。

波動的特徵皆來自此方程式：

波型以定速傳播

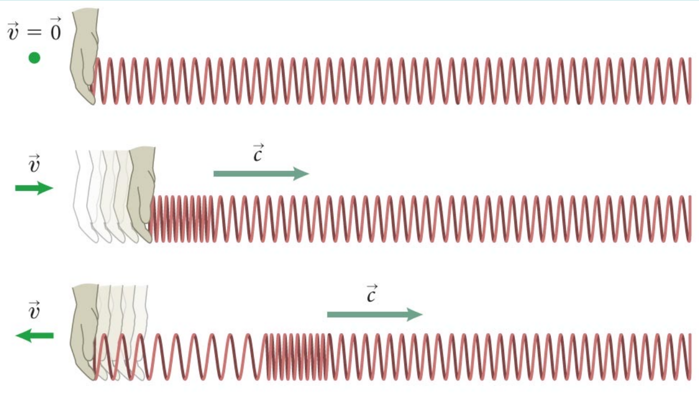
波型在傳播過程中不變形

疊加定律

以上結果適用於任何滿足波方程式的波動現象！

## 連續介質的波方程式Wave Equation

$$\frac{\partial^2 \phi}{\partial t^2} = v^2 \frac{\partial^2 \phi}{\partial x^2}$$



Initial Condition: 起始的弦位移$\phi(x,0)$，起始的弦垂直方向速度$\frac{\partial \phi}{\partial t}(x,0)$。

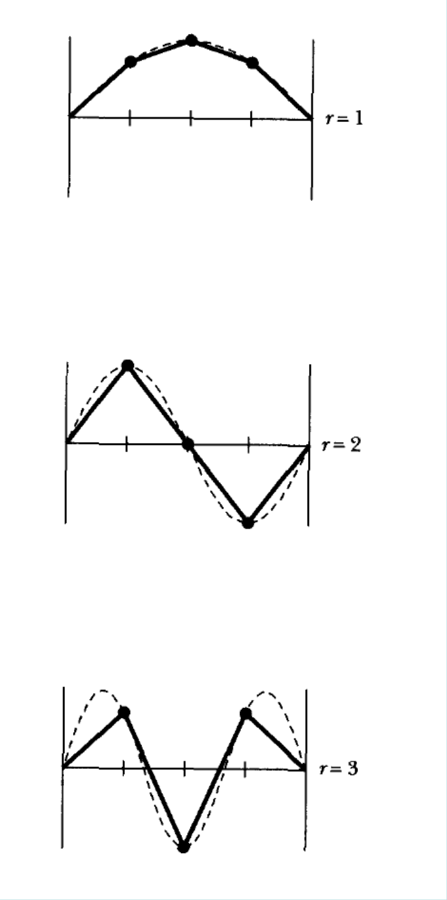
是不是要考慮邊界差別很大！

若不需考慮邊界、離開邊界很遠，介質中會有行進波的傳播 d'Alembert solution：

$$\phi(x,t) = f(x - vt) + g(x + vt)$$

若介質大小有限，連續介質就如大數目的彈簧組，有一系列模式振動。

彈簧組運動：一個本徵向量就對應一個可獨立振盪的模式。



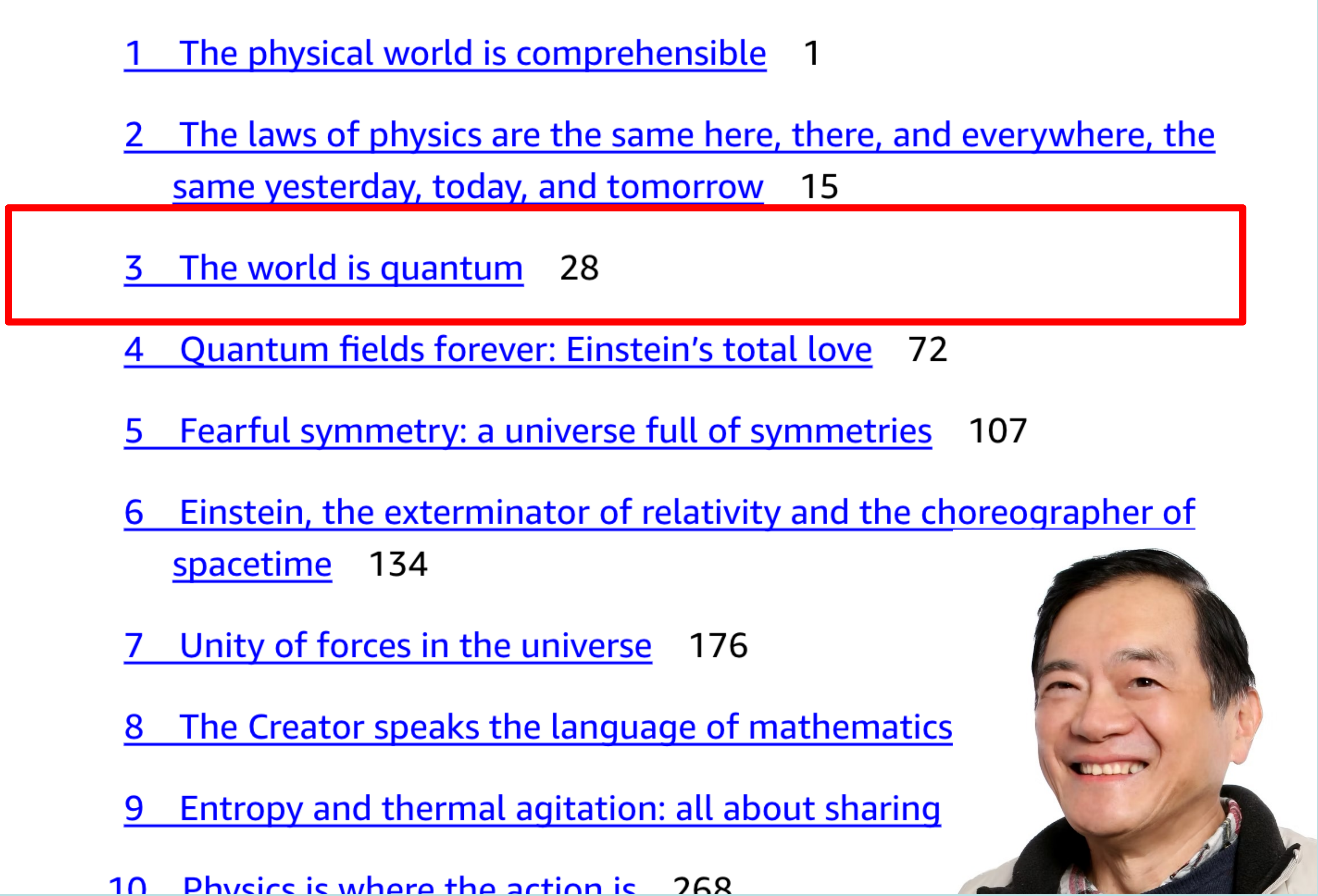
$$\boldsymbol{a}^{(1)}\sim\begin{pmatrix}\frac{1}{\sqrt{2}}\\1\\\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$\boldsymbol{a}^{(2)}\sim\begin{pmatrix}1\\0\\-1\end{pmatrix}$$

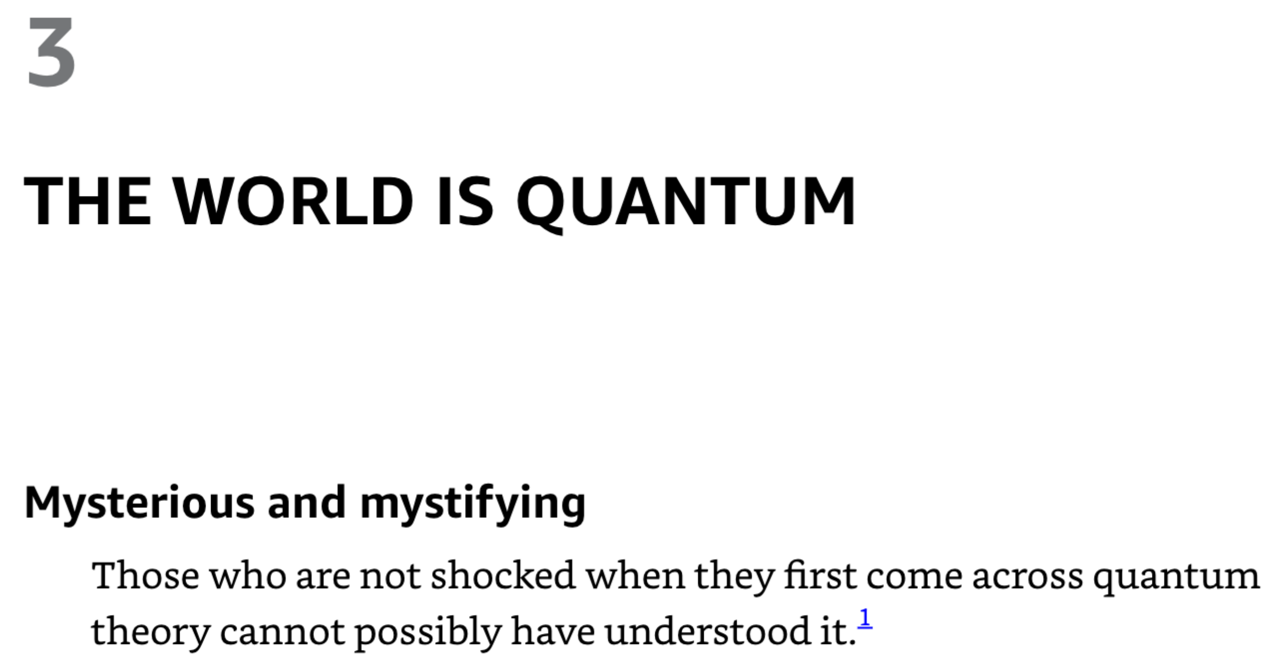
$$\boldsymbol{a}^{(3)}\sim\begin{pmatrix}\frac{1}{\sqrt{2}}\\-1\\\frac{1}{\sqrt{2}}\end{pmatrix}$$

The importance of wave equation goes beyond the above wave phenomena.

到此都是古典物理。



# 這是一個量子世界！



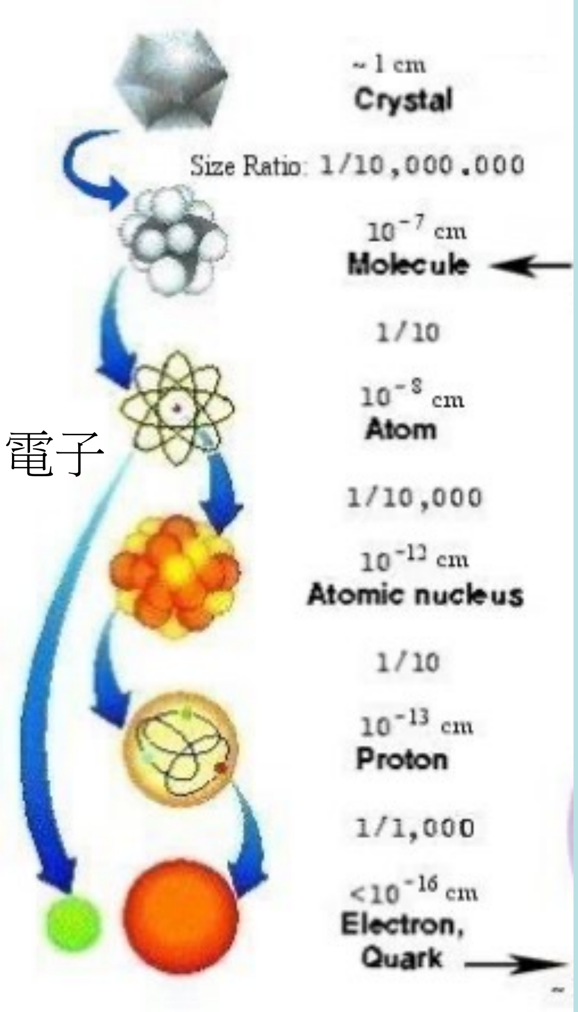
## 3

## THE WORLD IS QUANTUM

### Mysterious and mystifying

> Those who are not shocked when they first come across quantum theory cannot possibly have understood it.[1]

一直不斷分割下去，我們的確發現了一層層、越來越微小的物質組成成分。



電子

日常生活物質

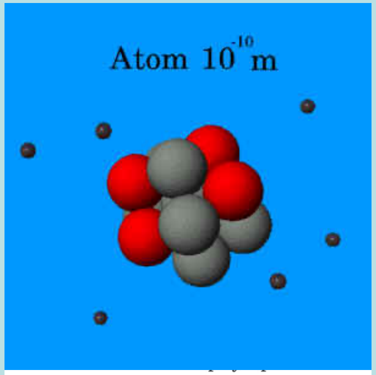
分子(千萬分之一公分)

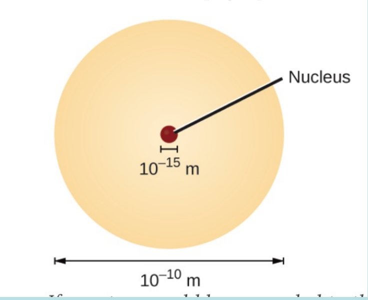
原子 (億分之一公分)

原子核

質子、中子

夸克





原子是由電子與原子核組成。

從原子、電子以下的微觀世界遵守的物理規則，稱為量子(Quantum)力學。

物質竟然是由極微小的、少數幾種、暫時無法分割之顆粒狀成分、所組成。

同一種粒子(如電子、某元素原子核)完全相同，有以上特徵的圖像就稱為量子。

原本物理學家期待微觀世界的物理規則與牛頓的物理沒有差別。
電子就是十分微小的、更像粒子的月亮、地球而已。
但二十世紀初的實驗竟然發現，量子力學與牛頓力學截然不同，十分詭異。

節目 知識好好玩

EP07｜電子的劈腿問題，它是一種波嗎？

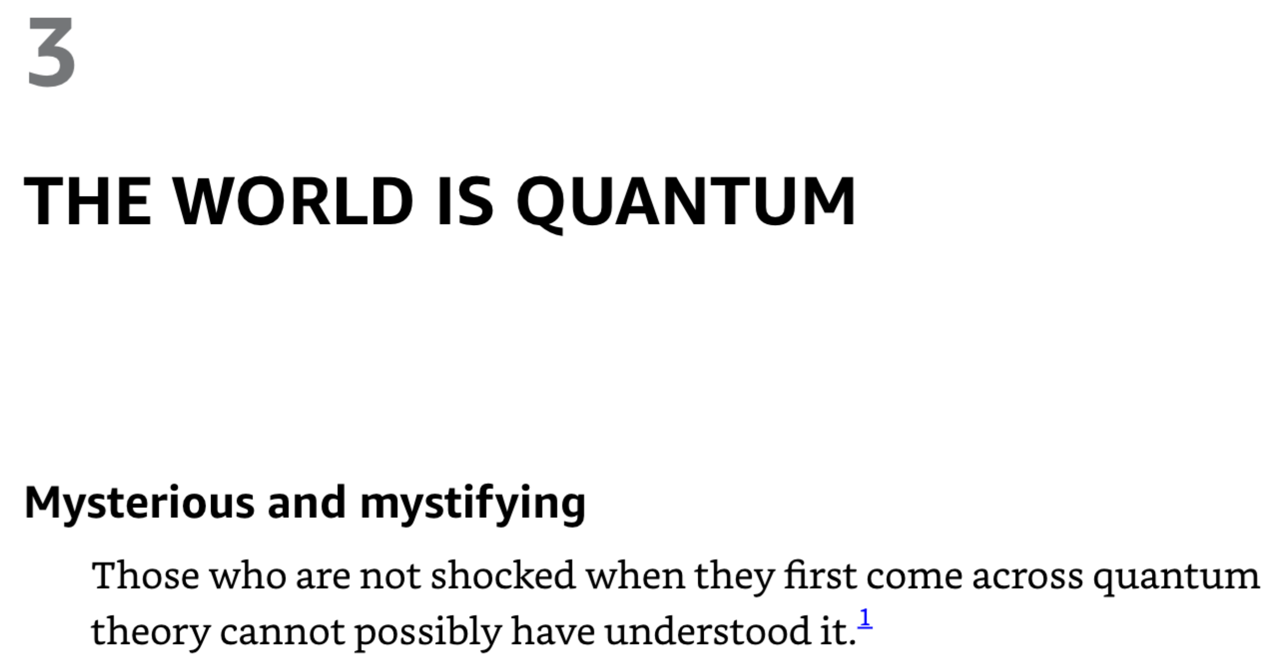
電子是粒子也是波！

主持人 | 張嘉泓

單曲長度 | 00:17:54　發布時間 | 2021-07-06

#張嘉泓 #物理好好玩 #波 #撞球 #粒子性 #薛丁格 #波動力學

Originally Physicists expect the physical law for the quantum world are no different from Newton’s mechanics. Electrons are no different from moon and earth. We now know they are totally different. Quantum world is bizarre.



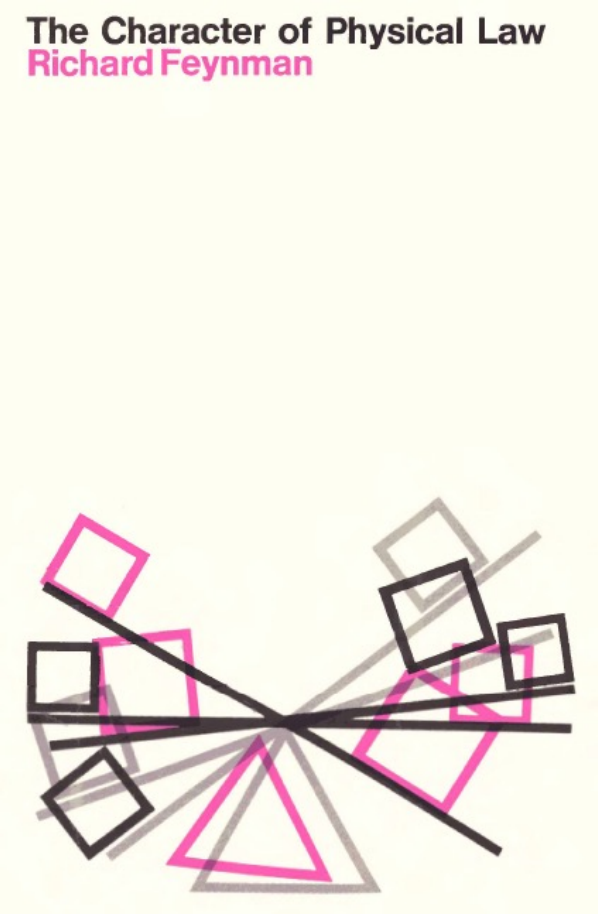
## 3

# THE WORLD IS QUANTUM

### Mysterious and mystifying

> Those who are not shocked when they first come across quantum theory cannot possibly have understood it.[1]

你第一次學量子物理如果沒有被嚇一跳，那你一定不了解它。

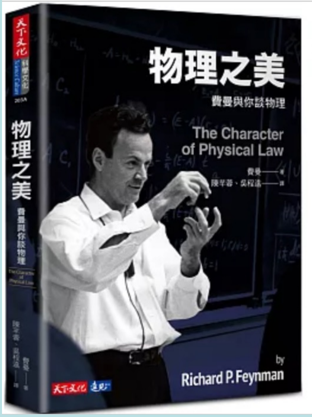


**I think I can safely say that nobody understands quantum mechanics. Feynman 1960’s.**

So do not take the lecture too seriously, feeling that you really have to understand what I am going to describe, but just relax and enjoy it.

**沒有人真正理解量子力學。 費曼 1960’s。**

所以對這演講別太認真，感覺非得了解我在說什麼。放輕鬆、就盡量享受吧！

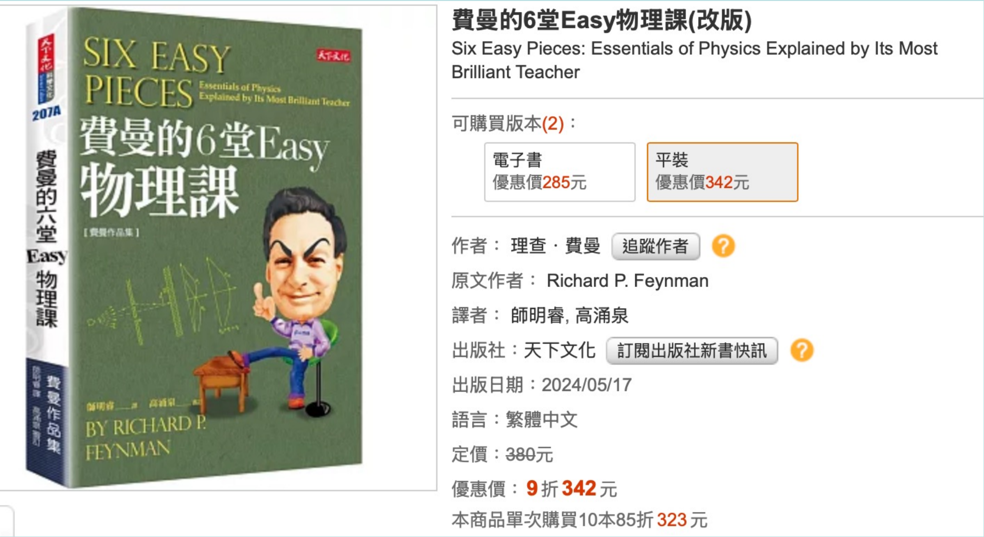


By Filipe Bastos Posted on 10/01/2020 Posted in Difficult Emotions, Emotional Intelligence, Self-knowledge

偉大的物理學家誠實的承認：

沒有人真正理解量子力學。

TAMIKO THIEL/WIKICOMMONS



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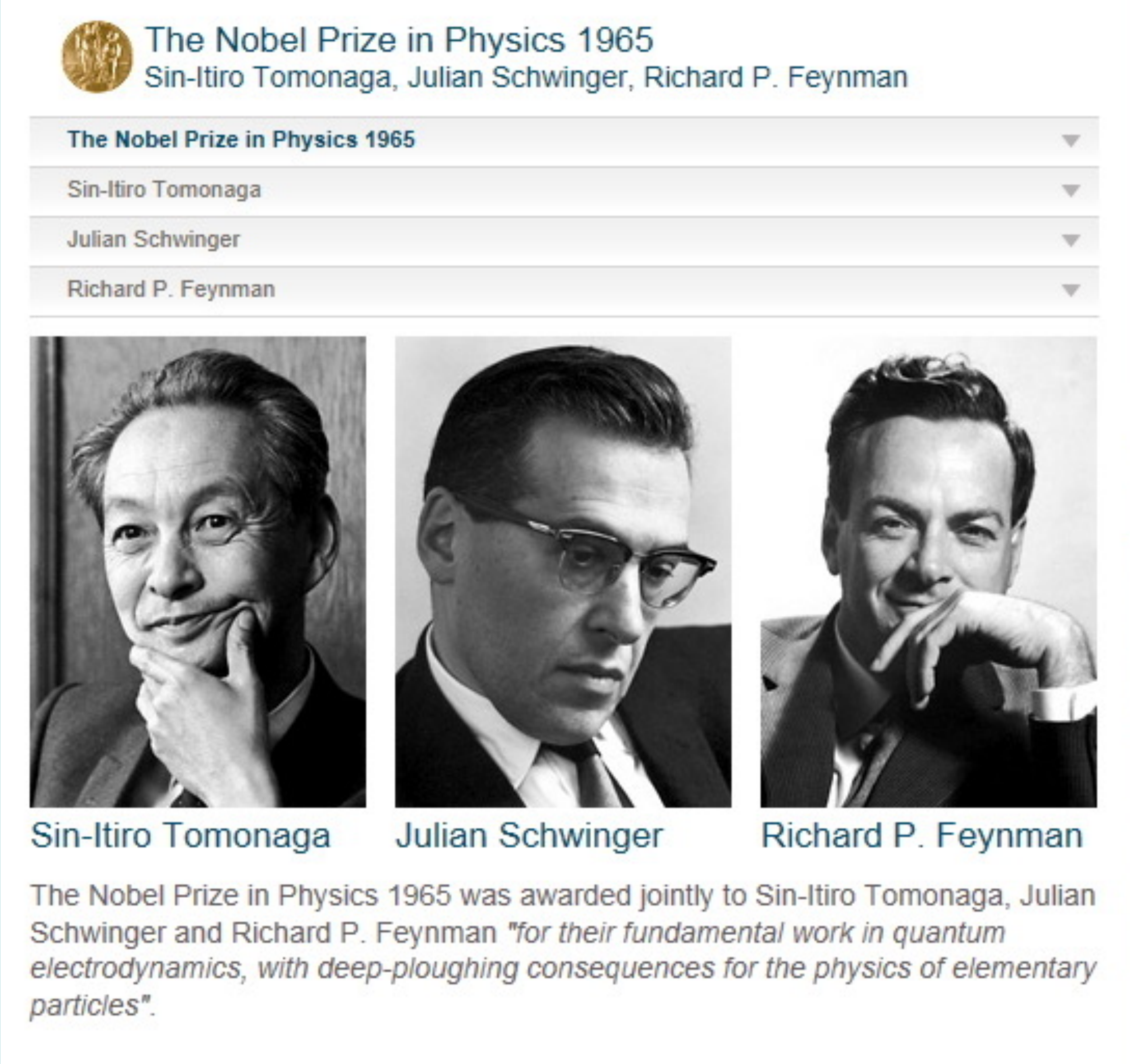
# 但量子力學極度正確！



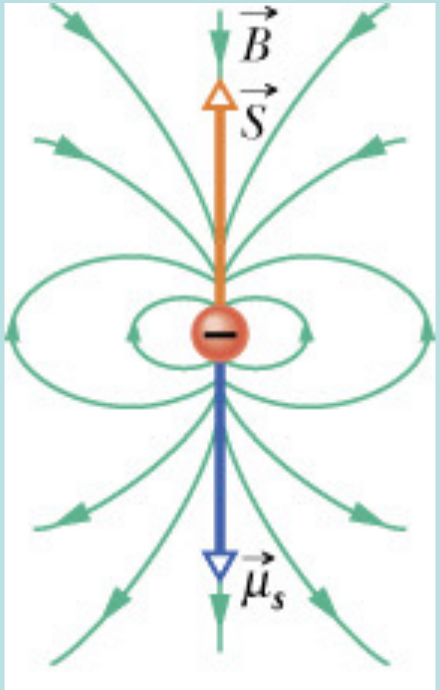
The Nobel Prize in Physics 1965
Sin-Itiro Tomonaga, Julian Schwinger, Richard P. Feynman

Sin-Itiro Tomonaga Julian Schwinger Richard P. Feynman

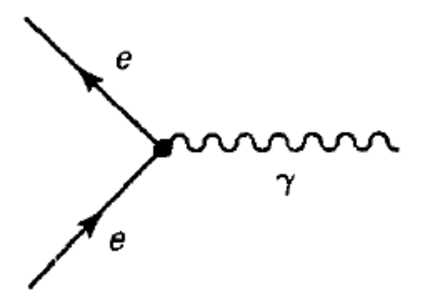
The Nobel Prize in Physics 1965 was awarded jointly to Sin-Itiro Tomonaga, Julian Schwinger and Richard P. Feynman *"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"*.

電子除了帶負電，本身也是一磁鐵。

量子電動力學對電子磁鐵的磁性強度的計算驚人地極度準確！



$$\vec{\mu}_s = -\frac{e}{m}\vec{s} = -g\frac{e}{2m}\vec{s}$$

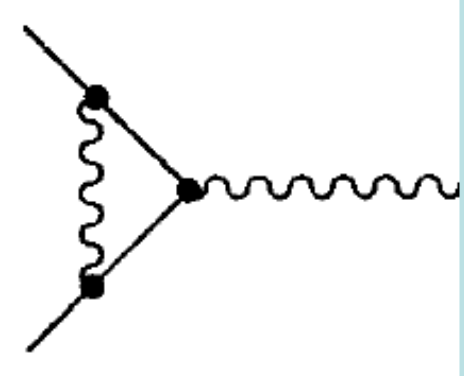


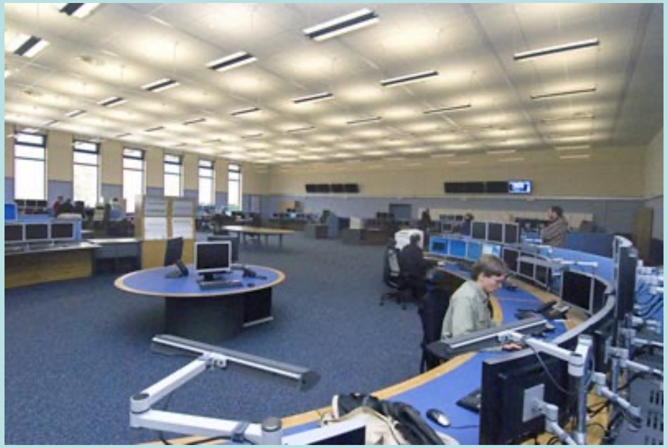
This first order diagram predict $g = 2$.

The following diagram gives the leading correction.

$$\frac{1}{2}g_{\text{theory}} = 1 + (1159652187.9 \pm 8.8) \times 10^{-12}$$

$$\frac{1}{2}g_{\text{experi}} = 1 + (1159652188.4 \pm 4.3) \times 10^{-12}$$





## FAST FACTS

**PROTON VELOCITY:**
99.9999991% of light speed

**PROTONS PER BUNCH:**
up to 100 billion

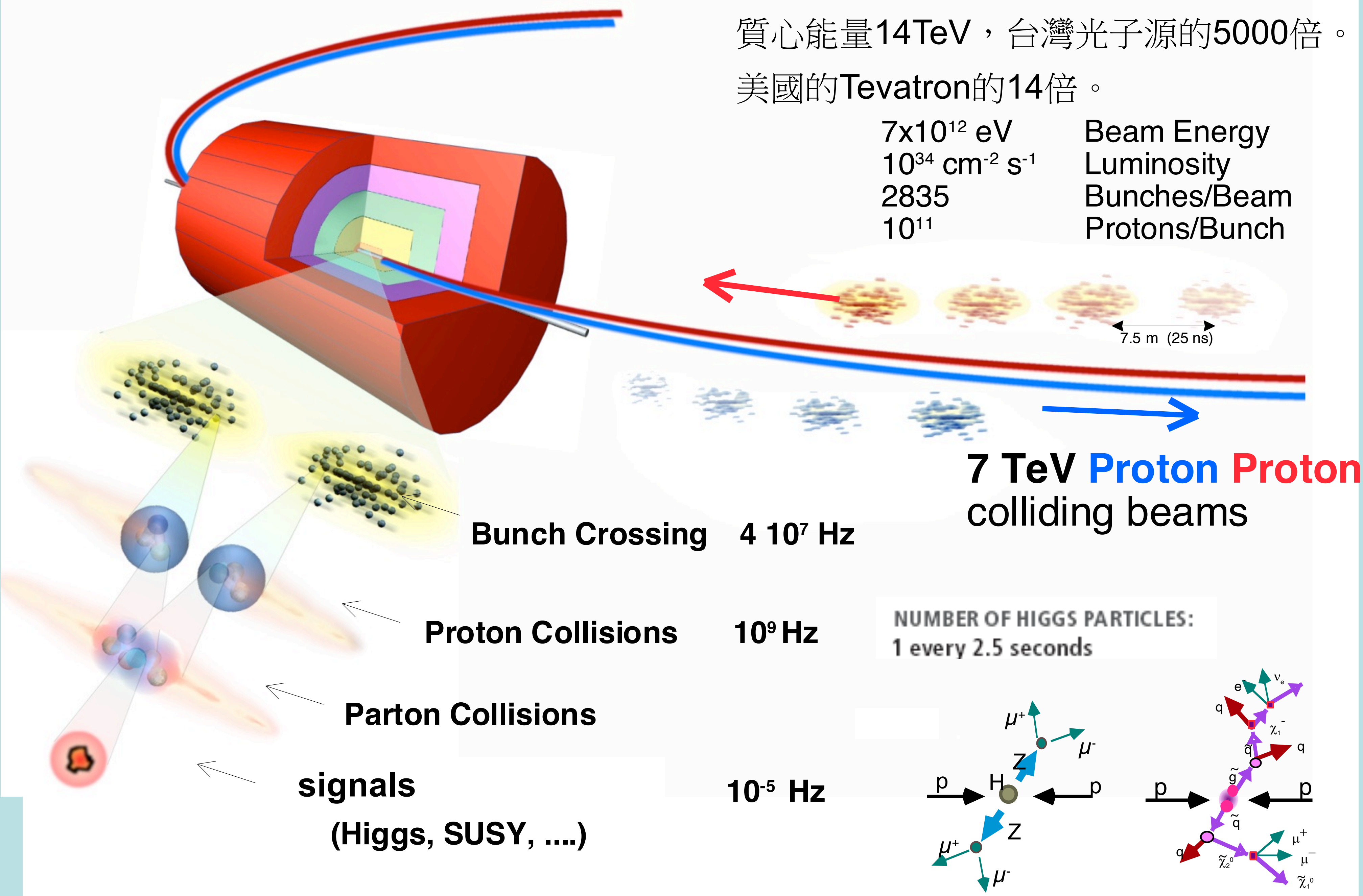
**NUMBER OF BUNCHES:**
up to 2,808

**BUNCH CROSSINGS PER SECOND:**
up to 31 million, at 4 locations

**COLLISIONS PER BUNCH CROSSING:**
up to 20

**DATA PER COLLISION:**
about 1.5 megabytes

**NUMBER OF HIGGS PARTICLES:**
1 every 2.5 seconds (at full beam luminosity and under certain assumptions about the Higgs)



質心能量14TeV，台灣光子源的5000倍。

美國的Tevatron的14倍。

| | |
|---|---|
| $7\text{x}10^{12}$ eV | Beam Energy |
| $10^{34}$ cm$^{-2}$ s$^{-1}$ | Luminosity |
| 2835 | Bunches/Beam |
| $10^{11}$ | Protons/Bunch |

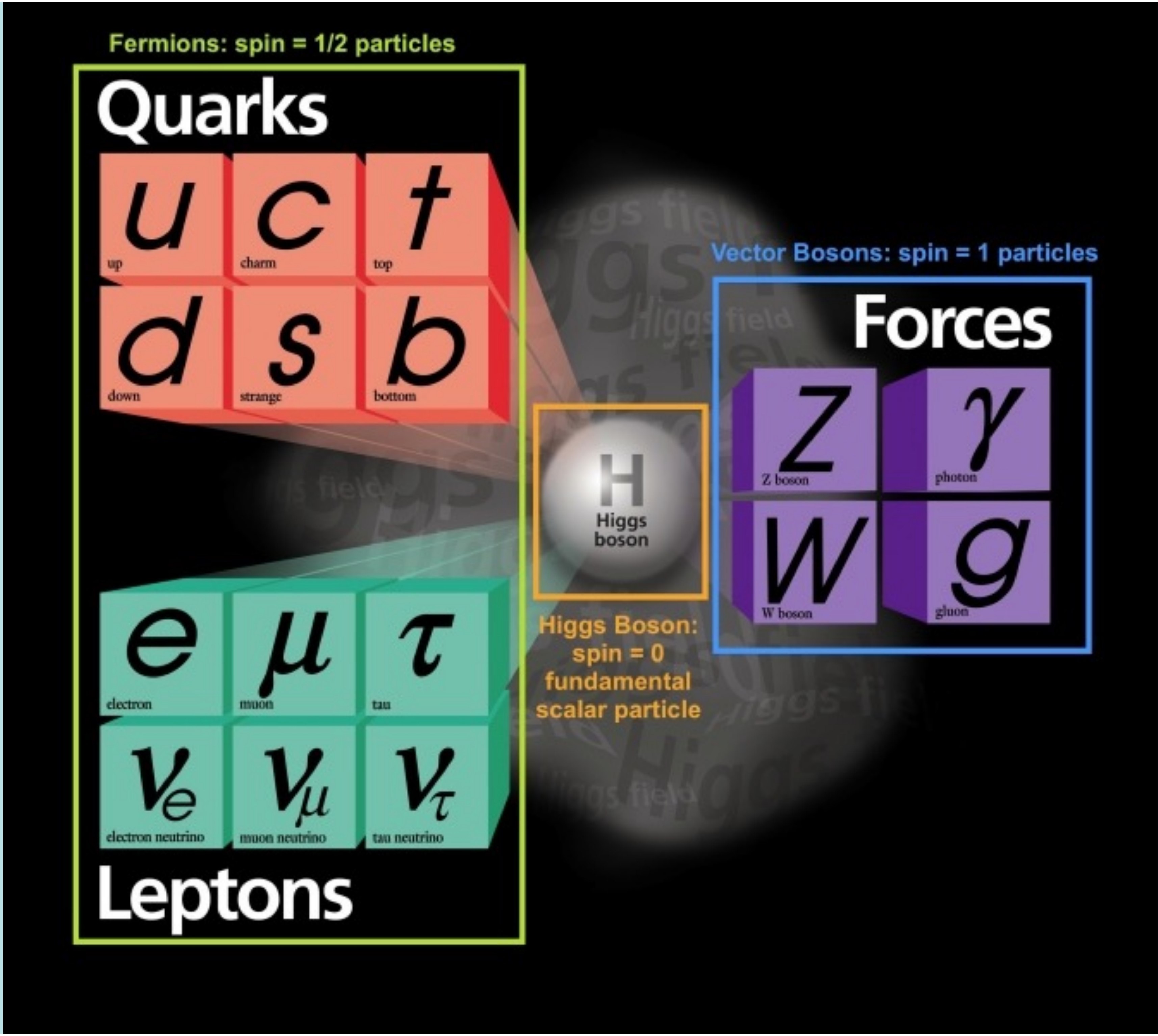
**7 TeV Proton Proton**

colliding beams

| | |
|---|---|
| **Bunch Crossing** | **$4\ 10^7$ Hz** |
| **Proton Collisions** | **$10^9$ Hz** |
| **Parton Collisions** | |
| **signals**<br>**(Higgs, SUSY, ....)** | **$10^{-5}$ Hz** |

NUMBER OF HIGGS PARTICLES:
1 every 2.5 seconds

**Selection of 1 event in 10,000,000,000,000**


Fermions: spin = 1/2 particles
Quarks
u
up
c
charm
t
top
d
down
s
strange
b
bottom
e
electron
μ
muon
τ
tau
$\nu_e$
electron neutrino
$\nu_\mu$
muon neutrino
$\nu_\tau$
tau neutrino
Leptons
H
Higgs boson
Higgs Boson: spin = 0 fundamental scalar particle
Vector Bosons: spin = 1 particles
Forces
Z
Z boson
γ
photon
W
W boson
g
gluon
Higgs field

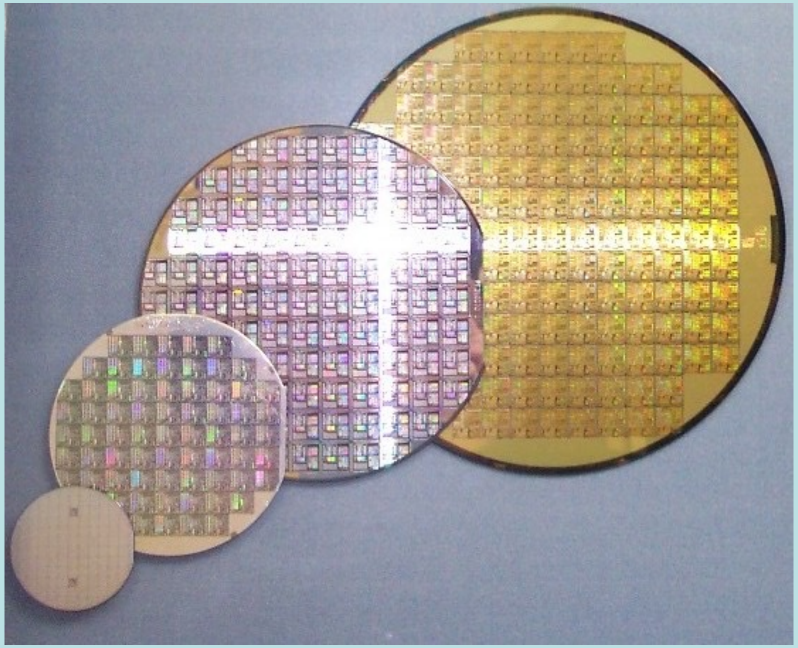
# 沒有量子物理就沒有半導體！你天天都在做實驗，證實量子力學是對的！

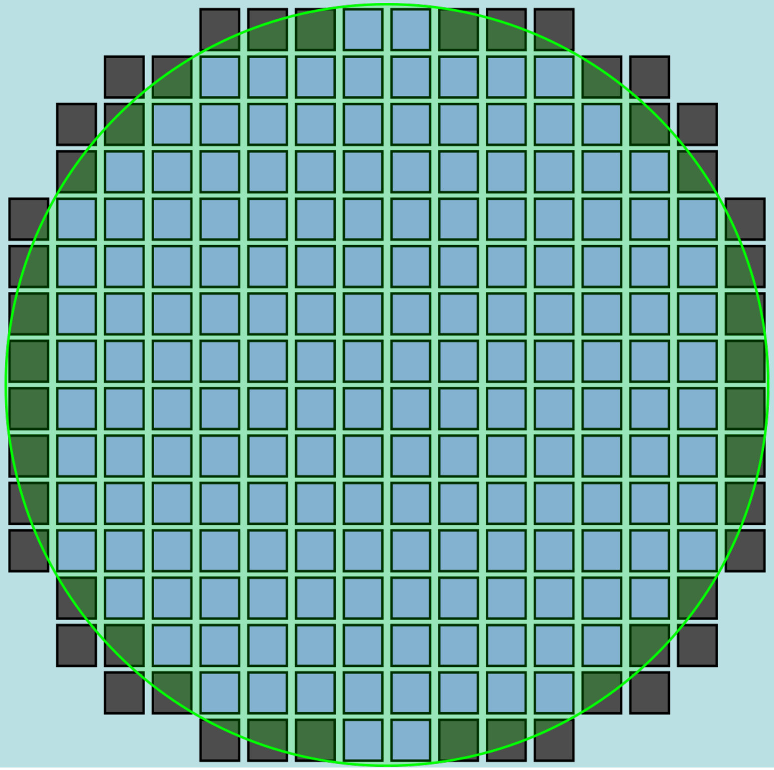
EP08｜台灣這美麗的矽島——聊一聊半導體物理

主持人｜張嘉泓

以上所描述的電晶體配置有一個很長的名字：金屬氧化物半導體、場效型電晶體 MOSFET，比起傳統的雙極型電晶體，場效電晶體更適合於平面的半導體大量製造，而且體積可以極度縮小。如上所述，場效電晶體的主體是由p型與n型半導體分布組成，這個分布可以規劃在二維平面上，於是由電晶體所組成的電路就可以在一片平板上展開。製作時會以矽為晶片基板，第一步先全部摻雜為p型半導體。接著先設計好藍圖，在平面上安排出n型半導體的位置。第三步，在基板上塗一層感光的光阻劑，然後將設計畫在一片光罩上，當光罩放置於基板上時，光罩的圖案會遮蓋規劃為p型半導體的區域。接著以紫外線曝照，消除掉光罩圖案未遮蓋處的光阻劑，於是規劃為n型半導體區域的基板就曝露在外。最後，將此曝露的部分滲進適當雜質，就得到所設計的n型半導體分布，而光阻劑覆蓋的部分則維持原來的p型。利用這樣的製程就可以在晶片上建構出你所設計的分布，其他如絕緣層、導電線路與電極也可以用類似的方式往上堆疊建構。

上一段描述是不是讓你感覺很枯燥、很機械化？那就對了，一個接著一個的步驟，有條不紊，如此，就可以大量機械化生產。而且平面化的設計，可以將電路非常節省空間地集中在一塊小晶片上，這就稱為積體電路。它的最大好處在於，只要你的藍圖光罩夠精細，電晶體的大小幾乎可以無限地縮小。如此我們才能把驚人的計算能力，置於一個日常生活能夠輕鬆攜帶的裝置。積體電路在1960年代出現，大概從






tsmc


RELIEF WRENCH LOCATED BEHIND THIS PANEL

EUV stands for extreme ultraviolet light, which produces the circuitry.

國科會外文學門 2025年度跨域人文系列講座｜第一場

# 02/22

週六 15:00-17:00

# 逃避雖然可恥，但卻有用
# 量子糾纏與薛丁格的貓

量子力學是真實微觀世界的物理規則，例如生活中無所不在的電子就必須遵守。它所預測的有些現象比科幻情節還奇特，更驚悚的是，量子力學似乎有時與我們從經驗歸納出來的科學原則相違背。因此，兩位量子革命的先驅愛因斯坦與薛丁格，後期反過來表達了非常尖銳的少數反對意見，提出了實在離譜的量子力學現象，這就是量子糾纏與薛丁格的貓。

面對如此尖銳又無法解決的質疑，絕大多數的物理學家採取了逃避的態度，說好聽是把問題放在括號內、存而不論，其實是置之不理。但他們務實地就已知的部分繼續前行，竟然成功得發展出了如繁花般的近代物理。現在從基本粒子以至半導體，我們已能對微觀世界、精確掌握並開發利用，令人嘆為觀止。逃避真是有用的，但問題還在括號內，沒有解決。

經過了半個世紀，當初的反對意見，並未被全然忽視，有一小群物理學家慢慢的、就其意義仔細琢磨。近幾年的實驗證實了，愛因斯坦覺得不可能的量子糾纏，竟然真的發生了。因此，反對意見並沒有推翻量子力學，反而指出了它比較幽微、難以想像、扭曲卻強大的特性。由此製造出的糾纏狀態可能會成爲未來量子電腦的基礎。

這樣的結果肯定不是愛因斯坦與薛丁格當初想像得到的。至於現在他們是否願意接納量子力學，那就不可知了。

**主講人｜張嘉泓**

國立臺灣師範大學物理學系副教授

**主持人｜黃宗慧**

國科會外文學門（文學二）召集人
國立臺灣大學外國語文學系教授

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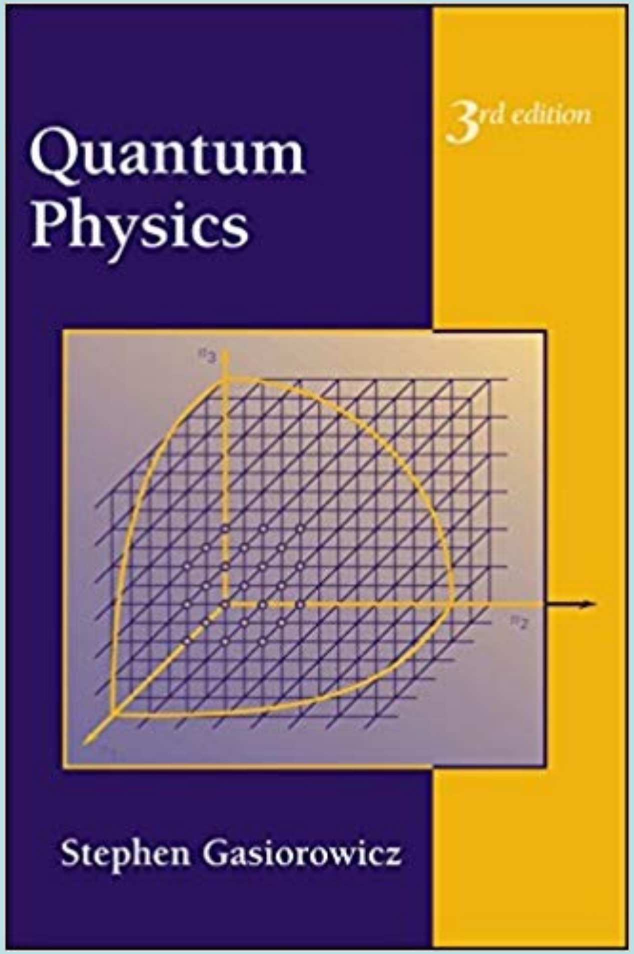
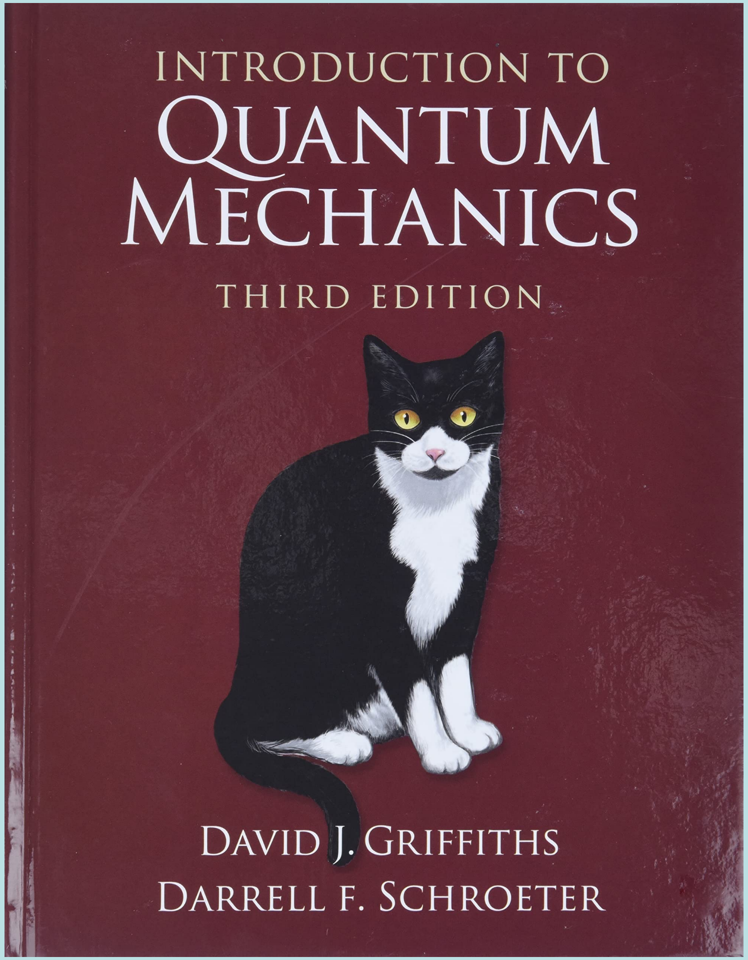
# 逃避雖然可恥，但卻有用：量子糾纏與薛丁格的貓

原是匈牙利文的諺語（Szégyen a futás, de hasznos）

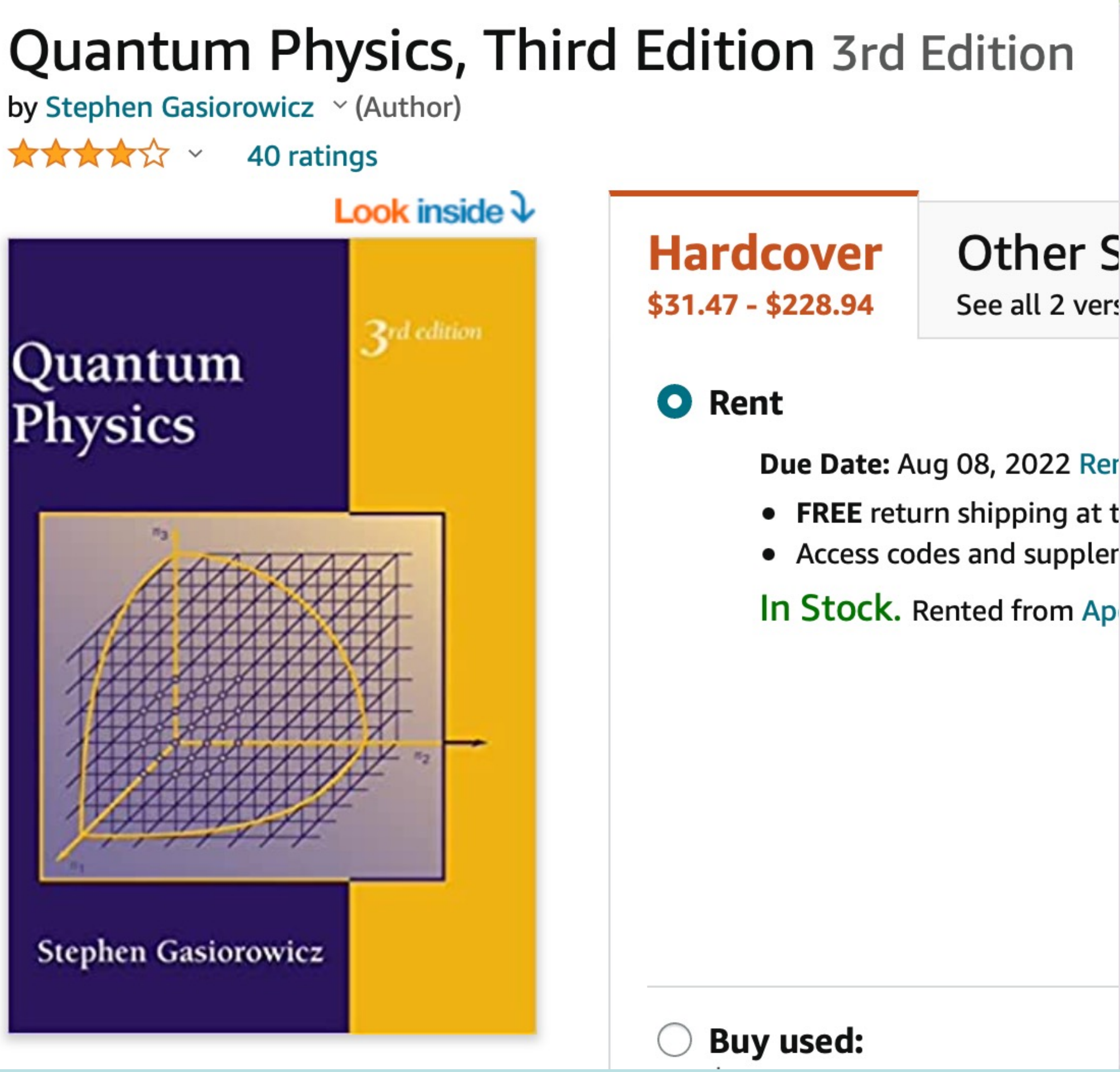
古典物理雖然重要，量子力學才是未來！

# 這堂數學物理的課其實是要偷渡量子物理到大二。





本課程將以近代量子物理面對的問題為核心，介紹量子力學基本概念，
闡述解決這些問題的數學方法。



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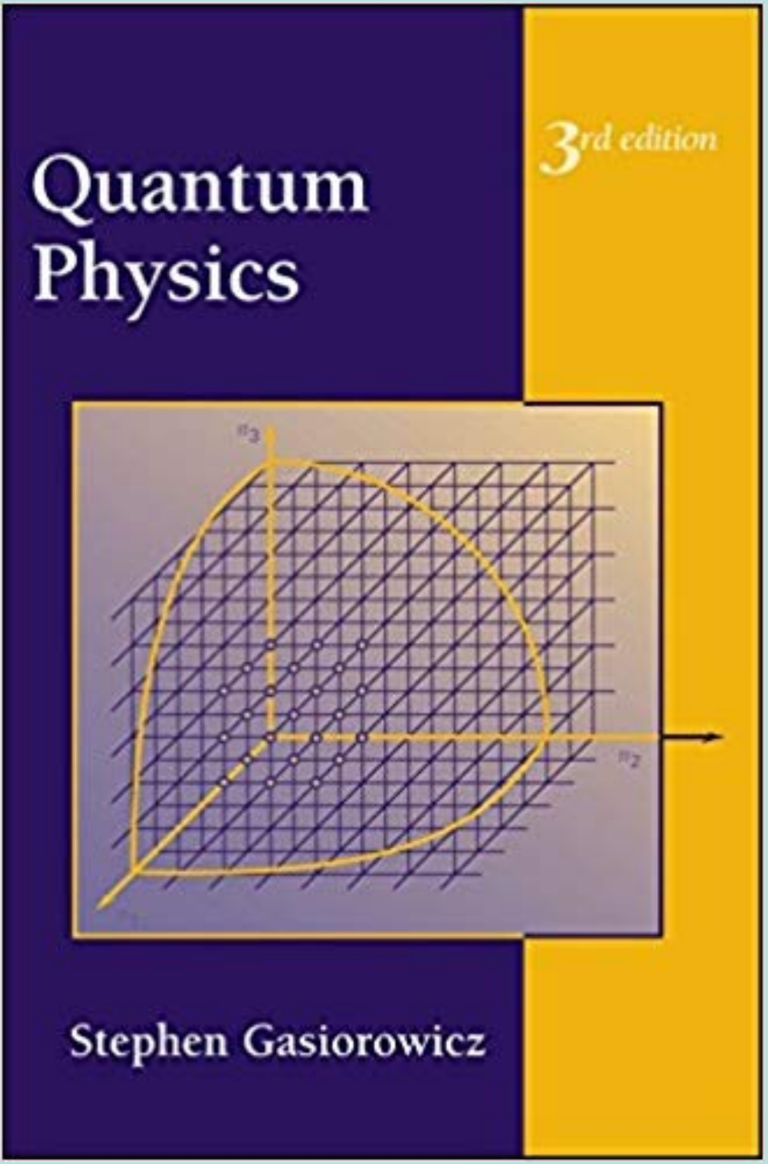

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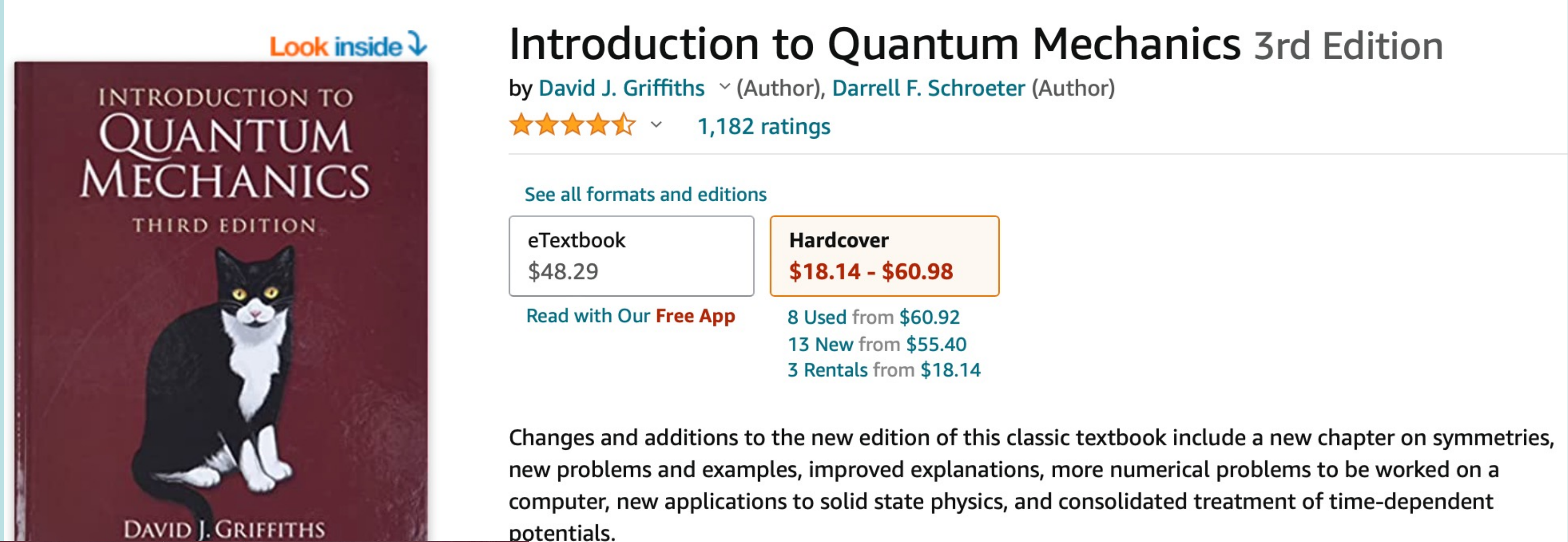

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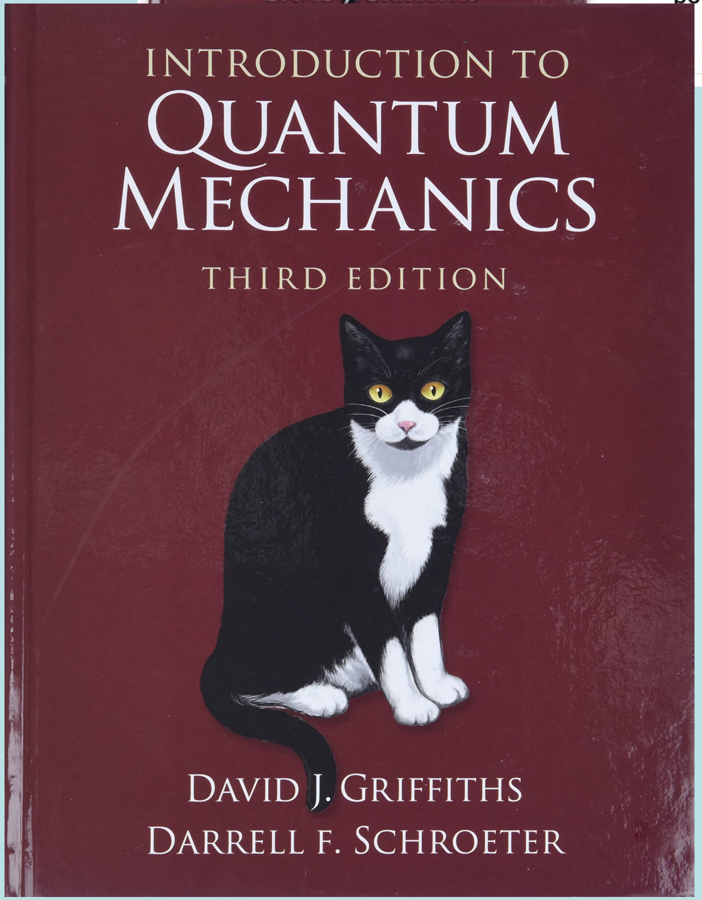

大四 量子力學導論

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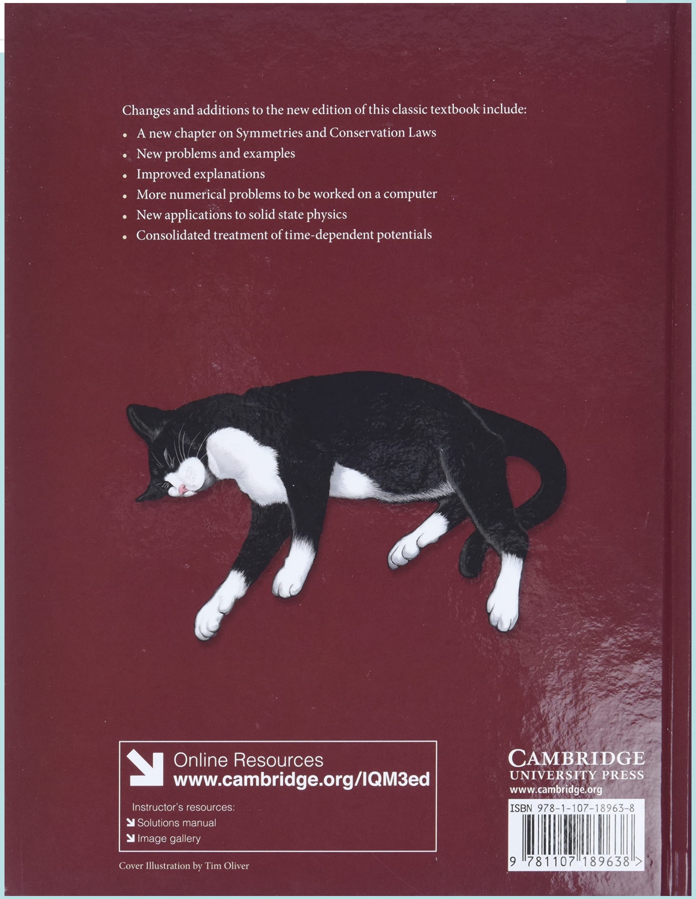

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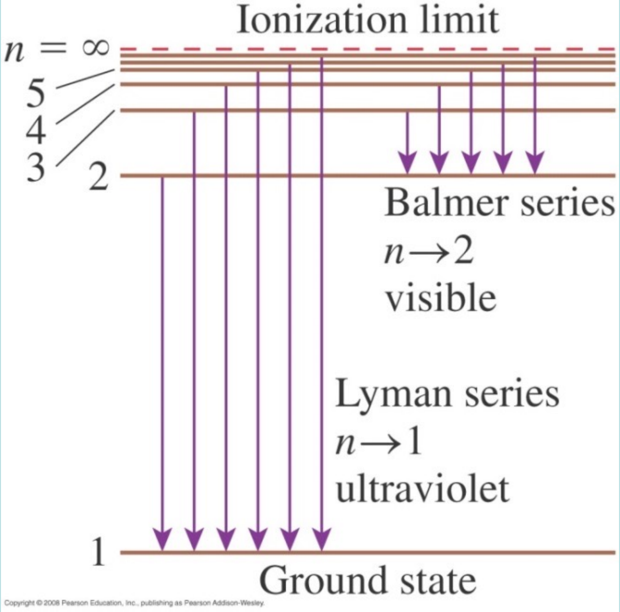
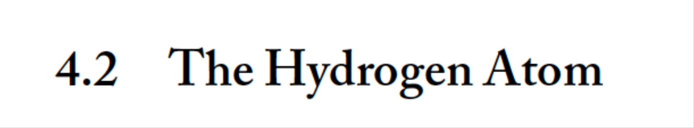
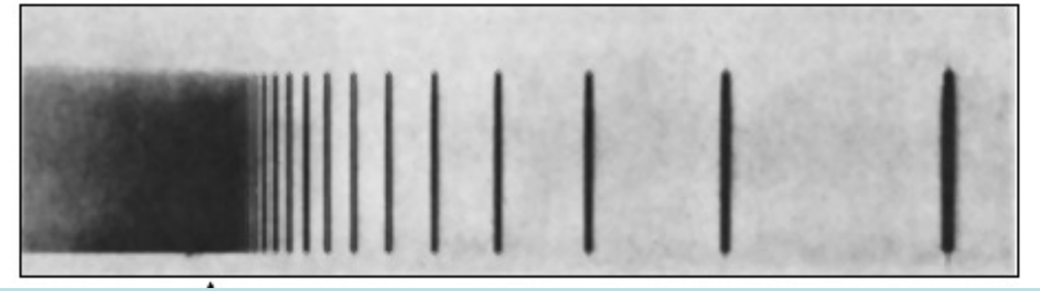
You may say: How bizarre could the quantum world be be? Try me.

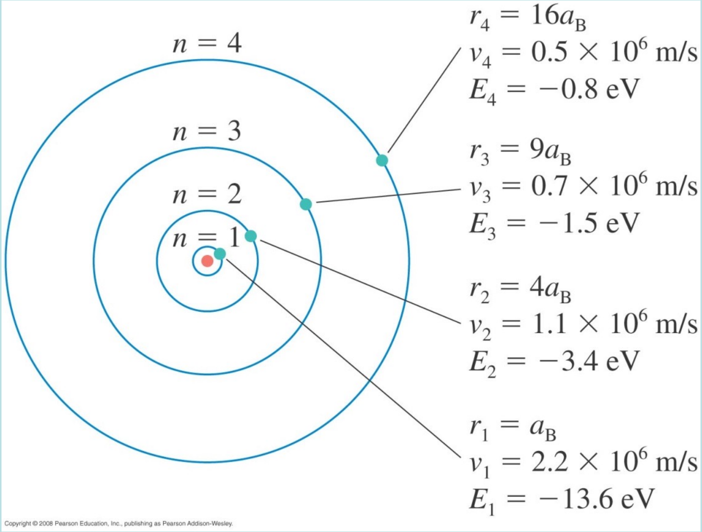
# 波爾原子模型的基本假設



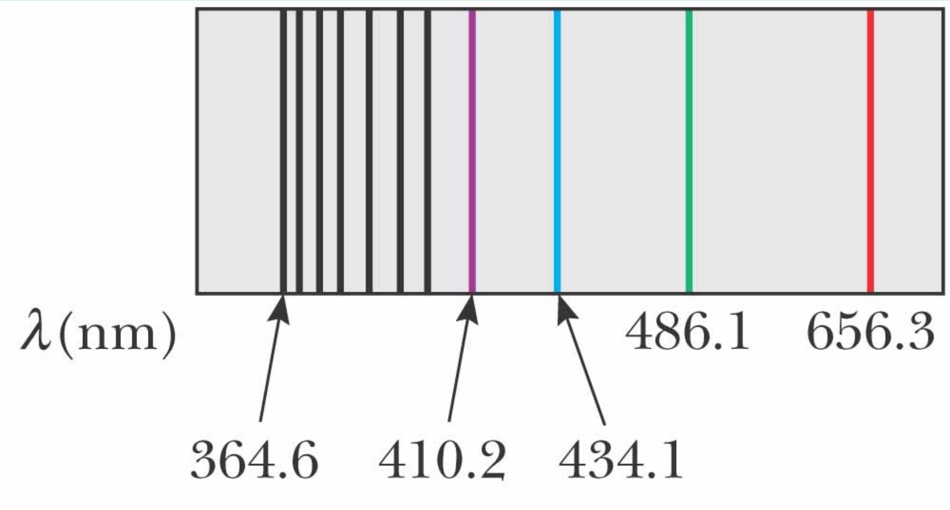


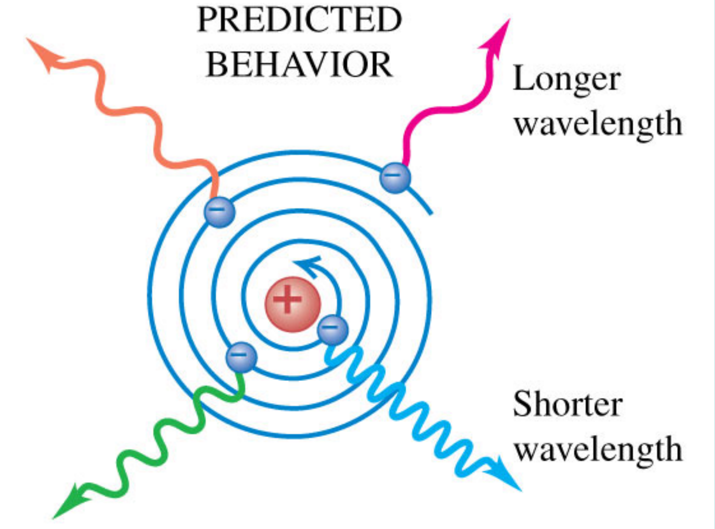
## 4.2 The Hydrogen Atom



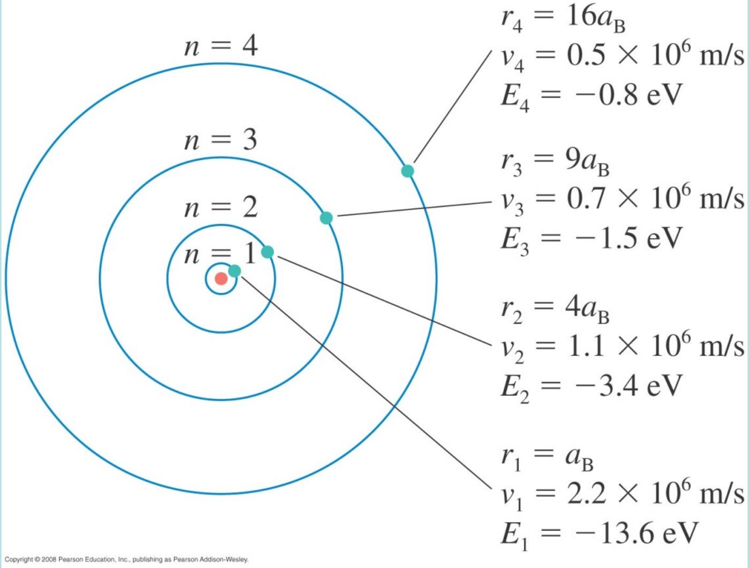


波爾的原子模型是一個極為正確又精確但肯定是錯誤的理論！





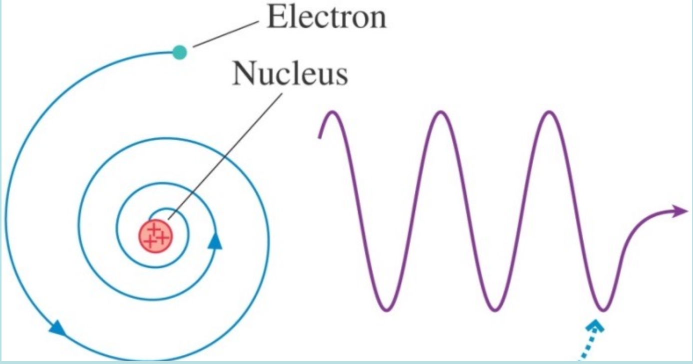
關鍵的原子內電子的穩定態是一個非常矛盾奧秘的觀念。



電子只能選擇符合量子化條件的軌道形成穩定態。

為什麼？量子化條件是從何而來？

為什麼基態軌道上的電子會穩定？加速的點電荷不是會放出電磁波的嗎？



畢竟這是巨觀觀察到的事實！

難道微觀世界與巨觀世界不同？

革命派的新物理令人迷惑！

舊政權的王子前來撥亂反正！

# Prince Louie Victor de Broglie 德布羅意

**Louis-Victor-Pierre-Raymond, 7th duc de Broglie (1892-1987)**

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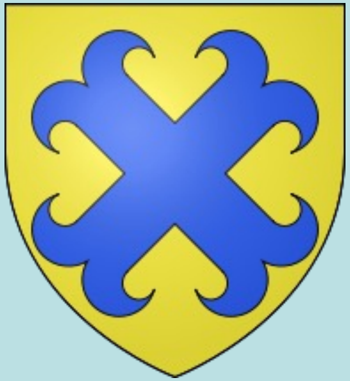
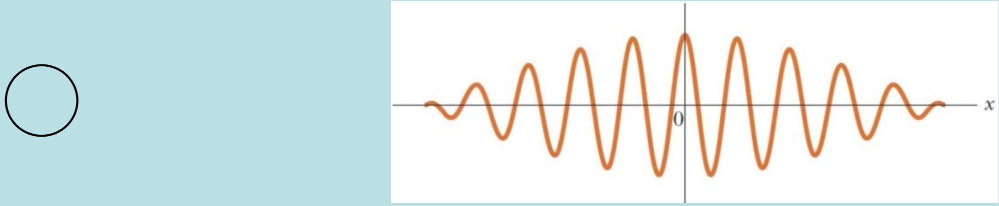


House of Broglie
since 1600’s

解決問題的辦法之一，就是先把它弄糟！

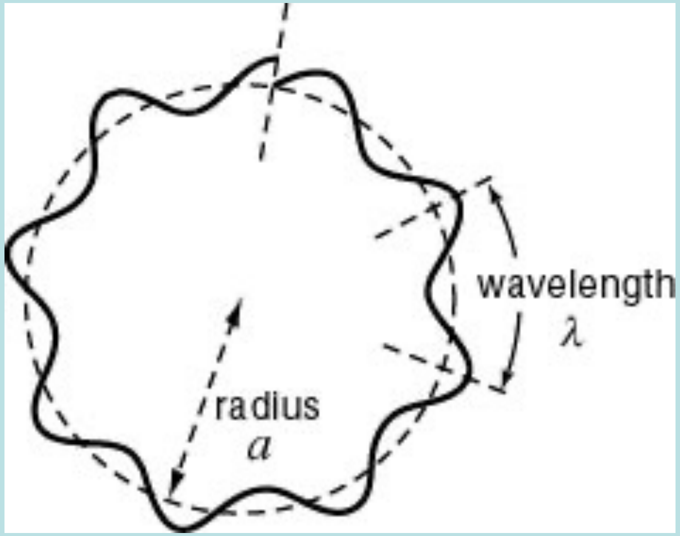


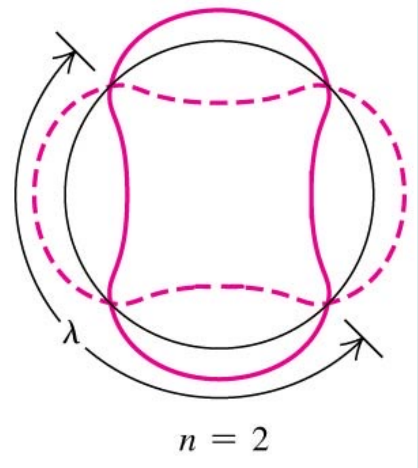
如果光又是粒子又是波，電子是不是也是如此？

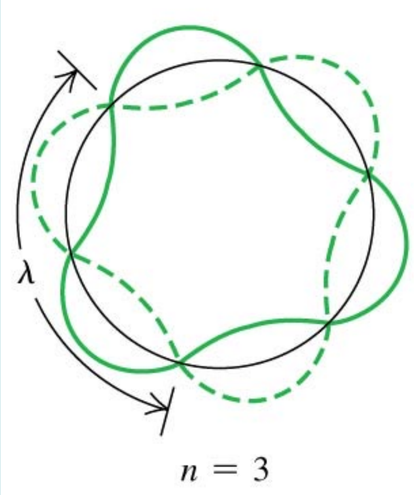
電子可能也是一種波（物質波 (1924)）。

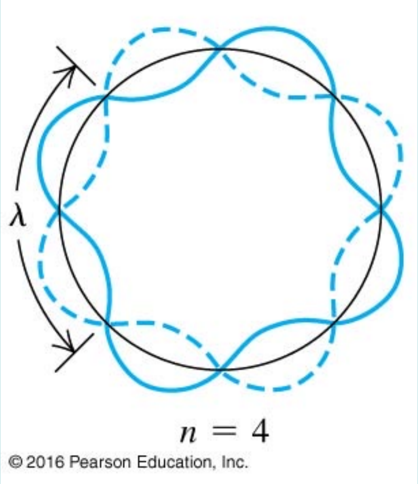
但更要緊的是：假設電子是波的好處是甚麼？

將原子中的電子波放在有限的圓形軌道上，波函數在圓周上必須一致，圓周必須是波長的整數倍值，因此電子的軌道不能任意！









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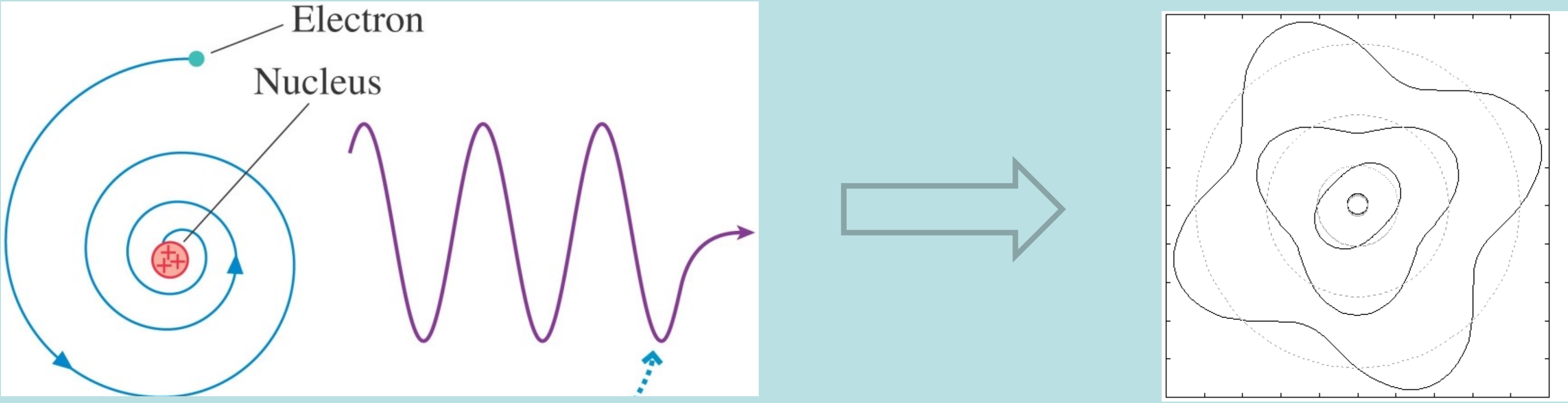
波爾模型的問題一下子得到解決！

原子中穩定態的電子為什麼不會放出電磁波的嗎？



原子中的電子是一個穩定的駐波！

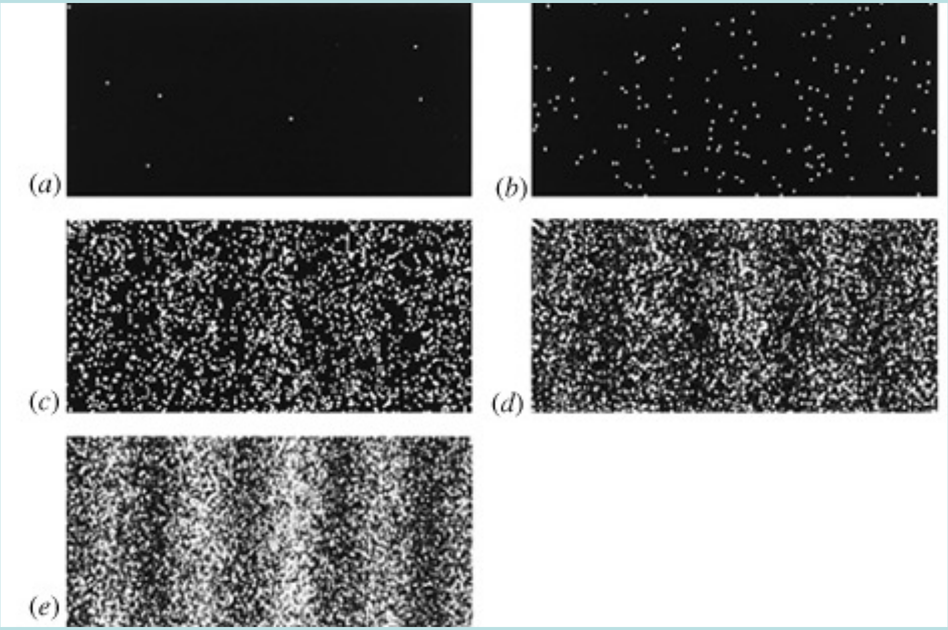
原來原子中的電子並沒有所謂軌跡，並沒有點電荷進行加速！

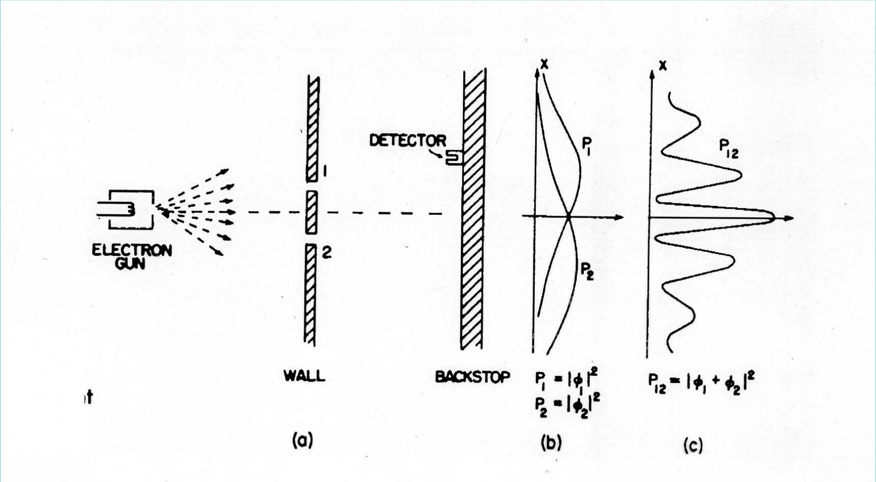
自然也就沒有釋放電磁波的問題！

這個解決方法原則上是對的！

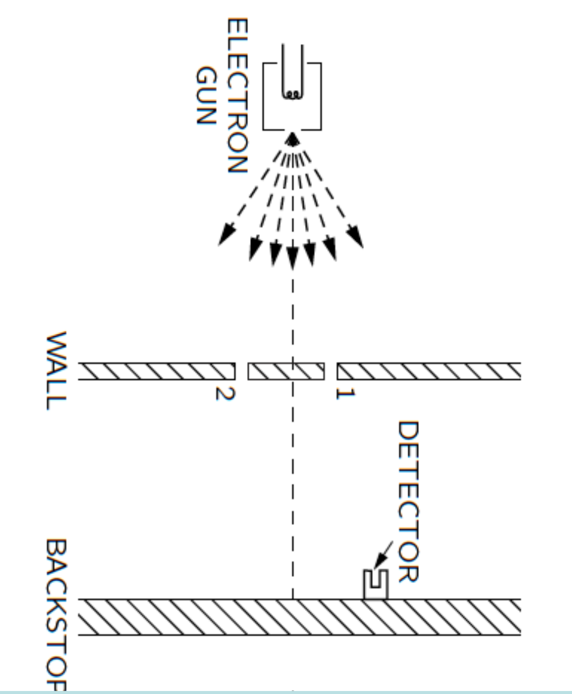
原子內有分離的穩定態的電子，這個事實顯示電子是具有波的特性的！

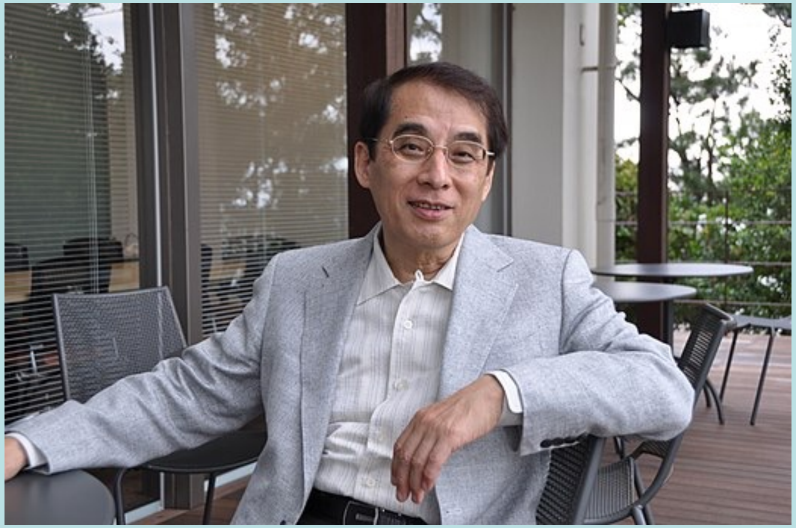
# 電子有干涉現象



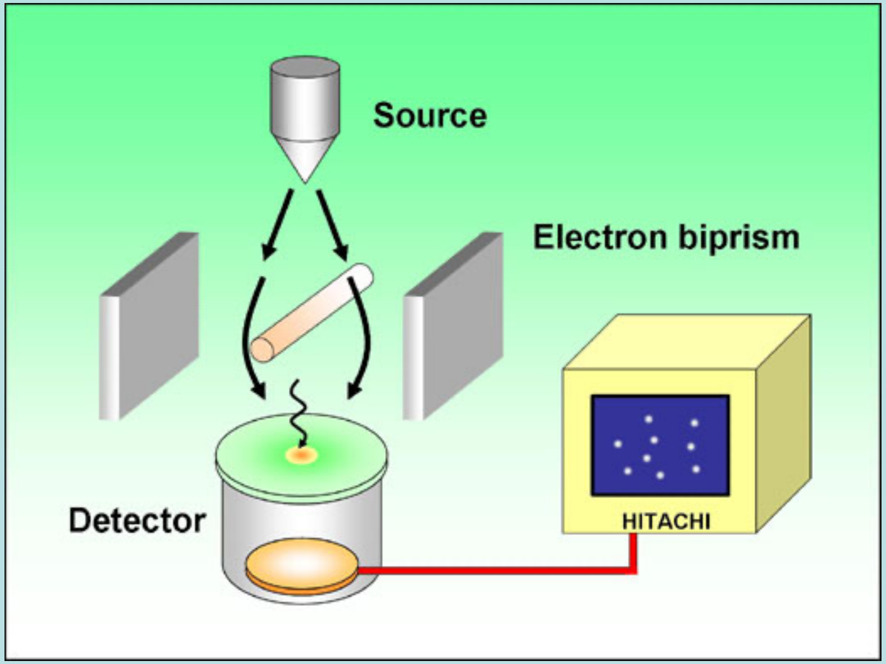


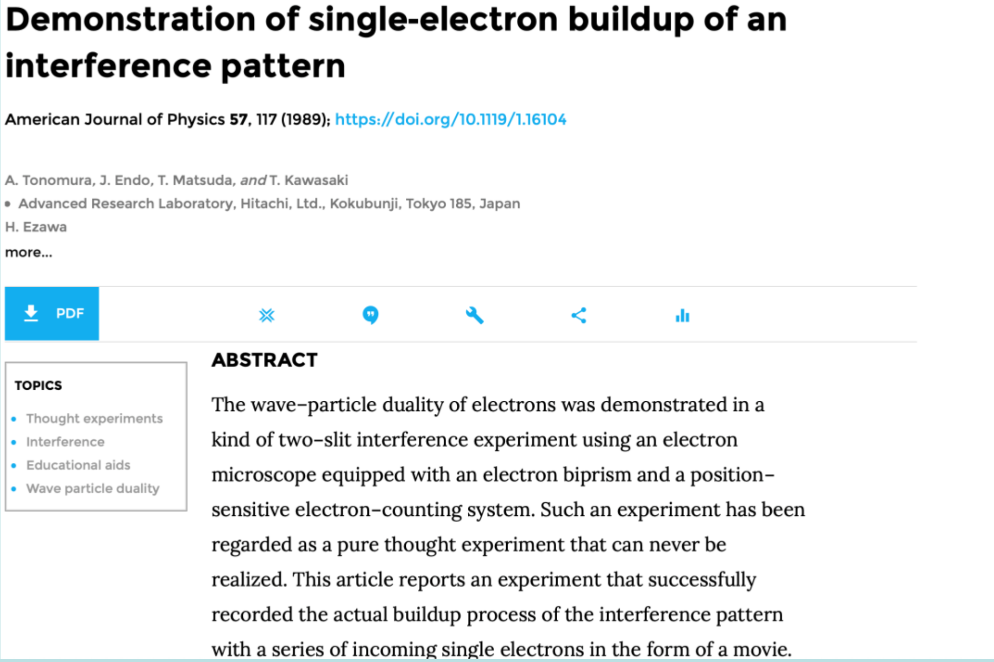
每一個點是一顆電子！





外村 彰 Tonomura Akira,
1942 – 2012





## Demonstration of single-electron buildup of an interference pattern

American Journal of Physics **57**, 117 (1989); https://doi.org/10.1119/1.16104

A. Tonomura, J. Endo, T. Matsuda, *and* T. Kawasaki
- Advanced Research Laboratory, Hitachi, Ltd., Kokubunji, Tokyo 185, Japan

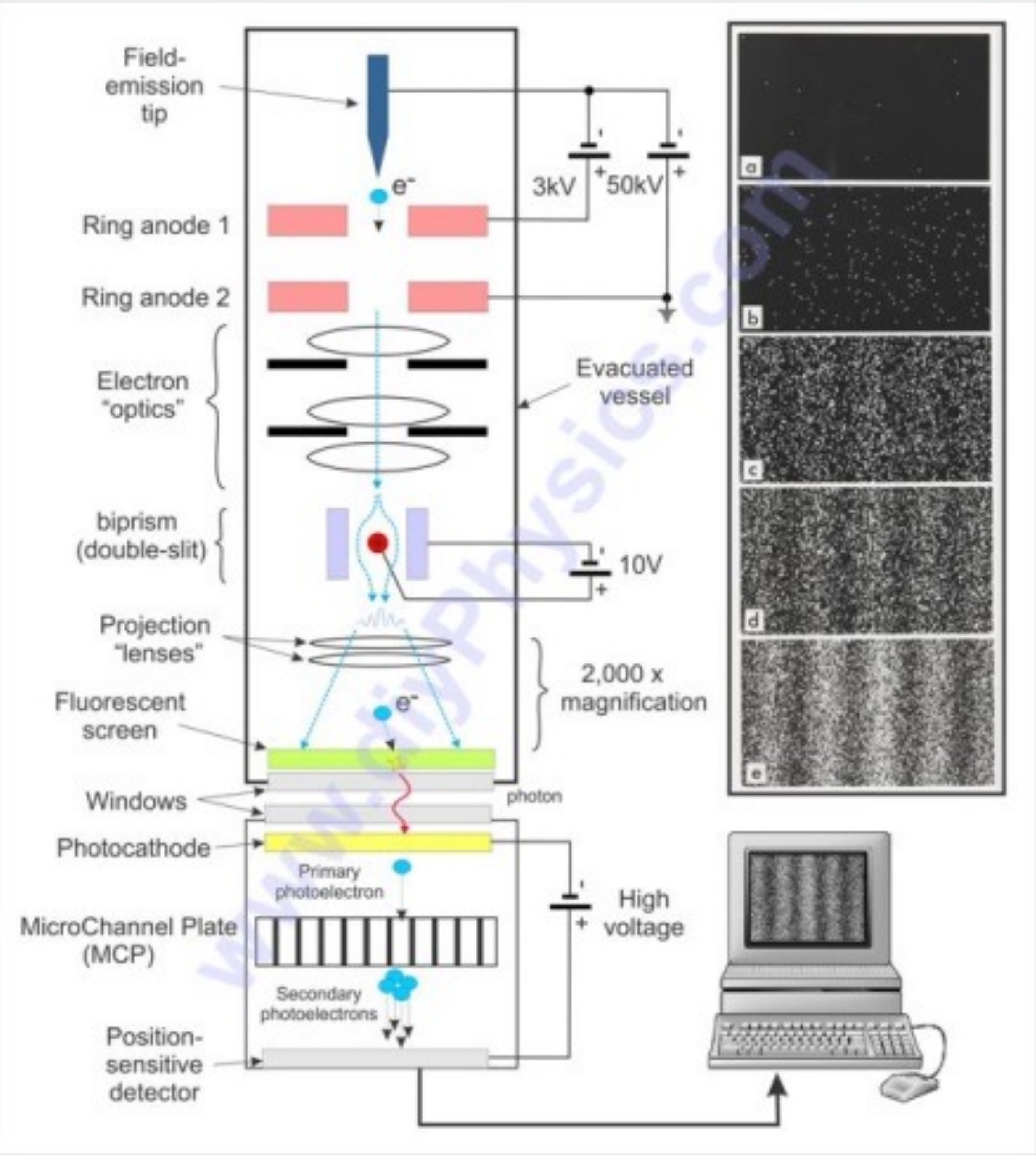
H. Ezawa
more...

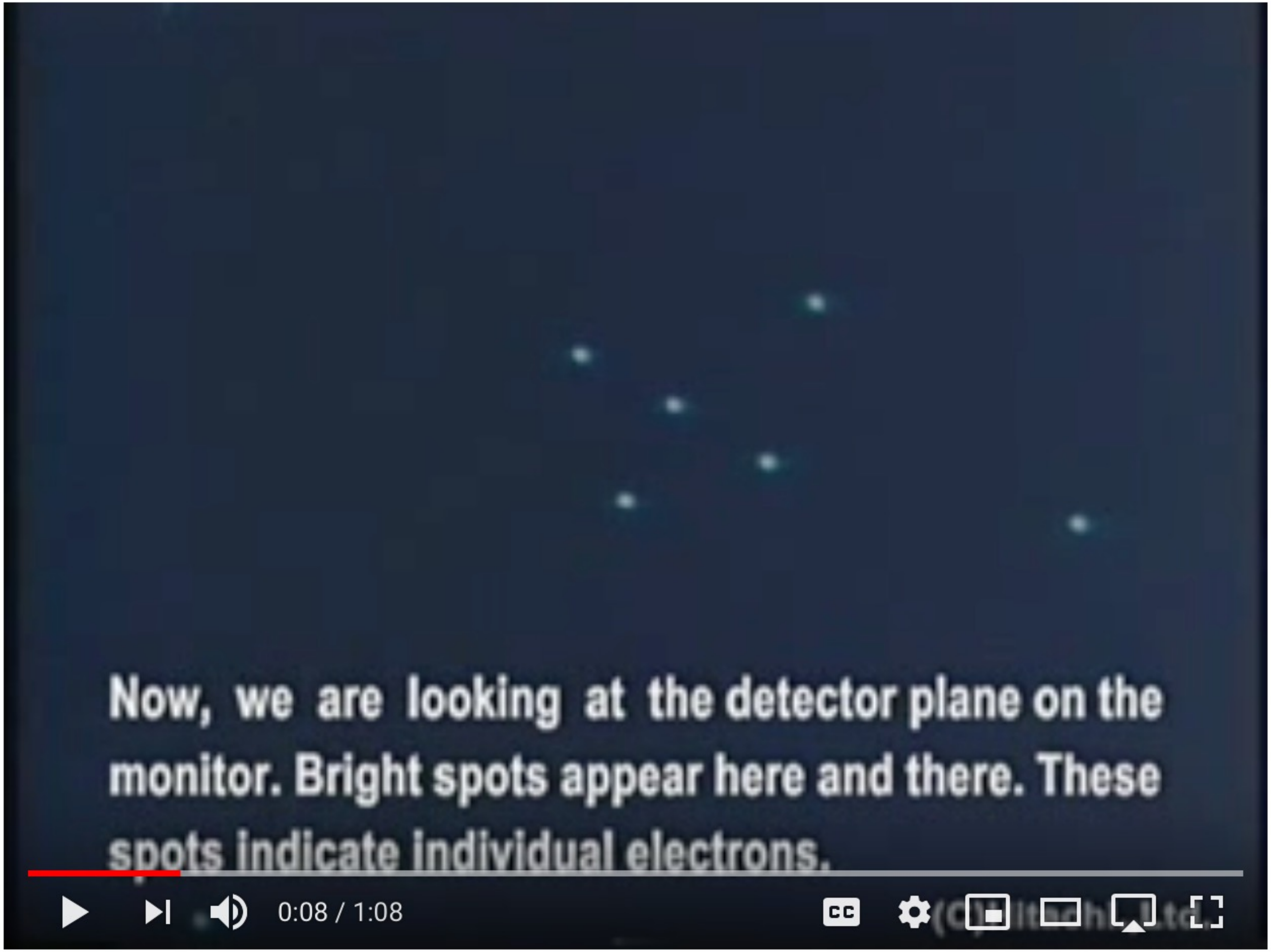
PDF

**TOPICS**
- Thought experiments
- Interference
- Educational aids
- Wave particle duality

**ABSTRACT**

The wave–particle duality of electrons was demonstrated in a kind of two–slit interference experiment using an electron microscope equipped with an electron biprism and a position–sensitive electron–counting system. Such an experiment has been regarded as a pure thought experiment that can never be realized. This article reports an experiment that successfully recorded the actual buildup process of the interference pattern with a series of incoming single electrons in the form of a movie.


Field-emission tip
e⁻
Ring anode 1
Ring anode 2
3kV
50kV
Electron "optics"
Evacuated vessel
biprism (double-slit)
10V
Projection "lenses"
2,000 x magnification
Fluorescent screen
e⁻
Windows
photon
Photocathode
Primary photoelectron
MicroChannel Plate (MCP)
High voltage
Secondary photoelectrons
Position-sensitive detector
a
b
c
d
e



Now, we are looking at the detector plane on the monitor. Bright spots appear here and there. These spots indicate individual electrons.

One electron double slit experiment by Akira Tonomura

Every wave has a wave equation.

Give me a wave equation for electrons.

要研究一個波，就要先得到它的波方程式！

要寫波方程式，得先有一個波函數。

假設電子波有一個波函數： $\Psi(x,t)$

這是甚麼東西？

薛丁格假設我們可以先不管它的意義是甚麼！


© 2004 Thomson - Brooks/Cole

E. Schrodinger 薛丁格

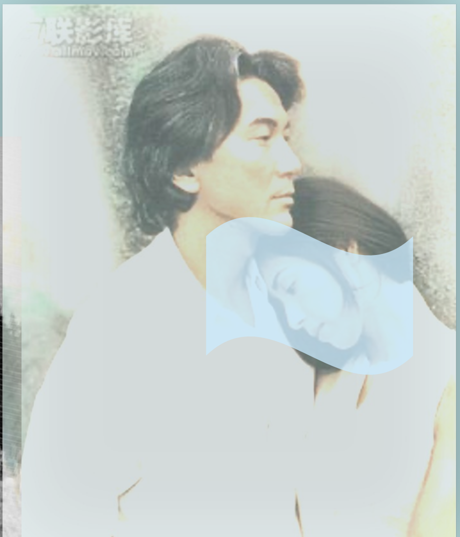
Villa Herwig, Arosa Switzerland 1925

薛丁格從無到有 from scratch猜出一個波方程式：

Schrodinger Wave Equation

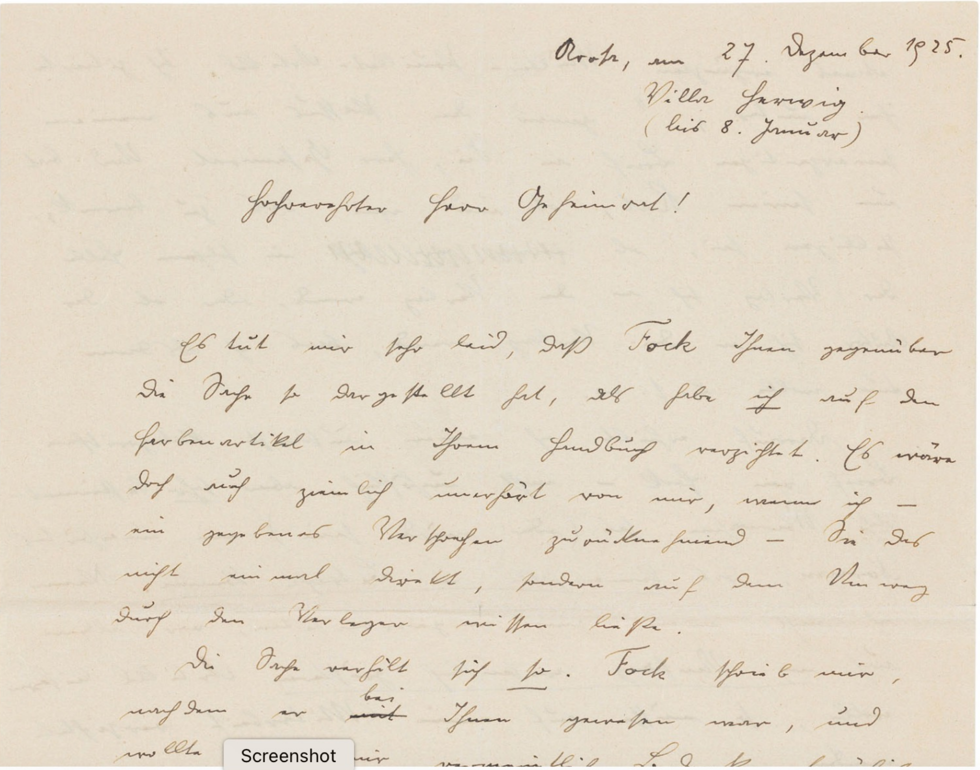
$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=i\hbar\frac{\partial\Psi}{\partial t}$$

注意：波函數是複數！



Schrödinger spent the Christmas break of 1925–1926 at the Villa Herwig at Arosa where he intended to relax and enjoy the skiing. His mind was, however, consumed by his recent researches at the expense of what should have been a period of relaxation. As he noted in a letter to Willy Wien of 27 December 1925,

> 'At the moment I am plagued by a new atomic theory . . . I believe I can write down a vibrating system – constructed in a comparatively natural manner and not by *ad hoc* assumptions – which has as its eigenfrequencies the term frequencies of the hydrogen atom.'

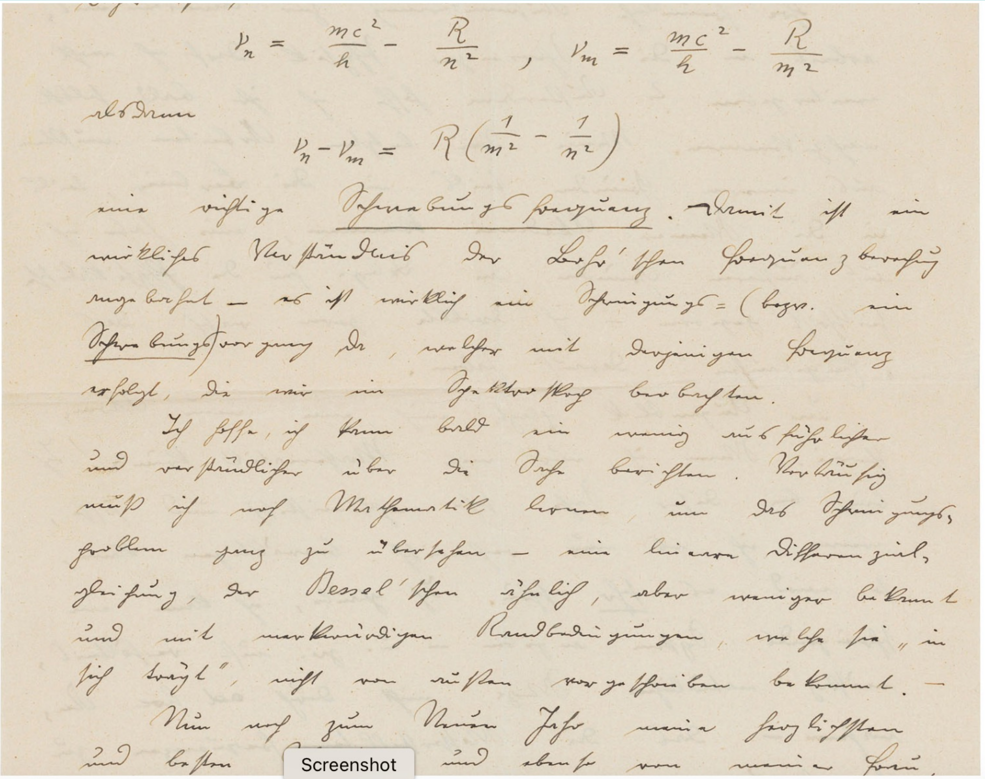


Arosa, am 27. Dezember 1925.
Villa Herwig
(bis 8. Jänner)

Hochverehrter Herr Geheimrat!

Es tut mir sehr leid, daß Fock Ihnen gegenüber die Sache so dargestellt hat, als habe ich auf den Farbenartikel in Ihrem Handbuch verzichtet. Es wäre doch auch ziemlich unerhört von mir, wenn ich – ein gegebenes Versprechen zurücknehmend – Sie das nicht einmal direkt, sondern auf dem Umweg durch den Verleger wissen ließe.

Die Sache verhält sich so. Fock schrieb mir, nachdem er ~~mit~~ bei Ihnen gewesen war, und wollte mir



$$\nu_n = \frac{mc^2}{h} - \frac{R}{n^2}, \quad \nu_m = \frac{mc^2}{h} - \frac{R}{m^2}$$

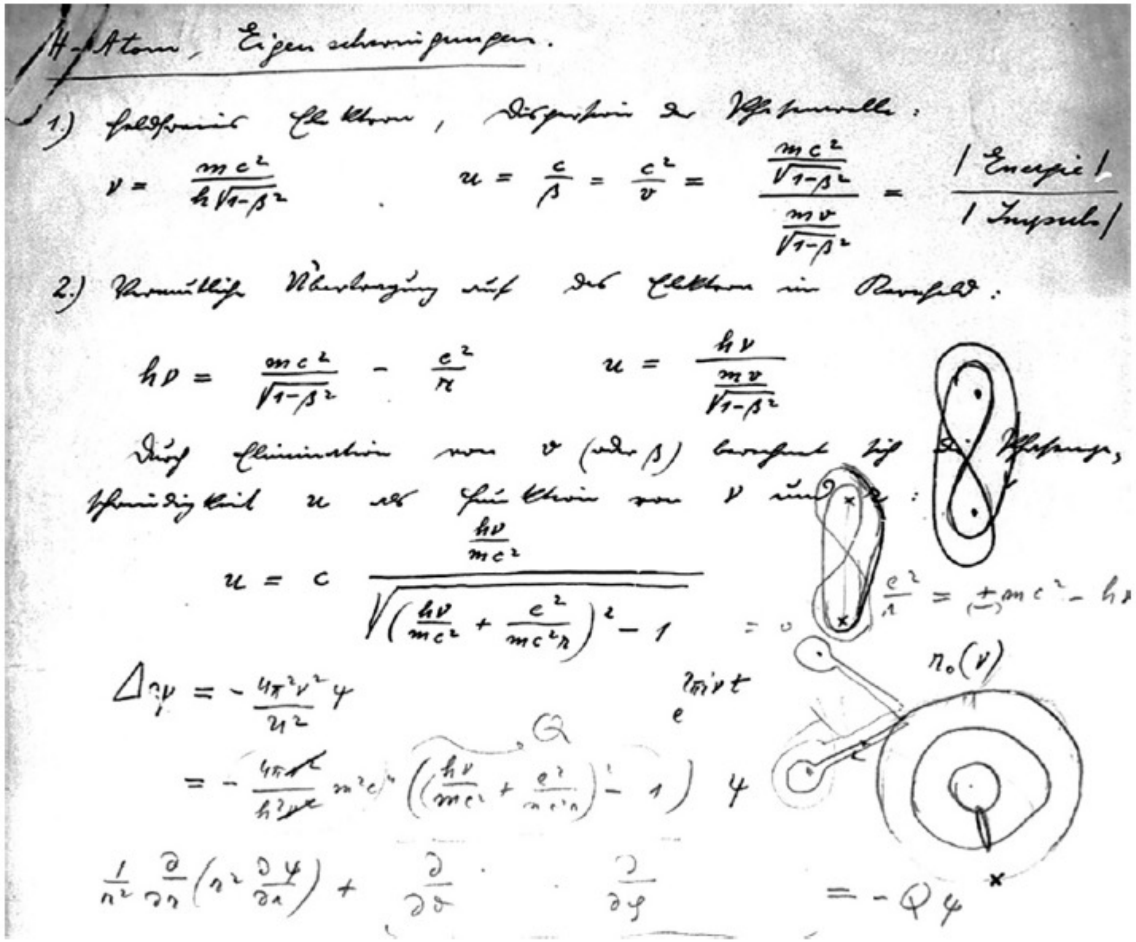
alsdann

$$\nu_n - \nu_m = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

eine richtige Schwebungsfrequenz. Damit ist ein wirkliches Verständnis der Bohr'schen Frequenzbedingung angebahnt – es ist wirklich ein Schwingungs= (bezw. ein Schwebungs)vorgang da, welcher mit derjenigen Frequenz erfolgt, die wir im Spektroskop beobachten.

Ich hoffe, ich kann bald ein wenig ausführlicher und verständlicher über die Sache berichten. Vorläufig muß ich noch Mathematik lernen, um das Schwingungsproblem ganz zu übersehen – eine lineare Differenzialgleichung, der Bessel'schen ähnlich, aber weniger bekannt und mit merkwürdigen Randbedingungen, welche sie „in sich trägt", nicht von außen vorgeschrieben bekommt. –

Nun noch zum Neuen Jahre meine herzlichsten und besten und ebenso von meiner Frau,



H-Atom, Eigenschwingungen.

1.) feldfreies Elektron, Dispersion der Phasenwelle:

$\nu = \frac{mc^2}{h\sqrt{1-\beta^2}}$ . $u = \frac{c}{\beta} = \frac{c^2}{v} = \frac{\frac{mc^2}{\sqrt{1-\beta^2}}}{\frac{mv}{\sqrt{1-\beta^2}}} = \frac{|\text{Energie}|}{|\text{Impuls}|}$

2.) Vermutliche Übertragung auf das Elektron im Kernfeld:

$h\nu = \frac{mc^2}{\sqrt{1-\beta^2}} - \frac{e^2}{r}$ $u = \frac{h\nu}{\frac{mv}{\sqrt{1-\beta^2}}}$

Durch Elimination von $v$ (oder $\beta$) berechnet sich die Fortpflanzungsgeschwindigkeit $u$ als Funktion von $\nu$ und $r$:

$$u = c\,\frac{\frac{h\nu}{mc^2}}{\sqrt{\left(\frac{h\nu}{mc^2}+\frac{e^2}{mc^2r}\right)^2-1}}$$

$= 0$

$\frac{e^2}{r} = \pm mc^2 - h\nu$

$r_0(\nu)$

$e^{2\pi i\nu t}$

$$\Delta\psi = -\frac{4\pi^2\nu^2}{u^2}\psi$$

$$= -\frac{4\pi^2\cancel{\nu^2}}{h^2\cancel{\nu^2}}\,\overbrace{m^2c^4\left(\left(\frac{h\nu}{mc^2}+\frac{e^2}{mc^2r}\right)^2-1\right)}^{Q}\,\psi$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{\partial}{\partial\vartheta} \qquad \frac{\partial}{\partial\varphi} \qquad = -Q\psi$$

**Fig. 7.1** The page from Schrödinger's notebook containing the first appearance of the wave equation for the hydrogen atom. The wave equation is just visible under the expression for $u$. The loop diagrams with intricate loops around two points refer to the path for a contour integration used to solve second-order differential equations by a method not in general use today. The lower doodle seems to be concerned with jumps between Bohr orbits. (Used by

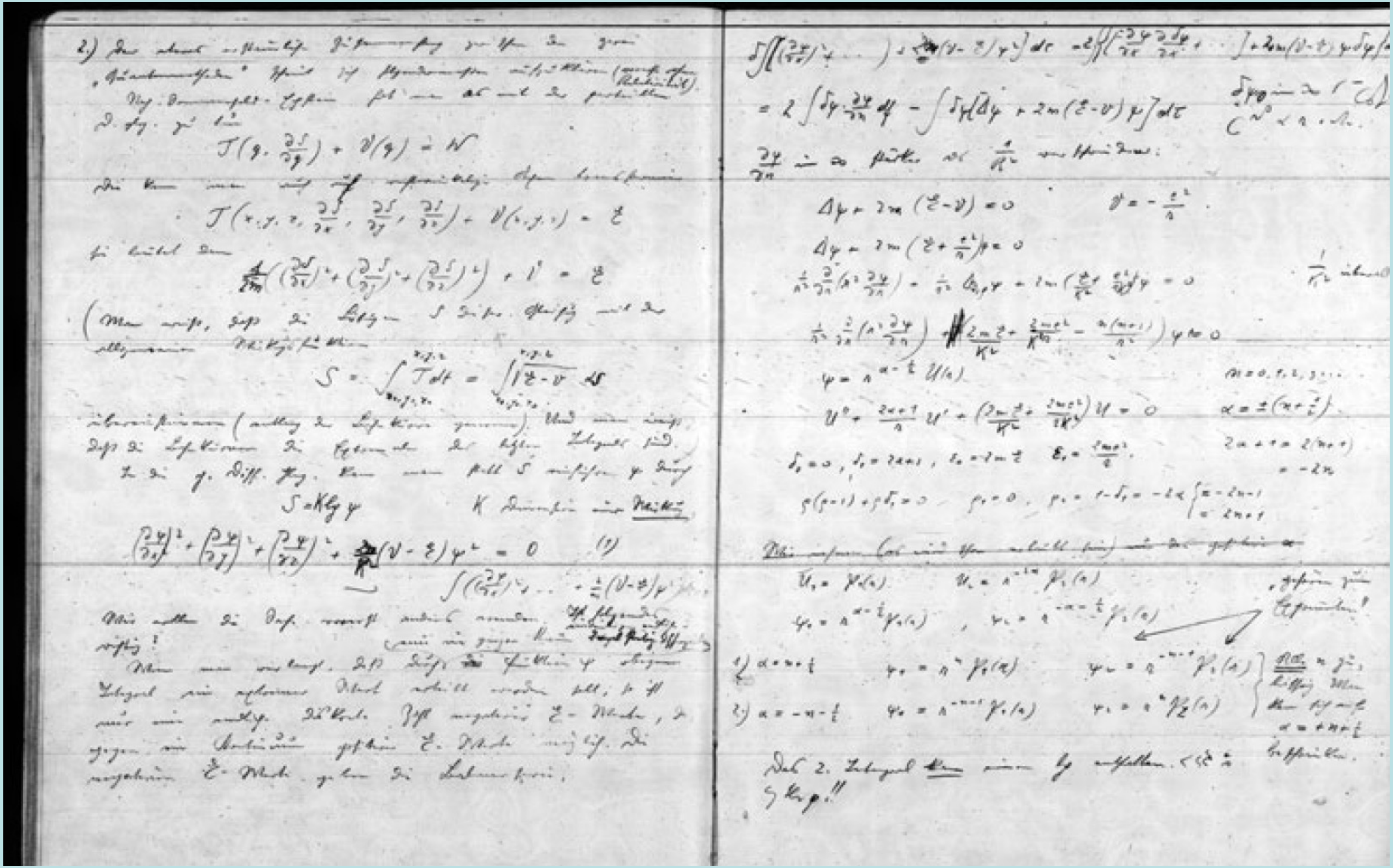
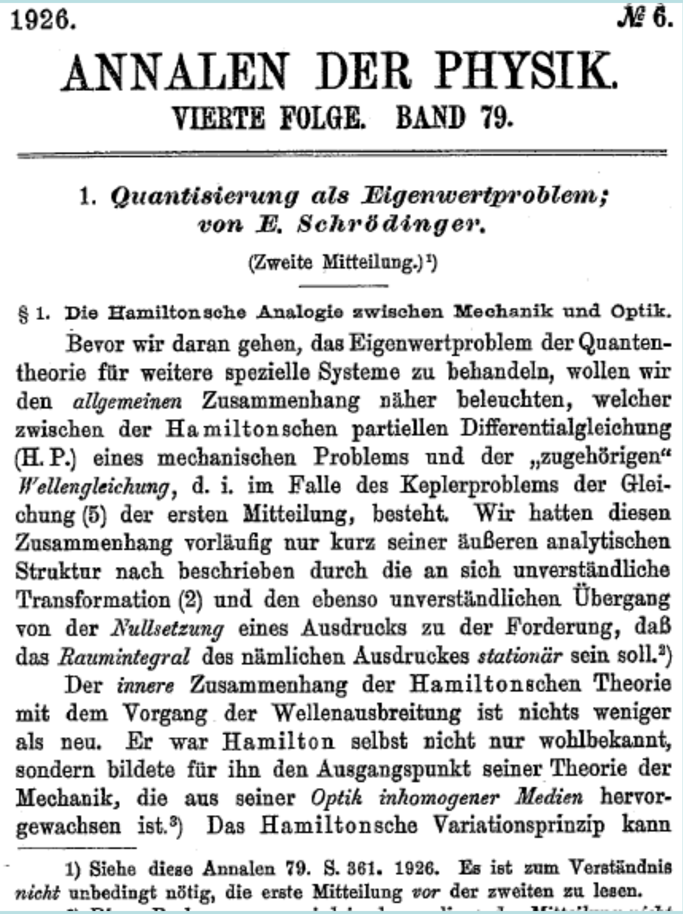


Abbildung 9. Double page from Schrödinger's notebook that very probably served as the basis for his first communication on wave mechanics (Schrödinger, 1926b) and can thus be dated to late 1925 or early 1926. On an earlier page of this notebook, Schrödinger had

# A total of four papers in 1926



1926. № 6.

# ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 79.

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## 1. *Quantisierung als Eigenwertproblem;* *von E. Schrödinger.*

(Zweite Mitteilung.)[1]

### § 1. Die Hamiltonsche Analogie zwischen Mechanik und Optik.

Bevor wir daran gehen, das Eigenwertproblem der Quantentheorie für weitere spezielle Systeme zu behandeln, wollen wir den *allgemeinen* Zusammenhang näher beleuchten, welcher zwischen der Hamiltonschen partiellen Differentialgleichung (H. P.) eines mechanischen Problems und der „zugehörigen“ *Wellengleichung*, d. i. im Falle des Keplerproblems der Gleichung (5) der ersten Mitteilung, besteht. Wir hatten diesen Zusammenhang vorläufig nur kurz seiner äußeren analytischen Struktur nach beschrieben durch die an sich unverständliche Transformation (2) und den ebenso unverständlichen Übergang von der *Nullsetzung* eines Ausdrucks zu der Forderung, daß das *Raumintegral* des nämlichen Ausdruckes *stationär* sein soll.[2]

Der *innere* Zusammenhang der Hamiltonschen Theorie mit dem Vorgang der Wellenausbreitung ist nichts weniger als neu. Er war Hamilton selbst nicht nur wohlbekannt, sondern bildete für ihn den Ausgangspunkt seiner Theorie der Mechanik, die aus seiner *Optik inhomogener Medien* hervorgewachsen ist.[3] Das Hamiltonsche Variationsprinzip kann

---

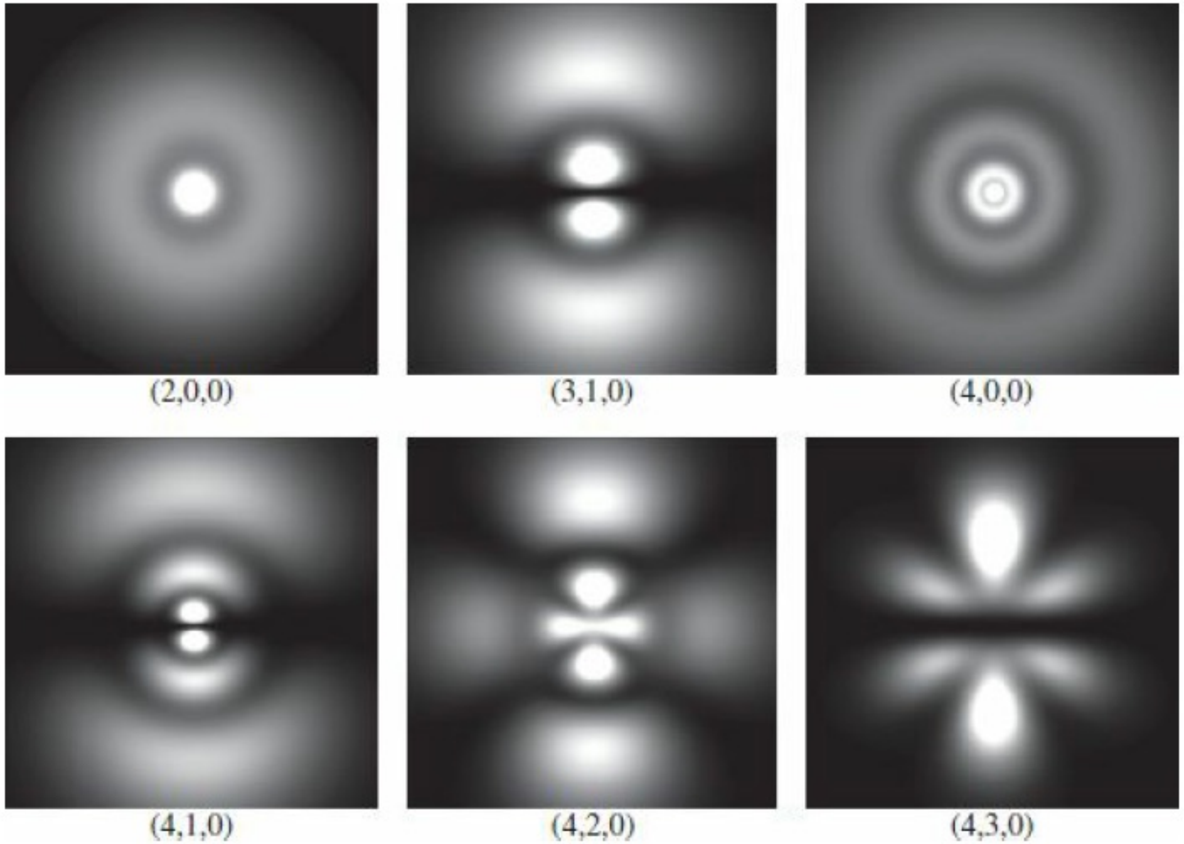
[1] Siehe diese Annalen 79. S. 361. 1926. Es ist zum Verständnis *nicht* unbedingt nötig, die erste Mitteilung *vor* der zweiten zu lesen.

無法如其他的波方程式由介質的性質推導！

根據少數的線索，猜出物質波的波動方程式。

Ψ

原子核周圍穩定的波，必需滿足波動微分方程式，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(4,1,0) (4,2,0) (4,3,0)

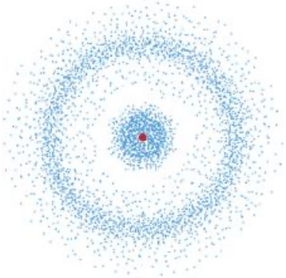
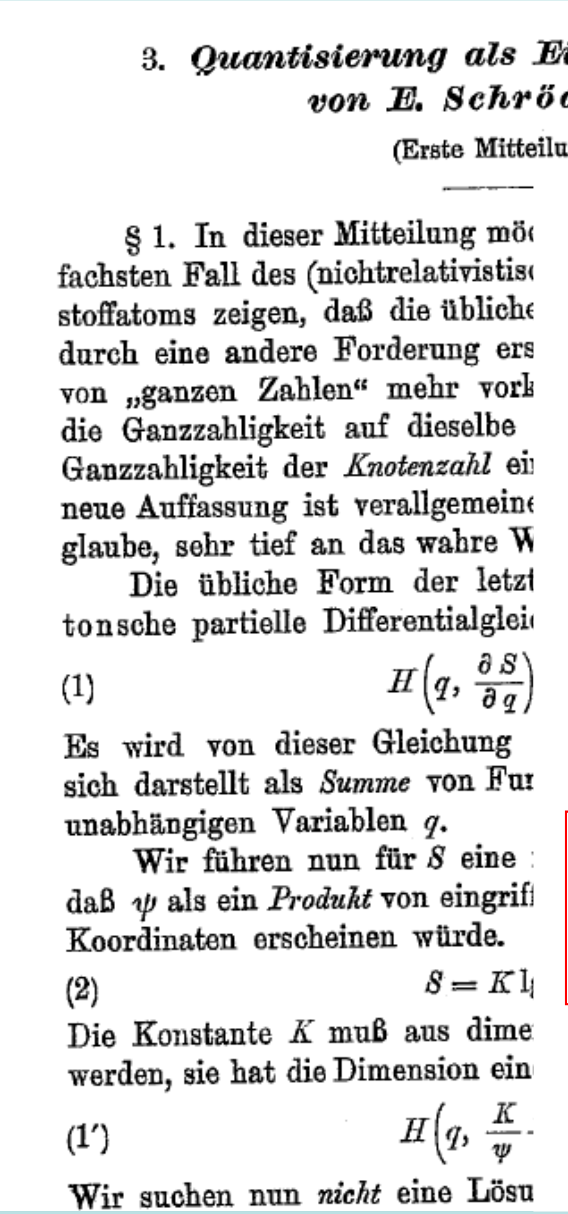


**Figure 4.8:** Density plots for the first few hydrogen wave functions, labeled by $(n, \ell, m)$. Printed by permission using "Atom in a Box" by Dauger Research. You can make your own plots by going to: http://dauger.com.



### 3. Quantisierung als Eigenwertproblem;
### von E. Schrödinger.

(Erste Mitteilung.)

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Koordinaten erscheinen würde.

(2) $S = K\, \mathrm{l}$

Die Konstante $K$ muß aus dime
werden, sie hat die Dimension ein

(1′) $H\left(q, \frac{K}{\psi}\right.$

Wir suchen nun *nicht* eine Lösu

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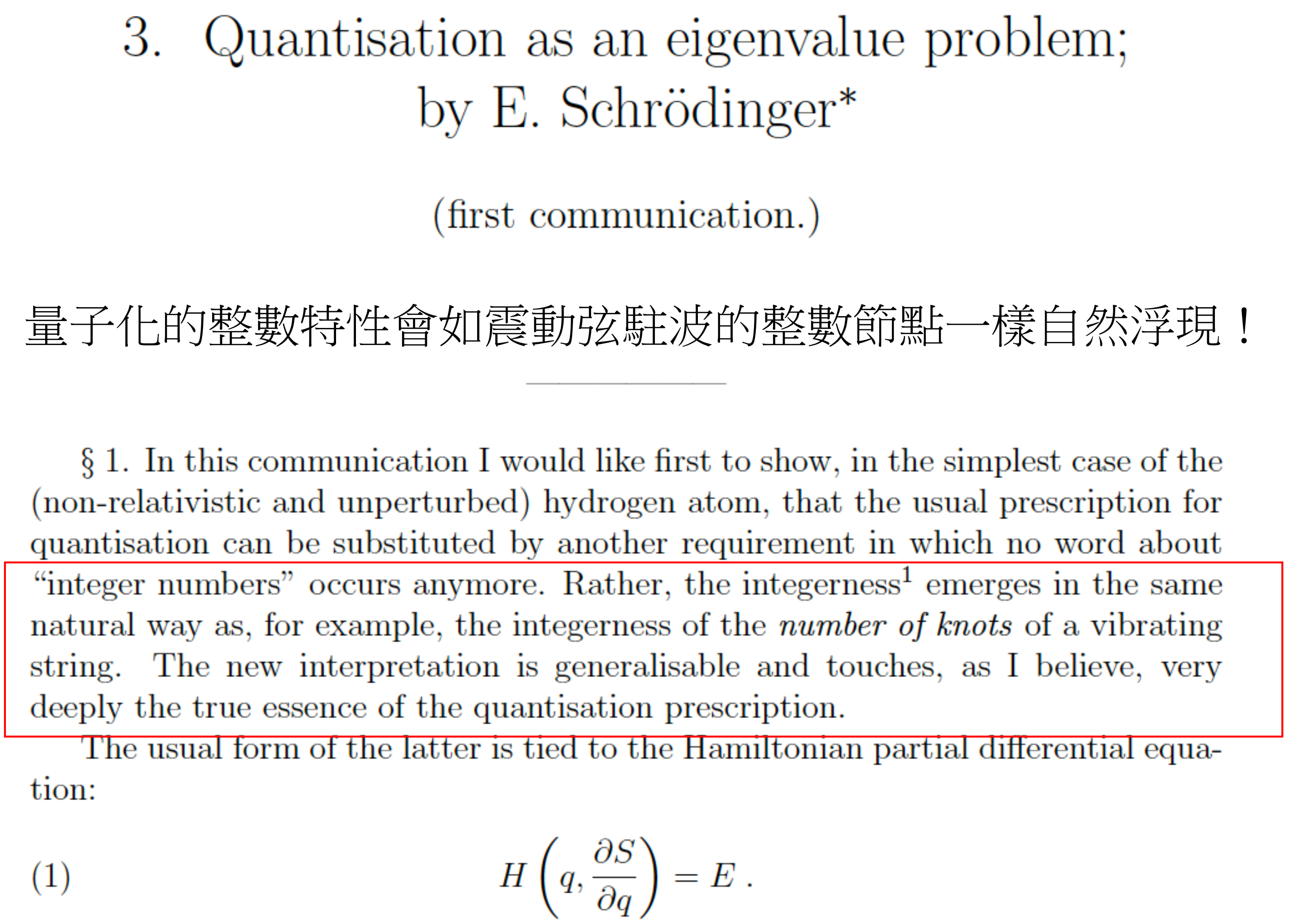

# 3. Quantisation as an eigenvalue problem;
## by E. Schrödinger*

(first communication.)

量子化的整數特性會如震動弦駐波的整數節點一樣自然浮現！

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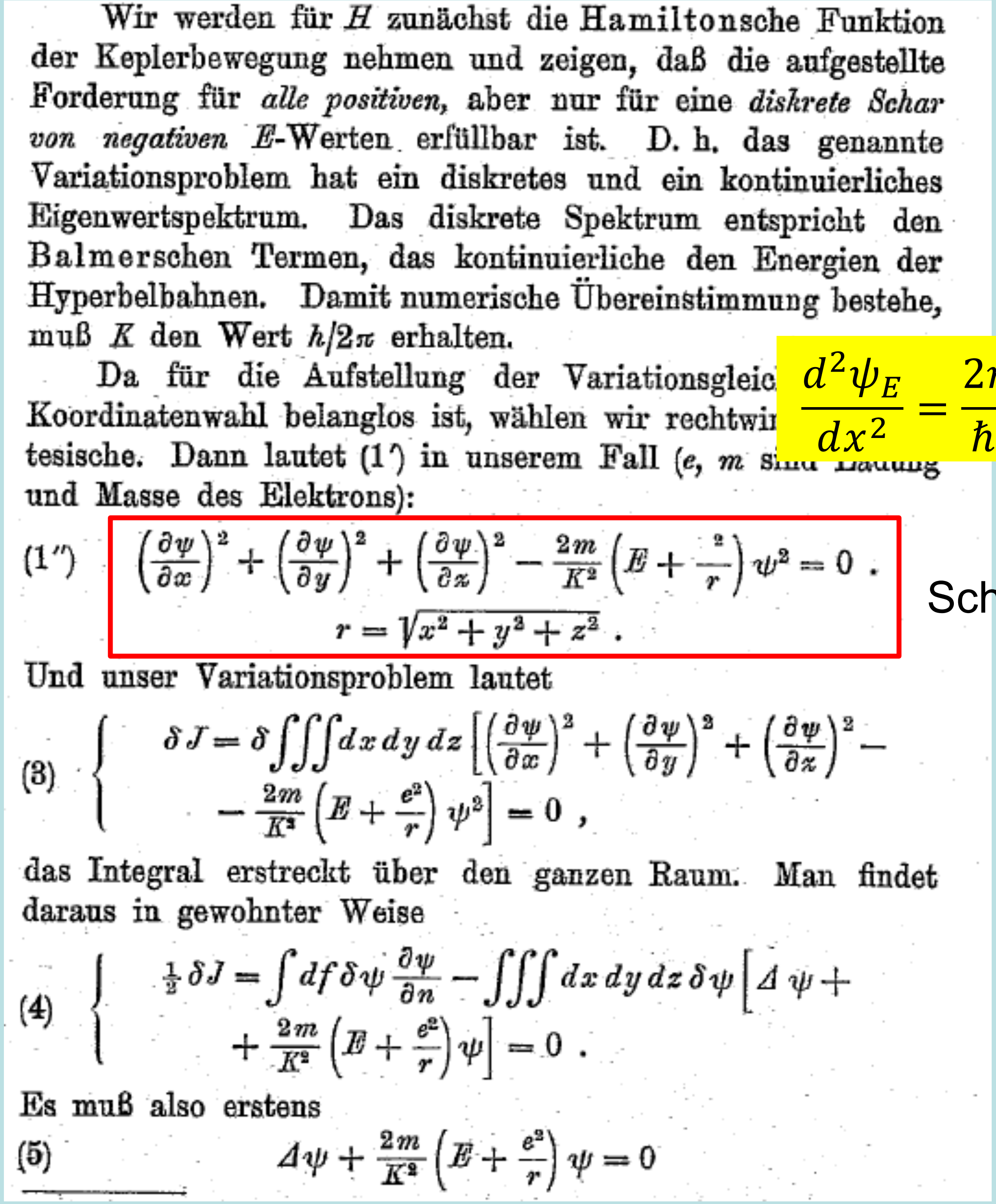
The usual form of the latter is tied to the Hamiltonian partial differential equation:

$$H\left(q, \frac{\partial S}{\partial q}\right) = E\,. \tag{1}$$



Wir werden für $H$ zunächst die Hamiltonsche Funktion der Keplerbewegung nehmen und zeigen, daß die aufgestellte Forderung für *alle positiven*, aber nur für eine *diskrete Schar von negativen* $E$-Werten erfüllbar ist. D. h. das genannte Variationsproblem hat ein diskretes und ein kontinuierliches Eigenwertspektrum. Das diskrete Spektrum entspricht den Balmerschen Termen, das kontinuierliche den Energien der Hyperbelbahnen. Damit numerische Übereinstimmung bestehe, muß $K$ den Wert $h/2\pi$ erhalten.

Da für die Aufstellung der Variationsgleic Koordinatenwahl belanglos ist, wählen wir rechtwi tesische. Dann lautet (1′) in unserem Fall ($e$, $m$ sind Ladung und Masse des Elektrons):

$$\frac{d^2\psi_E}{dx^2} = \frac{2m}{\hbar^2}[V(x) - E] \cdot \psi_E$$

$$(1'')\qquad \boxed{\begin{gathered}\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2 - \frac{2m}{K^2}\left(E + \frac{{}^2}{r}\right)\psi^2 = 0\ . \\ r = \sqrt{x^2 + y^2 + z^2}\ .\end{gathered}}$$

Schrodinger Equation

Und unser Variationsproblem lautet

$$(3)\qquad \left\{\begin{aligned} \delta J = \delta\iiint dx\,dy\,dz\Bigg[\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial y}\right)^2 + \left(\frac{\partial\psi}{\partial z}\right)^2 - \\ - \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi^2\Bigg] = 0\ , \end{aligned}\right.$$

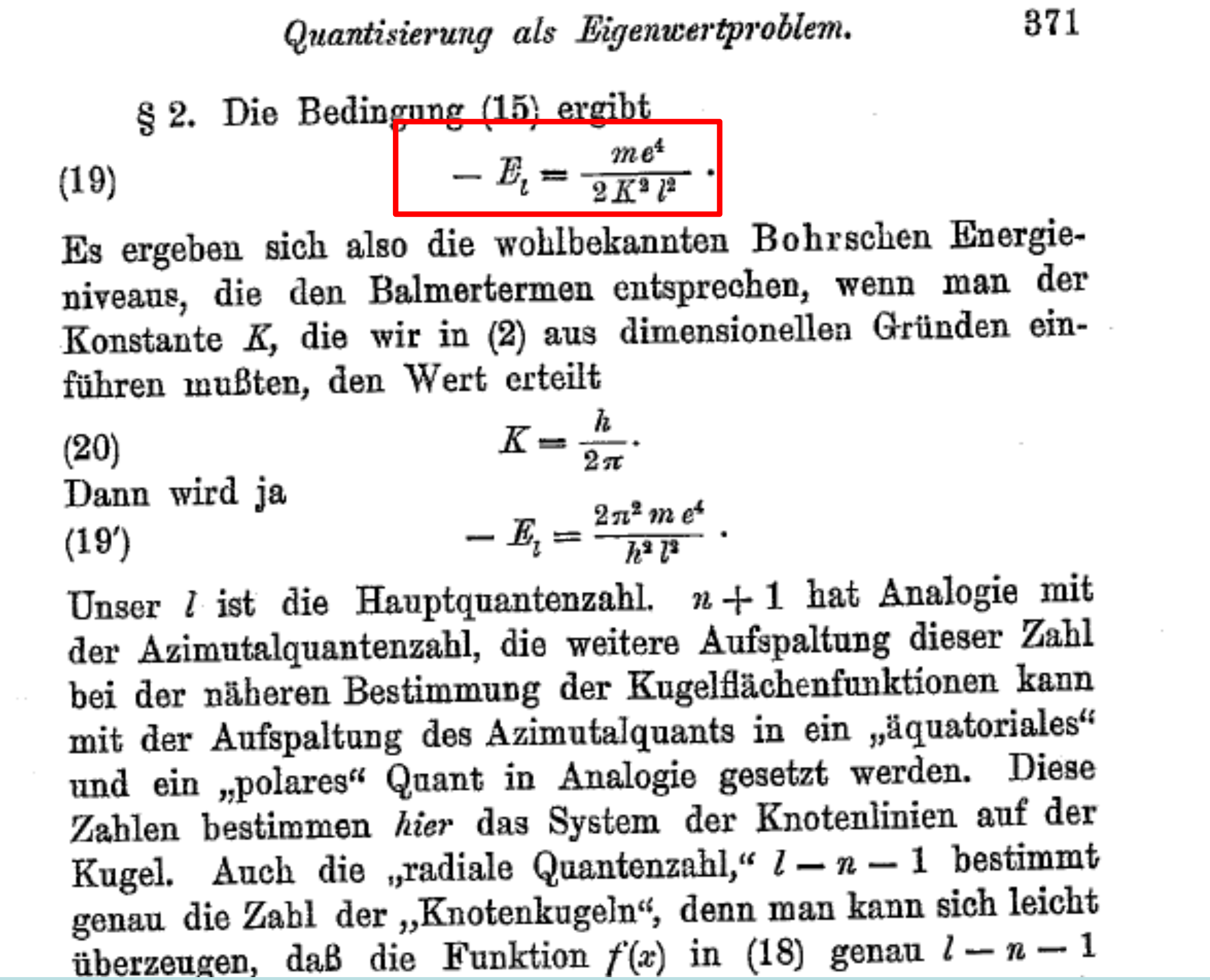
das Integral erstreckt über den ganzen Raum. Man findet daraus in gewohnter Weise

$$(4)\qquad \left\{\begin{aligned} \tfrac{1}{2}\delta J = \int df\,\delta\psi\,\frac{\partial\psi}{\partial n} - \iiint dx\,dy\,dz\,\delta\psi\Bigg[\Delta\psi + \\ + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi\Bigg] = 0\ . \end{aligned}\right.$$

Es muß also erstens

$$(5)\qquad \Delta\psi + \frac{2m}{K^2}\left(E + \frac{e^2}{r}\right)\psi = 0$$

此方程式只有對某些能量值$E$才有解！數學的本徵值問題。



*Quantisierung als Eigenwertproblem.* 371

§ 2. Die Bedingung (15) ergibt

$$\boxed{-E_l = \frac{m e^4}{2 K^2 l^2}} \tag{19}$$

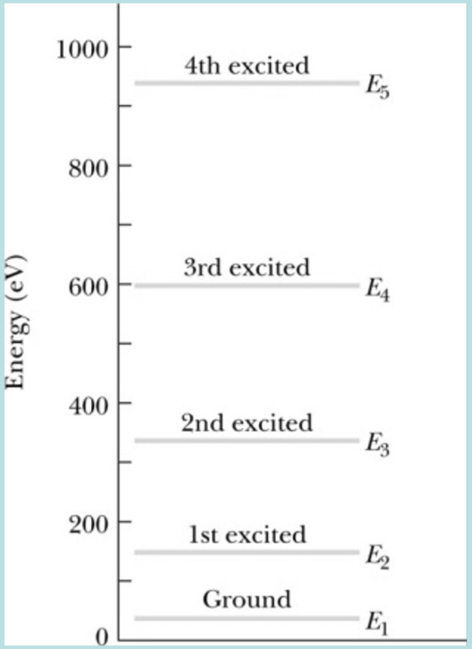
Es ergeben sich also die wohlbekannten Bohrschen Energieniveaus, die den Balmertermen entsprechen, wenn man der Konstante $K$, die wir in (2) aus dimensionellen Gründen einführen mußten, den Wert erteilt

$$K = \frac{h}{2\pi}. \tag{20}$$

Dann wird ja

$$-E_l = \frac{2\pi^2 m e^4}{h^2 l^2}. \tag{19'}$$

Unser $l$ ist die Hauptquantenzahl. $n+1$ hat Analogie mit der Azimutalquantenzahl, die weitere Aufspaltung dieser Zahl bei der näheren Bestimmung der Kugelflächenfunktionen kann mit der Aufspaltung des Azimutalquants in ein „äquatoriales“ und ein „polares“ Quant in Analogie gesetzt werden. Diese Zahlen bestimmen *hier* das System der Knotenlinien auf der Kugel. Auch die „radiale Quantenzahl,“ $l-n-1$ bestimmt genau die Zahl der „Knotenkugeln“, denn man kann sich leicht überzeugen, daß die Funktion $f(x)$ in (18) genau $l-n-1$



氫原子能階

薛丁格能嚴格地得到正確的氫原子能階是天大的成就！

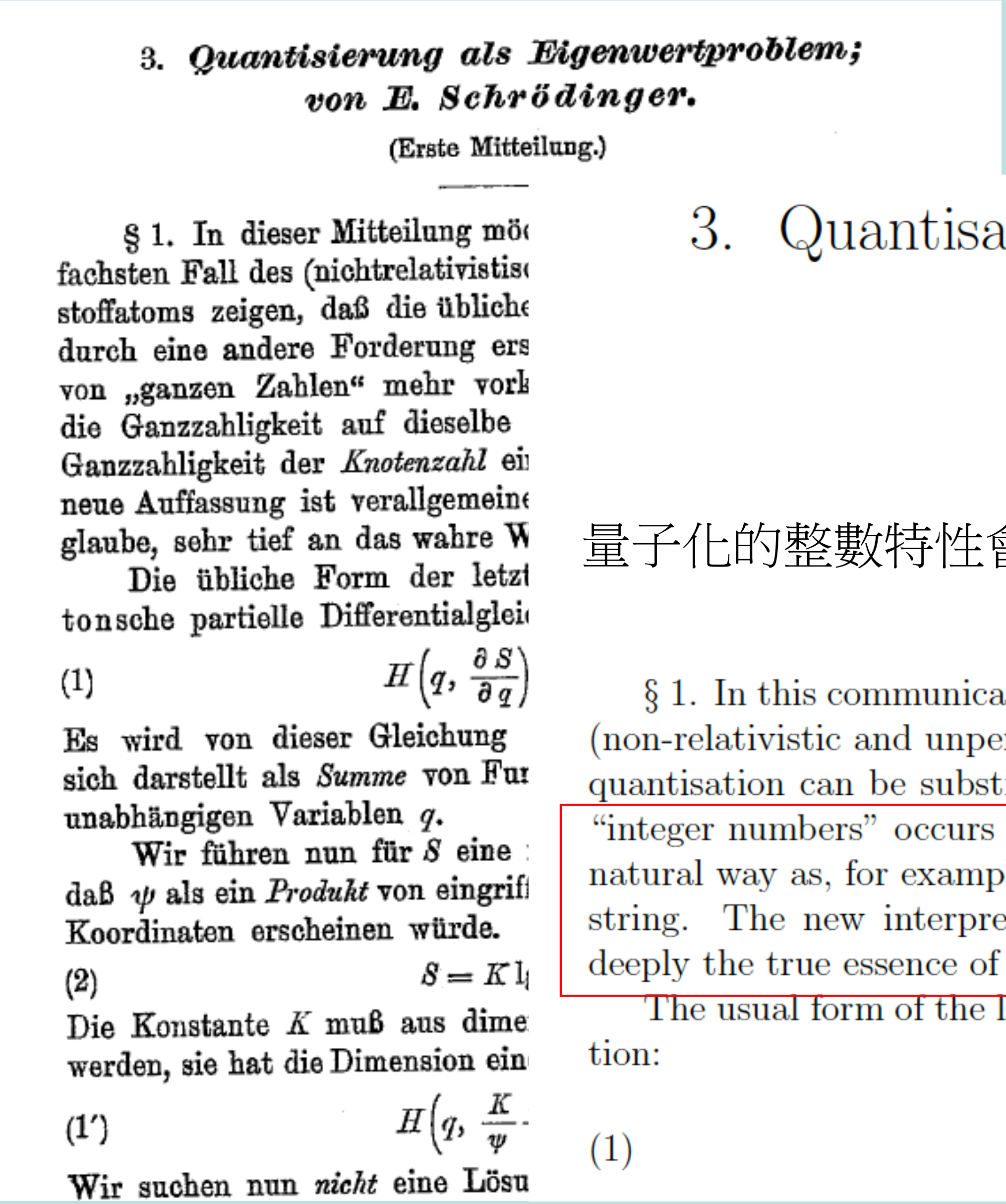
而且完全只用大家所熟悉的波的概念！

這個方法到現在都是解氫原子能階最簡單的方法。

能量的量子化原來是電子的波性的表現！

# Quantization as an Eigenvalue Problem 1926

# 能量的量子化作為（不過就是）一個本徵值問題



**3. *Quantisierung als Eigenwertproblem;***
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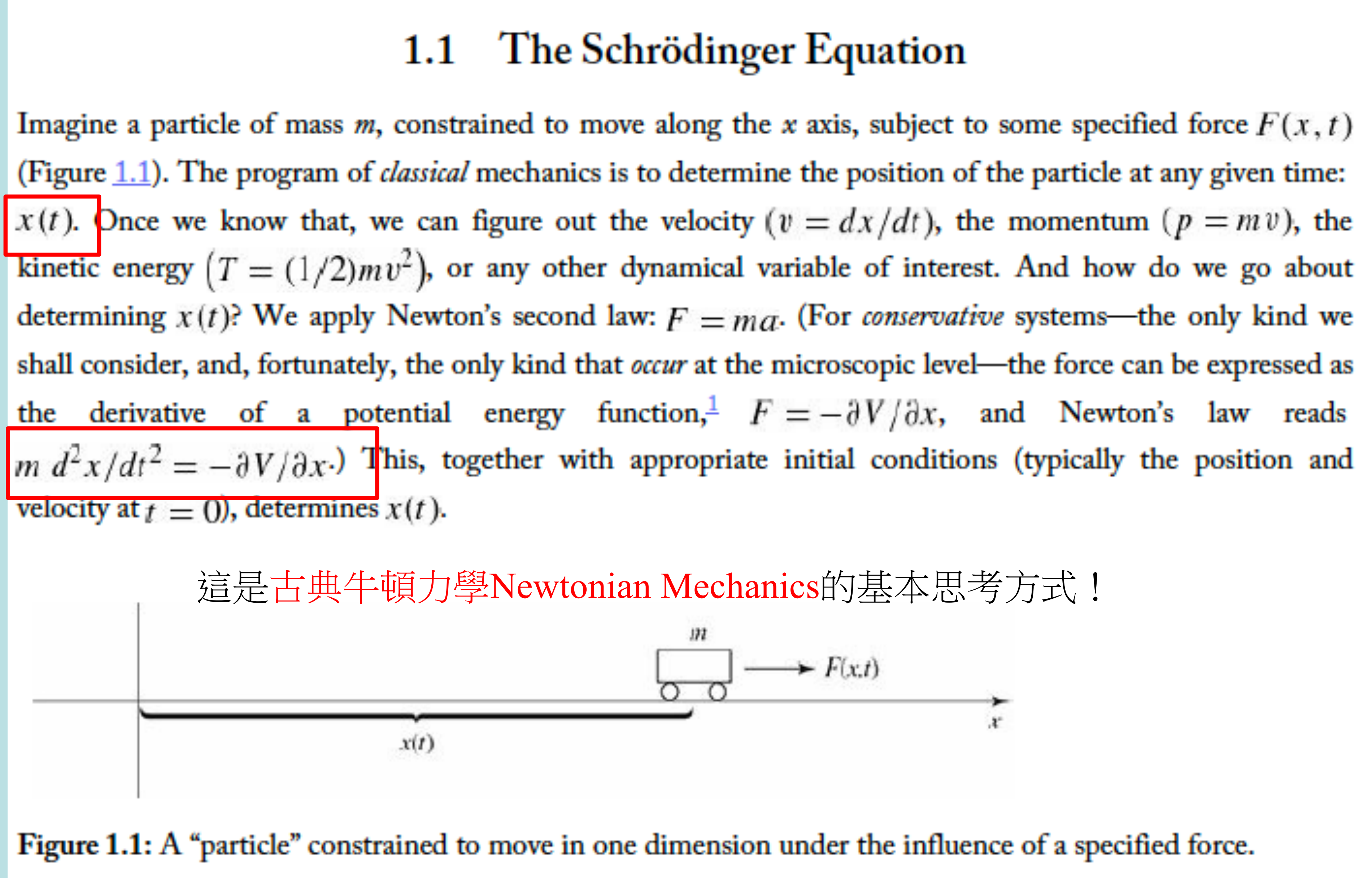
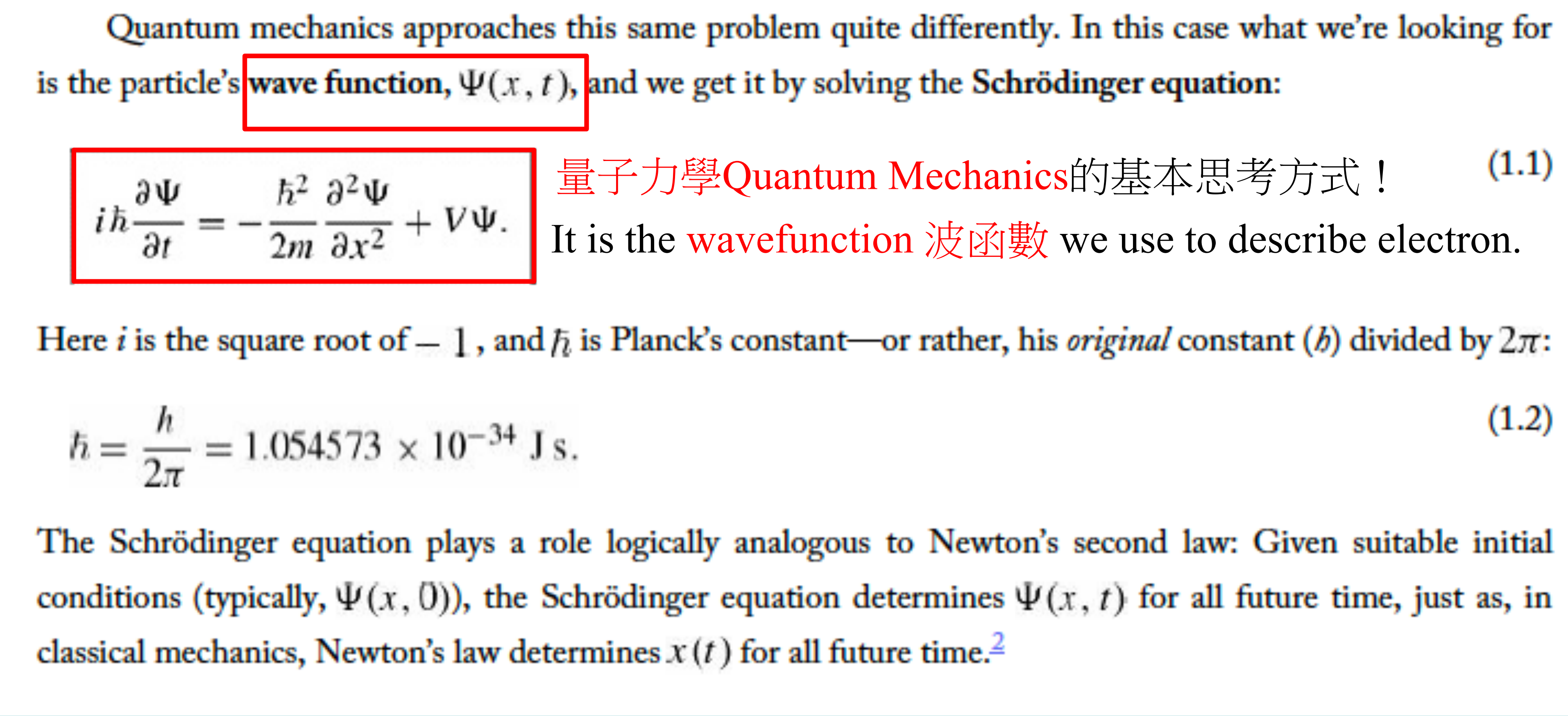


## 1.1 The Schrödinger Equation

Imagine a particle of mass $m$, constrained to move along the $x$ axis, subject to some specified force $F(x, t)$ (Figure 1.1). The program of *classical* mechanics is to determine the position of the particle at any given time: $\boxed{x(t).}$ Once we know that, we can figure out the velocity $(v = dx/dt)$, the momentum $(p = mv)$, the kinetic energy $\left(T = (1/2)mv^2\right)$, or any other dynamical variable of interest. And how do we go about determining $x(t)$? We apply Newton's second law: $F = ma$. (For *conservative* systems—the only kind we shall consider, and, fortunately, the only kind that *occur* at the microscopic level—the force can be expressed as the derivative of a potential energy function,[1] $F = -\partial V/\partial x$, and Newton's law reads $\boxed{m\, d^2x/dt^2 = -\partial V/\partial x.)}$ This, together with appropriate initial conditions (typically the position and velocity at $t = 0$), determines $x(t)$.

這是古典牛頓力學Newtonian Mechanics的基本思考方式！

**Figure 1.1:** A "particle" constrained to move in one dimension under the influence of a specified force.



Quantum mechanics approaches this same problem quite differently. In this case what we're looking for is the particle's **wave function,** $\Psi(x,t)$, and we get it by solving the **Schrödinger equation:**

$$\boxed{i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi.} \tag{1.1}$$

量子力學Quantum Mechanics的基本思考方式！

It is the wavefunction 波函數 we use to describe electron.

Here $i$ is the square root of $-1$, and $\hbar$ is Planck's constant—or rather, his *original* constant ($h$) divided by $2\pi$:

$$\hbar = \frac{h}{2\pi} = 1.054573 \times 10^{-34}\ \text{J s}. \tag{1.2}$$

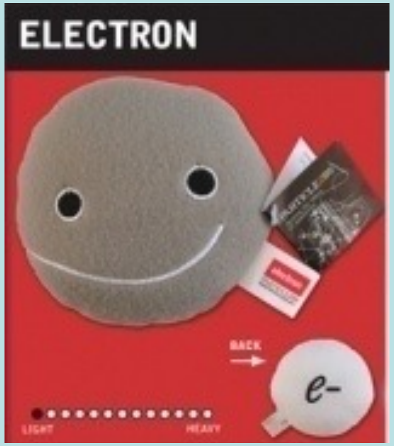
The Schrödinger equation plays a role logically analogous to Newton's second law: Given suitable initial conditions (typically, $\Psi(x,0)$), the Schrödinger equation determines $\Psi(x,t)$ for all future time, just as, in classical mechanics, Newton's law determines $x(t)$ for all future time.[2]

$x(t)$

$$m\frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x}$$

$\psi(x,t)$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$



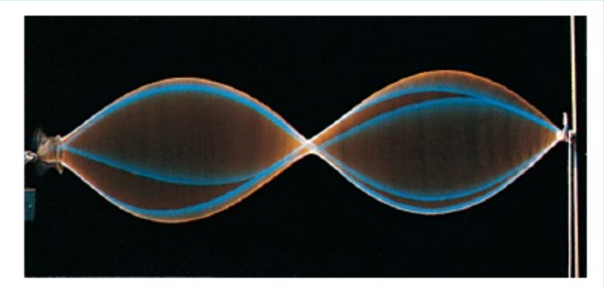
這是一個嶄新的時代開始！描述微觀世界的方法不同了！

It is the beginning of a new age!

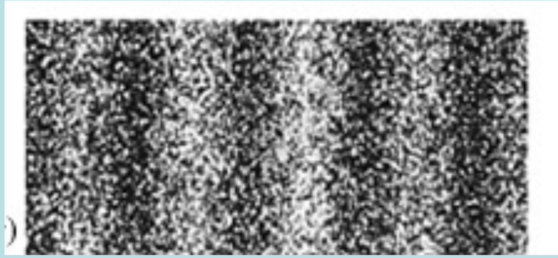
The way quantum world works is totally different.

此波方程式與古典波方程式，同是偏微分方成，式在數學上有許多相似之處！

Schrodinger Wave Equation is a Partial Differential Equation very similar to classical wave equation.



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

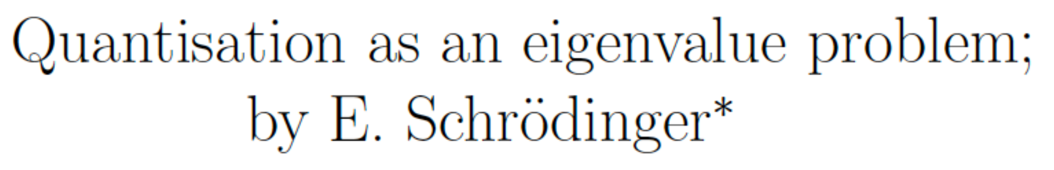


Scanned at the American Institute of Physics

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

The technique to solve them and their solution are very similar, too.

The technique: Separation of variables and Eigenvalue problem.



Quantisation as an eigenvalue problem;
by E. Schrödinger*

Eigenvalue problems 這個字再次重複出現！

數學物理的最重要洞見就是：這兩個解物理問題的絕招：

Differential Equation (Ordinary and Partial) 微分方程式

Vector and Matrix 向量與矩陣

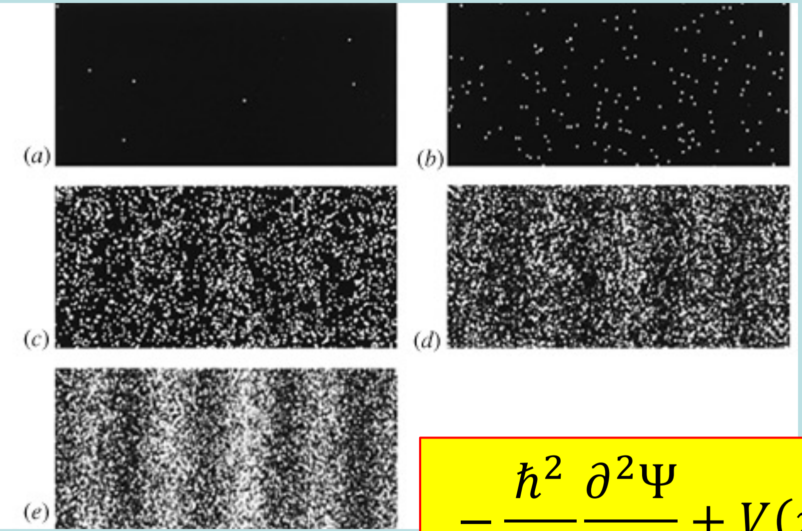
微分方程式與矩陣，竟然有一個共通的數學結構：

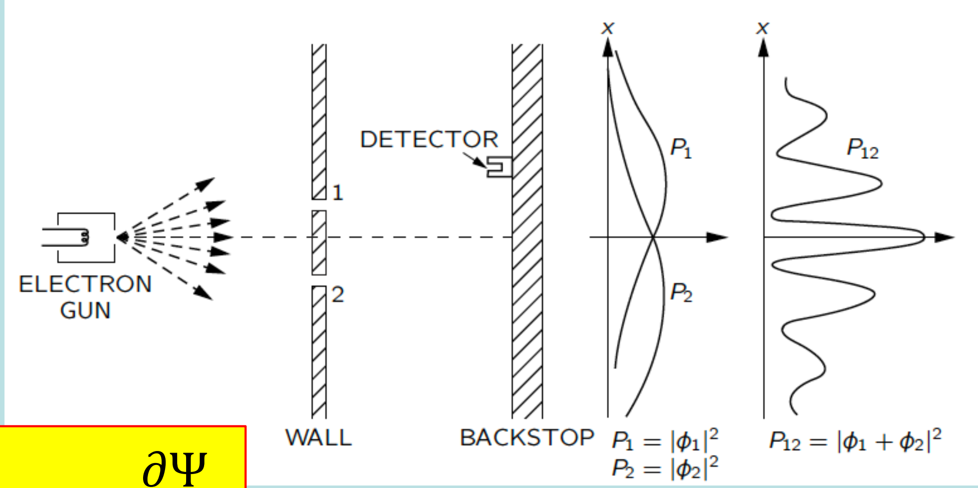
Eigenvalues and Eigenfunctions 本徵值與本徵函數

這個課的課名應該叫：“本徵值問題”

What is the wavefunction Ψ exactly? Could we observe it?

Since it is a complex number function, we can never observe it.





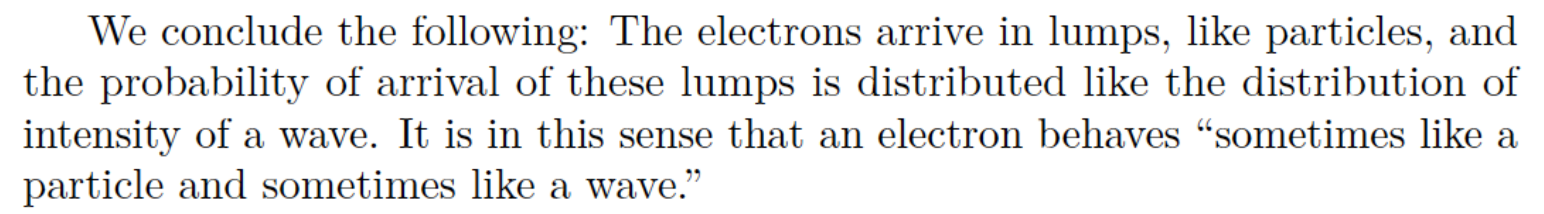
$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$P_{12} \neq P_1 + P_2$$

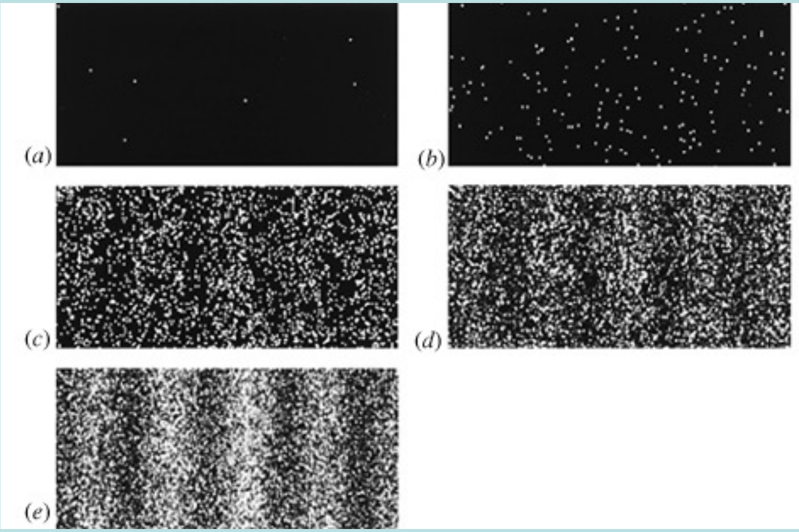
干涉的條紋分佈得用波函數的疊加說明。$\Psi_{12} = \Psi_1 + \Psi_2$

進一步證實了波的傳播是適用於電子在撞擊觀察屏幕前的運動。

屏幕上波函數的強度是正比於該處發現電子的機率！



We conclude the following: The electrons arrive in lumps, like particles, and the probability of arrival of these lumps is distributed like the distribution of intensity of a wave. It is in this sense that an electron behaves “sometimes like a particle and sometimes like a wave.”



然而，觀察到的電子都是呈現顆粒狀，

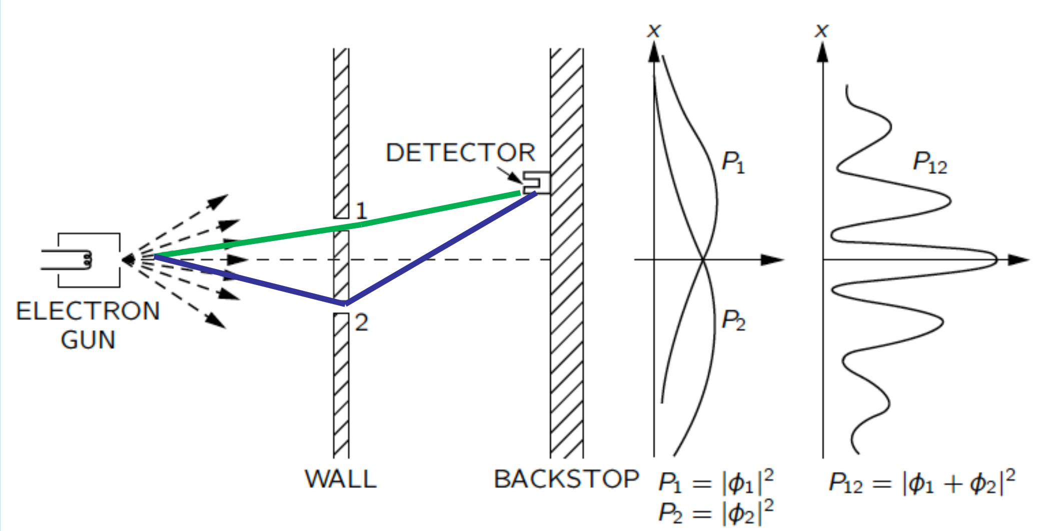
電子是如粒子般無法分割的！

我們似乎無可避免必須接受粒子狀的波，或波狀的粒子！

Electron is a particle and a wave.

It is impossible to be both particle and wave.

但，波性與粒子性是幾乎無法相容的！



一個顆粒一直都有一個特定的位置！

在此處就不能同時在彼處

劈腿不可行原則！

實驗結果 $P_{12} \neq P_1 + P_2$

每一個粒子不是從狹縫一就是從狹縫二通過。只能擇一

矛盾

如此你可以對到達屏幕的電子分類！

分類後可加總！因此：

$$P_{12} \cancel{= P_1} + P_2$$

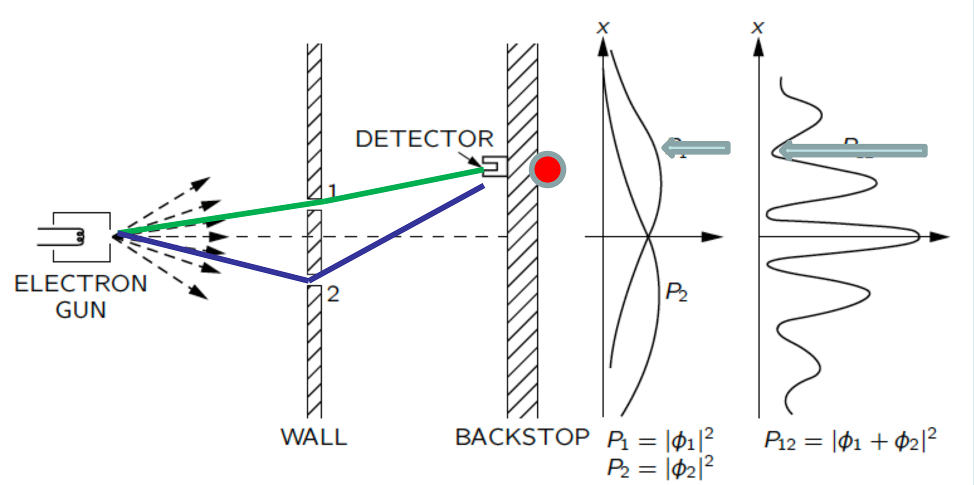
電子到達狹縫屏幕時沒有特定的位置可言！

電子有劈腿的行徑！

這是一個很詭異的情況！。

依照有特定位置粒子的常理：從狹縫一通過的電子並不在意狹縫二是否打開。

所以兩狹縫都開時，屏幕上任一處的粒子數必大於只開一個狹縫。



照理說 $P_{12} \geq P_1$

但在干涉暗點，我們發現：

$P_{12} < P_1$

狹縫二打開後，粒子數竟然減少！

從狹縫一通過的電子竟然知道狹縫二是否打開。電子竟能自己跟自己干涉。

單顆電子就可以感覺整個空間的狀態！它瀰漫在空間中，如波一般！

以上的問題對波是一點問題也沒有，波本來就可以有來自兩狹縫的貢獻。

波天生就是劈腿的。

在撞到屏幕前電子的狀態是兩個不相容的狀態$|1\rangle, |2\rangle$的疊加！

$$|1\rangle \quad + \quad |2\rangle \qquad \sim \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$$

【物理好好玩S1EP07】電子的劈腿問題，它是一種波嗎？

曾幾何時，我們的生命已一天都離不開電。電就是電子的流動，因此我們得認識電子。在四月的節目〈玩骰子的上帝——從放射性原子核談起〉中，我們已經介紹了，電子與原子核組合成了原子，而原子就組成所有的物質。但電子究竟是什麼？最自然的想像，我們都覺得電子是像撞球一樣的粒子。這樣的圖像很有用，因為電子的首要特徵，就是它如撞球一樣，完全不能分割。當然如果能量用得大一點，撞球還是可以被切碎的。但物理學家就算用極大的能量，到今天為止，還是無法將電子切開。換言之，我們從沒有看到過半個電子，而且無論你是在何處找到它，每一個電子都是一模一樣的。它的性質完全無法微調，輕一點、重一點都不可能，每一個都貨真價實。

撞球還有一個特點，每一個時刻它只有一個特定的位置，在這裏，就不能同時在那裡，在位置這件事上，它是不能劈腿的。二十世紀初的物理學家竟然發現，電子不完全是這樣。

**電子如果是波，劈腿就名正言順了**

最早提出電子有劈腿嫌疑的是一位法國的王子，德布羅意。德布羅意家族從十六世紀起就是法國的公爵，德布羅意是家中的老二，所以是王子，後來哥哥過世了，他才繼承了爵位。原本德布羅意是學歷史的，但受到哥哥的影響，改學物理。他的博士論文提出了一個駭人聽聞的想法，電子應該是一個波。請聽眾注意，這時沒有任何實驗證據顯

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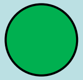
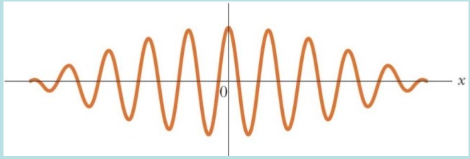
It is impossible to be both particle and wave. Unless…….

電子是一個怪異的東西，它又像粒子，又像波





可是這兩個不相容的圖像都必須捨去一些特性才能並存！

電子是粒子，但此粒子的位置與動量不能同時精確測量。

電子是波，只是電子的無法觀察測量，一測量，電子就以粒子的型式出現。

# 電子的真面目

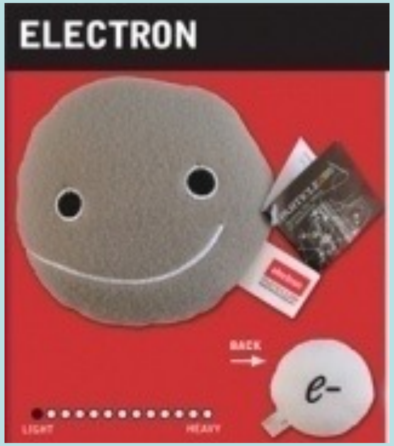
波的強度等於觀察時在該處發現此粒子的機率！

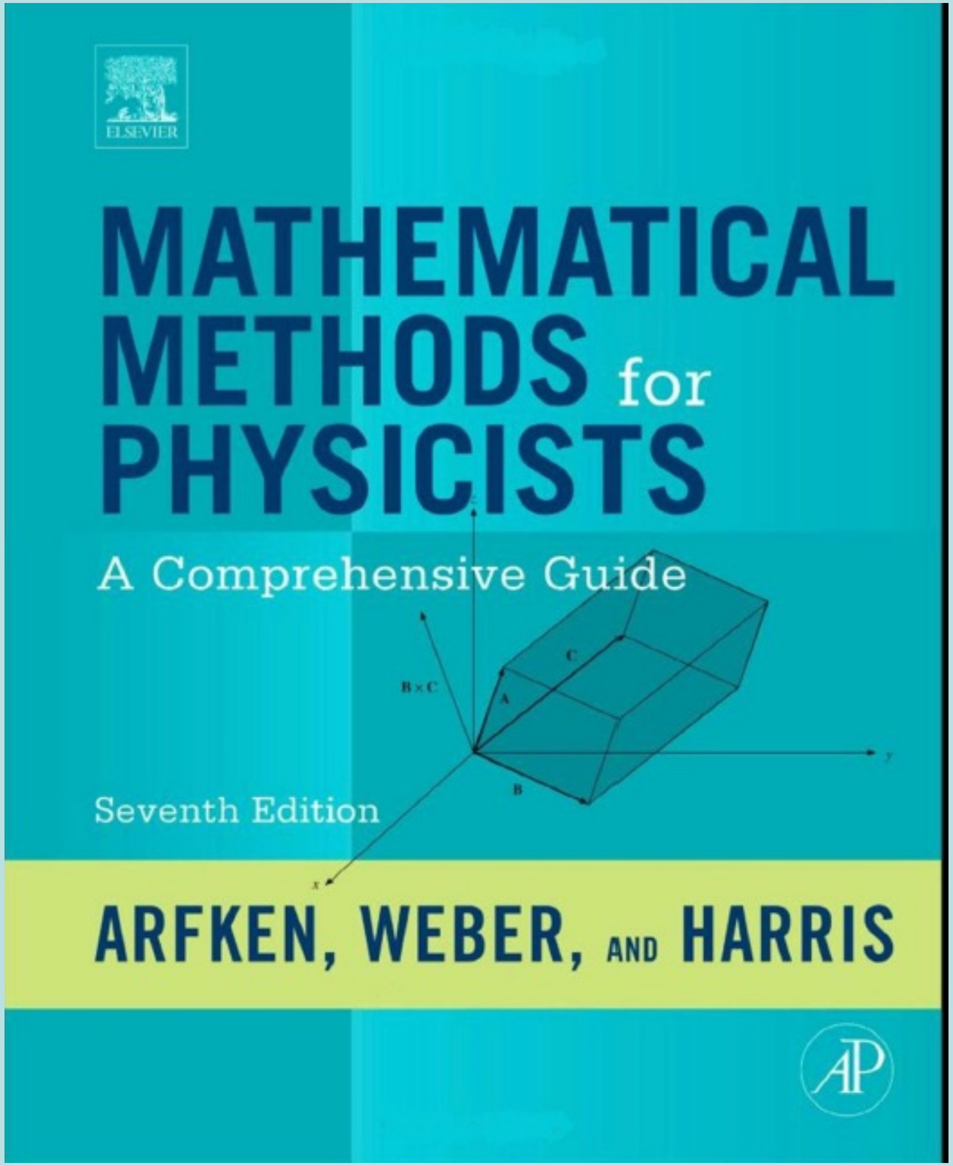
(未觀察時）
狀態的變化是以波方程式來計算

觀察時電荷及位置總是顆粒狀

由波函數$\psi(x,t)$來描述的粒子

（而不是位置函數 $x(t)$ ）





ELSEVIER

MATHEMATICAL
METHODS for
PHYSICISTS

A Comprehensive Guide

Seventh Edition

ARFKEN, WEBER, AND HARRIS

我們將先從古典普通物理中已經介紹過、需要較進階數學方法的問題開始暖身：

Ordinary Differential Equation 常微分方程式

1. Decay Equation, First order Linear Ordinary Differential Equation.

2. SHO, damped SHO and Forced Oscillation equation. Second order Linear ODE.

3. Fourier Series. Vectors.

4. Coupled SHO. Systems of second order ODE’s.

5. Vectors, Matrices, linear Equation and Eigenvector Eigenvalue problem.

Partial Differential Equation.

1. 3D Vector Calculus: Gradient, Divergence, Curl. Gauss’ Stokes Theorem.

2. Maxwell Equation by 3D vector calculus notations. Feynman Lectures V. 2 Ch. 1,2,3.

3. Electromagnetic Wave Equation. Partial Differential Equation.

4. Separation of variables. Fourier Series and Eigenvalue problems, similar with matrix.

Eigenvalue problems 這個字重複出現！

第二個數學主題：Linear Algebra 線性代數

量子物理

5. Quantum mechanics basics: Particle, wave and probability interpretation.

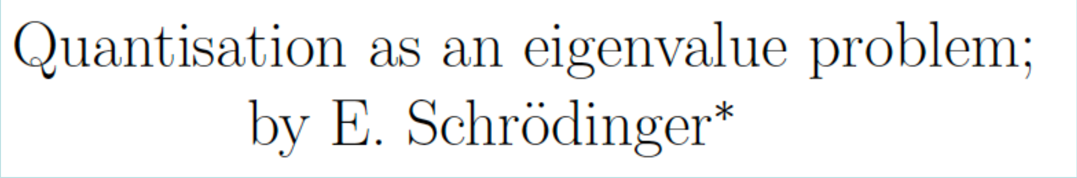
6. Schrödinger Wave Equation.

7. Separation of variables. Fourier Series and Eigenvalue problems, similar with matrix.

求解偏微分方程式、線性代數方程式與矩陣，竟然有共通的數學結構：

Eigenvalues and Eigenfunctions 本徵值與本徵函數，

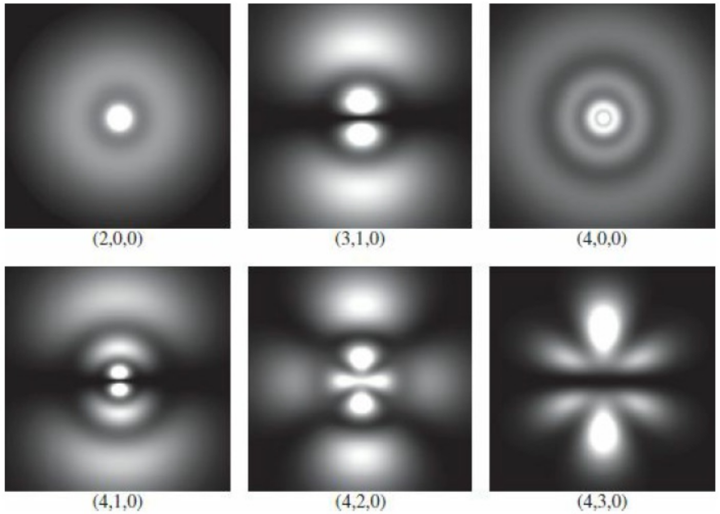
這個課的課名應該叫：“本徵值問題”


Quantisation as an eigenvalue problem;
by E. Schrödinger*

Linear Algebra 線性代數與偏微分方程式的本徵值問題。

氫原子內電子需要滿足的定態波方程式

## 4.2 The Hydrogen Atom


(2,0,0) (3,1,0) (4,0,0)

(4,1,0) (4,2,0) (4,3,0)

Figure 4.8: Density plots for the first few hydrogen wave functions, labeled by $(n, \ell, m)$. Printed by permission using "Atom in a Box" by Dauger Research. You can make your own plots by going to: http://dauger.com.

作了Separation of variables後，方程式化簡為Second order ordinary differential equations.

標準的求解方法稱為Method of power series.

得到的解與古典波方程式在導電圓球內的解幾乎相同，稱為：Special Functions.

了解Special Functions需要的數學工具就是複變函數Functions of complex variables

接著連結引入量子力學的薛定格波方程式，描述了解這個波方程式所需要的機率解釋，以及測不准原理扮演的角色。相關的解偏微分方式的方法，將帶領進入數學的分析。

很快我們就會發現解薛定格波方程式最好的方法就是利用定態，這是能量的本徵函數。於是整個課程的核心：本徵問題的介紹就可以開始。抽象的線性代數是接下來的重點，線性變換將都可以以矩陣表示，所以本徵問題在矩陣問題的表現將非常清晰。量子力學的自旋問題會是一個重要的例子。

接著本徵問題會被擴展到波函數空間，寫下氫原子內電子需要滿足的定態波方程式。以此具體問題出發，介紹特殊函數與處理的方法。

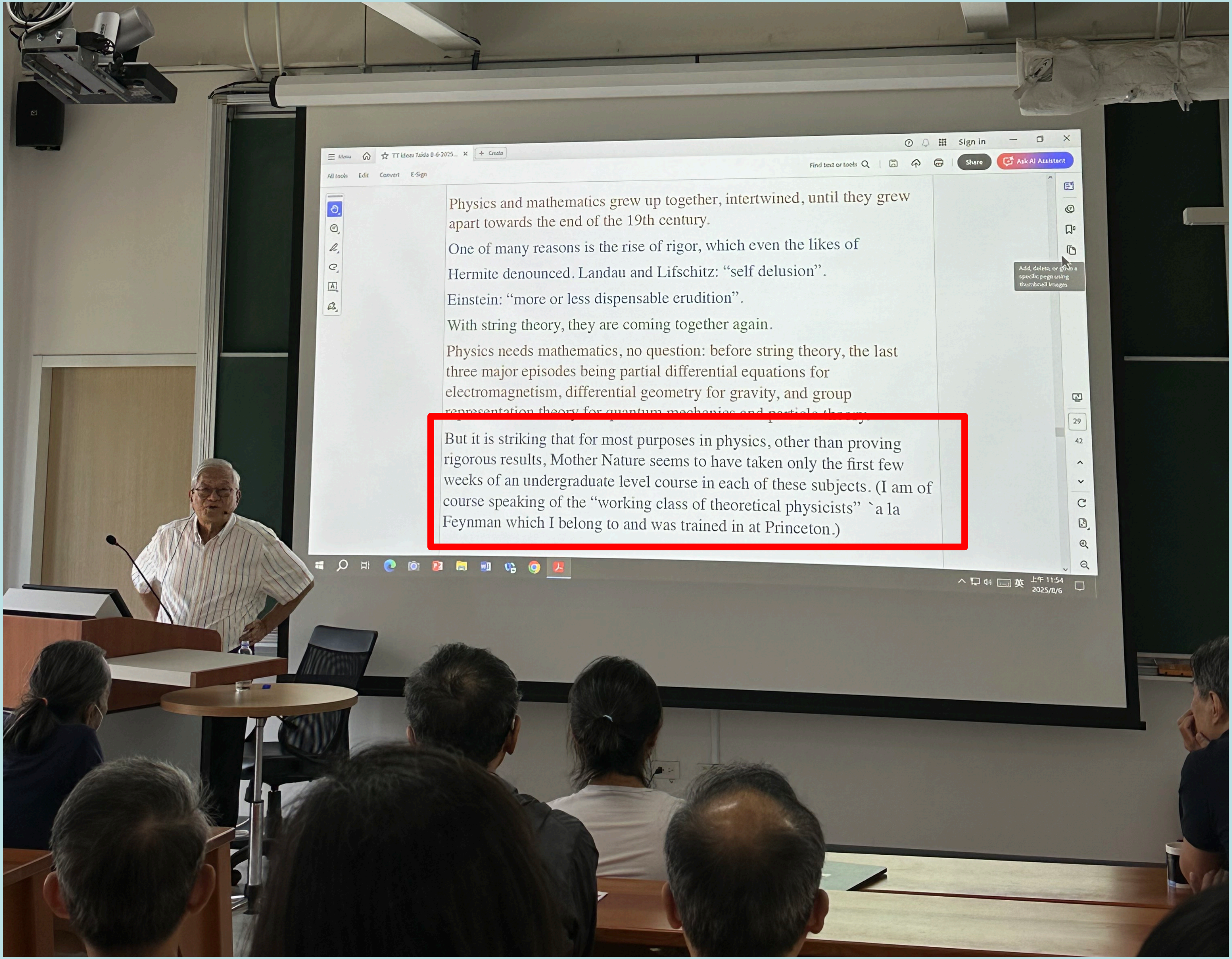
最後以複變函數的方法做結，說明複變函數可以對許多數學問題，包括特殊函數有非常有用的效果。

偏微分方程式 Partial Differential Equation

線性代數 Linear Algebra

複變函數 Functions of complex variables

但我們不會堅持完整的介紹這幾個科目。因為物理需要的沒那麼多！



Physics and mathematics grew up together, intertwined, until they grew apart towards the end of the 19th century.

One of many reasons is the rise of rigor, which even the likes of Hermite denounced. Landau and Lifschitz: "self delusion".

Einstein: "more or less dispensable erudition".

With string theory, they are coming together again.

Physics needs mathematics, no question: before string theory, the last three major episodes being partial differential equations for electromagnetism, differential geometry for gravity, and group representation theory for quantum mechanics and particle theory.

But it is striking that for most purposes in physics, other than proving rigorous results, Mother Nature seems to have taken only the first few weeks of an undergraduate level course in each of these subjects. (I am of course speaking of the "working class of theoretical physicists" `a la Feynman which I belong to and was trained in at Princeton.)