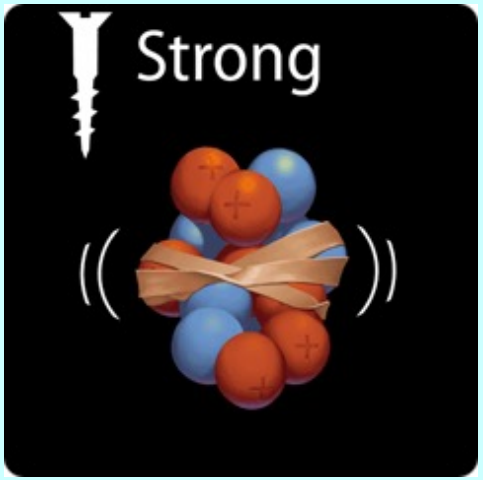
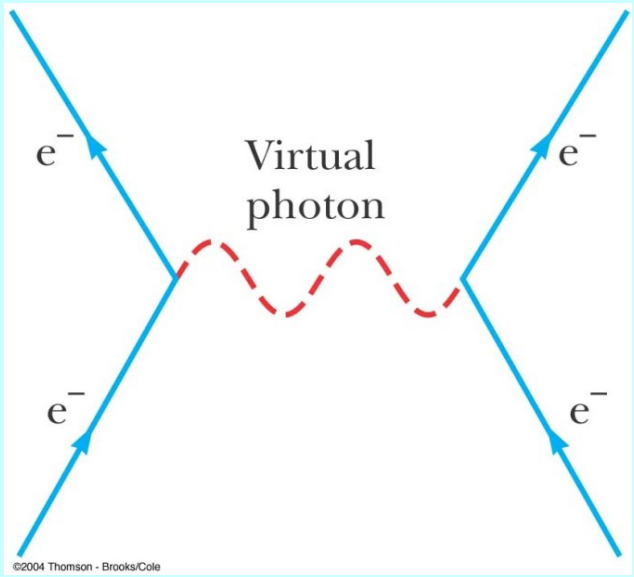
歡迎掃描 QR Code
填寫問卷分享心得！

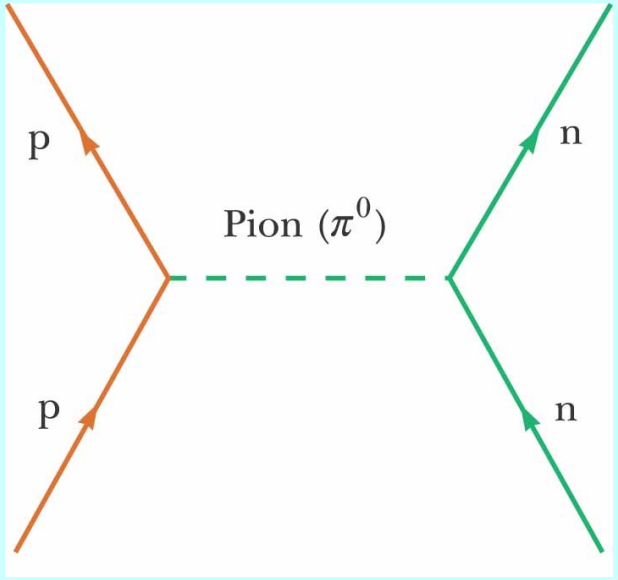
強作用力真是奇特……..

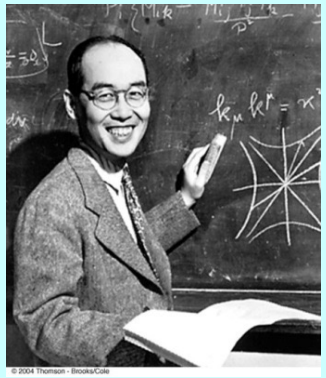


Strong is Colorful 強是色彩繽紛的。

Pions π







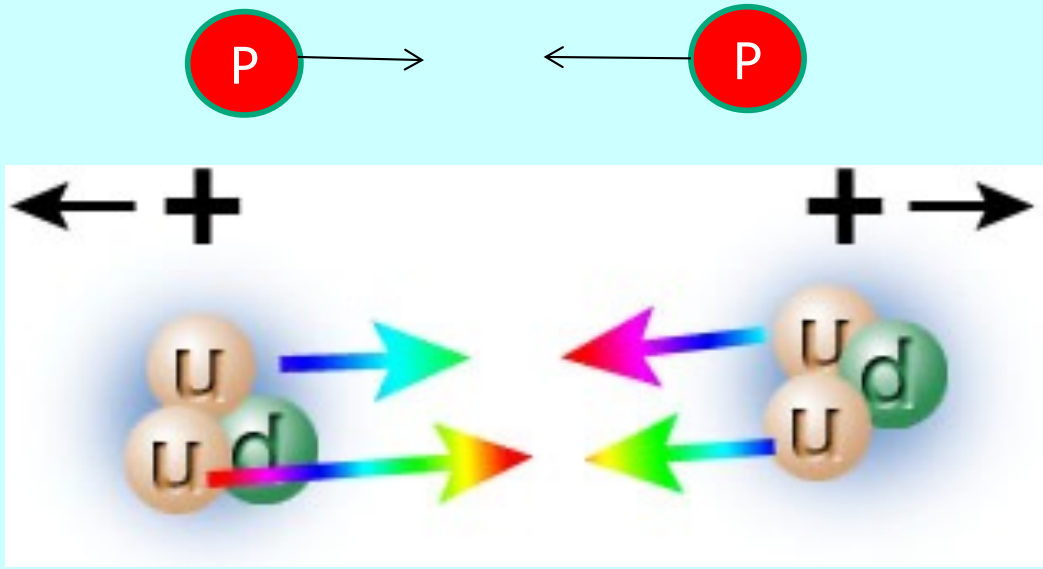
湯川建議有一個新粒子如光子媒介電磁力一樣，媒介核力。

This is ground breaking since pion would be first fundamental particle not existing in nature.

This idea of mediation particle as interaction become a paradigm.

他進一步預測這個新粒子的質量介於電子與質子之間：~100Mev.

以π子為媒介粒子的圖像並不是基本的，也不是特別有用。



如果質子是由更基本的夸克所組成.....那麼

質子質子間的核力，分解開後應是介於夸克與夸克間的力。

這個夸克之間很強的作用，將夸克束縛在微小的重子與介子中。

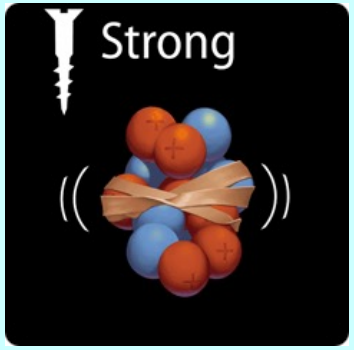
而且更糟的是：量子場論依賴微擾計算。

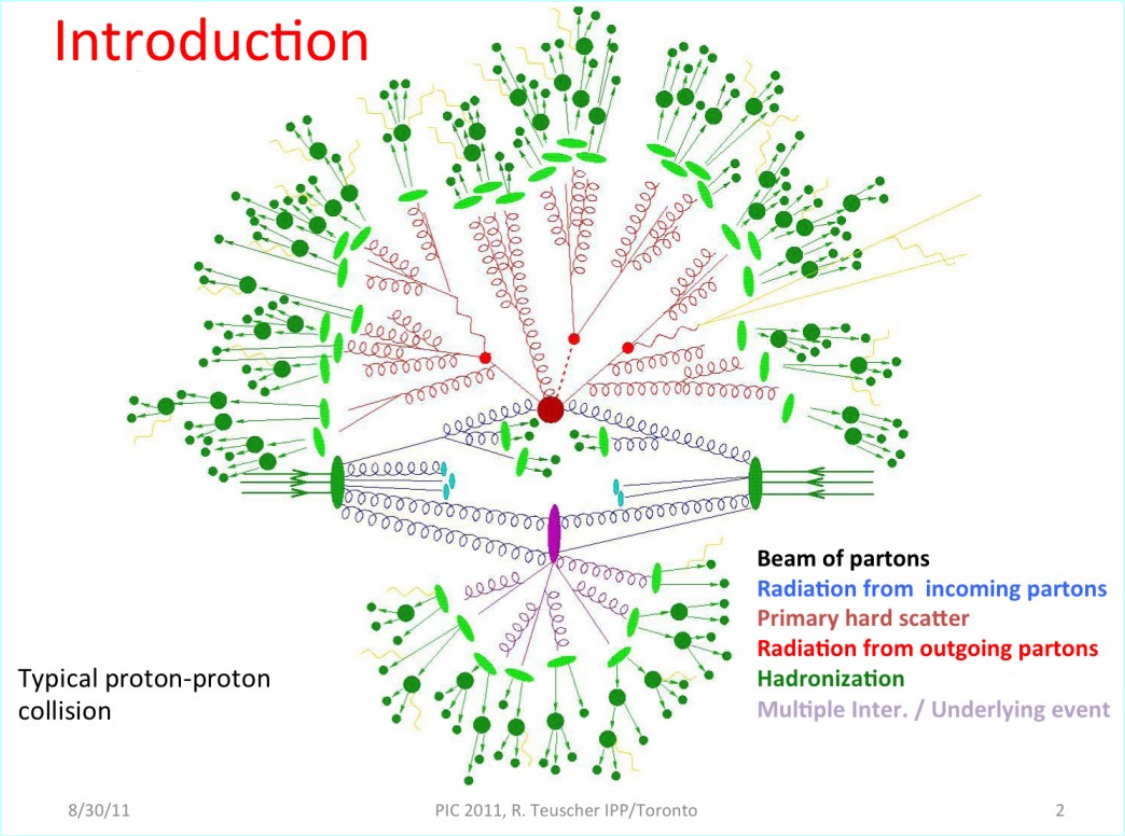
若是弱交互作用（$g \ll 1$），越複雜的圖的貢獻越小（$g^4 \ll g^2$），

對特定的精確度，只需有限的圖即可。

但強交互作用是強交互作用！ $g \gg 1$。

Feynman Diagrams的微擾展開，是既發散，又非微擾。

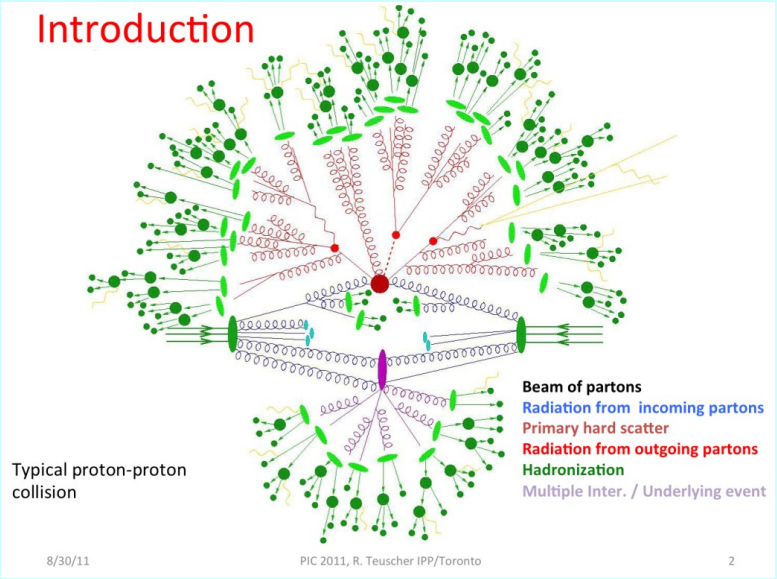




質子碰撞的費曼圖

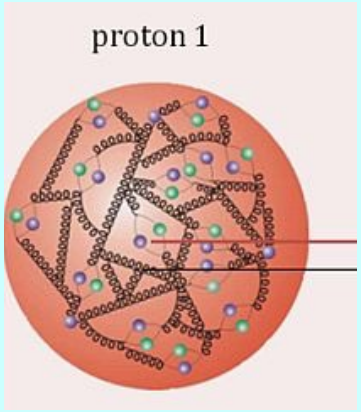
這樣的圖除了示意之外，

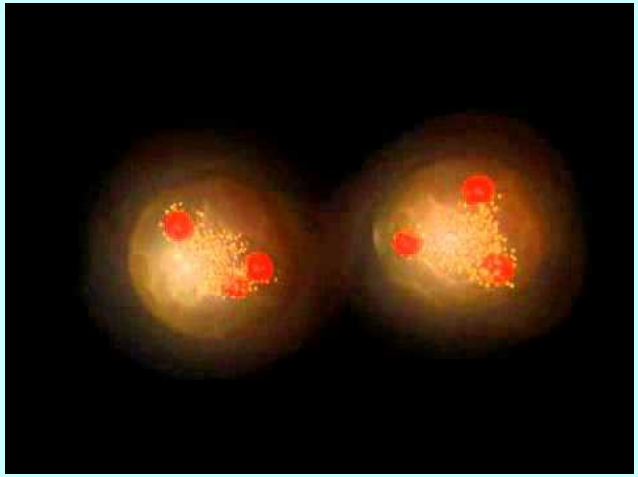
其實沒有甚麼計算的誠意。

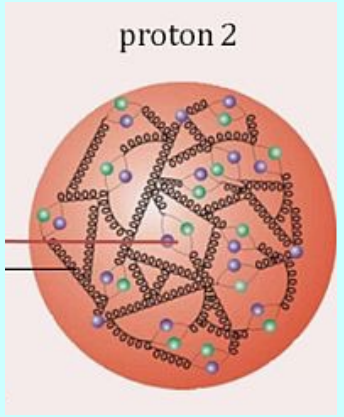


質子撞擊同時，組成成分Parton還正在與其他Parton一直進行強交互作用。

黨外競爭同時黨內互打不停，想像兩團泥巴撞擊，

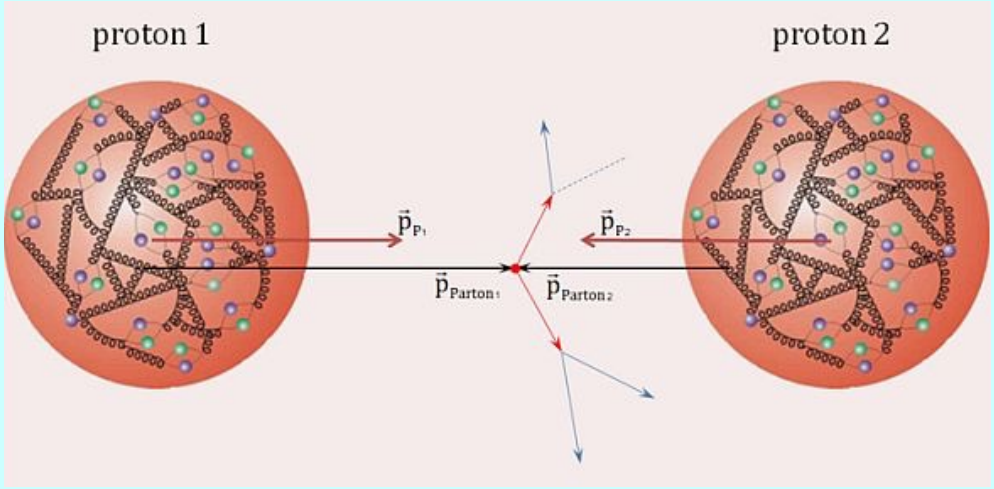


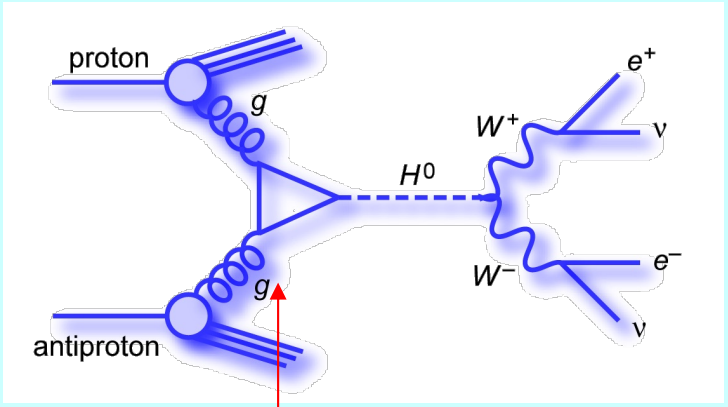




但實驗數據意外顯示：當能量很大時，質子中的夸克與膠子近似是自由的。

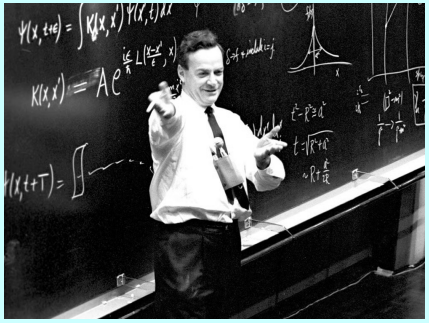
在撞擊的瞬間，只有一個成分參與，其他的組成成分完全沒有發生作用。





單將上場！大軍退開。

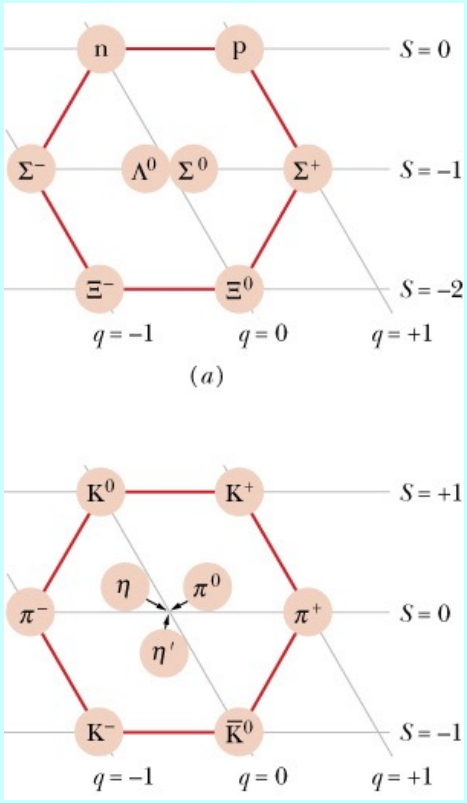
主將對決！

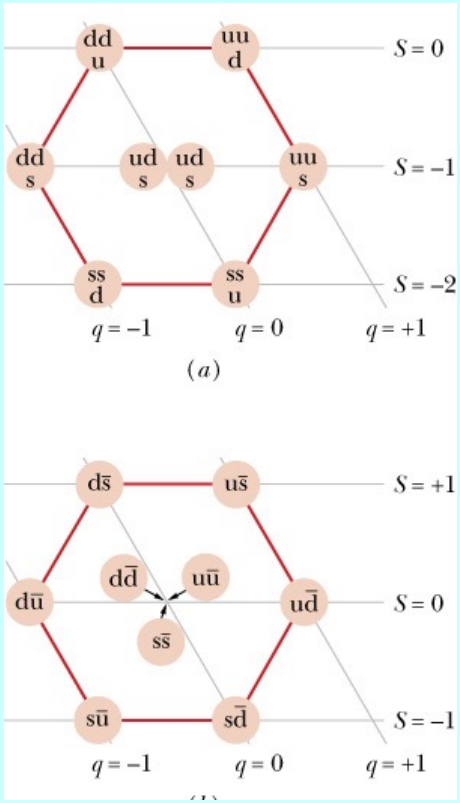


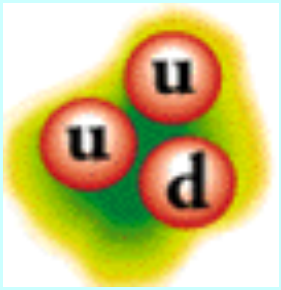
難道強交互作用的強弱 Coupling Constant 還因人設事，看情況而定？

研究強作用應該針對夸克如何被束縛於強子（重子與介子）之中！

重子

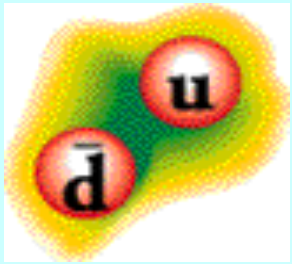






由三個夸克構成

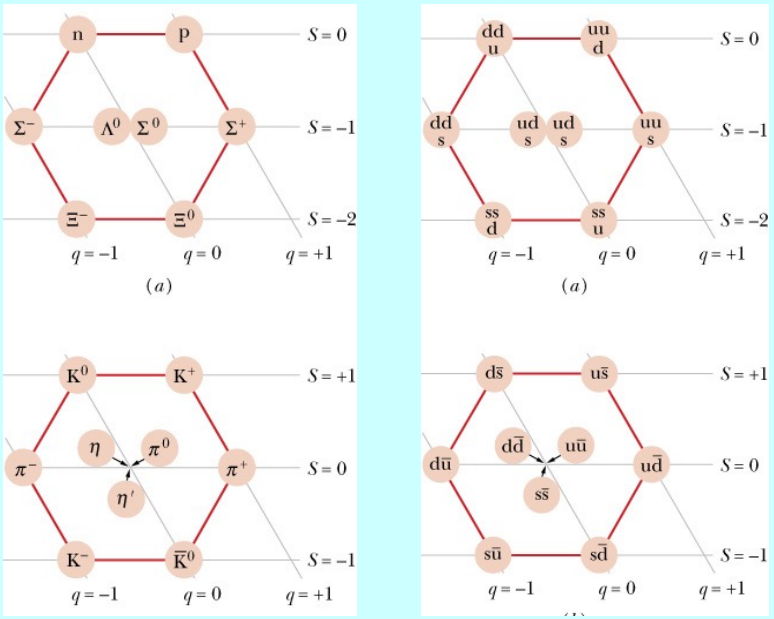
介子



由一個夸克及一個反夸克構成

夸克被束縛只有這兩者組合，為什麼？

為什麼兩個夸克與一個反夸克，或四個夸克不能形成束縛態呢？

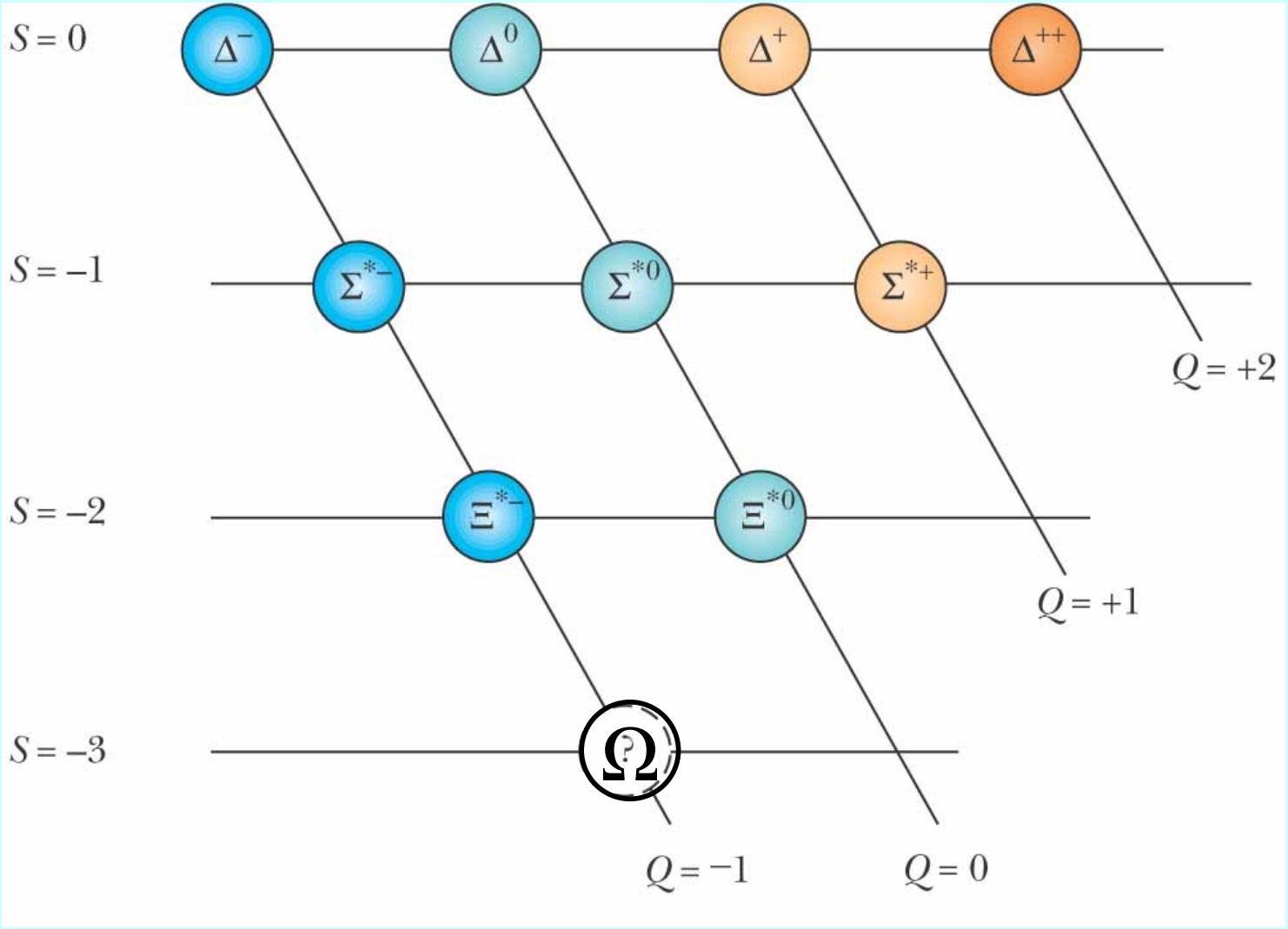


圖中口味迥異的強子質量相近。將夸克束縛於強子中的力應該與口味無關。

兩個夸克間是斥力(沒見過兩個夸克的束縛態)，那為什麼三個夸克就能形成束縛態？

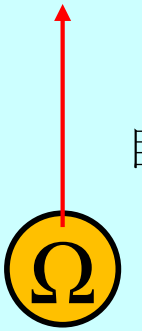
除非夸克有個未發現的性質(量子數)，否則很難雙夸克態是排斥而三夸克態是束縛。

（此兩狀態中的雙夸克具有不一樣的新量子數）

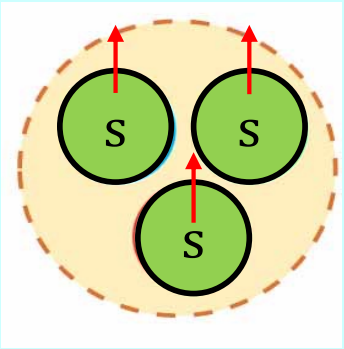


另一個線索來自自旋3/2的重子奇怪行為！

Another evidence!



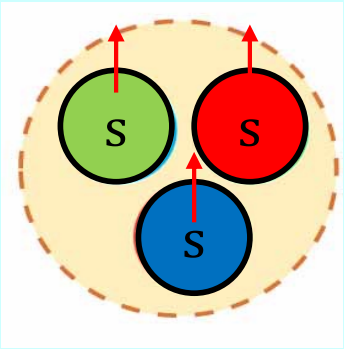
自旋3/2



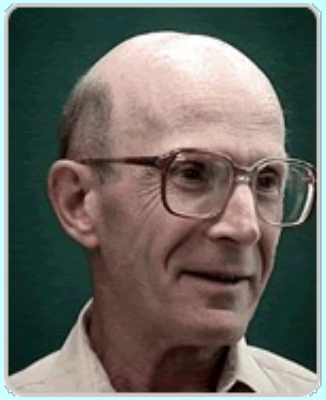
組成Ω的夸克所有性質完全相同，

違反 Exclusion Principle

除非這三個夸克有不同之處！



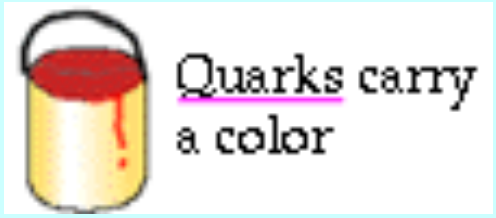
顏色不同！

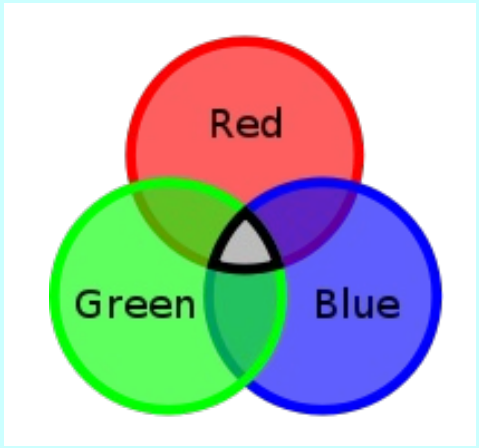


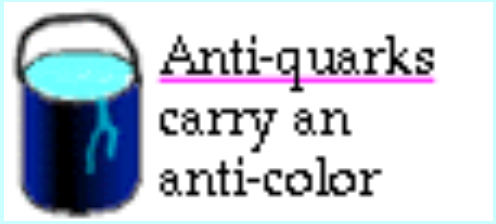
Greenberg 1964

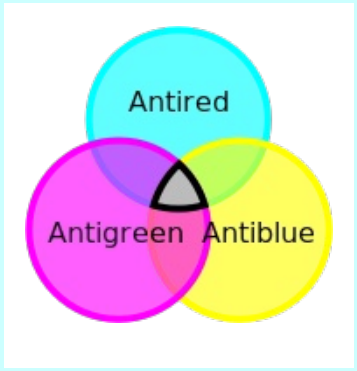
這個新的性質被稱為色彩或顏色:Color。

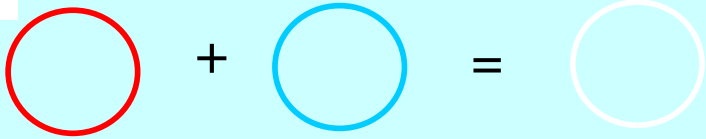
每一種風味的夸克還可以帶有顏色，假設顏色共有三種。











紅加反紅等於無色，以白代表。

PHYSICAL REVIEW VOLUME 139, NUMBER 4B 23 AUGUST 1965

Three-Triplet Model with Double $SU(3)$ Symmetry*

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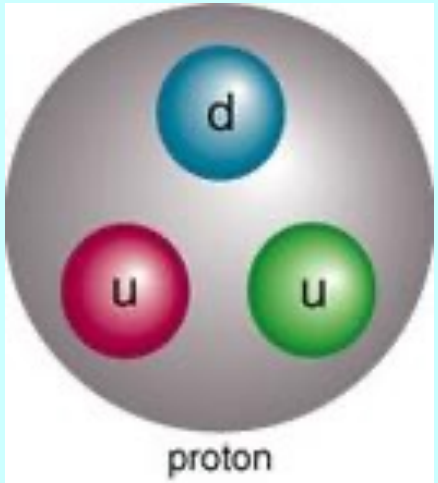
(Received 12 April 1965)

With a view to avoiding some of the kinematical and dynamical difficulties involved in the single-triplet quark model, a model for the low-lying baryons and mesons based on three triplets with integral charges is proposed, somewhat similar to the two-triplet model introduced earlier by one of us (Y. N.). It is shown that in a $U(3)$ scheme of triplets with integral charges, one is naturally led to three triplets located symmetrically about the origin of I_3-Y diagram under the constraint that the Nishijima–Gell-Mann relation remains intact. A double $SU(3)$ symmetry scheme is proposed in which the large mass splittings between different representations are ascribed to one of the $SU(3)$, while the other $SU(3)$ is the usual one for the mass splittings within a representation of the first $SU(3)$.

夸克同時有三種口味，及三種顏色！因此有兩個獨立的$SU(3)$。

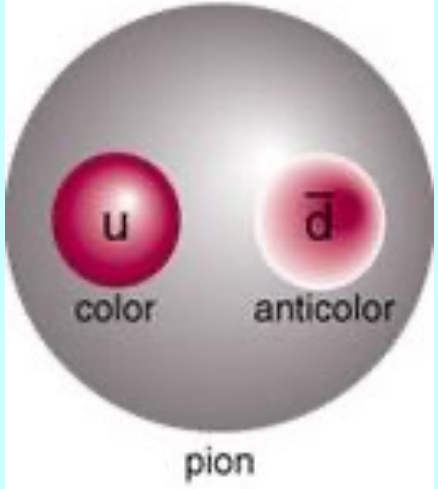
重子中的三個夸克的顏色使他們的作用力是吸引的！



如果假設.....

重子中的三個夸克各帶一種不同的顏色。

紅加藍加綠等於無色



介子中的反夸克帶著夸克顏色的反色。

紅加反紅也等於無色。

這兩種組合剛好就是最簡單的兩種（由三個色彩所組成的）無色組合

猜想：自然界存在可觀測的夸克束縛態Bound State必須是無色的！

顏色決定了把夸克束縛在一起的交互作用！姑稱為色力！

自然的想像：夸克進行色力交互作用時會變色！

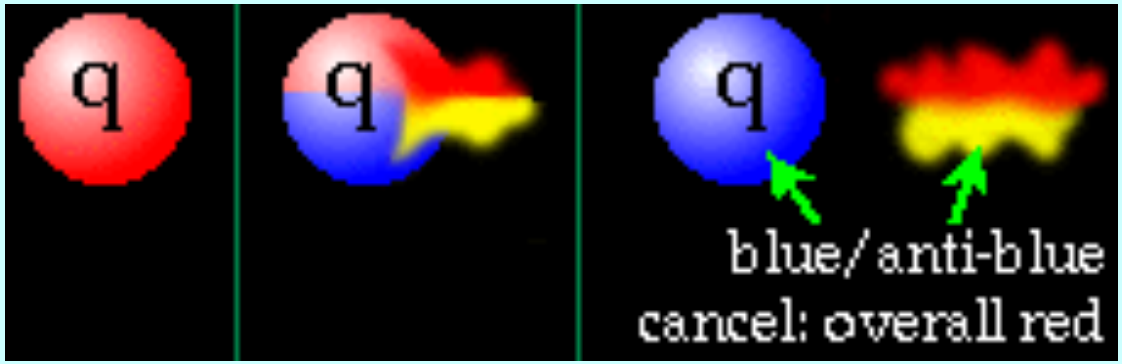
帶色的夸克變色時，應該會放出一個同時帶色與反色的粒子。

例如帶紅色的夸克可以放出一個帶反藍色與紅色的粒子，然後變藍色，。

這個帶反藍色與紅色的粒子被另一藍色夸克吸收後，會將它變成紅色。

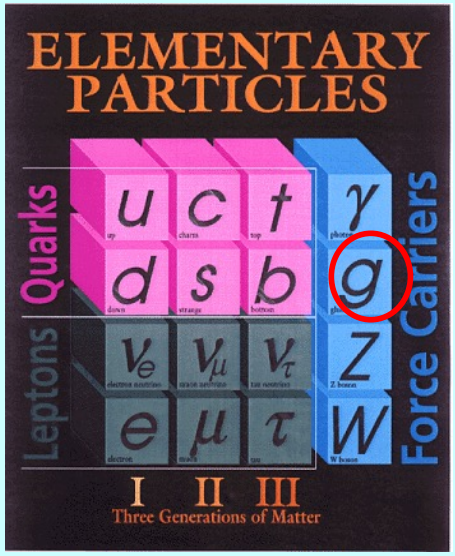
注意夸克進行色力交互作用變色時，會保持口味。

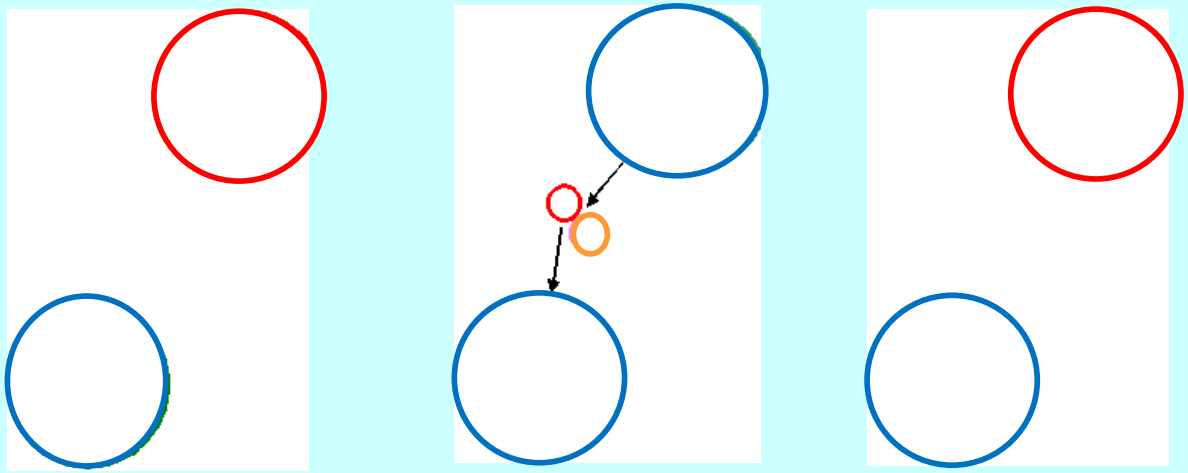
這個帶色與反色的粒子如光子一樣，是一個自旋為 1 的波色子，稱為膠子 Gluon。



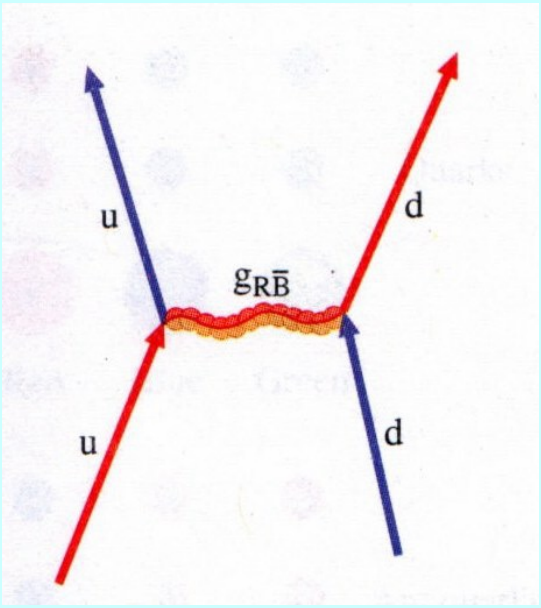
膠子就是把夸克黏在一起的色力的媒介子！

在色力作用中（強交互作用）中，夸克改變自己的顏色，放出一個媒介子膠子。



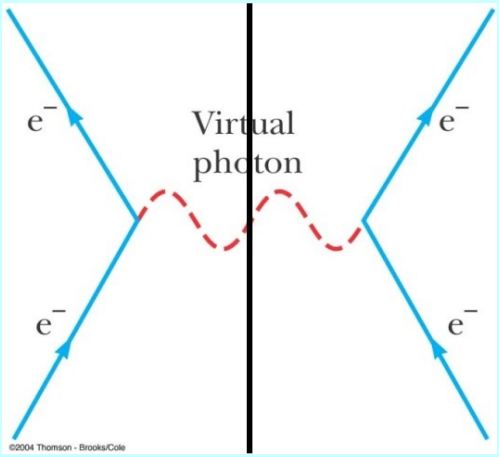


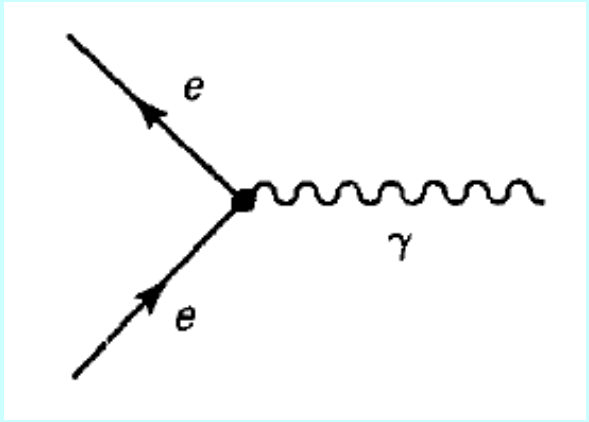
夸克發射膠子可以很容易轉化成 Feynman 圖。



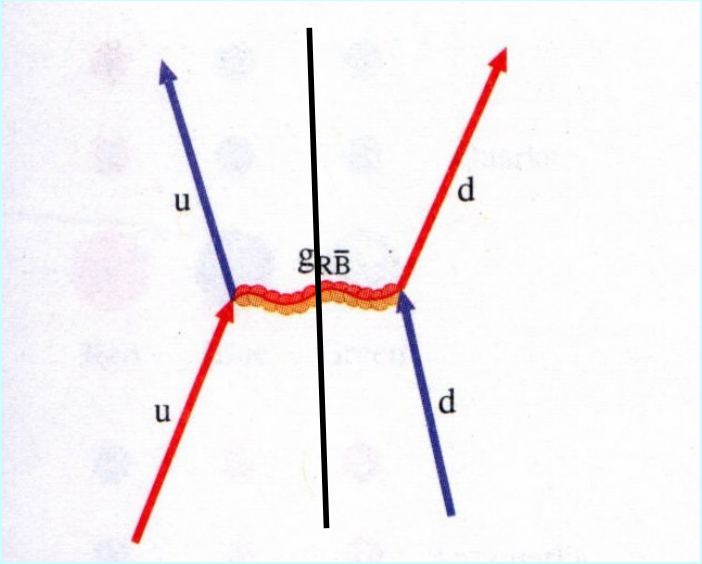
如同電子交換光子以進行電磁交互作用，

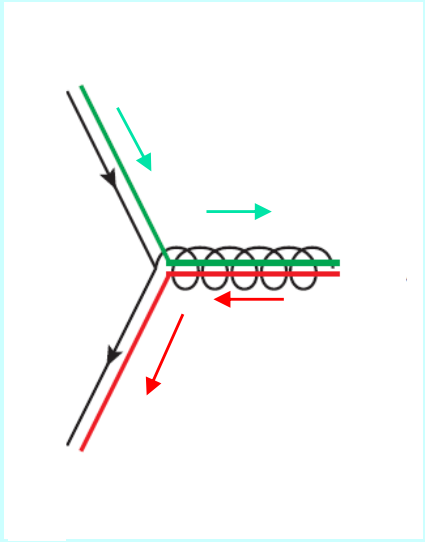
夸克透過交換膠子進行色力交互作用





如同電磁作用可以拆成元件，夸克的色作用也可以拆成元件：





基本色力交互作用

色力交互作用與電若作用如此類似，大膽猜想，膠子就是規範波色子！

量子色動力學 QCD Quantum Chromodynamics

Strong interaction is a $SU(3)$ gauge theory.

What are the triplets of strong $SU(3)$? Color triplets of quarks.

$$\Psi = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

$$U = e^{-iA} = e^{-\frac{i}{2}\sum_{i=1}^{8}\lambda^i\alpha^i}$$

$SU(3)$ U are 3×3 special unitary matrices.

Hermitian and Traceless 的3×3矩陣A有八個線性獨立的分量 α_{1-8} 任意連續變數。

These satisfy $[T^a, T^b] = i\varepsilon^{abc}T^c$. For SU(3) the generators are often written in a standard basis $T^a = \frac{1}{2}\lambda^a$, with λ^3 and λ^8 diagonal (the **Gell-Mann matrices**):

$$\lambda^1 = \begin{pmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & \\ i & 0 & \\ & & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} & & 1 \\ & 0 & \\ 1 & & \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & & -i \\ & 0 & \\ i & & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & & \\ & 0 & -i \\ & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}.$$

$SU(2)$ Gauge symmetry

$$\Psi(x) \rightarrow \exp\left(-\frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\alpha}(x)\right)\Psi(x)$$

$$\boldsymbol{\alpha}(x) = \left(\alpha^1(x), \alpha^2(x), \alpha^3(x)\right)$$

$$\left(W_\mu^1, W_\mu^2, W_\mu^2\right) \equiv \boldsymbol{W}_\mu$$

$SU(3)$ Gauge symmetry

$$\Psi(x) \rightarrow \exp\left(-\frac{i}{2}\sum_{i=1}^{8}\lambda^i\alpha^i(x)\right)\Psi(x)$$

$$\alpha^{i=1\cdots8}(x)$$

$$G_\mu^{i=1\cdots8}(x)$$

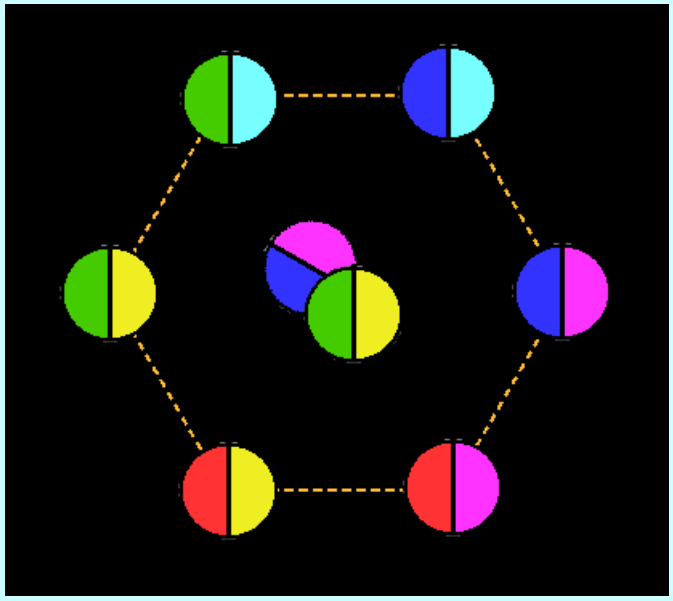
Gluon

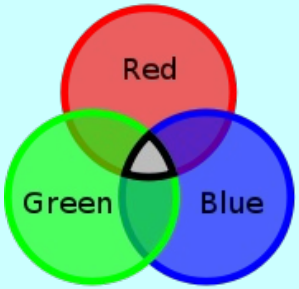
$$\left\{\begin{array}{ll} |1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} & |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ |2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} & |6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2} \\ |3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} & |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ |4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2} & |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \end{array}\right\}$$

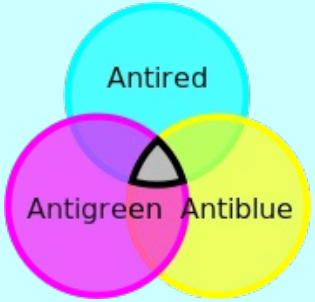
在地化 Localization

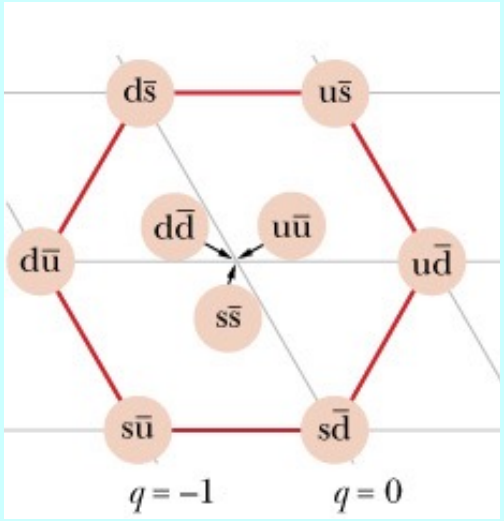
膠子共有八種，帶一個顏色及一個反色，剛好也是色彩 $SU(3)$的Octet。

Gluons, combinations of color and anticolor, form an octet of color $SU(3)$.

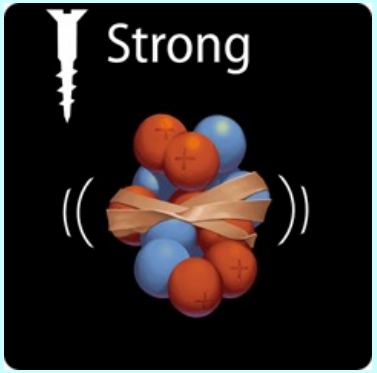






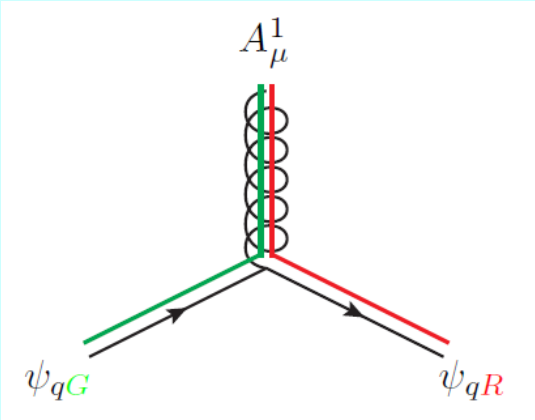


$$\left\{\begin{array}{ll} |1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} & |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2} \\ |2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} & |6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2} \\ |3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} & |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2} \\ |4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2} & |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6} \end{array}\right\}$$



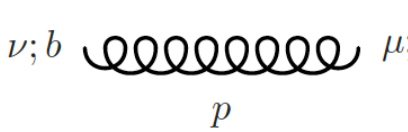
色力交互作用的基本元件

夸克放出膠子，改變顏色！但保持原來口味。

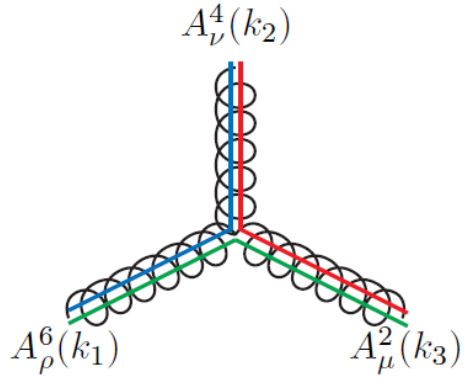


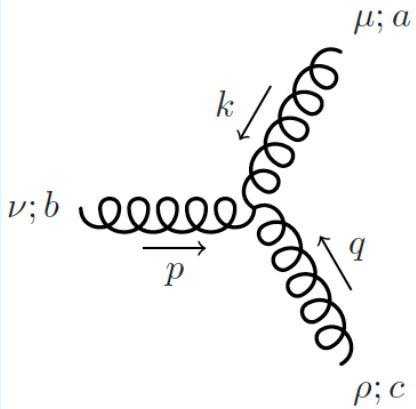
$$\frac{1}{4}\boldsymbol{G}_{\mu\nu}\cdot\boldsymbol{G}^{\mu\nu}=\frac{1}{4}\sum_{i=1}^{3}G^{i}_{\mu\nu}G^{i\mu\nu}$$

$$\boldsymbol{G}_{\mu\nu}=\partial_\mu\boldsymbol{W}_\nu-\partial_\nu\boldsymbol{W}_\mu+g(\boldsymbol{W}_\mu\times\boldsymbol{W}_\nu)$$



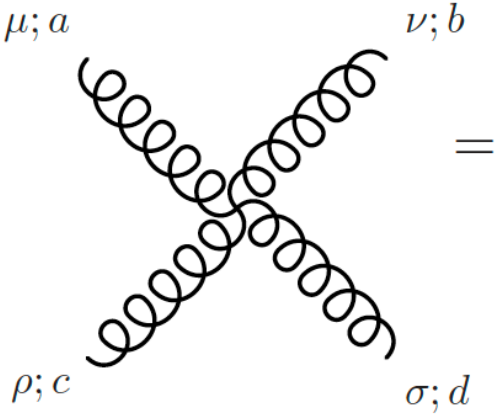
$$= i\frac{-g^{\mu\nu}+(1-\xi)\frac{p^\mu p^\nu}{p^2}}{p^2+i\varepsilon}\delta^{ab}.$$





$$= gf^{abc}[g^{\mu\nu}(k-p)^\rho+g^{\nu\rho}(p-q)^\mu+g^{\rho\mu}(q-k)^\nu]$$

膠子可以放出膠子，改變顏色！

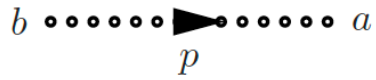


$$\begin{aligned}= -ig^2\times\big[&f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})\\&+f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\sigma}g^{\nu\rho})\\&+f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})\big]\end{aligned}$$

To quantize non-abelian gauge theory, it is convenient to introduce a scalar fermion.

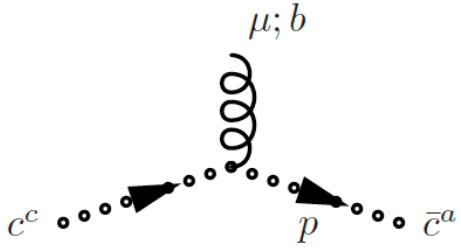
One ghost field for each gauge boson.

ghost propagator is



$$= \frac{i\delta^{ab}}{p^2 + i\varepsilon}.$$

The ghost vertex Feynman rule is



$$= -gf^{abc}p^{\mu}.$$

The transition amplitude for the decay of A: $A \to B + C$

the amplitude of BC in the long future which evolves from A in the long past.

$$\langle BC|U(\infty,-\infty)|A\rangle$$

The transition amplitude for the scattering of A: $A + A \to B + B$

$$\langle BB|U(\infty,-\infty)|AA\rangle$$

The amplitude is the corresponding matrix element of the operator $U(\infty,-\infty)$.

$S \equiv U(\infty,-\infty)$ S operator is the key object in particle physics!

To calculate amplitudes we make an important, and slightly dodgy, assumption:

Initial and final states are eigenstates of the free theory

This means that we take the initial state $|i\rangle$ at $t \to -\infty$, and the final state $|f\rangle$ at $t \to +\infty$, to be eigenstates of the free Hamiltonian H_0. At some level, this sounds plausible: at $t \to -\infty$, the particles in a scattering process are far separated and don't feel the effects of each other. Furthermore, we intuitively expect these states to be eigenstates of the individual number operators N, which commute with H_0, but not $H_{\rm int}$. As the particles approach each other, they interact briefly, before departing again, each going on its own merry way. The amplitude to go from $|i\rangle$ to $|f\rangle$ is

$$\lim_{t_\pm \to \pm\infty} \langle f|\, U(t_+,t_-)\,|i\rangle \equiv \langle f|\, S\,|i\rangle \qquad (3.26)$$

where the unitary operator S is known as the S-matrix. (S is for scattering). There are a number of reasons why the assumption of non-interacting initial and final states is shaky:

- Obviously we can't cope with bound states. For example, this formalism can't describe the scattering of an electron and proton which collide, bind, and leave as a Hydrogen atom. It's possible to circumvent this objection since it turns out that bound states show up as poles in the S-matrix.

- More importantly, a single particle, a long way from its neighbors, is never alone in field theory. This is true even in classical electrodynamics, where the electron sources the electromagnetic field from which it can never escape. In quantum electrodynamics (QED), a related fact is that there is a cloud of *virtual* photons surrounding the electron. This line of thought gets us into the issues of renormalization — more on this next term in the "AQFT" course. Nevertheless, motivated by this problem, after developing scattering theory using the assumption of non-interacting asymptotic states, we'll mention a better way.

S operator can not be solved exactly.

But it can be calculated in a perturbative expansion.

Perturbation expansion of U to solve the equation:

$$\frac{d}{dt}U(t,t_0) = -i\,H_{\mathrm{I}} \cdot U(t,t_0)$$

例如 $\mathcal{L}_{\mathrm{Int}} = \lambda\phi^4(x) + g \cdot \bar{\psi}(x)\psi(x)\phi(x)$

Solve it by a perturbation expansion in small parameters like λ, g in H_{I}.

$$U = U^{(0)} + U^{(1)} + \cdots \qquad U^{(n)} \propto \lambda^n \text{ or } g^n$$

$U^{(0)} = U(\lambda = 0) = 1$ No interaction 無交互作用時，粒子狀態不變。

To leading order in λ, g :

$$\frac{d}{dt}U^{(1)} = -i\,H_{\mathrm{I}}U^{(0)} = -iH_{\mathrm{I}} \qquad H_{\mathrm{I}} \propto \lambda \text{ or } g$$

$$U^{(1)}(t,t_0) = -i\int_{t_0}^{t} dt'\,H_{\mathrm{I}}(t')$$

This is called Born Approximation.

$$U^{(1)}(\infty,-\infty) = S^{(1)} = -i\int_{-\infty}^{\infty} dt'\,H_{\mathrm{I}}(t')$$

In leading order, S equals time integration of the interaction Hamiltonian H_{I}.

S operator 在領先項等於Interaction Hamiltonian H_{I} 對時間的積分！

The interaction Hamiltonian H_{I} can be written in terms Hamiltonian density.

$$S^{(1)} = -i \int_{-\infty}^{\infty} dt\, H_{\mathrm{I}}(t) = -i \int_{-\infty}^{\infty} dt \int d^3\vec{x}\, \mathcal{H}_{\mathrm{I}}(\vec{x}, t) = i \int_{-\infty}^{\infty} d^4x\, \mathcal{L}_{\mathrm{I}}(x)$$

To leading order, S matrix equals

the spacetime integration of the interaction Lagrangian density.

It is Lorentz invariant if the interaction Lagrangian density is invariant.

Vertex

In ABC model, every particle corresponds to a field:

$$A \to \phi_A(x) \equiv A(x) \text{ etc}$$

Add the following interaction term in the Lagrangian:

$$\mathcal{L}_{\mathrm{I}}(x) = g\, A(x)B(x)C(x)$$

The transition amplitude for A decay: $A(\vec{p}_1) \to B(\vec{p}_2) + C(\vec{p}_3)$ can be computed as :

$$\langle BC|U(\infty,-\infty)|A\rangle = \langle BC|S|A\rangle$$

To leading order:

$$S = i\int_{-\infty}^{\infty} d^4x\, \mathcal{L}_{\mathrm{I}}(x) = i\int_{-\infty}^{\infty} d^4x\, g\, A(x)B(x)C(x)$$

$$\langle B(\vec{p}_2)\, C(\vec{p}_3)|S|A(\vec{p}_1)\rangle = \left\langle B(\vec{p}_2)\, C(\vec{p}_3)\middle|\left[ig\int_{-\infty}^{\infty} d^4x\, g\, A(x)B(x)C(x)\right]\middle|A(\vec{p}_1)\right\rangle$$

$$\hat{A} = \int \frac{d^3\vec{p}}{\sqrt{2\omega_p}(2\pi)^3}\left(\hat{a}_{\vec{p}}e^{-ipx} + \hat{a}^\dagger_{\vec{p}}e^{ipx}\right)$$

$$\hat{B} = \int \frac{d^3\vec{p}'}{\sqrt{2\omega_{p'}}(2\pi)^3}\left(\hat{b}_{\vec{p}'}e^{-ip'x} + \hat{b}^\dagger_{\vec{p}'}e^{ip'x}\right)$$

$$\hat{C} = \int \frac{d^3\vec{p}''}{\sqrt{2\omega_{p''}}(2\pi)^3}\left(\hat{c}_{\vec{p}''}e^{-ip''x} + \hat{c}^\dagger_{\vec{p}''}e^{ip''x}\right)$$

There are 8 terms, but only the one that match particles in process will survive.

$$\sim\left(\hat{a}_{\vec{p}} + \hat{a}^\dagger_{\vec{p}}\right)\cdot\left(\hat{b}_{\vec{p}'} + \hat{b}^\dagger_{\vec{p}'}\right)\cdot\left(\hat{c}_{\vec{p}''} + \hat{c}^\dagger_{\vec{p}''}\right)$$

$\hat{a}_{\vec{p}}$

$$\langle B(\vec{p}_2)\, C(\vec{p}_3)\left|\left[ig\int_{-\infty}^{\infty} d^4x\; g\,\hat{A}(x)\hat{B}(x)\hat{C}(x)\right]\right|A(\vec{p}_1)\rangle$$

$\hat{b}^\dagger_{\vec{p}'}$

$\hat{c}^\dagger_{\vec{p}''}$

$$\hat{a}_{\vec{p}}|A(\vec{p}_1)\rangle = |0\rangle(2\pi)^3\sqrt{2\omega_p}\cdot\delta^3(\vec{p}-\vec{p}_1)$$

$$\sim\langle 0|0\rangle\sim 1$$

$$\langle BC(\vec{p}_3)|\hat{c}^\dagger_{\vec{p}''} = \langle B|(2\pi)^3\sqrt{2\omega_{p''}}\cdot\delta^3(\vec{p}''-\vec{p}_3)$$

$$\langle B(\vec{p}_2)|\hat{b}^\dagger_{\vec{p}'} = \langle 0|(2\pi)^3\sqrt{2\omega_{p'}}\cdot\delta^3(\vec{p}'-\vec{p}_2)$$

其他的項都是零！All the rest terms vanish.

$$\int \frac{d^3\vec{p}}{\sqrt{2\omega_p}(2\pi)^3}\left[e^{-ipx}\delta^3(\vec{p}-\vec{p}_1)\right]$$

$$\int \frac{d^3\vec{p}'}{\sqrt{2\omega_{p'}}(2\pi)^3}\left[e^{ip'x}\delta^3(\vec{p}'-\vec{p}_2)\right]$$

$$\int \frac{d^3\vec{p}''}{\sqrt{2\omega_{p''}}(2\pi)^3}\left[e^{ip''x}\delta^3(\vec{p}''-\vec{p}_3)\right]$$

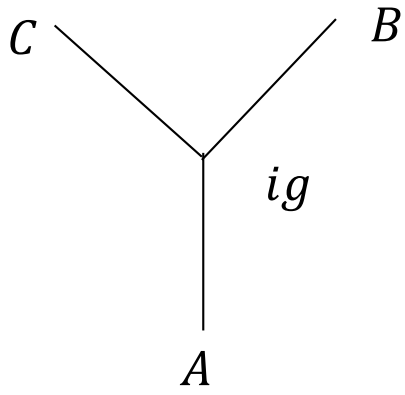
$$\left\langle B(\vec{p}_2)\, C(\vec{p}_3)\left|\left[ig\int_{-\infty}^{\infty} d^4x\; g\, A(x)B(x)C(x)\right]\right|A(\vec{p}_1)\right\rangle$$

$\hat{a}_{\vec{p}_1}$ $\hat{b}^{\dagger}_{\vec{p}_2}$ $\hat{c}^{\dagger}_{\vec{p}_3}$ This is the only surviving term.

The remaining numerical factor is:

$$ig\int d^4x\, e^{-ip_1x}\; e^{ip_2x}\, e^{ip_3x} = ig\int d^4x\, e^{-i(p_1-p_2-p_3)x} = ig\delta^4(p_1-p_2-p_3)$$

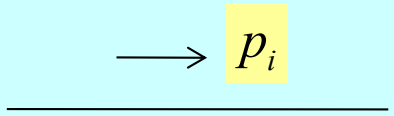
Momentum Conservation



The amplitude is represented by this diagram.

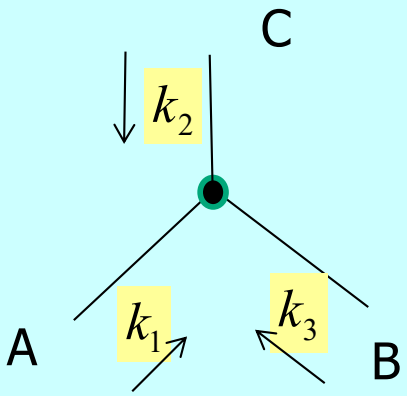
For a toy ABC model

Three scalar particle with masses m_A, m_B ,m_C

External Lines 

1

Lines for each kind of particle with appropriate masses.

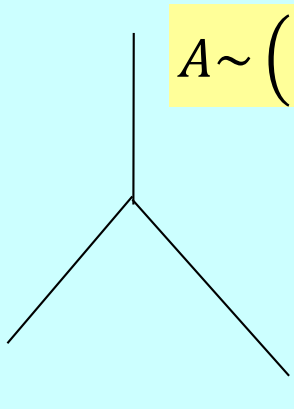
Vertex 

$-ig$

$(2\pi)^4 \cdot \delta^4(k_1 + k_2 + k_3)$

The configuration of the vertex determine the interaction of the model.

$\mathcal{L}_{\mathrm{I}}(x) = g\, A(x)B(x)C(x)$



$A \sim \left(\hat{a}_{\vec{p}} + \hat{a}^{\dagger}_{\vec{p}}\right)$

$C \sim \left(\hat{c}_{\vec{p}\,''} + \hat{c}^{\dagger}_{\vec{p}\,''}\right)$

$B \sim \left(\hat{b}_{\vec{p}\,'} + \hat{b}^{\dagger}_{\vec{p}\,'}\right)$

Interaction Lagrangian

vertex

Every field operator in $\mathcal{L}_{\mathrm{I}}$ corresponds to one leg in the vertex.

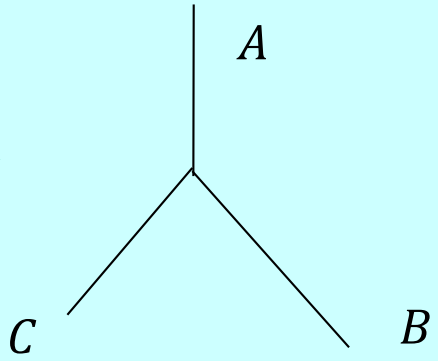
Every field is a linear combination of a and a^{+} $\sim \left(\hat{a}_{\vec{p}} + \hat{a}^{\dagger}_{\vec{p}}\right)$

Every leg of a vertex can either annihilate or create a particle!

This diagram is the combination of 8 diagrams!

$$\mathcal{L}_{\mathrm{I}}(x) = g\, A(x)B(x)C(x)$$

$$i \int_{-\infty}^{\infty} d^4x\, g\, A(x)B(x)C(x)$$



Interaction Lagrangian ⟺ vertex

The Interaction Lagrangian is integrated over the whole spacetime.

Interaction could happen anywhere anytime.

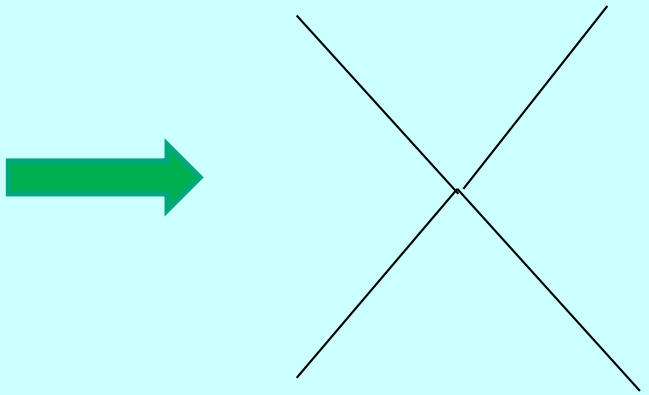
The amplitudes at various spacetime need to be added up.

$$ig \int d^4x\, e^{-ip_1 x}\, e^{ip_2 x}\, e^{ip_3 x} = ig \int d^4x\, e^{-i(p_1 - p_2 - p_3)x} = ig\delta^4(p_1 - p_2 - p_3)$$

The integration yields a momentum conservation.

In momentum space, the factor for a vertex is simply a constant.

$\mathcal{L}_I = \lambda\phi^4$



$\phi \sim \left(\hat{a}_{\vec{p}} + \hat{a}^{\dagger}_{\vec{p}}\right)$

Interaction Lagrangian

Vertex

Every field operator in the interaction corresponds to one leg in the vertex.

Every leg of a vertex can either annihilate or create a particle!

The whole series of operator U can be explicitly written in this notation:

$$U^{(n)}(t,t_0) = \frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1 dt_2 \cdots dt_n \, T[H_{\mathrm{I}}(t_1)\cdot H_{\mathrm{I}}(t_1)\cdots H_{\mathrm{I}}(t_n)]$$

The whole series can be summed into an exponential:

$$U(t,t_0) = \sum_{n=1}^{\infty} \frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1 dt_2 \cdots dt_n \, T[H_{\mathrm{I}}(t_1)\cdot H_{\mathrm{I}}(t_1)\cdots H_{\mathrm{I}}(t_n)]$$

$$= \sum_{n=1}^{\infty} T\frac{(-i)^n}{n!}\int_{t_0}^{t} dt_1 dt_2 \cdots dt_n \, [H_{\mathrm{I}}(t_1)\cdot H_{\mathrm{I}}(t_1)\cdots H_{\mathrm{I}}(t_n)]$$

$$= \sum_{n=1}^{\infty} T\frac{(-i)^n}{n!}\cdot\left(\int_{t_0}^{t} dt \, [H_{\mathrm{I}}(t)]\right)^n = T\left[\exp\left(-i\int_{t_0}^{t} dt \, [H_{\mathrm{I}}(t)]\right)\right]$$

Freeman Dyson, 1923

The proof is so simple. From Tong's notes.

$$U(t,t_0) = 1 - i\int_{t_0}^{t} dt'\, H_I(t') + (-i)^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt''\, H_I(t')H_I(t'') + \ldots \qquad (3.23)$$

Proof: The proof of Dyson's formula is simpler than explaining what all the notation means! Firstly observe that under the T sign, all operators commute (since their order is already fixed by the T sign). Thus

$$\begin{aligned} i\frac{\partial}{\partial t}\, T\exp\left(-i\int_{t_0}^{t} dt'\, H_I(t')\right) &= T\left[H_I(t)\exp\left(-i\int_{t_0}^{t} dt'\, H_I(t')\right)\right] \\ &= H_I(t)\, T\exp\left(-i\int_{t_0}^{t} dt'\, H_I(t')\right) \qquad (3.24) \end{aligned}$$

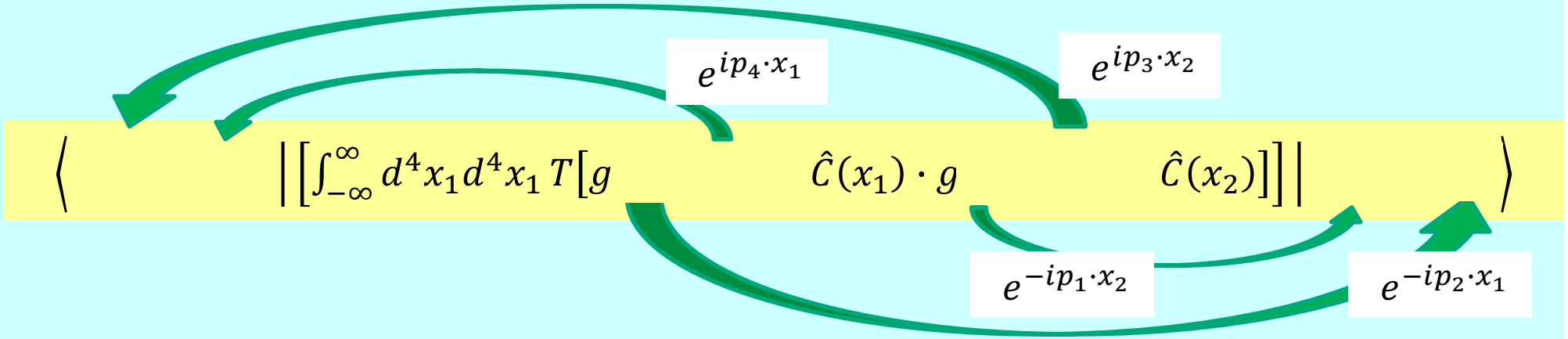
since t, being the upper limit of the integral, is the latest time so $H_I(t)$ can be pulled out to the left. □

Before moving on, I should confess that Dyson's formula is rather formal. It is typically very hard to compute time ordered exponentials in practice. The power of the formula comes from the expansion which is valid when H_I is small and is very easily computed.

$$S^{(2)} = U^{(2)}(\infty, -\infty) = -\frac{1}{2} \int_{-\infty}^{\infty} d^4x_1 \int_{-\infty}^{\infty} d^4x_2 \; T[\mathcal{L}_{\mathrm{I}}(x_1) \cdot \mathcal{L}_{\mathrm{I}}(x_2)]$$

Amplitude for scattering $A + A \rightarrow B + B$

$$\langle B(\vec{p}_3)\, B(\vec{p}_4) | S | A(\vec{p}_1) A(\vec{p}_1) \rangle$$



$$= \int_{-\infty}^{\infty} d^4x_1 d^4x_1 \; e^{-i(p_1 - p_3)\cdot x_2} \cdot e^{-i(p_2 - p_4)\cdot x_1} \langle 0 | T[\hat{C}(x_1)\hat{C}(x_2)] | 0 \rangle$$

Fourier Transformation

Propagator between x_1 and x_2

p_1-p_3 pour into C at x_2 p_2-p_4 pour into C at x_1

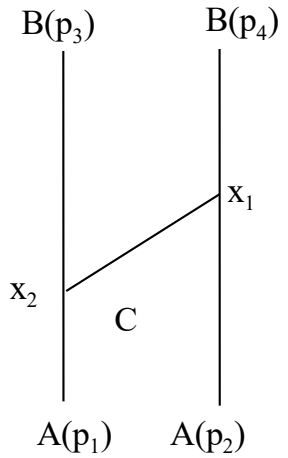
Why is it called propagator?

$t_1 > t_2$

$$\langle 0|T[\hat{C}(x_1)\hat{C}(x_2)]|0\rangle = \langle 0|\hat{C}(x_1)\hat{C}(x_2)|0\rangle \sim \langle 0|(a+a^\dagger)_{x_1} \cdot (a+a^\dagger)_{x_2}|0\rangle$$

A particle is created at x_2 and later annihilated at x_1.

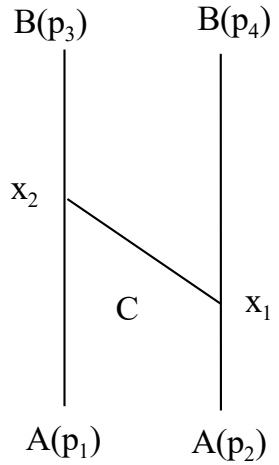
It is indeed a propagator?



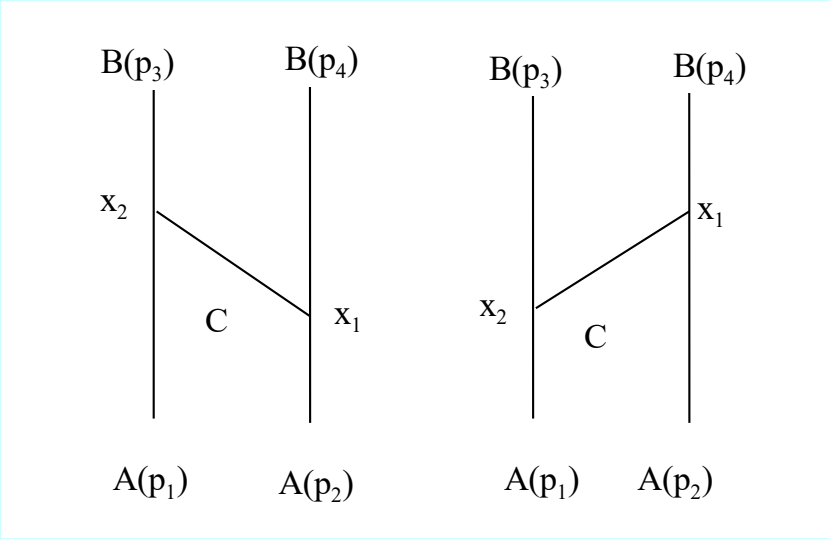
$t_2 > t_1$

$$\langle 0|T[\hat{C}(x_1)\hat{C}(x_2)]|0\rangle = \langle 0|\hat{C}(x_2)\hat{C}(x_1)|0\rangle \sim \langle 0|(a+a^\dagger)_{x_2} \cdot (a+a^\dagger)_{x_1}|0\rangle$$

A particle is created at x_1 and later annihilated at x_2.



$$\langle B(\vec{p}_3)\, B(\vec{p}_4)|S|A(\vec{p}_1)A(\vec{p}_1)\rangle = \int_{-\infty}^{\infty} d^4x_1 d^4x_1\; e^{-i(p_1-p_3)\cdot x_2} \cdot e^{-i(p_2-p_4)\cdot x_1} \langle 0|T\left[\hat{C}(x_1)\hat{C}(x_2)\right]|0\rangle$$



This construction ensures causality of the process.

It is actually the sum of two possible but exclusive processes.

Every Interaction vertex is integrated over the whole spacetime.

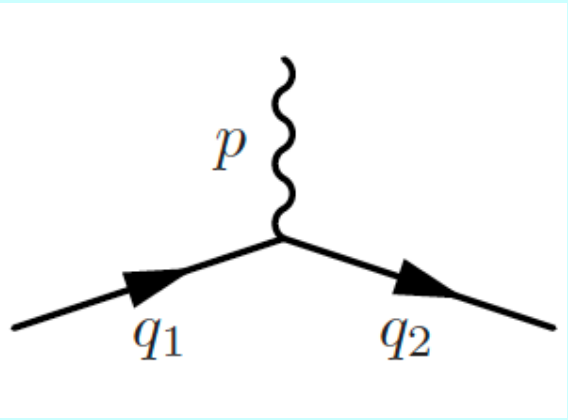
Interaction could happen anywhere anytime.

And amplitudes need to be added up.

$$\psi(x) = \int \frac{d^3\vec{p}}{\sqrt{2\omega_p}(2\pi)^3}\left(\hat{a}_{\vec{p}s}u_{\vec{p}s}e^{-ipx} + \hat{b}^\dagger_{\vec{p}}v_{\vec{p}s}e^{ipx}\right)$$

$$\bar{\psi}(x) = \int \frac{d^3\vec{p}'}{\sqrt{2\omega_p}(2\pi)^3}\left(\hat{b}_{\vec{p}'s}\bar{v}_{\vec{p}',s}e^{-ip'x} + \hat{a}^\dagger_{\vec{p}'}\bar{u}_{\vec{p}'s}e^{ip'x}\right)$$

$$A^\mu(x) = \int \frac{d^3\vec{p}''}{\sqrt{2\omega_p}(2\pi)^3}\left(\hat{c}_{\vec{p}''s}\varepsilon^\mu_{\vec{p}''s}e^{-ip''x} + \hat{c}^\dagger_{\vec{p}''}\varepsilon^{\mu*}_{\vec{p}''s}e^{ip''x}\right)$$



$$\sim\left(\hat{b}_{\vec{p}'} + \hat{a}^\dagger_{\vec{p}'}\right)\cdot\left(\hat{a}_{\vec{p}} + \hat{b}^\dagger_{\vec{p}}\right)\cdot\left(\hat{c}_{\vec{p}''} + \hat{c}^\dagger_{\vec{p}''}\right)$$

There are 8 terms, but only the one that match particles in process will survive.

$$\langle B(\vec{q}_2)\, C(\vec{p})\left|\left[ig\int_{-\infty}^{\infty} d^4x\ \bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)\right]\right|A(\vec{q}_1)\rangle$$

$\hat{a}^\dagger_{q_2}$ $\hat{a}_{\vec{q}_1}$ $\hat{a}^\dagger_{\vec{p}}$ This is the only surviving term.

The remaining numerical factor is: $\left[ig\int_{-\infty}^{\infty} d^4x\ e^{iq_2x}e^{-iq_1x}e^{-ipx}\left(\bar{u}_{\vec{q}_2s}\gamma^\mu u_{\vec{q}_1s}\right)\varepsilon^\mu_{\vec{p}s}\right]$

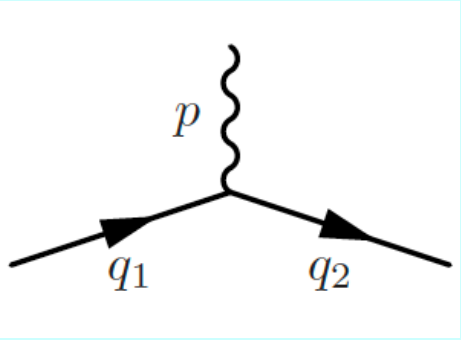
Momentum Conservation

$$ig\int d^4x\, e^{-ip_1x}\ e^{ip_2x}\, e^{ip_3x} = ig\int d^4x\, e^{-i(p_1-p_2-p_3)x} = ig\delta^4(p_1-p_2-p_3)$$

The amplitude is represented by this diagram.

$ig\left(\bar{u}_{\vec{q}_2s}\gamma^\mu u_{\vec{q}_1s}\right)\varepsilon^\mu_{\vec{p}s}$ $\xrightarrow[1PI]{} ig\gamma^\mu$

$$\mathcal{L}_{\mathrm{I}} = \bar{\psi}(x)\gamma^{\mu}\psi(x)A_{\mu}(x)$$



Interaction Lagrangian

Vertex $ig\gamma^{\mu}$

Every field operator in the interaction corresponds to one leg in the vertex.

Every leg of a vertex can either annihilate or create an antiparticle!

This diagram is the combination of 6 diagrams!

The Interaction Lagrangian is integrated over the whole spacetime.

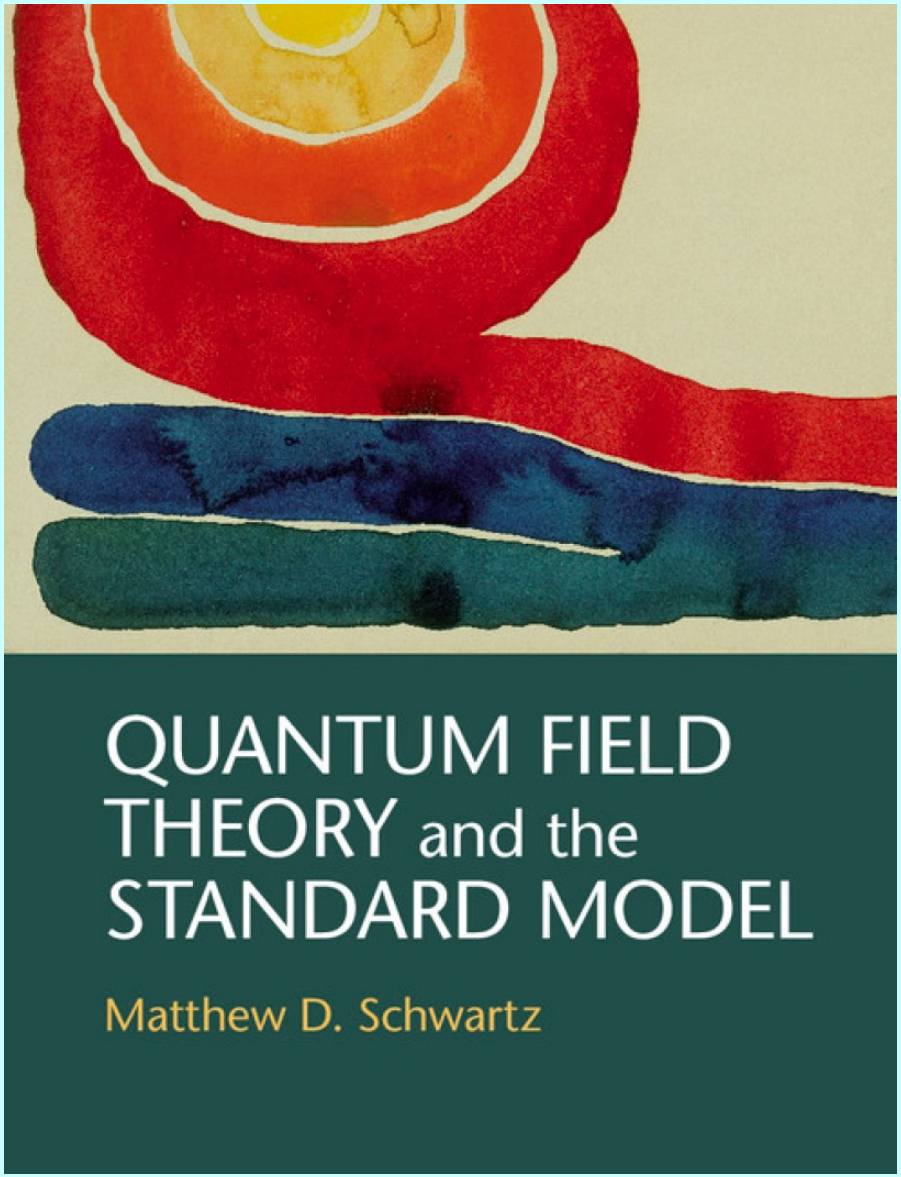
Interaction could happen anywhere anytime.

The amplitudes at various spacetime need to be added up.

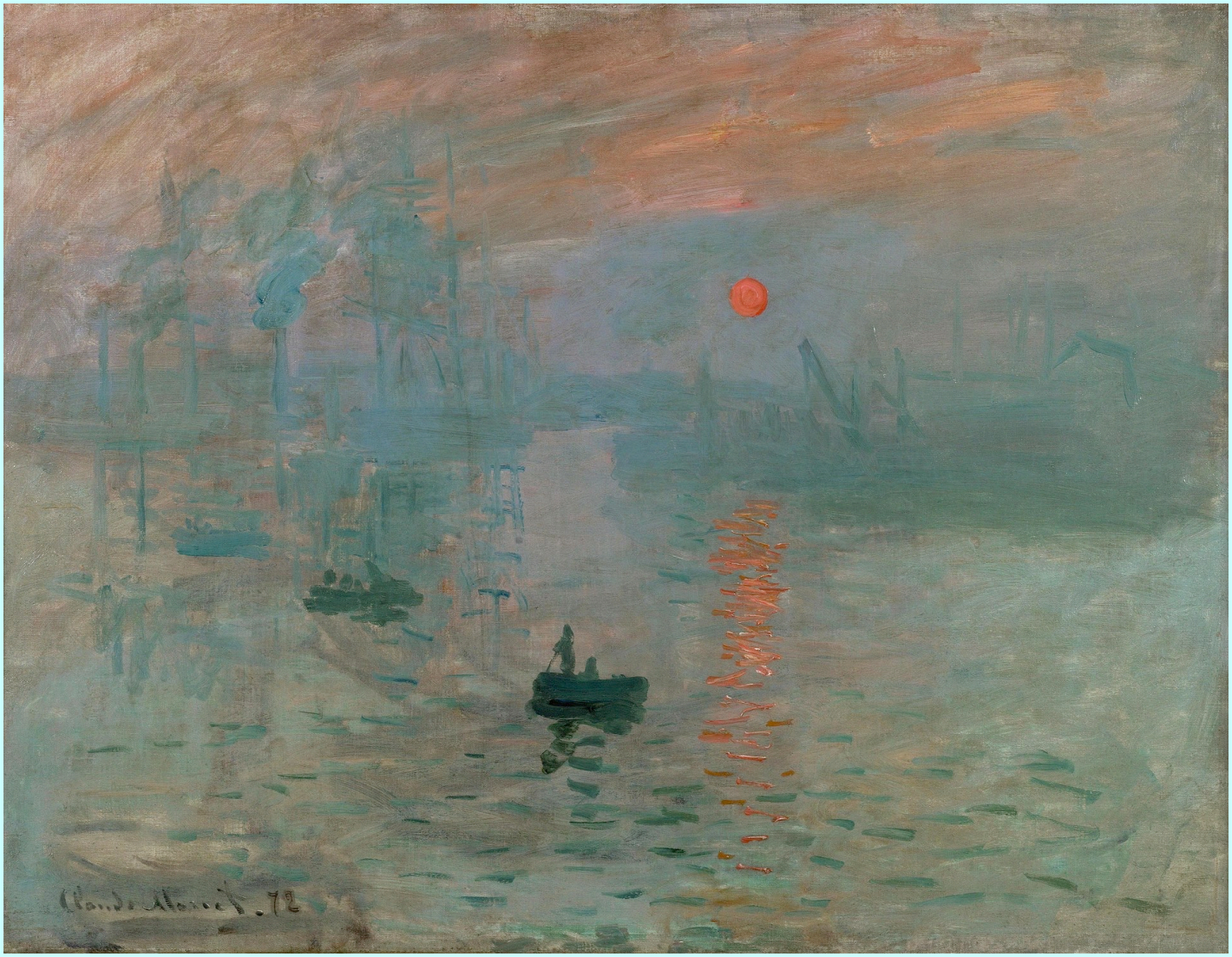
The integration yields a momentum conservation.

In momentum space, the factor for a vertex is simply a constant.

重整化群 Renormalization Group



彌漫於萬物深處，沉靜無波的希格斯場



印象、日出，莫內 Le Havre 勒哈芙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1a.1 Local Quantum Field Theory

Local field theory is a useful idealization. The only way we know to describe the quantum mechanical interactions of a finite number of types of particles in ordinary space-time is in a local quantum field theory characterized by a local Lagrangian density, with the interactions described by products of fields at the same space-time point.[1]

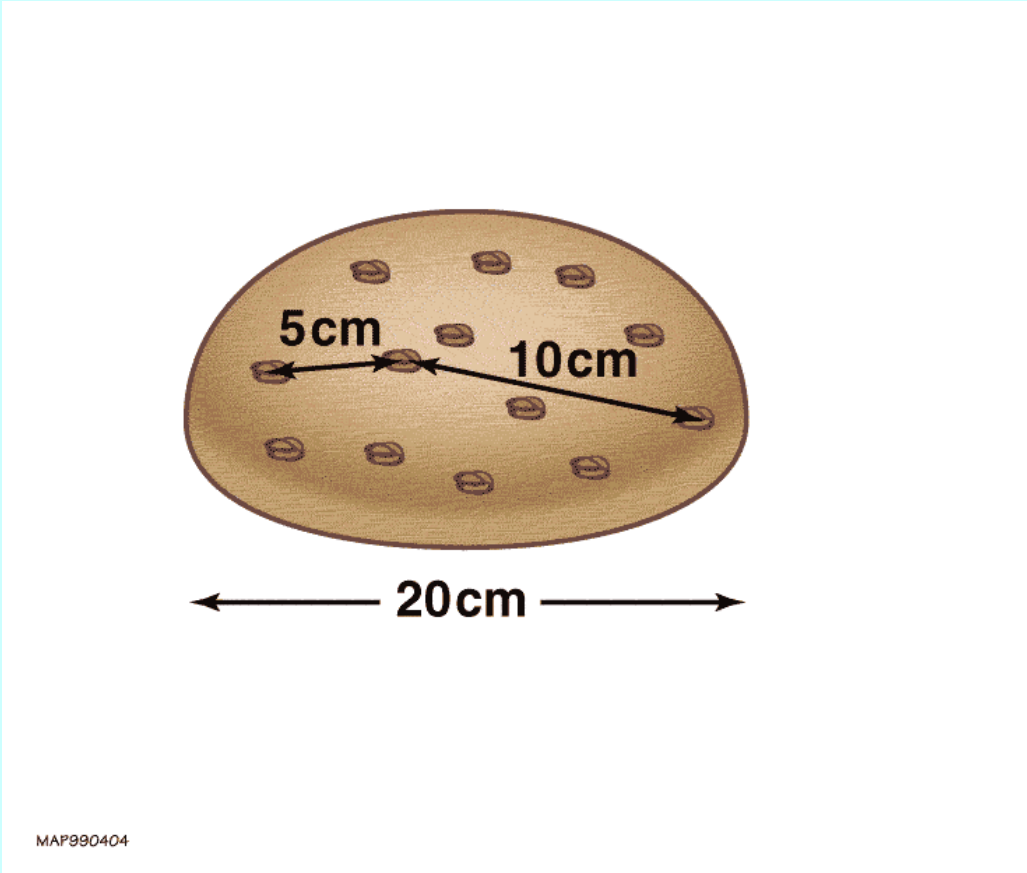
We strongly suspect that this picture is only approximate. It would be astonishing if our naive picture of the space-time continuum, or quantum mechanics, or anything else that the human mind can imagine, continued to make sense down to arbitrarily small distances. However, relativistic quantum mechanics appears to be an excellent approximation at the distance we can probe today. It is hard to see how deviations from the rules of relativistic quantum mechanics could have failed to show up in any of the myriad of experiments at high energy accelerators around the world, unless they are hidden because they are associated with physics at very tiny distances. Thus we are justified in adopting local quantum field theory as a provisional description of physics at the distances we can probe for the foreseeable future.

Naïve QFT does not make sense.

Intrinsically QFT needs to be modified to be even an approximation of nature.

This modification is regularization (make it finite) and renormalization (remove ∞).

Renormalization group is a group, trivial group: scaling 縮放.

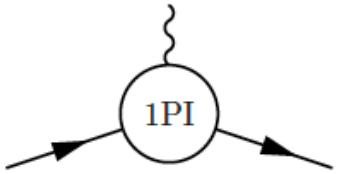


Renormalization group should have been called renormalization ambiguity.

Starting point is calculation of 1PI diagrams. We'll take QED vertex as example.

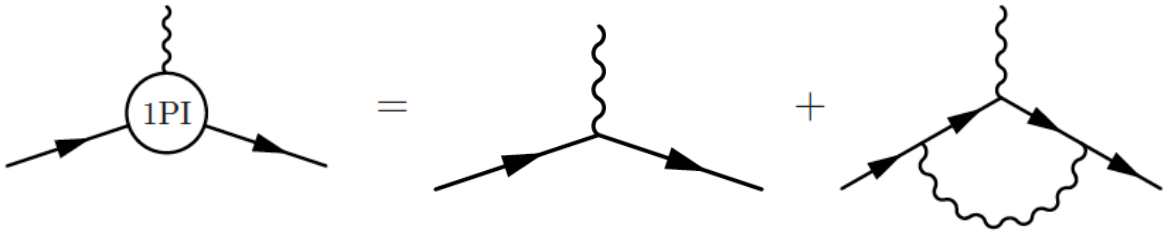
It could be proven: if 1PI is finite, we are safe, ie. all green functions are finite.

$$-ie_R\Gamma^\mu =$$



.

1PI diagrams usually starts with a coupling g, a perturbation parameter.



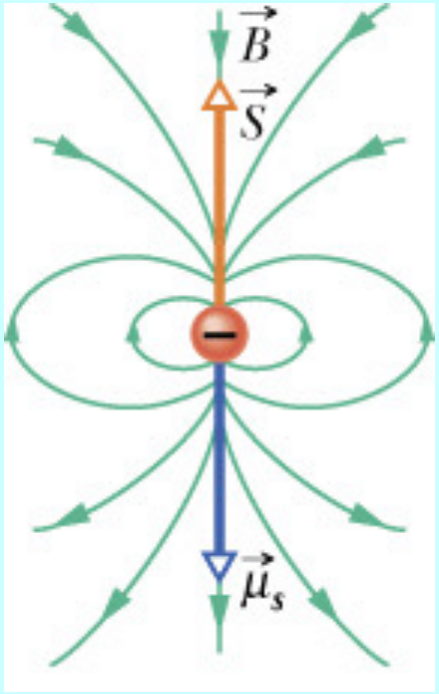
$$\Gamma^\mu(p) = F_1(p^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m_e}p_\nu F_2(p^2).$$

Part F_2 of the contribution with magnetic moment Dirac structure $\sigma_{\mu\nu}$ is finite.

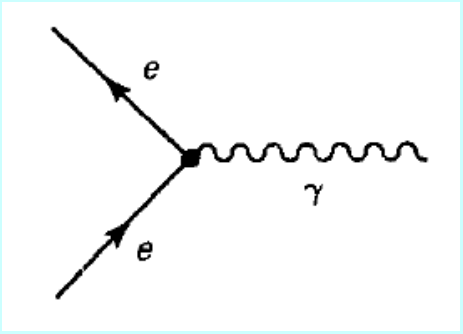
$$F_2(p^2) = \frac{e_R^2}{4\pi^2}\int_0^1 d^3x\,\delta(x+y+z-1)\,\frac{z(1-z)m_R^2}{(1-z)^2m_R^2 - xyp^2} + \mathcal{O}(e_R^4). \qquad (19.46)$$

In particular, $F_2(0) = \frac{\alpha}{2\pi}$, which led to a prediction for the anomalous magnetic moment of the electron: $g-2 = 2F_2(0) = \frac{\alpha}{\pi}$. Since this correction was finite, no counterterm was needed.

量子電動力學對電子的磁偶極的計算驚人地極度準確！



$$\vec{\mu}_s = -\frac{e}{m}\vec{s} = -g\frac{e}{2m}\vec{s}$$

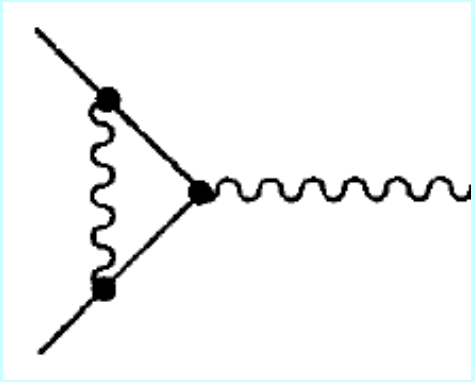


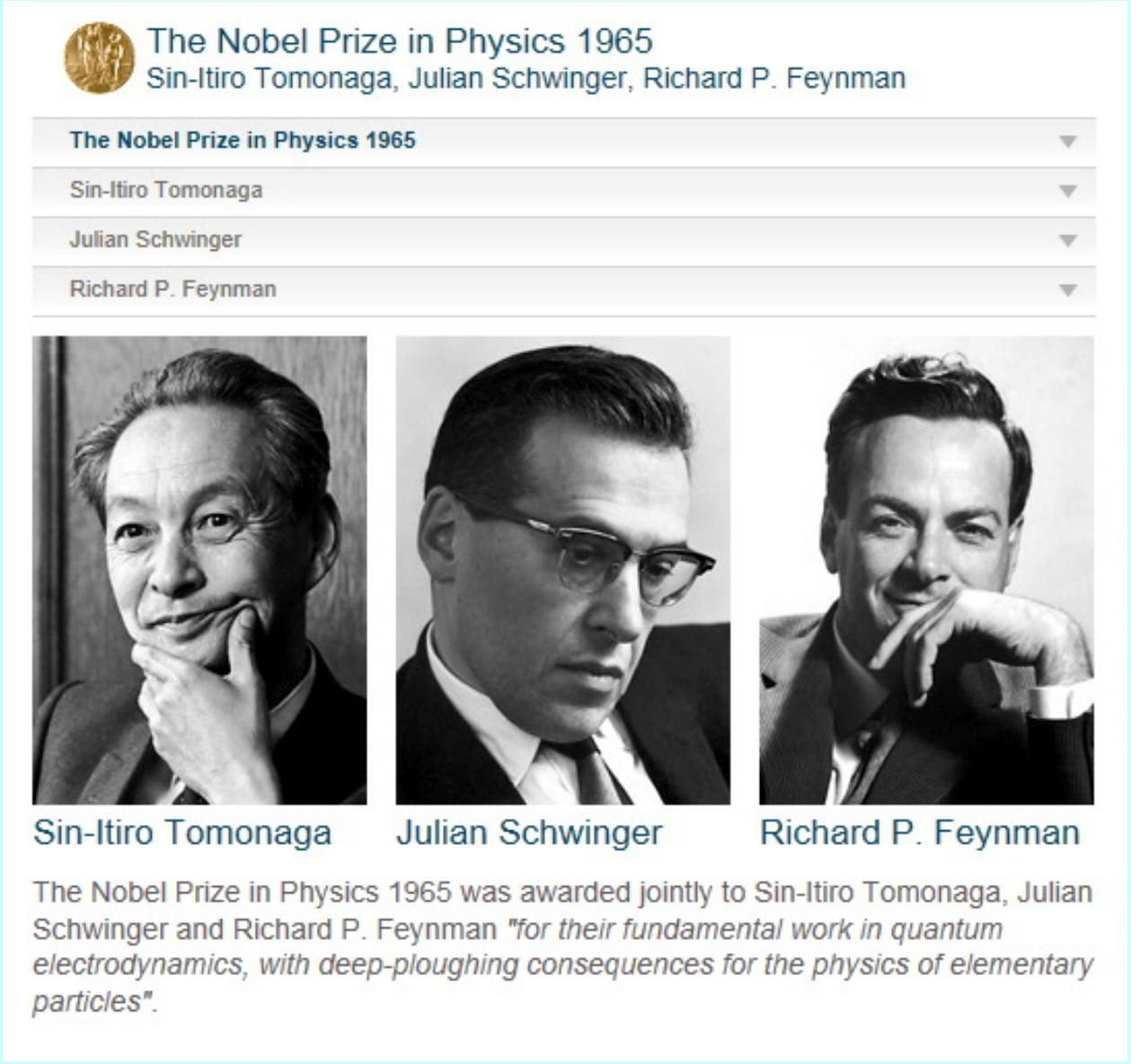
This first order diagram predict $g = 2$.

The following diagram gives the leading correction.

$$\frac{1}{2}g_{\text{theory}} = 1 + (1159652187.9 \pm 8.8) \times 10^{-12}$$

$$\frac{1}{2}g_{\text{experi}} = 1 + (1159652188.4 \pm 4.3) \times 10^{-12}$$





Schwinger use Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$ to calculate. Not sure it is Lorentz invariant.

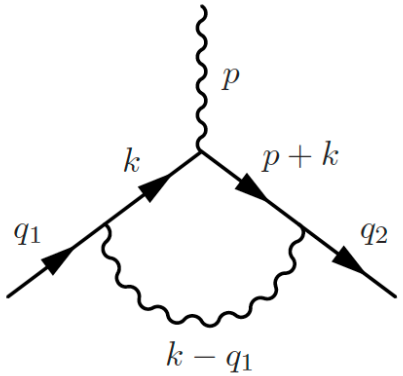
Feynman starts with a manifestly Lorentz formalism to calculate. But nobody understand what he was computing.

RG in Cutoff Regurization 重整化群在截止動量正規化

Schwartz 19.3

Part F_1 with current Dirac structure γ^μ is divergent.

$$f(p^2) = -2ig^2 \int \frac{d^4k}{(2\pi)^4} \int dx\, dy\, dz\, \delta(x+y+z-1) \times \frac{k^2 - 2(1-x)(1-y)p^2 - 2(1-4z+z^2)m_R^2}{[k^2 - (m_R^2(1-z)^2 - xyp^2)]^3}.$$



The part k^2 is logarithmically divergent $\frac{k^5}{k^6}\,dk$ as $k \to \infty$.

Introduce a Pauli-Villar momentum Cutoff Λ regularization:

$$\Delta = (1-z)^2 m_R^2 - xyp^2$$

You set an upper limit on k integration at $k^2 = \Lambda^2$.

$$f(p^2) = \frac{g^2}{8\pi^2} \int_0^1 dx\, dy\, dz\, \delta(x+y+z-1) \left[\ln \frac{z\Lambda^2}{\Delta} + \frac{p^2(1-x)(1-y) + m_R^2(1-4z+z^2)}{\Delta} \right].$$

Set $p^2 = 0$, the divergence can be written as:

$$\Gamma \sim \gamma^\mu \frac{g^2}{8\pi^2} \ln\left(\frac{\Lambda}{m}\right)$$

$$\int_0^\Lambda \frac{k^5}{k^6}\,dk = \int_0^\Lambda \frac{1}{k}\,dk \sim \ln \Lambda$$

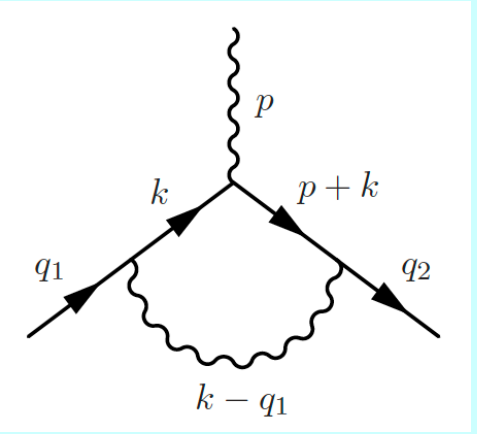
This Λ is not cutoff in Wilson's RG of condense matter theory.

In condense matter theory, Λ is physical, indicating separations of lattice.

In particle physics, there is no lattice.

Λ is a temporary remedy to regulate infinity so that we can at least calculate.

$$\Gamma \sim g^2 \int \frac{d^4k}{(2\pi)^4} \left(\gamma^\alpha \frac{i}{\not p + \not k - m} \gamma^\mu \frac{i}{\not k - m} \gamma^\beta \right) \frac{-i g_{\alpha\beta}}{(k - q_1)^2}$$



$$= g^2 \int \frac{d^4k}{(2\pi)^4} \left(\gamma^\alpha \frac{\not p + \not k + m}{(p+k)^2 - m^2} \gamma^\mu \frac{\not k + m}{k^2 - m^2} \gamma^\beta \right) \frac{g_{\alpha\beta}}{(k - q_1)^2}$$

$$= g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(p+k)^2 - m^2](k^2 - m^2)(k - q_1)^2} \cdot \left[\gamma^\alpha (\not p + \not k + m) \gamma^\mu (\not k + m) \gamma^\beta g_{\alpha\beta} \right]$$

$$f(p^2) = -2ig^2 \int \frac{d^4k}{(2\pi)^4} \int dx\, dy\, dz\, \delta(x + y + z - 1) \times \frac{k^2 - 2(1-x)(1-y)p^2 - 2(1 - 4z + z^2)m_R^2}{[k^2 - (m_R^2(1-z)^2 - xyp^2)]^3}.$$

B Appendix B Regularization

B.1 Integration parameters

To evaluate loop integrals in quantum field theory, it is often helpful to introduce Feynman or Schwinger parameters.

B.1.1 Feynman parameters

Feynman parameters are based on a number of easily verifiable mathematical identities. The simplest is

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[A + (B - A)x]^2} = \int_0^1 dx\, dy\, \delta(x + y - 1) \frac{1}{[xA + yB]^2}. \tag{B.1}$$

Other useful identities are

$$\frac{1}{AB^n} = \int_0^1 dx\, dy\, \delta(x + y - 1) \frac{n y^{n-1}}{[xA + yB]^{n+1}}, \tag{B.2}$$

$$\frac{1}{ABC} = \int_0^1 dx\, dy\, dz\, \delta(x + y + z - 1) \frac{2}{[xA + yB + zC]^3}. \tag{B.3}$$

These are useful because they let us complete the square in the denominator. For example,

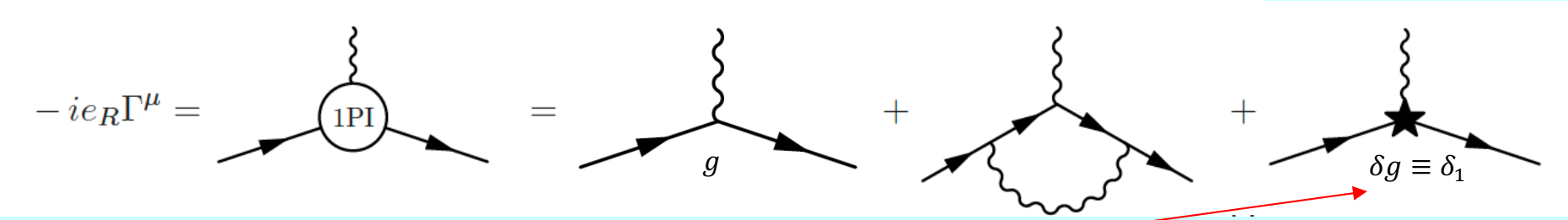
$$\begin{aligned} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(k-p)^2} &= \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx \frac{1}{[k^2 + x((k-p)^2 - k^2)]^2} \\ &= \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[(k - xp)^2 - \Delta]^2}, \end{aligned} \tag{B.4}$$

where $\Delta = -p^2 x(1 - x)$. Then we can shift $k \to k + xp$ leaving an integral that only depends on k^2.

We need our calculation to be rid of Λ eventually. Λ is unphysical and infinite.

$$\Gamma \sim \frac{g^2}{8\pi^2}\ln\left(\frac{\Lambda}{m}\right)$$

I hide the gamma matrix γ^μ.



The recipe is to add counter term $\delta g \equiv g\delta_1$ to cancel Λ in one loop 1PI.

$$\delta_1 = -\frac{g^2}{8\pi^2}\left(\frac{1}{2}\ln\frac{\Lambda^2}{m_R^2} + \frac{9}{4} + \ln\frac{m_\gamma^2}{m_R^2}\right)$$

for on-shell subtraction.

Now 1PI is Λ independent or finite as $\Lambda \to \infty$.

Noting leading term is g, we justify this step by saying δg is there in the beginning.

The counter term δg is combined with perturbation parameter g to form bare g_0.

$$g_0 = g + g\delta_1 = Z_g g \qquad Z_g = 1 + \delta_1$$

Bare g_0 is the parameter appearing in the full $\mathcal{L}$. But it's not perturbation parameter.

This is legitimation by redefinition, technically called renormalization.

$$\mathcal{L} \supset ig^0\bar{\psi}\gamma^\mu A^\mu\psi = ig\bar{\psi}\gamma^\mu A^\mu\psi + ig\delta_1\bar{\psi}\gamma^\mu A^\mu\psi$$

It works! We can prove: if 1PI is finite, we are safe, ie. all green functions are finite.

The trouble is if the purpose of counter terms is just to cancel the divergence in 1PI Γ, there is a freedom in choosing how much to cut or cancel.

You are free to replace m by any μ. Any μ gives as a competent a δ_1 to serve the job.

$$\delta_1 = -\frac{g^2}{8\pi^2}\ln\left(\frac{\Lambda}{\mu}\right)$$

Choosing different μ means cutting more or cutting less from one loop 1PI Γ.

$$g_0 = g + g\delta_1\left(\frac{\Lambda}{\mu}\right)$$

Since μ is not a real parameter of the theory, g_0 is not μ dependent.

Hence the perturbation parameter $g(\mu)$ unexpectedly is μ dependent.

$$g_0(\Lambda) = g(\mu) + g\delta_1\left(\frac{\Lambda}{\mu}\right)$$

Without field renormalization, dependencies will cancel between $\delta_1\left(\frac{\Lambda}{\mu}\right)$ and $g(\mu)$.

We usually define a beta function and calculate it in perturbation expansion:

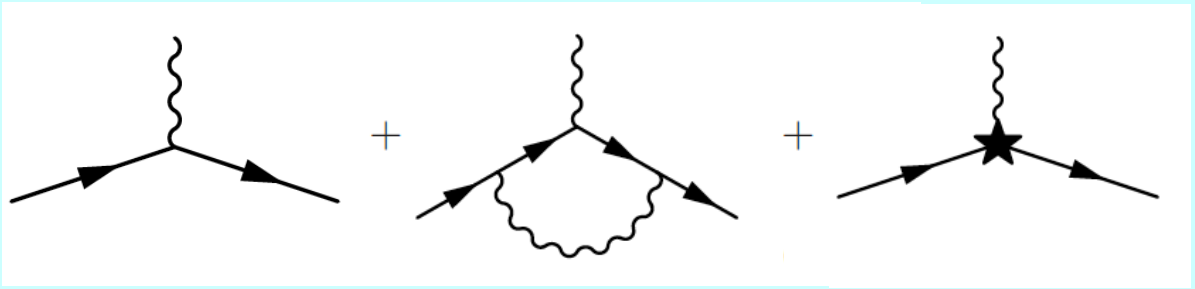
$$\beta = \frac{d}{d\ln\mu}g(\mu) = \mu\frac{dg}{d\mu} = -\mu\frac{d(g\delta_1)}{d\mu}$$

We calculate g's μ dependence by counter terms.

There is an inevitable renormalization calculation ambiguity: we can choose any μ.

Choose different μ means cutting more or cutting less from one loop 1PI diagram.

After cancellation from counter terms, 1-loop 1PI will contain $\ln \mu$:



$$\Gamma = g(\mu) + \quad + \delta g \sim g(\mu) + cg^3 \ln\left(\frac{p}{\Lambda}\right) + cg^3 \ln\left(\frac{\Lambda}{\mu}\right) \sim g(\mu) + cg^3 \ln\left(\frac{p}{\mu}\right)$$

p denotes roughly the typical energy of the process, like electron mass m.

To have a small log so that perturbation is workable, you better choose $\mu \sim p$.

$g(p)$ then will be the coupling you feel, called running coupling constant.

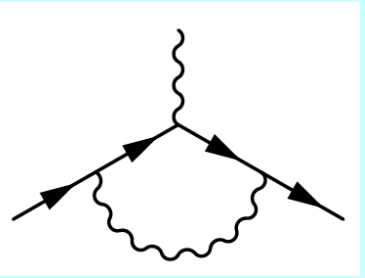
The coupling constant you feel depends on the energy scale of your experiments.

Calculate μ dependence of $g(\mu)$ is extremely important.

The practical calculation is usually done in DRMS instead of using cutoff Λ.

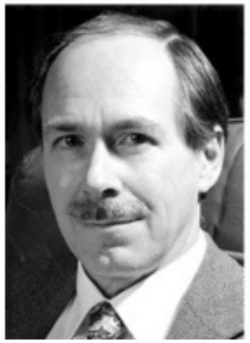
Assume $d < 4, \varepsilon = 4 - d > \to 0$. The integral is convergent $\frac{k^{1+d}}{k^6}\, dk\, \frac{k^{5-\varepsilon}}{k^6}\, dk$ as $k \to \infty$.

$$= -2ig^2\mu^{4-d} \int \frac{d^dk}{(2\pi)^d} \int dx\, dy\, dz\, \delta(x+y+z-1)$$
$$\times \frac{\left(2-\frac{4}{d}\right)k^2 - 2(1-x)(1-y)p^2 - 2(1-4z+z^2)m_R^2}{\left(k^2-\Delta+i\varepsilon\right)^3},$$



This is a kind of regularization, called Dimensional Regularization (DR).

The Nobel Prize in Physics 1999



Gerardus 't Hooft
Prize share: 1/2

Martinus J.G. Veltman
Prize share: 1/2

The Nobel Prize in Physics 1999 was awarded jointly to Gerardus 't Hooft and Martinus J.G. Veltman *"for elucidating the quantum structure of electroweak interactions in physics"*

$$f(p^2) = -2ig^2 \int \frac{d^4k}{(2\pi)^4} \int dx\, dy\, dz\, \delta(x+y+z-1)$$
$$\times \frac{k^2 - 2(1-x)(1-y)p^2 - 2(1-4z+z^2)m_R^2}{[k^2 - (m_R^2(1-z)^2 - xyp^2)]^3}.$$

B.3.4 k^μ integrals

We will often have integrals with factors of momenta, such as $k^\mu k^\nu$, in the numerator:

$$F^{\mu\nu}(\Delta) = \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2-\Delta)^n}. \tag{B.50}$$

These can be simplified using a trick. Since the integral is a tensor under Lorentz transformations but only depends on the scalar Δ, it must be proportional to the only tensor around, $g^{\mu\nu}$. Then, just by dimensional analysis, we must get the same thing as in an integral with $k^\mu k^\nu$ replaced by $ck^2 g^{\mu\nu}$ for some number c. Contracting with $g^{\mu\nu}$, we see that $c = \frac{1}{4}$ or more generally $c = \frac{1}{d}$. Therefore,

$$\int \frac{d^dk}{(2\pi)^d} \frac{k^\mu k^\nu}{(k^2-\Delta)^n} = \frac{1}{d} g^{\mu\nu} \int \frac{d^dk}{(2\pi)^d} \frac{k^2}{(k^2-\Delta)^n}. \tag{B.51}$$

If there is just one factor of k^μ in the numerator, for example in

$$F(p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{k\cdot p}{(k^2-p^2)^4}, \tag{B.52}$$

then the integrand is antisymmetric under $k \to -k$. Since we are integrating over all k, the integral must vanish. So we will only need to keep terms with even powers of k in the numerator.

Assume $d < 4, \varepsilon = 4 - d > \to 0$. The integral is convergent $\frac{k^{1+d}}{k^6}\, dk\, \frac{k^{5-\varepsilon}}{k^6}\, dk$ as $k \to \infty$.

$$= -2i\, g^2 \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \int dx\, dy\, dz\, \delta(x + y + z - 1)$$
$$\times \frac{\left(2 - \frac{4}{d}\right) k^2 - 2(1-x)(1-y)p^2 - 2(1 - 4z + z^2) m_R^2}{\left(k^2 - \Delta + i\varepsilon\right)^3},$$

But in $d < 4$, you need to add a scale $\mu^{4-d} = \mu^{\varepsilon}$ to compensate and μ is arbitrary!

This introduces an arbitrary scale as earlier. We'd call it renormalization scale.

The only part to-be-divergent as $\varepsilon \to 0$ is k^2. We'll show it generates a pole in ε.

$$\mu^{\varepsilon} \left(2 - \frac{4}{d}\right) \int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - \Delta + i\varepsilon)^3} = -\frac{i}{16\pi^2} \left(\frac{2}{\varepsilon} + \ln \frac{\mu^2}{\Delta} + \cdots.\right)$$

The pole in ε diverges as $\varepsilon \to 0, d \to 4$.

$$\Delta = (1 - z)^2 m^2 - xyp^2$$

In Minimal Subtraction (MS) scheme, the counter term will cut just the pole.

$$\delta_1 = -\frac{g^2}{16\pi^2} \frac{2}{\varepsilon}$$

Now calculation: integrand depends only on k^2. Angle factor can be integrated.

$$\mu^{\varepsilon}\left(2-\frac{4}{d}\right)\int\frac{d^dk}{(2\pi)^d}\frac{k^2}{(k^2-\Delta+i\varepsilon)^3}=\mu^{\varepsilon}\left(2-\frac{4}{d}\right)\frac{\Omega(d)}{(2\pi)^d}\int\frac{k^{2+d}}{(k^2-\Delta+i\varepsilon)^3}\frac{dk}{k}$$

$$\int\frac{k^{2+d}}{(k^2-\Delta+i\varepsilon)^3}\frac{dk}{k}\xrightarrow[\text{Euclidean}]{}\to i\int\frac{k^{2+d}}{(k^2+\Delta)^3}\frac{dk}{k}=i\,\Delta^{-\frac{\varepsilon}{2}}\int\frac{y^{2+d}}{(1+y^2)^3}\frac{dy}{y}$$

$k\equiv y\sqrt{\Delta}$

This integral equals some Gamma function Γ. But we need to first maneuver.

$$x=\frac{y^2}{1+y^2},y^2=\frac{x}{1-x},1+y^2=\frac{1}{1-x}$$

$$2\ln y=\ln x-\ln(1-x)$$

Arfken p617

$$\beta(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}=\int_0^1 dx(1-x)^{a-1}x^{b-1}$$

$$2\frac{dy}{y}=dx\left(\frac{1}{x}+\frac{1}{1-x}\right)=\frac{dx}{x(1-x)}$$

$$=i\Delta^{-\frac{\varepsilon}{2}}\int\frac{y^{2+d}}{(1+y^2)^3}\frac{dy}{y}=i\Delta^{-\frac{\varepsilon}{2}}\int\left(\frac{x}{1-x}\right)^{1+\frac{d}{2}}(1-x)^3\frac{dx}{2x(1-x)}$$

$$=i\frac{\Delta^{-\frac{\varepsilon}{2}}}{2}\int x^{\frac{d}{2}}(1-x)^{1-\frac{d}{2}}dx=i\frac{\Delta^{-\frac{\varepsilon}{2}}}{2}\frac{\Gamma\left(1+\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)}{\Gamma(3)}$$

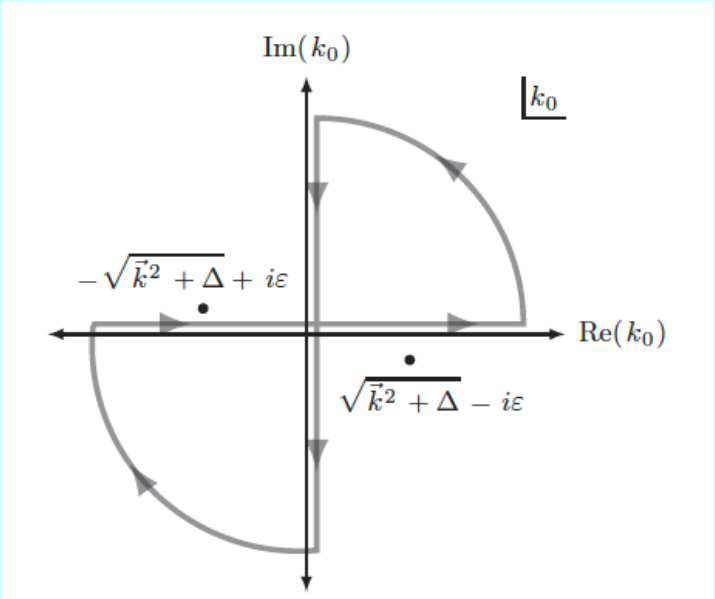
This is divergent.

B.2 Wick rotations

After introducing Feynman parameters and completing the square, one is often left with an integral over a loop momentum k^μ in Minkowski space. Once the $i\varepsilon$ factors are put in for Feynman propagators, 1-loop integrals often appear as

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - \Delta + i\varepsilon)^n}. \tag{B.11}$$

Assuming $\Delta > 0$ (you can check that Wick rotation still works for $\Delta < 0$ in Problem B.1), this integral has poles at $k_0 = \sqrt{\vec{k}^2 + \Delta} - i\varepsilon$ and $k_0 = -\sqrt{\vec{k}^2 + \Delta} + i\varepsilon$, as shown in Figure B.1. Since the poles are in the top-left and bottom-right quadrants of the k_0 complex plane, the integral over the figure-eight contour shown vanishes. Thus, the integrals over the real axis and the imaginary axis are equal and opposite. Therefore, we can substitute $k_0 \to ik_0$ so that $k^2 \to -k_0^2 - \vec{k}^2 = -k_E^2$, where $k_E^2 = k_0^2 + \vec{k}^2$ is the Euclidean momentum. This



for $d < 4$

$$\pi^{n/2} = \left(\int_{-\infty}^{\infty} e^{-p^2}\, dp\right)^n = \int e^{-p^2}\, d^n p = \Omega(n) \int_0^{\infty} e^{-p^2}\, p^{n-1}\, dp$$

$$= \frac{1}{2}\Omega(n) \int_0^{\infty} (p^2)^{\frac{n}{2}-1}\, e^{-p^2}\, d(p^2) = \frac{1}{2}\Omega(n)\, \Gamma\left(\frac{n}{2}\right)$$

$$\Omega(d) = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

Put $\Omega(d)$ in:

$$\int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2 - \Delta + i\varepsilon)^3} = \frac{\Omega(d)}{(2\pi)^d} \int \frac{k^{2+d}}{(k^2 - \Delta + i\varepsilon)^3} \frac{dk}{k}$$

$$= \frac{\Omega(d)}{(2\pi)^d} \frac{\Delta^{-\frac{\varepsilon}{2}}}{2} \frac{\Gamma\left(1 + \frac{d}{2}\right)\Gamma\left(2 - \frac{d}{2}\right)}{\Gamma(3)}$$

$$= \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{\frac{\varepsilon}{2}}} \frac{\Gamma\left(1 + \frac{d}{2}\right)\Gamma\left(2 - \frac{d}{2}\right)}{2\Gamma\left(\frac{d}{2}\right)}$$

$$= \frac{d}{4} \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{\frac{\varepsilon}{2}}} \Gamma\left(2 - \frac{d}{2}\right)$$

$$\Omega(d) = \frac{2\pi^{d/2}}{\Gamma\left(\frac{d}{2}\right)}$$

$$\Gamma(z+1) = z\Gamma(z)$$

$$\Gamma\left(1 + \frac{d}{2}\right) = \frac{d}{2}\Gamma\left(\frac{d}{2}\right)$$

$$\mu^{\varepsilon}\left(2-\frac{4}{d}\right)\cdot\frac{d}{4}\frac{i}{(4\pi)^{d/2}}\frac{1}{\Delta^{\frac{\varepsilon}{2}}}\Gamma\left(2-\frac{d}{2}\right)$$

$$\left(\frac{4-2\varepsilon}{4+\varepsilon}\right)\cdot\frac{4-\varepsilon}{4}=\left(\frac{1-\frac{1}{2}\varepsilon}{1+\frac{1}{4}\varepsilon}\right)\cdot\left(1-\frac{1}{4}\varepsilon\right)$$

$$\sim(1-\varepsilon)$$

Write all in terms of $\varepsilon = 4-d \to 0$

$$=\mu^{\varepsilon}\left(\frac{4-2\varepsilon}{4+\varepsilon}\right)\cdot\frac{4-\varepsilon}{4}\frac{i}{(4\pi)^{2-\varepsilon/2}}\frac{1}{\Delta^{\frac{\varepsilon}{2}}}\Gamma\left(\frac{\varepsilon}{2}\right)$$

$$\sim(1-\varepsilon)\cdot\frac{i}{16\pi^2}\frac{\left(\sqrt{4\pi}\mu\right)^{\varepsilon}}{\Delta^{\frac{\varepsilon}{2}}}\Gamma\left(\frac{\varepsilon}{2}\right)$$

$$\frac{\left(\sqrt{4\pi}\mu\right)^{\varepsilon}}{\Delta^{\frac{\varepsilon}{2}}}=e^{\frac{\varepsilon}{2}\ln\frac{4\pi\mu^2}{\Delta}}$$

$$\sim(1-\varepsilon)\cdot\frac{i}{16\pi^2}\left(1+\frac{\varepsilon}{2}\ln\frac{4\pi\mu^2}{\Delta}\right)\cdot\left(\frac{2}{\varepsilon}-\gamma_E\right)$$

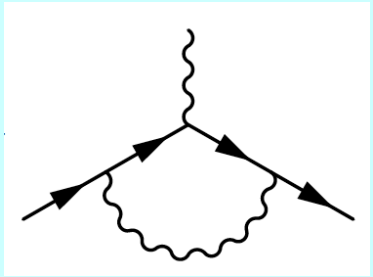
$$\Gamma(\varepsilon)=\frac{1}{\varepsilon}-\gamma_E+\mathcal{O}(\varepsilon)$$

$$=-\frac{i}{16\pi^2}\left(\frac{2}{\varepsilon}+\ln\frac{\mu^2}{\Delta}+\cdots.\right)$$

In Minimal Subtraction (MS) scheme, the counter term will cut just the pole.

$$\delta_1=-\frac{g^2}{16\pi^2}\frac{2}{\varepsilon}$$

negative!



Usually we just consult tables.

The integrals over Euclidean k_E are straightforward:

$$\int dk_E \frac{k_E^a}{(k_E^2+\Delta)^b} = \Delta^{\frac{a+1}{2}-b} \frac{\Gamma\left(\frac{a+1}{2}\right)\Gamma\left(b-\frac{a+1}{2}\right)}{2\Gamma(b)}. \tag{B.34}$$

Equations (B.22), (B.28) and (B.34) can be combined into a general formula:

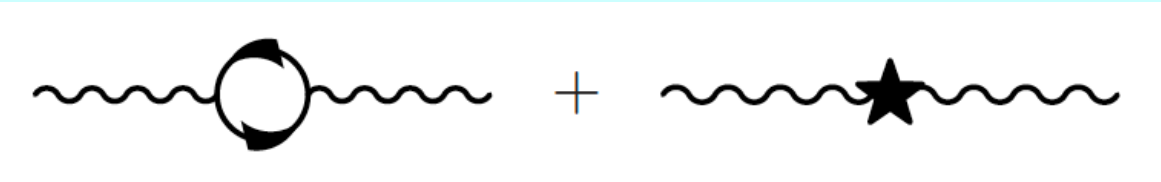
$$\int \frac{d^dk}{(2\pi)^d} \frac{k^{2a}}{(k^2-\Delta)^b} = i(-1)^{a-b} \frac{1}{(4\pi)^{d/2}} \frac{1}{\Delta^{b-a-\frac{d}{2}}} \frac{\Gamma\left(a+\frac{d}{2}\right)\Gamma\left(b-a-\frac{d}{2}\right)}{\Gamma(b)\Gamma(\frac{d}{2})}. \tag{B.35}$$

$$\int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-\Delta+i\varepsilon)^2} = \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{2-\frac{d}{2}}} \Gamma\left(\frac{4-d}{2}\right), \tag{B.36}$$

$$\int \frac{d^dk}{(2\pi)^d} \frac{k^2}{(k^2-\Delta+i\varepsilon)^2} = -\frac{d}{2} \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{1-\frac{d}{2}}} \Gamma\left(\frac{2-d}{2}\right), \tag{B.37}$$

$$\int \frac{d^dk}{(2\pi)^d} \frac{k^2}{(k^2-\Delta+i\varepsilon)^3} = \frac{d}{4} \frac{i}{(4\pi)^{d/2}} \frac{1}{\Delta^{2-\frac{d}{2}}} \Gamma\left(\frac{4-d}{2}\right), \tag{B.38}$$

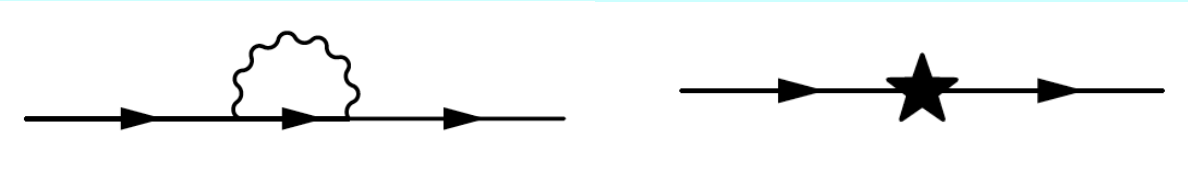
Photon vacuum polariztion



$$\delta_3 = \frac{g^2}{16\pi^2}\left(-\frac{8}{3\varepsilon}\right)$$

Electron self energy

negative!



$$\delta_2 = \delta_1 = \frac{g^2}{16\pi^2}\left(-\frac{2}{\varepsilon}\right)$$

negative!

QED Lagrangian in terms of bare parameters and fields.

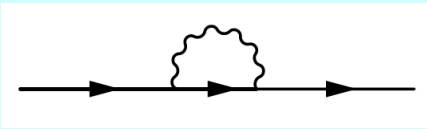
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^{0}F^{0\mu\nu} + \bar{\psi}^{0}i\gamma^{\mu}\partial_{\mu}\psi^{0} + ig^{0}\bar{\psi}^{0}\gamma^{\mu}A^{\mu}\psi^{0}$$

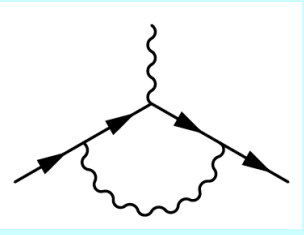
It is equal to $\mathcal{L}$ in terms of perturbation parameters and field plus counter terms.

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi + ig\bar{\psi}\gamma^{\mu}A^{\mu}\psi \; -\frac{1}{4}\delta_3 F_{\mu\nu}F^{\mu\nu} + \delta_2\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi + ig\delta_1\bar{\psi}\gamma^{\mu}A^{\mu}\psi$$

$$= -\frac{1}{4}(1+\delta_3)F_{\mu\nu}F^{\mu\nu} + (1+\delta_2)\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi + ig(1+\delta_1)\bar{\psi}\gamma^{\mu}A^{\mu}\psi$$

$$= -\frac{1}{4}Z_3 F_{\mu\nu}F^{\mu\nu} + Z_2\bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi + igZ_1\bar{\psi}\gamma^{\mu}A^{\mu}\psi$$





The bare parameters and bare fields are thus related to perturbation parameters and fields.

$$\psi^{0} = \sqrt{Z_2}\psi, \qquad A_{\mu}^{0} = \sqrt{Z_3}A_{\mu}, \qquad g^{0} = g\frac{Z_1}{Z_2\sqrt{Z_3}} \equiv gZ_g = g\left(1+\delta_1-\delta_2-\frac{1}{2}\delta_3\right)$$

If $d < 4, \varepsilon = 4 - d > \to 0$

$$g_0 = Z_g \mu^{\frac{\varepsilon}{2}} g \qquad Z_g = 1 + \delta_1 - \delta_2 - \frac{1}{2}\delta_3 = 1 - \frac{1}{2}\delta_3 = 1 + \frac{g^2}{12\pi^2}\frac{1}{\varepsilon}$$

g_0 is μ independent.

$$\mu \frac{d}{d\mu} g_0 = \mu \frac{d}{d\mu}\left(\mu^{\frac{\varepsilon}{2}} Z_g g\right) = \mu^{\frac{\varepsilon}{2}} Z_g \left(g\frac{\varepsilon}{2} + g\frac{\mu}{Z_g}\frac{dZ_g}{d\mu} + \mu\frac{dg}{d\mu}\right) = 0$$

β

$$\beta = -g\frac{\varepsilon}{2} - g\frac{\mu}{Z_g}\frac{dZ_g}{d\mu}$$

To leading order in g, $Z_g = 1$: $\beta^{(1)} = \mu\frac{dg}{d\mu} = -g\frac{\varepsilon}{2}$

To next-to-leading order: $\beta^{(3)} = -g\frac{\mu}{Z_g}\frac{dZ_g}{d\mu} = -g\left(\mu\frac{dg}{d\mu}\right)(2g)\frac{1}{12\pi^2}\frac{1}{\varepsilon}$

Z_g only depends on μ through coupling constant g. We plug in $\beta^{(1)} = -g\frac{\varepsilon}{2}$.

$\beta = \frac{g^3}{12\pi^2}$ β solely depends on the poles in counter terms.

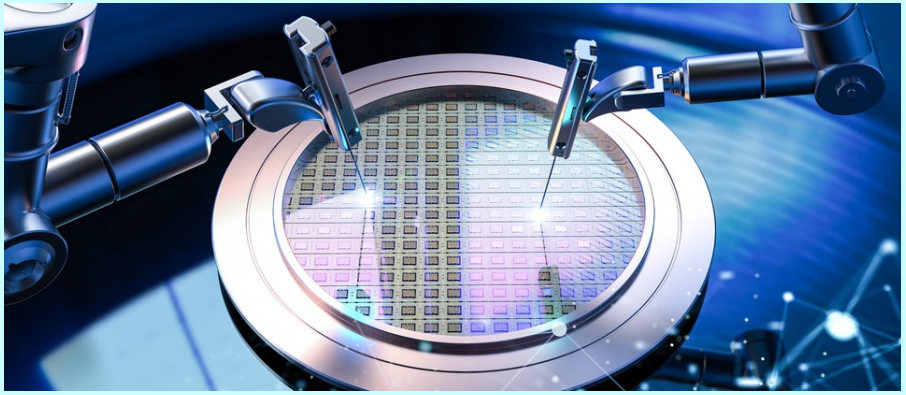
$$\beta = \frac{d}{d\ln\mu} g(\mu) = \frac{g^3}{12\pi^2}$$

$$\frac{d}{d\ln\mu}\alpha = \frac{\alpha^2}{3\pi} \qquad \alpha \equiv \frac{g^2}{4\pi}$$

$$\alpha = 3\pi \frac{1}{\ln\left(\frac{\mu}{\Lambda_{\mathrm{QED}}}\right)} \qquad \Lambda_{\mathrm{QED}} = 10^{286}\ \mathrm{eV}$$

Lesson: QED will eventually die: Landau Pole. But that is a long way off.

A better lesson: Different scales, different tools.



$$\psi^0 = \sqrt{Z_2}\psi, \qquad A_\mu^0 = \sqrt{Z_3}A_\mu, \qquad g^0 = gZ_g$$

$$\langle\Omega|T\psi(x_1)\cdots\psi(x_n)|\Omega\rangle = Z_2^{-n/2}\langle\Omega|T\psi^0(x_1)\cdots\psi^0(x_n)|\Omega\rangle$$

$$\langle\Omega|T\psi^0(x_1)\cdots\psi^0(x_n)|\Omega\rangle = Z_2^{n/2}\langle\Omega|T\psi(x_1)\cdots\psi(x_n)|\Omega\rangle = Z_2^{\frac{n}{2}}G_n(p,g,\mu)$$

This is a relation between bare field Greens function and renormalized Green function.

23.4.3 Renormalization group equation for Green's functions

Again, the left-hand side with bare fields is μ independent.

$$\mu\frac{d}{d\mu}G_n^0 = 0 = Z_2^{\frac{n}{2}}G_n(p,g,\mu) = Z_2^{\frac{n}{2}}\left(\mu\frac{\partial}{\partial\mu} + \frac{n}{2}\frac{\mu}{Z_2}\frac{dZ_2}{d\mu} + \beta\frac{\partial}{\partial g}\right)G_n(p,g,\mu)$$

$$\left(\mu\frac{\partial}{\partial\mu} + \frac{n}{2}\gamma_2 + \beta\frac{\partial}{\partial g}\right)G_n(p,g,\mu) = 0$$

Anomalous dimension

This equation is known variously as the **Callan–Symanzik equation**, the **Gell-Mann–Low equation**, the **'t Hooft–Weinberg equation** and the **Georgi–Politzer equation**. (The

$$0 = \mu \frac{d}{d\mu} G^{(0)}_{n,m}$$
$$= Z_3{}^{\frac{n}{2}} Z_2{}^{\frac{m}{2}} \left(\mu \frac{\partial}{\partial \mu} + \frac{n}{2} \frac{\mu}{Z_3} \frac{dZ_3}{d\mu} + \frac{m}{2} \frac{\mu}{Z_2} \frac{dZ_2}{d\mu} + \mu \frac{\partial e_R}{\partial \mu} \frac{\partial}{\partial e_R} + \mu \frac{\partial m_R}{\partial \mu} \frac{\partial}{\partial m_R} \right) G_{n,m}. \tag{23.74}$$

Defining

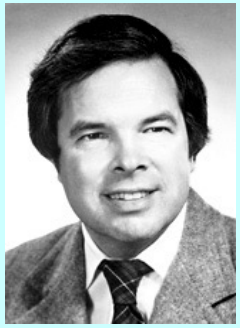
$$\gamma_3 = \frac{\mu}{Z_3} \frac{dZ_3}{d\mu}, \quad \gamma_2 = \frac{\mu}{Z_2} \frac{dZ_2}{d\mu}, \tag{23.75}$$

this reduces to

$$\left(\mu \frac{\partial}{\partial \mu} + \frac{n}{2} \gamma_3 + \frac{m}{2} \gamma_2 + \beta \frac{\partial}{\partial e_R} + \gamma_m m_R \frac{\partial}{\partial m_R} \right) G_{n,m} = 0. \tag{23.76}$$

This equation is known variously as the **Callan–Symanzik equation**, the **Gell-Mann–Low equation**, the **'t Hooft–Weinberg equation** and the **Georgi–Politzer equation**. (The differences refer to different schemes, such as $\overline{\text{MS}}$ or the on-shell physical renormalization scheme.)

Wilson's renormalization group



Ken Wilson 1963-2013

In condense matter theory, there is usually a physical Λ.

Λ is usually related to the separation of lattice.

Condense matter doesn't have an infinity problem.

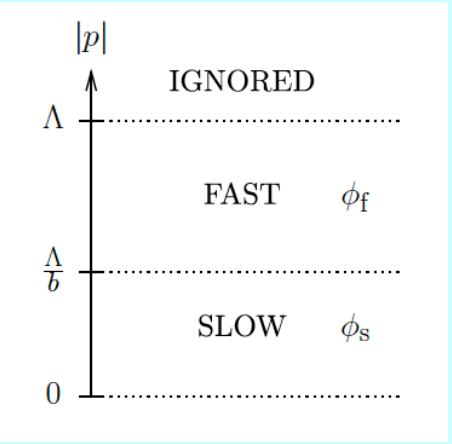
In configurations, assume we can separate the fast ϕ_f and slowing changing ϕ_s by $|p|$.

$$Z[\Lambda] = N \int_{\Lambda/b} (D\phi_s) e^{-\int d^d x_E \, \mathcal{L}_E[\phi_s]} \int_{\Lambda/b}^{\Lambda} (D\phi_f) \, e^{-\int d^d x_E \, \mathcal{L}_I[\phi_f,\phi_s]}$$

ϕ_f could also be said to be the more detailed, with finer pixels.

Let's say we can integrate over ϕ_f first without over ϕ_s.

$$\int_{\Lambda/b}^{\Lambda} (D\phi_f) \, e^{-\int d^d x_E \, \mathcal{L}_E[\phi_f,\phi_s]} = e^{-\int d^d x_E \, \delta\mathcal{L}_E[\phi_s]}$$



Most likely, the effects of ϕ_f can be written as a correction to $\mathcal{L}_E[\phi_s]$.

$$\int_{\Lambda/b} (D\phi_s) e^{-\int d^d x_E \, (\mathcal{L}_E[\phi_s] + \delta\mathcal{L}_E[\phi_s])}$$

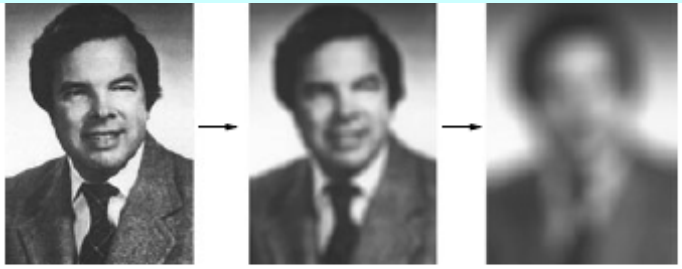


Fig. 34.3 Removing the largest Fourier components results in a loss of detail.

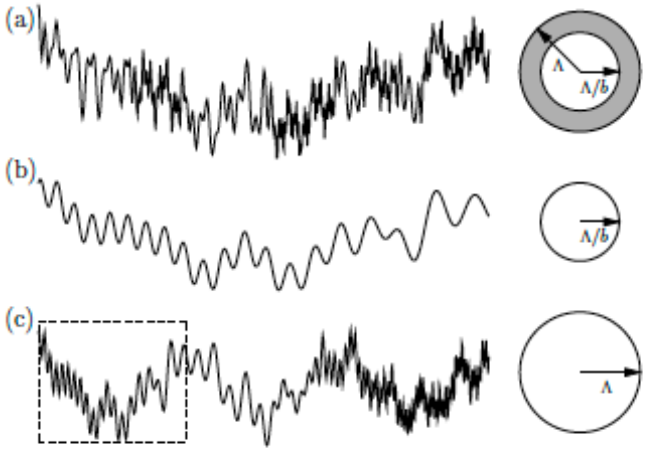


Fig. 34.4 The renormalization group process in momentum space. (a) The field $\phi(x)$ before the process. (b) The result of integrating out the largest momentum states (step I) is to remove the high spatial frequency components. (c) The field after rescaling (steps II and III). Note that the region in the dashed box is identical to the entire trace in (b).

We can rescale so that $\Lambda/b \to \Lambda$. It is equivalent to $p \to p' = bp$.

Fields need be rescaled too.

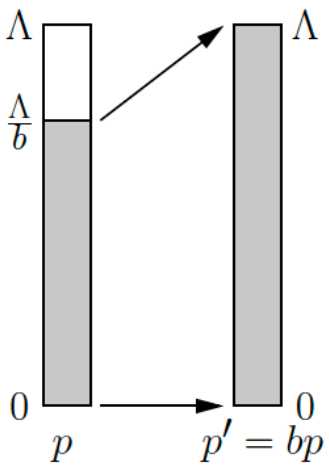


Fig. 34.6 Rescaling the momentum.

[7]The subject is notorious for its arcane terminology, and d_ϕ is known as the **anomalous dimension**.

[8]Sending ℓ to infinity corresponds to looking at the infrared behaviour of the couplings (since large length is the same as small energy). We could send $\ell \to 0$, if we want the ultraviolet behaviour.

Step II: We relabel the momenta. We do this by expressing our theory in terms of scaled momenta $p' = pb$. Since $b > 1$ this has the effect of stretching the momentum space out over the range we had originally, as shown in Fig. 34.6. This is seen by examining the limit of the integral, which is $p'/b = \Lambda/b$ or $p' = \Lambda$.

Step III: Finally we scale the fields. We select the term that we imagine will be most important in determining the scaling behaviour and require it to be *invariant* with the scaling of step II. We use this to rescale the field $\tilde{\phi}(p) = \tilde{\phi}(p'/b) = b^{d-d_\phi}\tilde{\phi}'(p')$, where d_ϕ is chosen to leave the important term invariant.[7]

$$\int_{\Lambda} (D\phi')e^{-\int d^d x_E \; \mathcal{L}'_E[\phi']}$$

out once, we repeat it an
to carry out the procedure
ting it. That is, we do the
of the coupling constants
nuous flow of the coupling
ation group method again
and again.

By removing the fastest varying fields we are looking at the physics at lower momentum, or equivalently at longer length scale. We can therefore ask how the physics changes as we send the length scale of interest ℓ to infinity. One way to do this is to set $b = e^\ell$ and search for flow equations of the form

$$\frac{dg_i}{d\ell} = \beta_i(g_1, g_2, ..., g_N). \qquad (34.13)$$

This is another form of the Gell-Mann–Low equation.[8]

So now we work in $\mathcal{L}'_E$ with possible new terms and changed coupling constants.

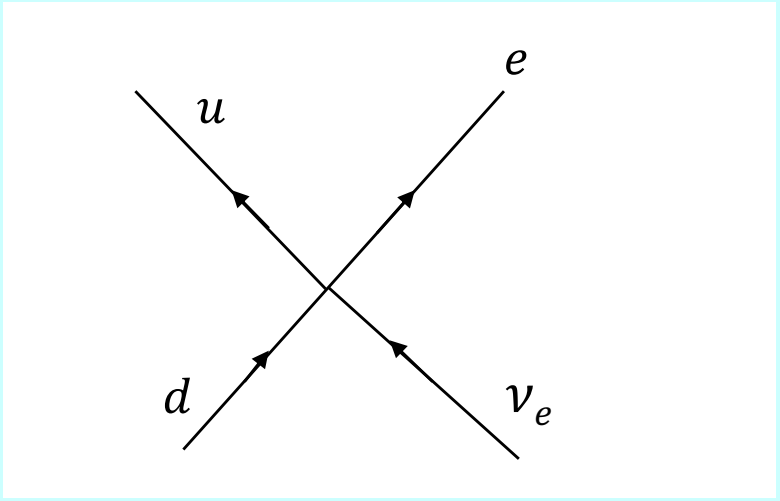
But momenta $p' = bp$ corresponds to a smaller p in the original theory.

$$\int_{\Lambda/b}^{\Lambda} (D\phi_f)\, e^{-\int d^d x_E\ \mathcal{L}_E[\phi_f,\phi_s]} = e^{-\int d^d x_E\ \delta\mathcal{L}_E[\phi_s]}$$

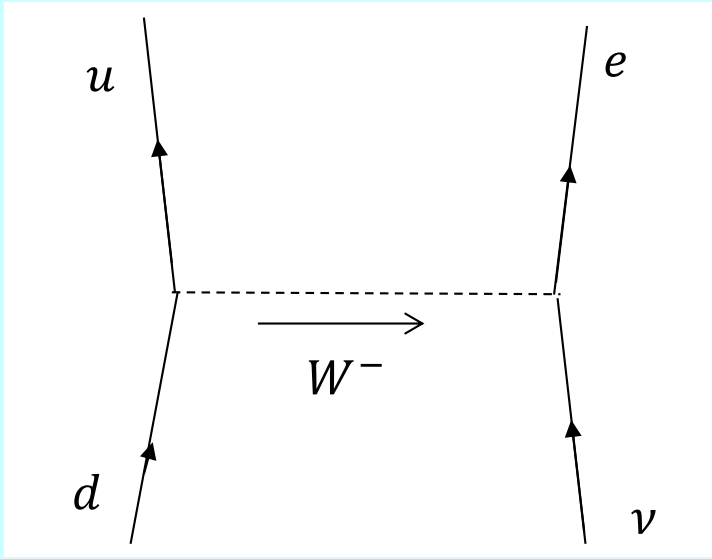
By integrating out fast changing field configuration,

their effects will be incorporated into parameter changes or new interactions.

Wilson's RG is actually Effetive Field Theory EFT of particle physics.



$\approx$



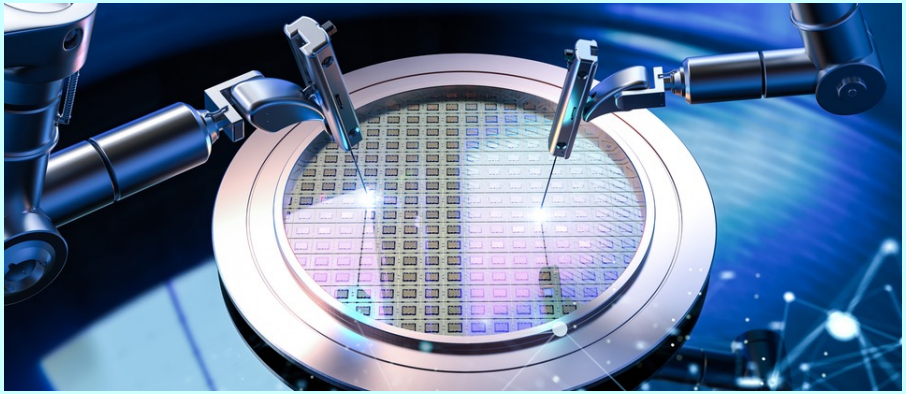
$p^2 \ll M_W^2$

$$-\frac{i}{M_W^2} \quad \Leftarrow \quad \frac{i}{p^2 - M_W^2}$$

If we integrate out W, we must add a 4-vertex to account for its effects.

在低能量時，Fermi 的理論是一個很有效的理論 Effective Field Theory。

Different scales, different tools.

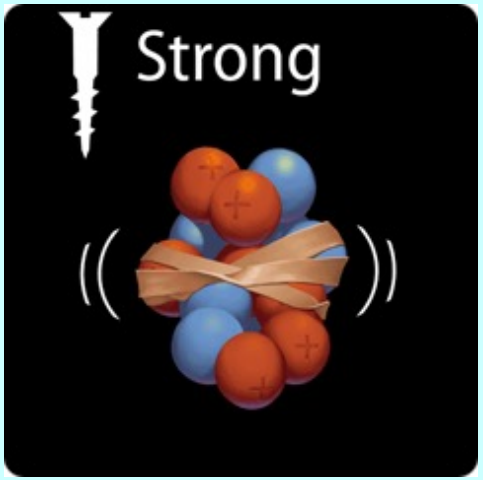


It would be stupid to dig ground using semiconductor manufacturing tools.

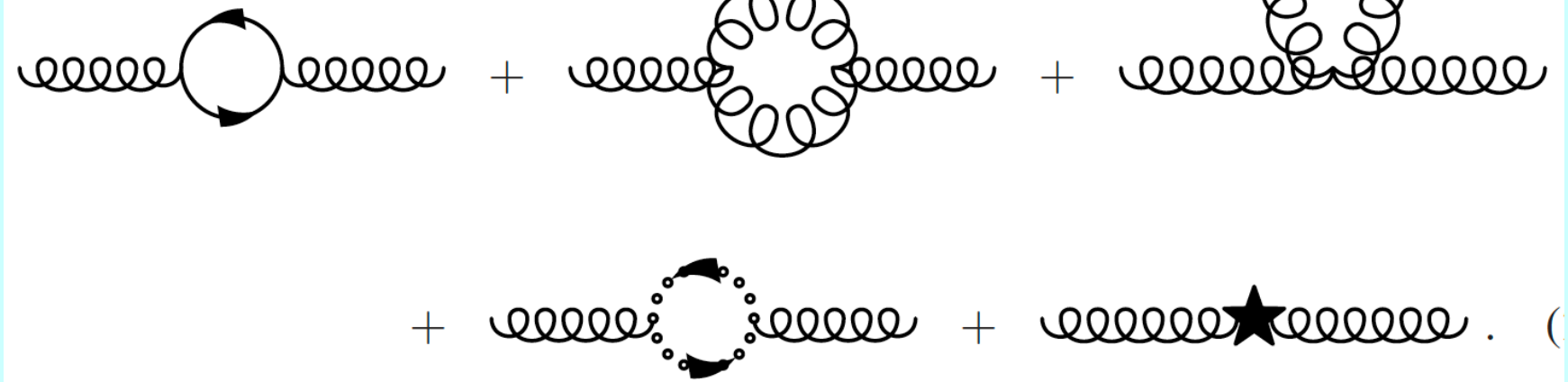
Wilson's RG and Effetive Field Theory are telling us:

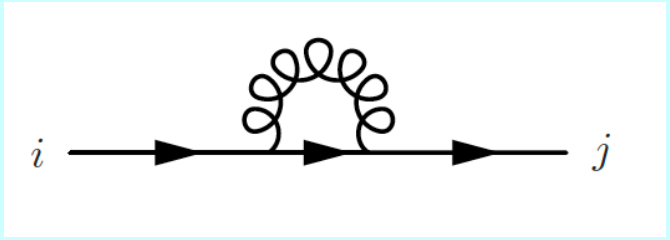
at different energy (details), efficient way to do science may be different effective theory.

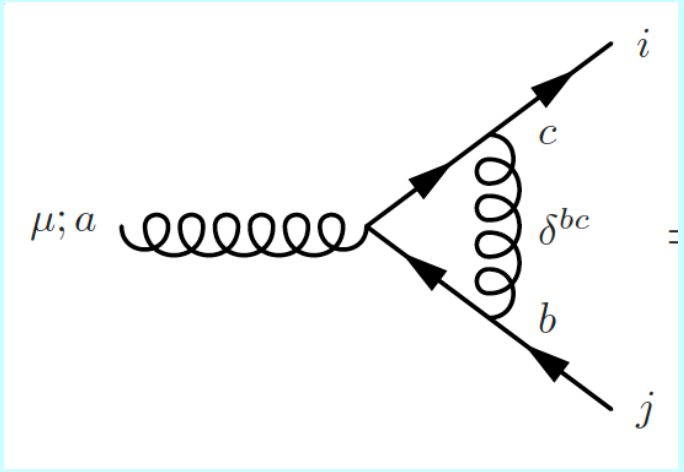
但強作用力真是奇特.....

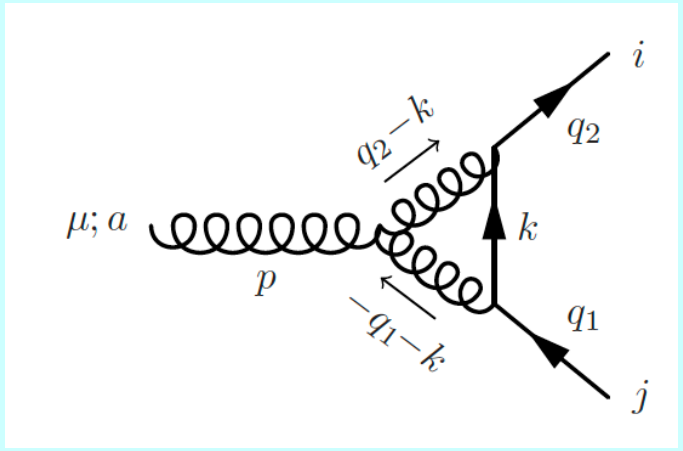


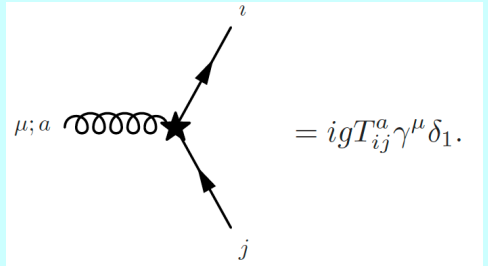
en by the graphs











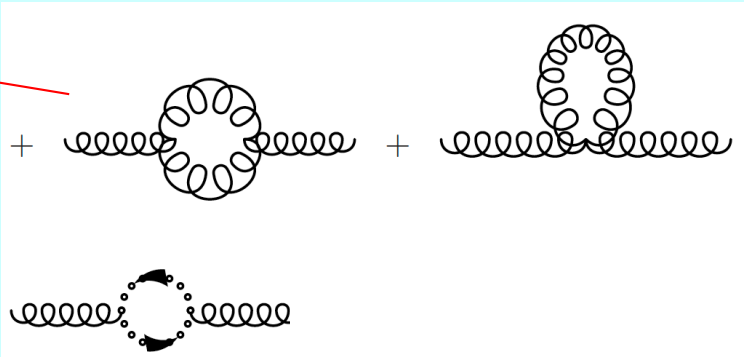
$$\delta_1 = \frac{1}{\varepsilon}\left(\frac{g^2}{16\pi^2}\right)\left[-2C_F - 2C_A\right.$$

Take $\xi = 1$

$$\delta_2 = \frac{1}{\varepsilon}\left(\frac{g^2}{16\pi^2}\right)\left[-2C_F\right.$$

Positive!

$$\delta_3 = \frac{1}{\varepsilon}\left(\frac{g^2}{16\pi^2}\right)\left[\frac{10}{3}C_A - \frac{8}{3}n_f T_F\right.$$



β will change sign.

$$\beta = -g\frac{\varepsilon}{2} - g\mu\frac{d}{d\mu}\left(\delta_1 - \delta_2 - \frac{1}{2}\delta_3\right)$$

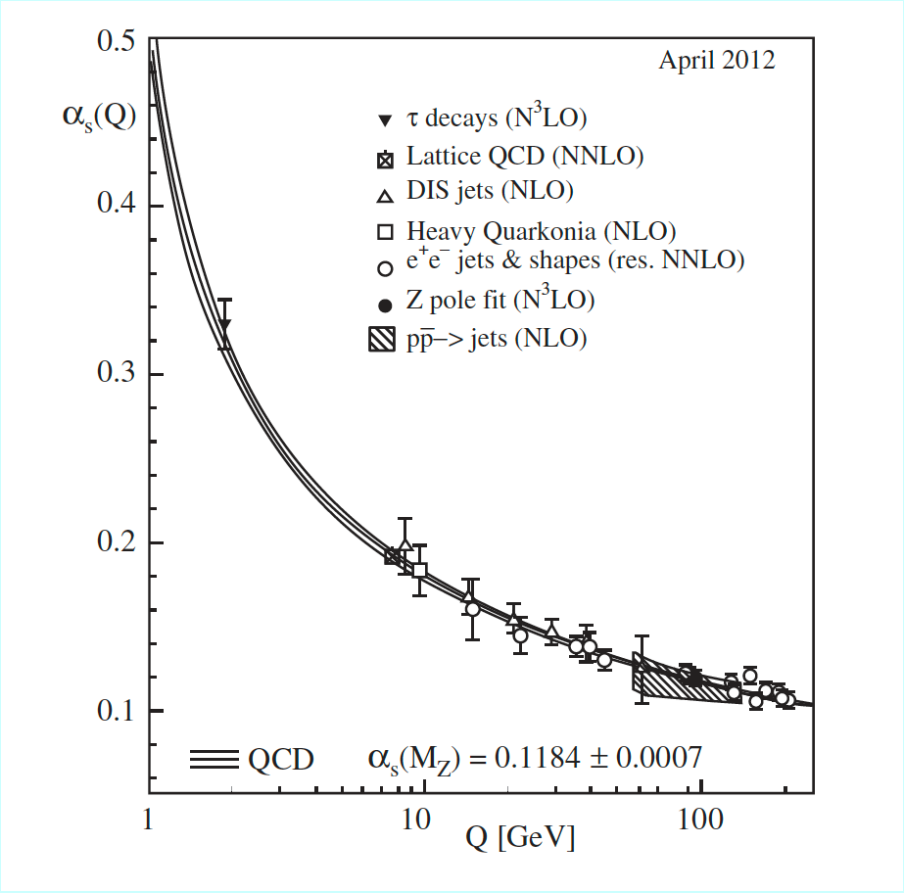
$$\beta = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}C_A - \frac{4}{3}n_f T_F\right) \quad \Longleftrightarrow \quad \beta = \frac{g^3}{12\pi^2}$$

Negative! $C_A = 3, T_F = \frac{1}{2}$

QED positive

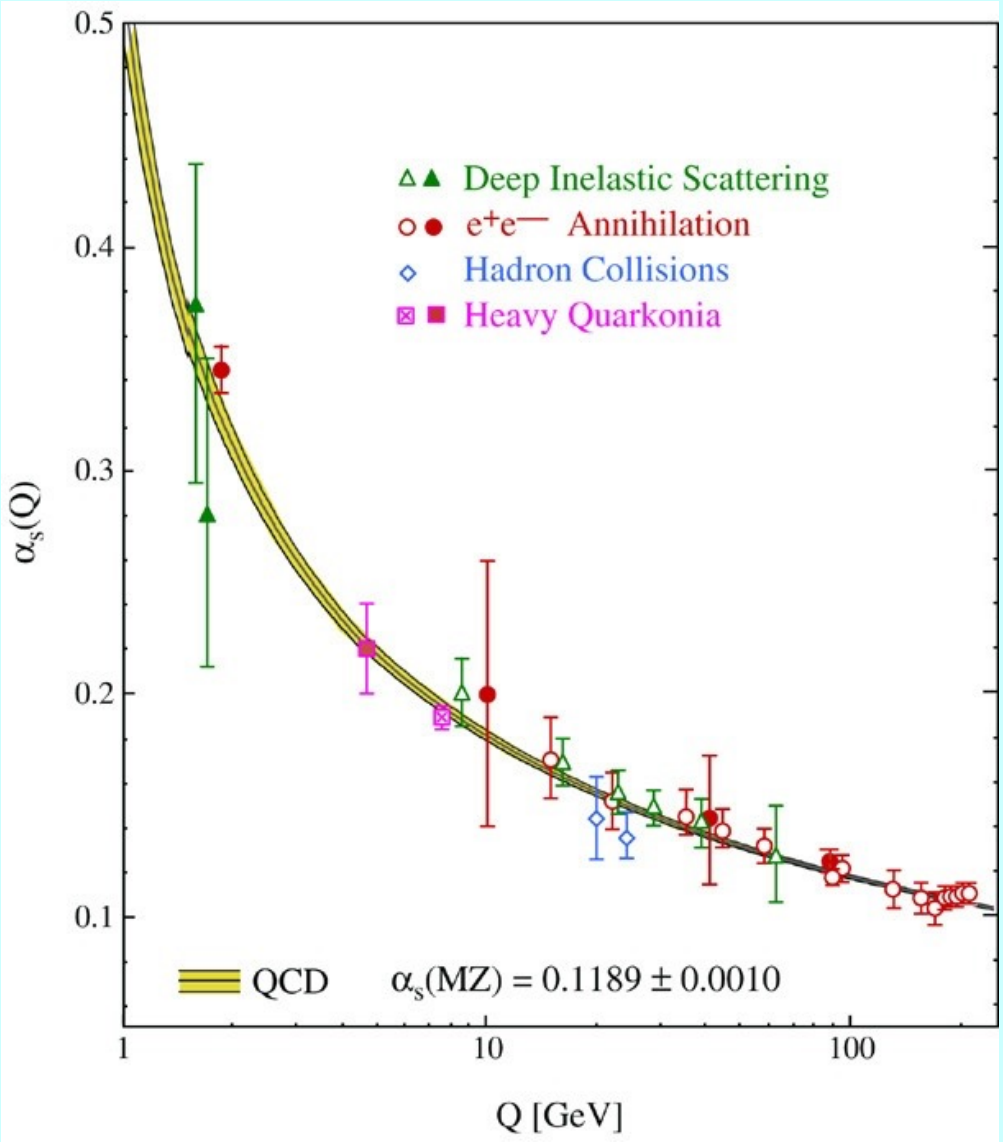
$$\frac{d}{d\ln\mu}\alpha = \frac{\alpha^2}{2\pi}\beta_0 \qquad \beta_0 = 11 - \frac{2}{3}n_f$$

$$\alpha = \frac{2\pi}{\beta_0}\frac{1}{\ln\left(\frac{\mu}{\Lambda_{QCD}}\right)}$$



強交互作用強度與距離的關係竟然與電磁作用相反。

Asymptotic Freedom 漸進自由



距離越近，能量越大，強作用的耦合常數越小！

Sidney Coleman

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

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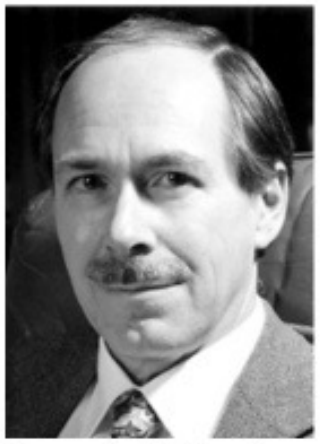
The Nobel Prize in Physics 2004

David J. Gross
Prize share: 1/3

H. David Politzer
Prize share: 1/3

Frank Wilczek
Prize share: 1/3

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction".*



Gerardus 't Hooft

The Nobel Prize in Physics 1999

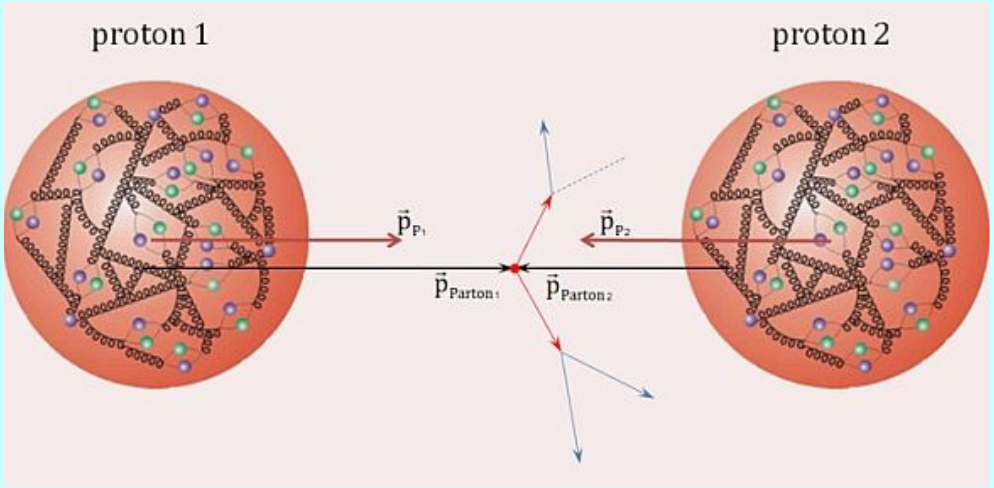
Gerardus 't Hooft
Prize share: 1/2

Martinus J.G. Veltman
Prize share: 1/2

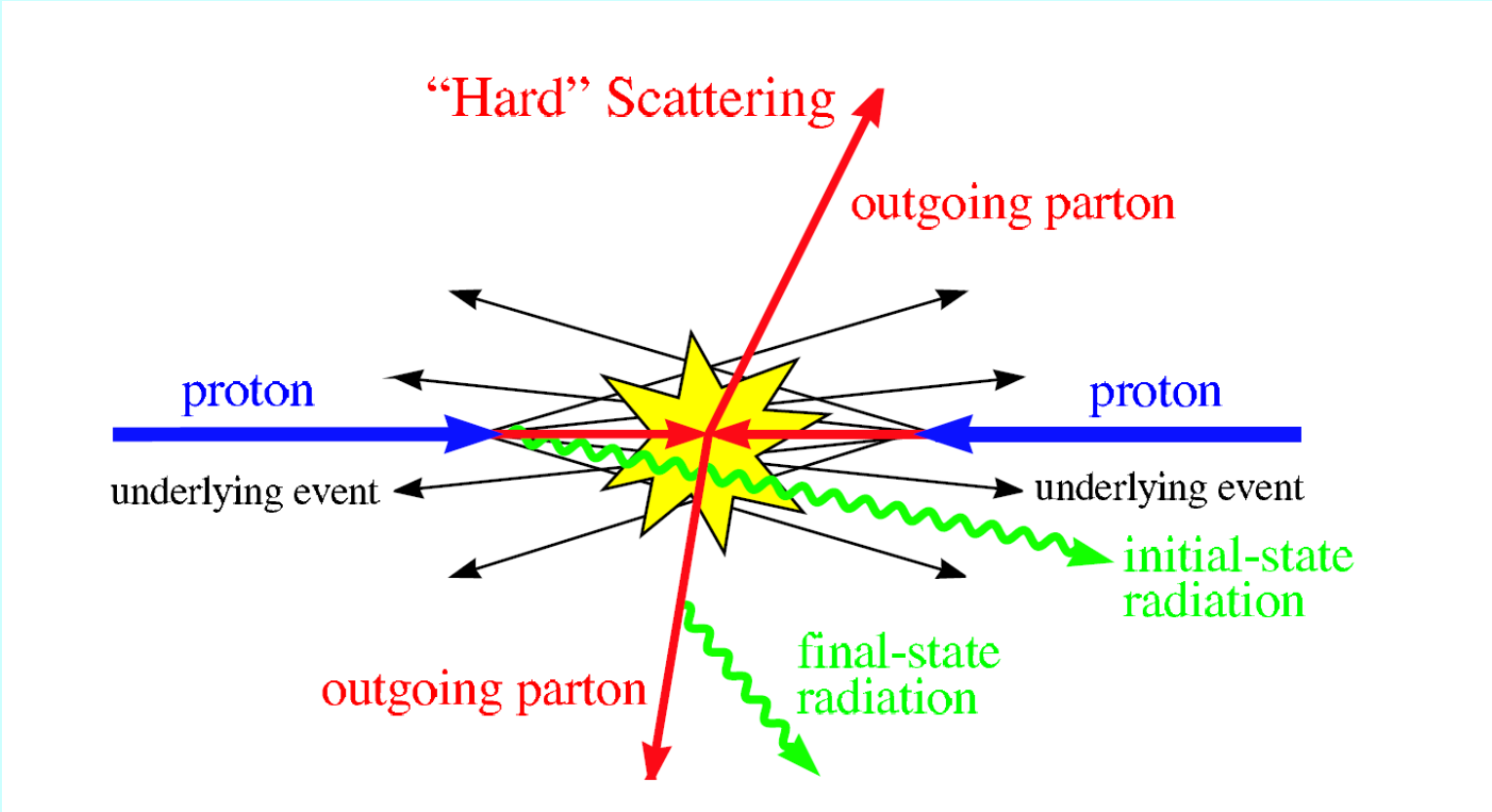
The Nobel Prize in Physics 1999 was awarded jointly to Gerardus 't Hooft and Martinus J.G. Veltman *"for elucidating the quantum structure of electroweak interactions in physics"*

當能量很大時，質子中的夸克與膠子近似是自由的。

在撞擊發生的瞬間，其他的組成成分完全沒有發生作用。



主將對決！



$$\sigma(AB \to F\ X) = \sum_{a,b} \int dx_1 dx_2\ P_{a/A}(x_1, Q^2) P_{b/B}(x_2, Q^2)\ \hat{\sigma}(ab \to F),$$

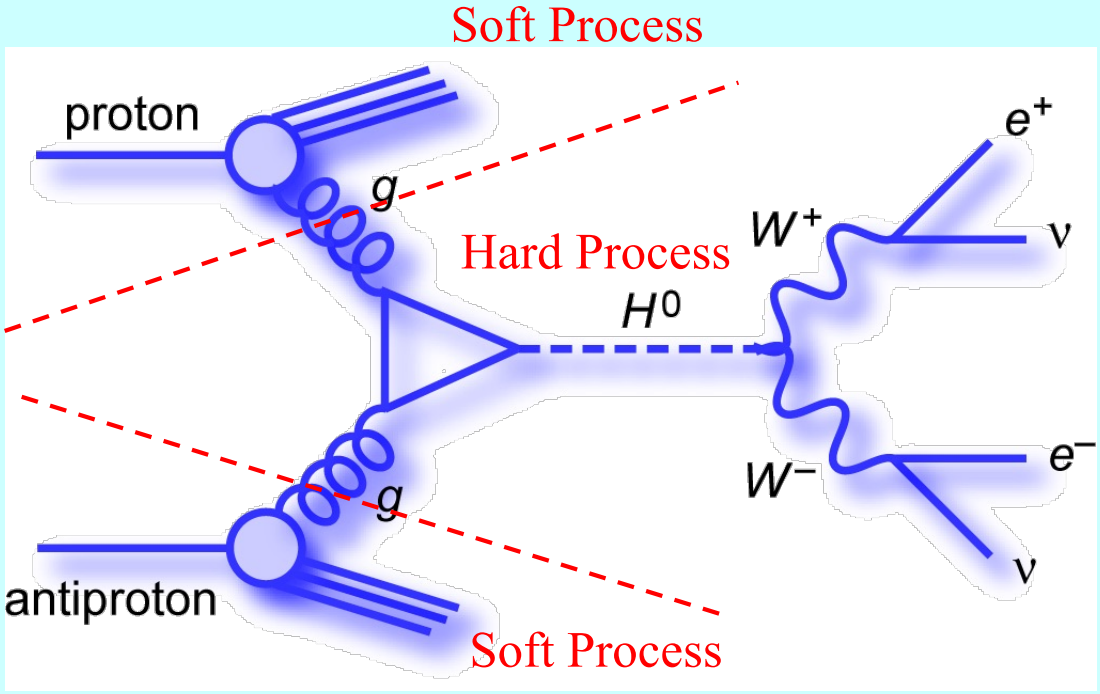
質子分析為組成成分Parton（及Parton在撞擊後組成其他強子）的Soft過程，

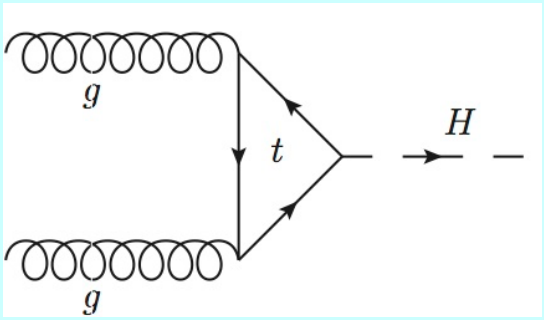
與Partons互相撞擊的Hard過程完全分離！

Soft過程，例如Fragmentation，可以從其他反應數據歸納得出。

Hard過程則可以計算！ Factorization Theorem

QCD at high energy is Perturbative！ Perturbative QCD！





以費因曼圖計算Parton的Hard Process是完全合法的！

若考慮量子效應，真空事實上很像一個電介質！

電子正子對可以一直產生又消逝。

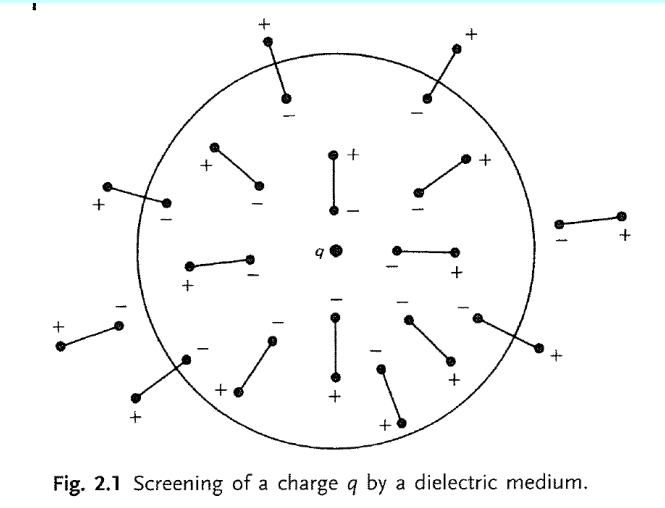
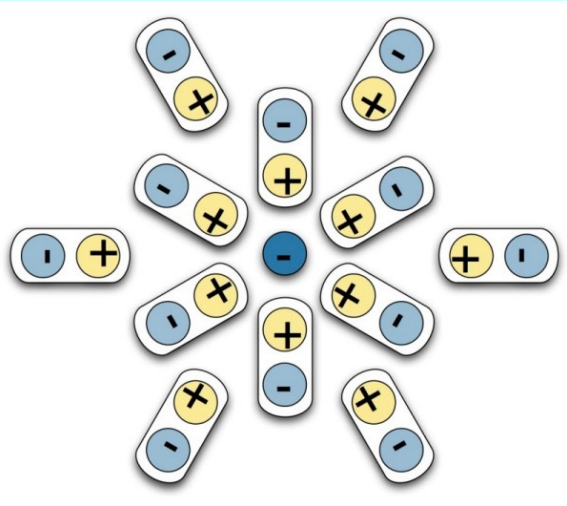
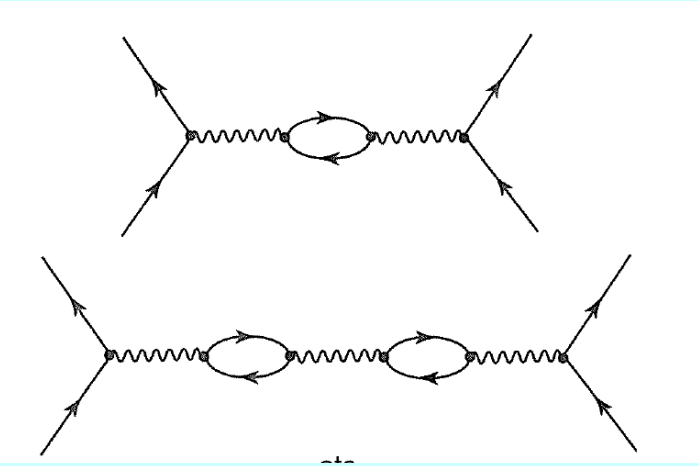


Fig. 2.1 Screening of a charge q by a dielectric medium.



若在一電荷旁，電子正子對中的電子與正子會分離，稱為Vacuum Polarization。

如此電荷的大小會感覺與距離有關（越近越強）！



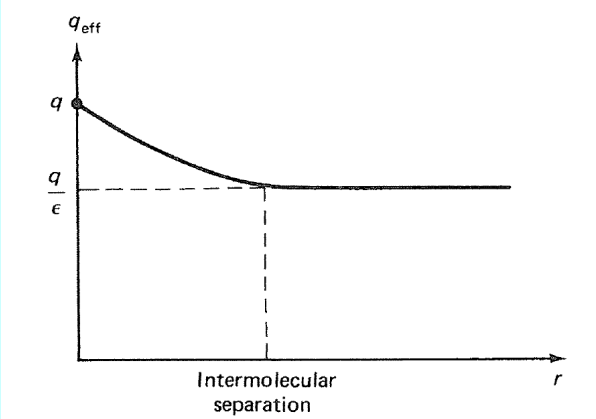


Fig. 2.2 Effective charge as a function of distance.

PAPERS | DECEMBER 01 1981

Asymptotic freedom as a spin effect

N. K. Nielsen

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Asymptotic freedom as a spin effect

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(Received 25 August 1980; accepted 26 November 1980)

It is shown how both the qualitative and the quantitative features of the asymptotic freedom of quantum chromodynamics can be understood in a rather intuitive way. The starting point is the spin of the gluon, which because of the gluon self-coupling makes the vacuum behave like a paramagnetic substance. Combining this result with Lorentz invariance, we conclude that the vacuum exhibits dielectric antiscreening and hence asymptotic freedom. The calculational techniques are with some minor modifications those of the Landau theory on the diamagnetic properties of a free-electron gas.

$\mu > 1$ $\mu\varepsilon = 1$ $\varepsilon < 1$

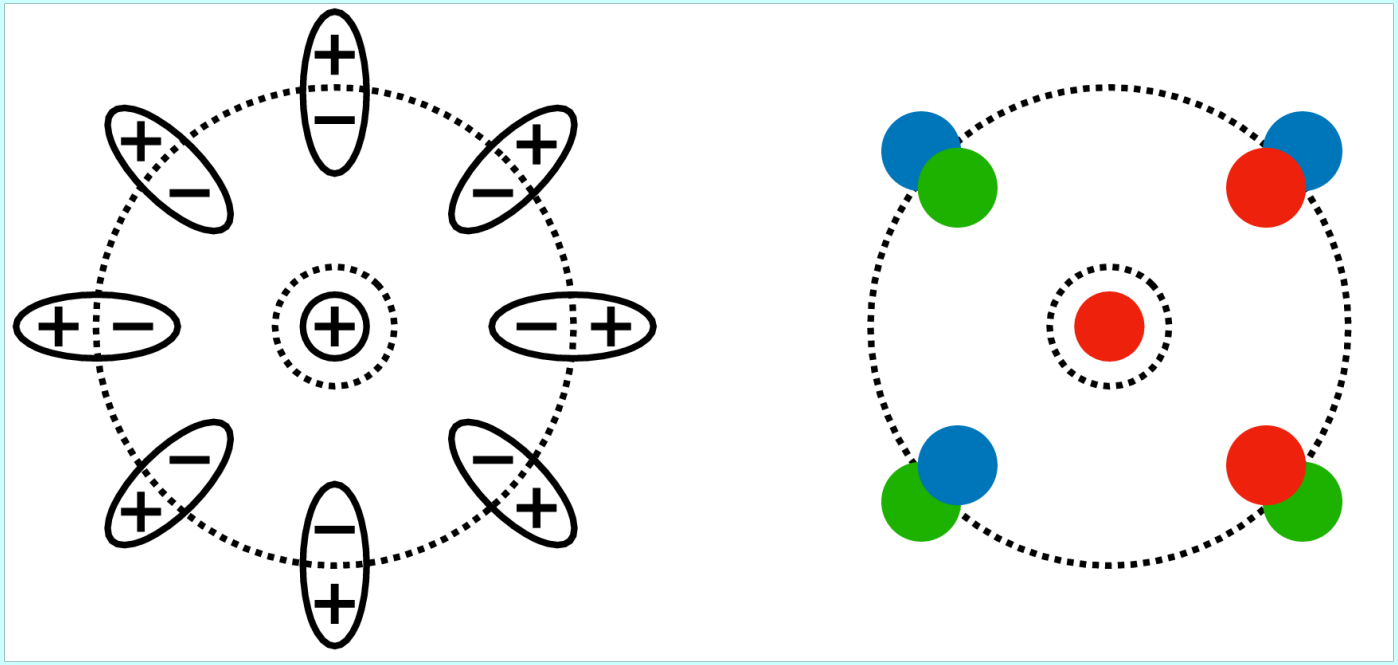
$$V = \frac{1}{4\pi\varepsilon(r)}\frac{q_1 q_2}{r}$$ The potential between two stationary charges is calculable.

$\varepsilon(r)$ is smaller than 1 and $\varepsilon(r) \to 1$ increases as $r \to 0$ for QCD.

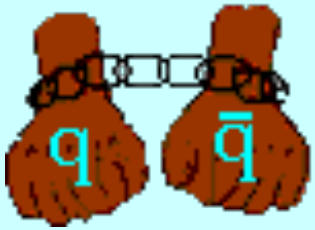
It means vacuum polarization, which becomes less as $r \to 0$, increase color.

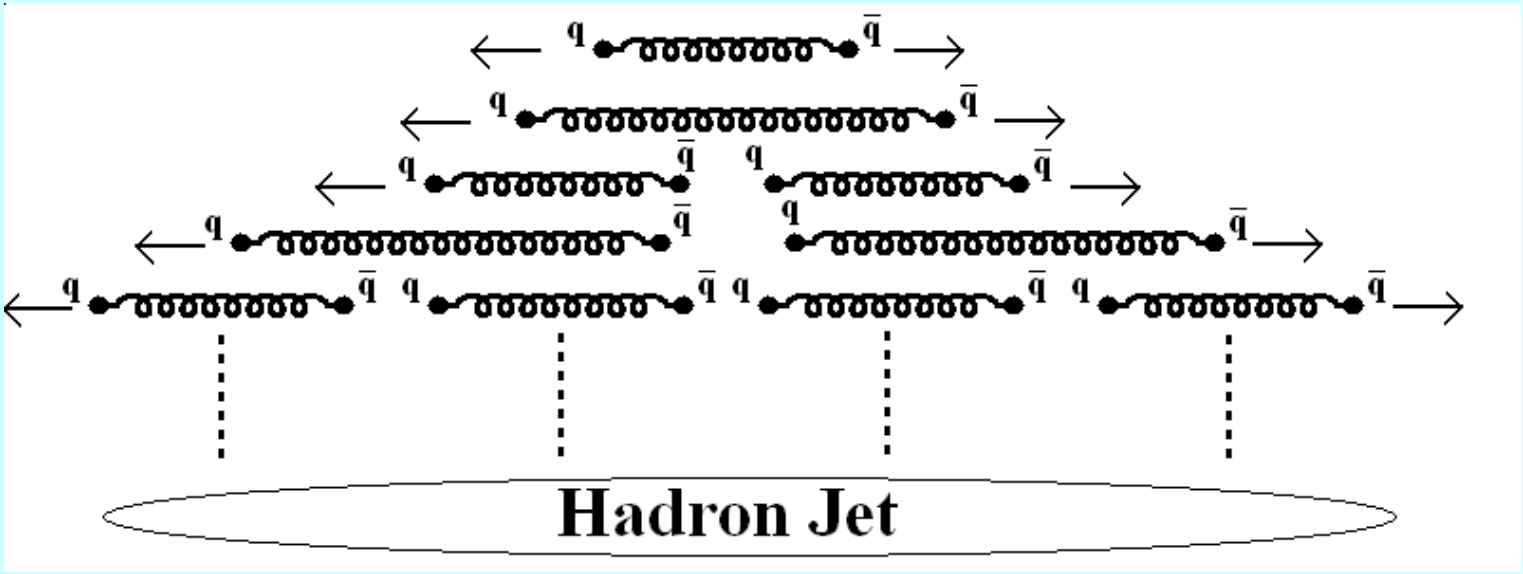
This is due to the attractive force between gluon of identical color.

Interaction, ie. $V(r)$ and coupling constant, decreases as $r \to 0$

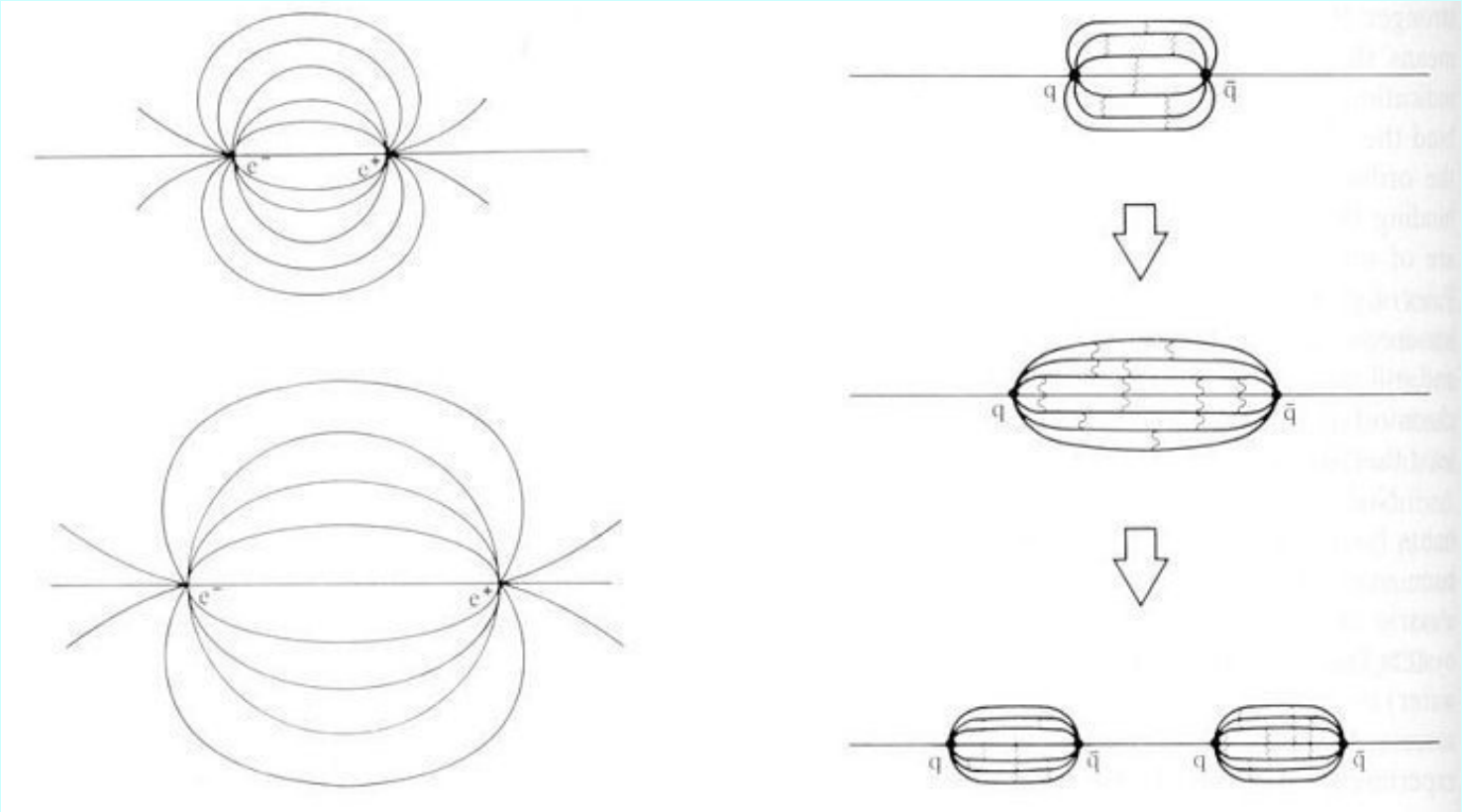


Screening vs Anti-screening. A cartoon representation of the screening effect in QED ($b < 0$) for which the effective electric charge increases closer to the bare charge and the anti-screening effect for asymptotically free theories ($b > 0$) such as QCD in which the net colour (red in this case) decreases closer to the bare particle.


q
q̄

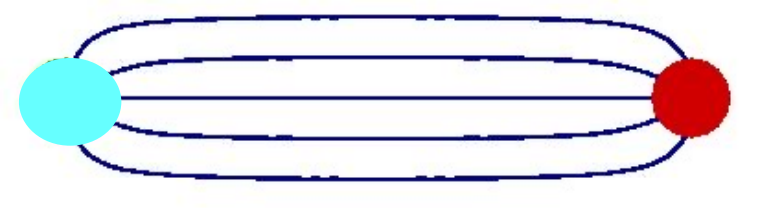

Hadron Jet

但色力非常特別！



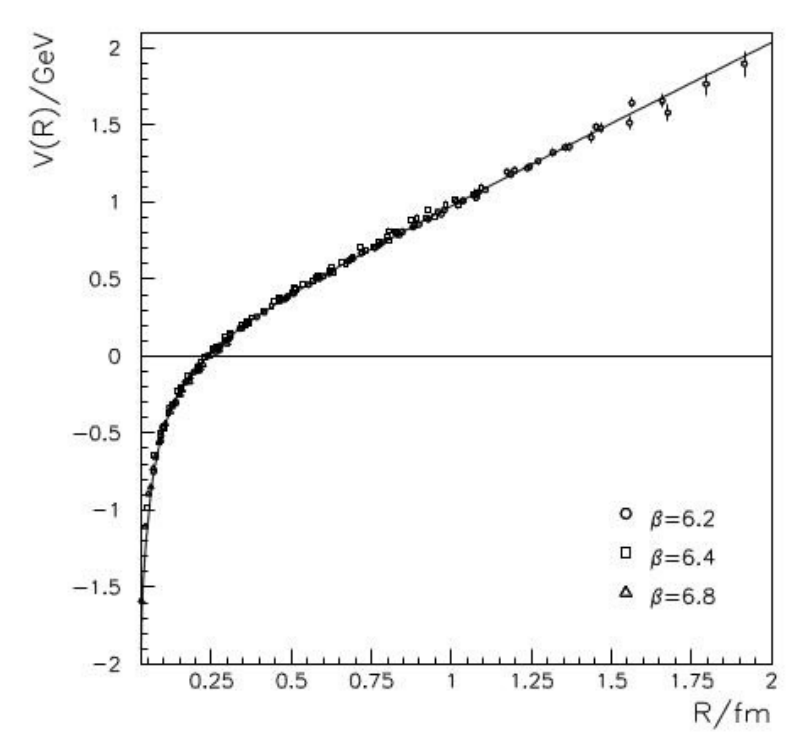
膠子彼此有交互作用，色力場線也似乎會彼此吸引。

遠距離時場線形成管狀·

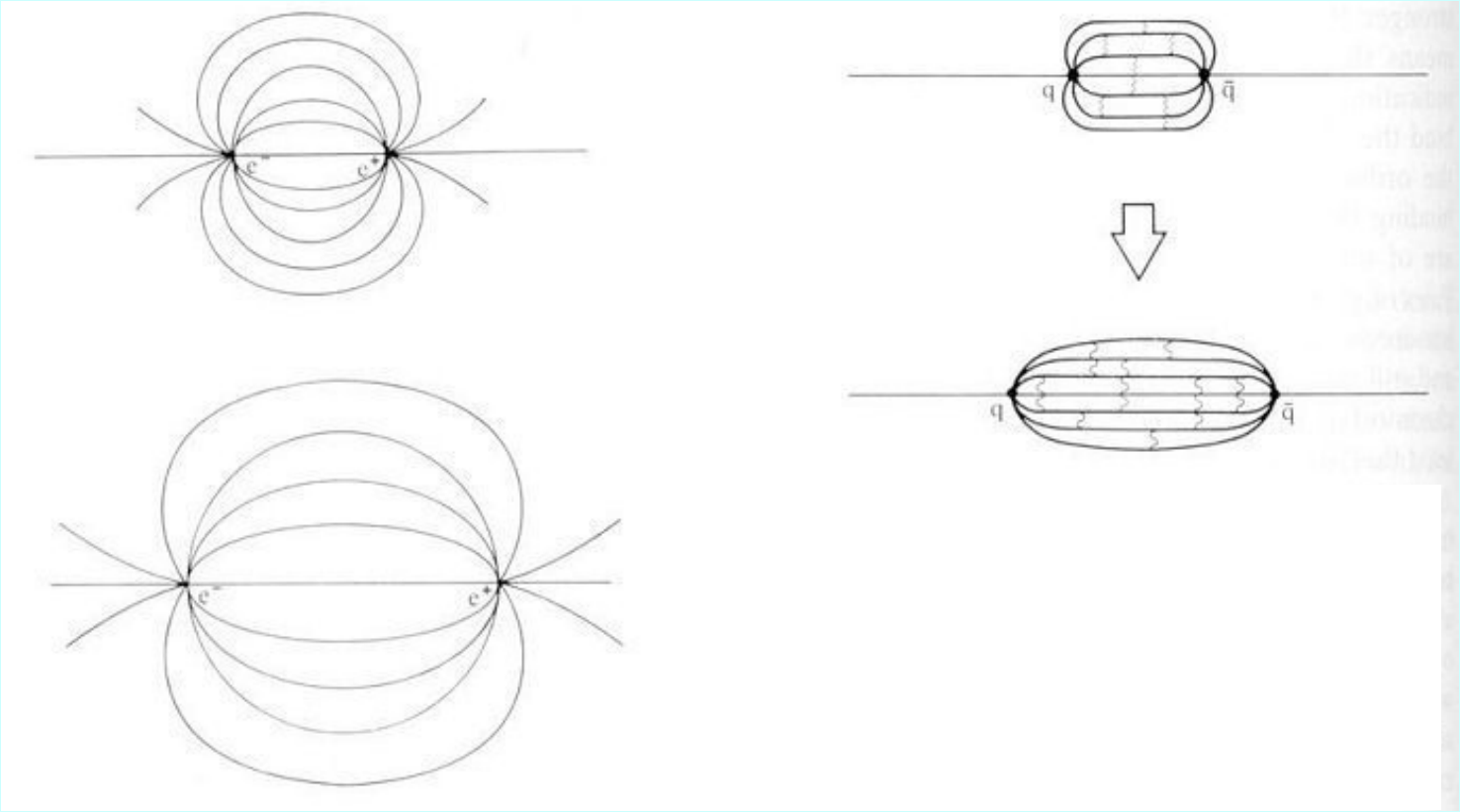


強交互作用的近似位能，近距離是平方反比，

遠距卻是線性關係！ $U \approx ar \quad r \to \infty$

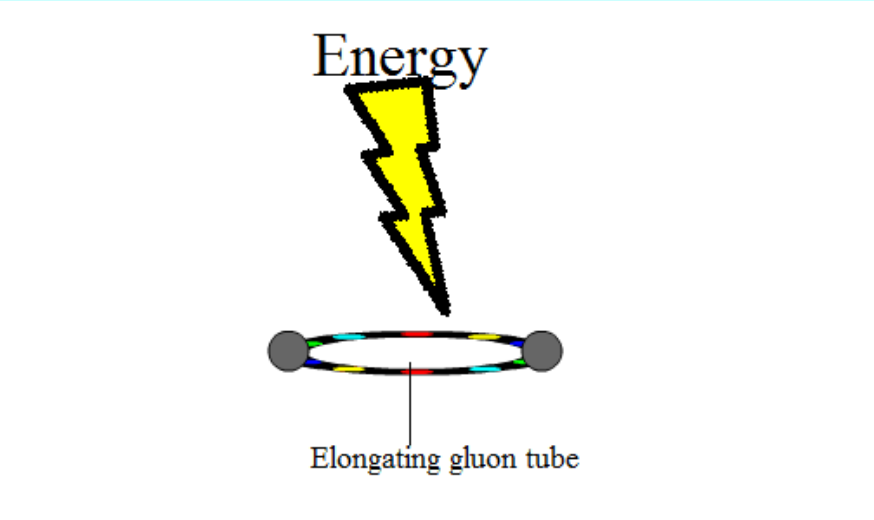


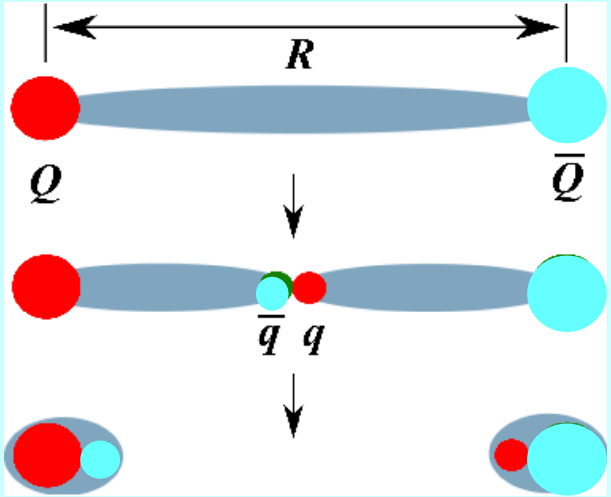
強交互作用正比於位能的微分，不隨距離而減小！



電磁作用隨距離增加而減小

強交互作用不隨距離而減小！



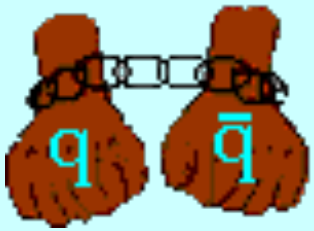


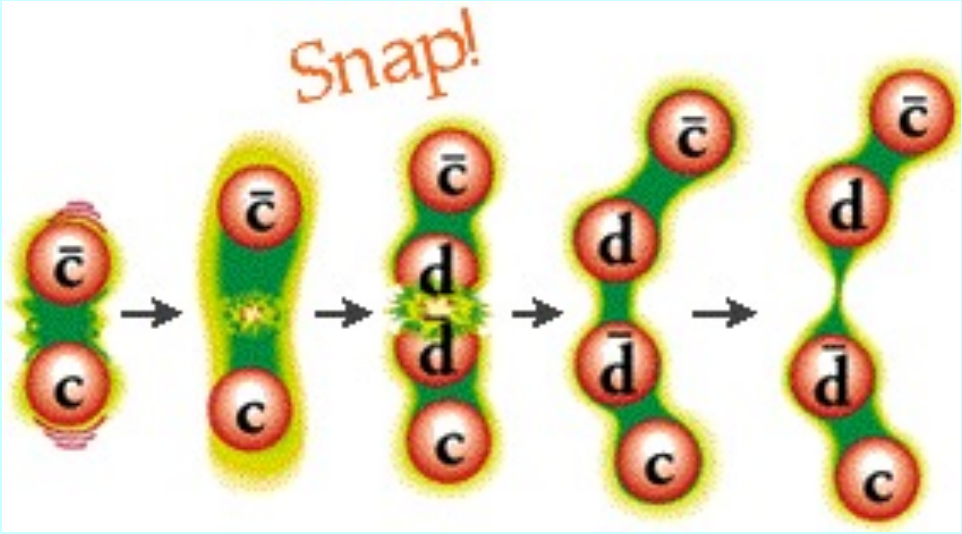
如果持續輸入能量，嘗試將一介子中的夸克與反夸克拉開。

色力位能會一直增加，直到足夠產生一個夸克加一個反夸克！

如此產生的兩個夸克又與原來的夸克配對，各自形成無色的強子。

因此，夸克永遠無法被單獨分離！





Quark Confinement 夸克囚禁

自然界不存在可觀測的有色粒子！

一個孤立的帶色夸克的色力作用位能極大，

能量可以產生許多夸克或膠子直到其顏色被中和為止。

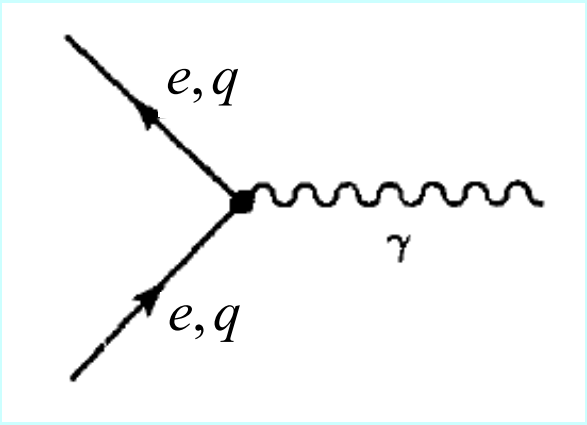
無色的組合能量較低！

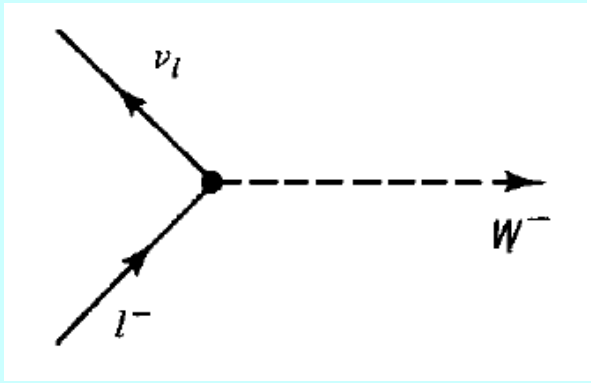
強交互作用強到無法見天日！

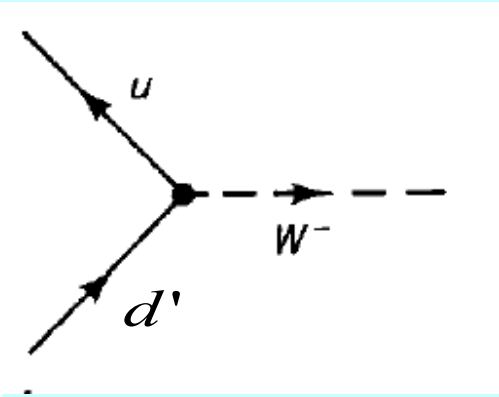
自然界無法觀察測量到帶顏色的物質！

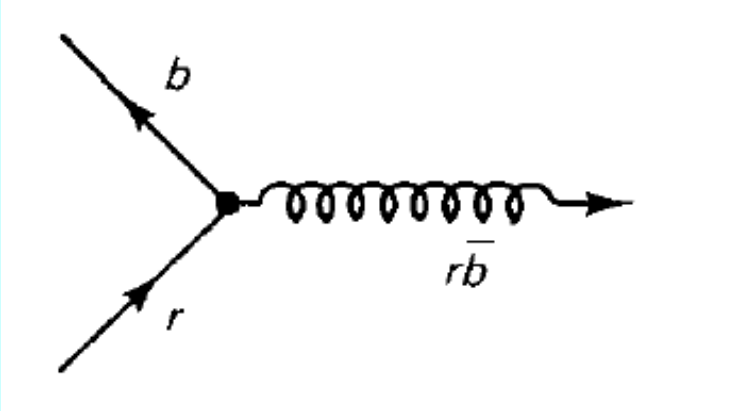
基本交互作用交點

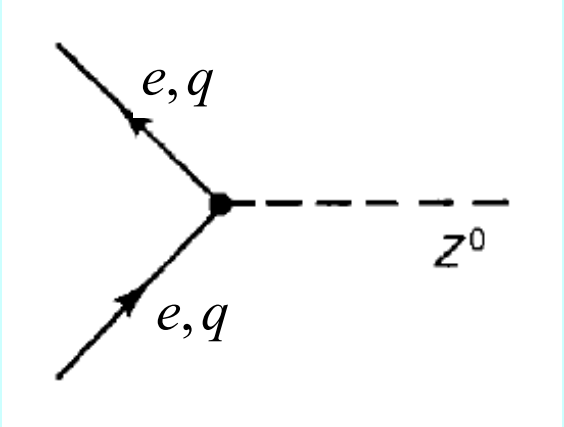
基本粒子的基本交互作用











基本交互作用可以由很簡單的元件組成：基本粒子放出一個媒介粒子。

放出媒介粒子後，不只改變基本粒子的動量，也可能改變粒子的身分！

基本交互作用的性質由媒介粒子決定。