Homework IV

1. Find the eigenvalues and eigenvectors (normalized to length one) of the following matrices:

   A. $\boldsymbol{B} = \begin{pmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{pmatrix}$

   B. $\boldsymbol{C} = \begin{pmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{pmatrix}$

   C. You can find the eigenvectors are the same as

   $$\boldsymbol{A} = \begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$$

   which we discussed in detail in class. And checking the corresponding eigenvalues, we will find that $\boldsymbol{B} = \boldsymbol{A}^m$ and $\boldsymbol{C} = \boldsymbol{A}^n$**.** Find m, n.

   D. Find the matrix $\boldsymbol{U}$ that will diagonalize $\boldsymbol{B} = \begin{pmatrix} 0.7 & 0.45 \\ 0.3 & 0.55 \end{pmatrix}$:

   $$\boldsymbol{U}^{-1} \cdot \boldsymbol{B} \cdot \boldsymbol{U} = \boldsymbol{\Lambda} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

   with $\lambda_{1,2}$ the two eigenvalues of $\boldsymbol{B}$.

2. Find the eigenvalues of the following matrices: $\boldsymbol{A}$ and $\boldsymbol{B}$ and $\boldsymbol{AB}$ and $\boldsymbol{BA}$

   $$\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$$

   Are the eigenvalues of $\boldsymbol{AB}$ equal the eigenvalues of $\boldsymbol{A}$ times the eigenvalue of $\boldsymbol{B}$**?**

3. The electron spin corresponds to a 2×2 matrix. When the spin is along the direction $\hat{n} = (\sin\theta, 0, \cos\theta)$, the matrix is: $S_n = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$. Find the eigenvalues and eigenvectors (normalized to length one) of this matrix.

   Hint: Ignore the $\frac{\hbar}{2}$ first and do it for $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$. Multiplying the eigenvalues found by $\frac{\hbar}{2}$ would give you the eigenvalues of S_n. Eigenvectors are the same.

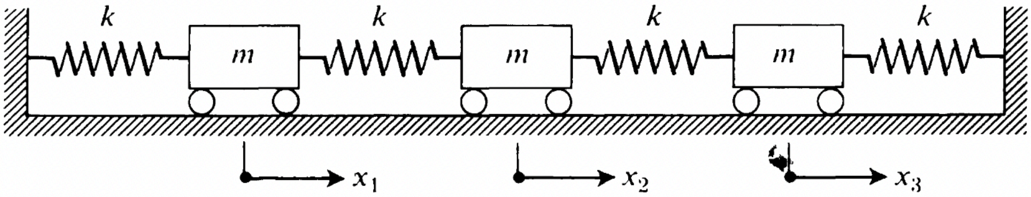

Figure 10.13

Consider the problem of three statically coupled masses (Fig. 10.13) analogous to those in Section 10.1, Example 1. Let x_1, x_2, and x_3 be their displacements from equilibrium positions.

a) Set up the equations of motion on the basis of Newton's second law.
b) Evaluate the potential energy of the system and show that it reduces to

$$V = kx_1^2 + kx_2^2 + kx_3^2 - kx_1x_2 - kx_2x_3.$$

d) Show that the characteristic frequencies can be obtained by solving

$$\det \begin{vmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0.$$

e) Find the characteristic frequencies and the normal modes.