Homework III

1. First consider the homogeneous ODE:

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 3x = 0$$

   A. Find the general solutions x_{h} of this ODE.

   B. Consider the inhomogeneous ODE:

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 3x = 12e^{-2t}$$

   Find one solution. Hint: $x \propto e^{-2t}$ would work.

   C. Find the general solutions x_{inh} of the inhomogeneous ODE.

   D. Assume the initial condition $x(0) = 0, \frac{dx}{dt}(0) = 0$, find the solution $x(t)$.

2.

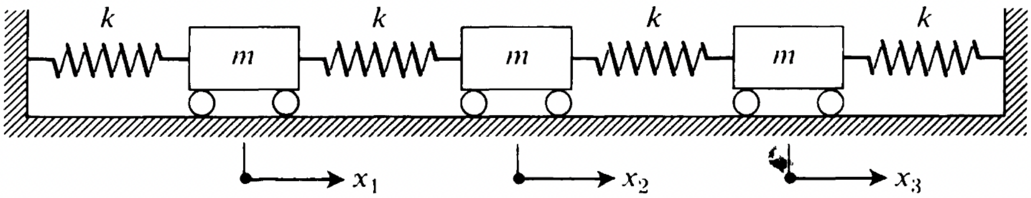

Figure 10.13

Consider the problem of three statically coupled masses (Fig. 10.13) analogous to those in Section 10.1, Example 1. Let x_1, x_2, and x_3 be their displacements from equilibrium positions.

a) Set up the equations of motion on the basis of Newton's second law.
b) Evaluate the potential energy of the system and show that it reduces to

$$V = kx_1^2 + kx_2^2 + kx_3^2 - kx_1x_2 - kx_2x_3.$$

d) Show that the characteristic frequencies can be obtained by solving

$$\det \begin{vmatrix} 2-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0.$$

e) Find the characteristic frequencies and the normal modes.