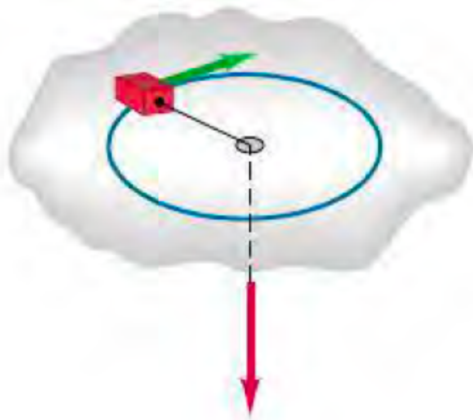
习題五

1.

10.40 • **CP** A small block on a frictionless, horizontal surface has a mass of 0.0250 kg. It is attached to a massless cord passing through a hole in the surface (**Fig. E10.40**). The block is originally revolving at a distance of 0.300 m from the hole with an angular speed of 2.85 rad/s. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.150 m. Model the block as a particle. (a) Is the angular momentum of the block conserved? Why or why not? (b) What is the new angular speed? (c) Find the change in kinetic energy of the block. (d) How much work was done in pulling the cord?

Figure **E10.40**



10.40. **IDENTIFY** and **SET UP:** $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where r is the distance of the block from the hole.

$$mr_1^2\omega_1 = mr_2^2\omega_2$$

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \omega_1 = \left(\frac{0.300 \text{ m}}{0.150 \text{ m}}\right)^2 (1.95 \text{ rad/s}) = 7.8 \text{ rad/s}$$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$$v_1 = r_1\omega_1 = (0.300 \text{ m})(1.95 \text{ rad/s}) = 0.585 \text{ m/s}$$

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0240 \text{ kg})(0.585 \text{ m/s})^2 = 0.00411 \text{ J}$$

$$K_2 = \frac{1}{2}mv_2^2$$

$$v_2 = r_2\omega_2 = (0.150 \text{ m})(7.8 \text{ rad/s}) = 1.17 \text{ m/s}$$

$$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0240 \text{ kg})(1.17 \text{ m/s}^2)^2 = 0.01404 \text{ J}$$

$$\Delta K = K_2 - K_1 = 0.01404 \text{ J} - 0.00411 \text{ J} = 0.00993 \text{ J} = 99.3\text{mJ}.$$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.00993$ J.

EVALUATE: Smaller r means smaller I. $L = I\omega$ is constant so ω increases and K increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

對於理想氣體：

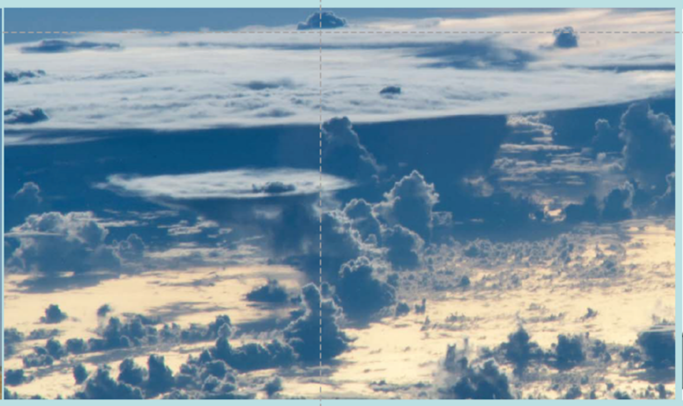
熱力學第零定律　　　熱力學第一定律

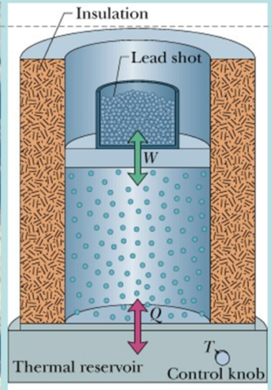
$$T=\frac{PV}{nR}$$

$$E_{\text{int}}=\frac{3}{2}nRT,\frac{5}{2}nRT,3nRT \qquad E_{\text{int}}=nc_VT$$

利用這兩個式子，即能判斷氣體與外界是否達到熱平衡，

以及計算在達到熱平衡的熱過程中進行的、見不到的熱量交換Q。



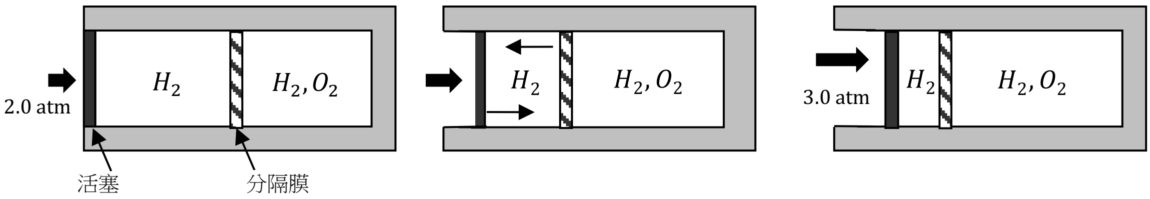


2. 考慮一個容器，左方有一可活動之活塞(圖中黑色方塊)。起始時，活塞右方空間總體積為V_0。體積V_0又以一半透膜(圖中斜線方塊，膜的厚度可以忽略)分割為大小相同的兩室(體積各是$0.5V_0$)（下圖左）。容器內充滿氬氣，氬氣可以自由通過此半透膜，起始時活塞上之壓力為 2.0 atm，氬氣溫度為 27˚C。膜右方的空間另有氧氣，氧氣無法通過半透膜。半透膜可左右活動，但一直維持使來自右方空間氧氣的分壓為 1.0 atm。

   實驗者開始增加活塞上的壓力，將活塞慢慢向右移動，在此過程中，氬氣溫度增加，同時半透膜會向左移動，如下圖中所示。觀察發現，當活塞與膜之間的空間體積為$0.2V_0$時，活塞上的壓力是 3.0 atm，如下圖右。假設此過程發生夠慢，氬氣與氧氣一直處於**熱平衡狀態**。本題中所有氣體皆可視為理想氣體。

A. 此時膜的右方空間的體積與V_0的比為多少？(10)

B. 此時氧氣溫度是多少˚C？(10)



解答：設右室的體積為xV_0。根據熱力學第零定律，在右室中達成熱平衡的氫氣與氧氣溫度相等，而分隔膜對氫氣無影響，因此左右兩室的氫氣體溫度與壓力皆相等。

對氫氣來說：$\frac{T_f}{P_f V_f} = \frac{T_i}{P_i V_i}$，$\frac{T_f}{3.0(0.2+x)V_0} = \frac{300}{2.0V_0}$，$T_f = 450(0.2 + x)$。

對氧氣來說：$\frac{T_f}{P_f V_f} = \frac{T_i}{P_i V_i}$，$\frac{T_f}{1.0xV_0} = \frac{300}{1.0 \cdot 0.5V_0}$，$T_f = 600x$。

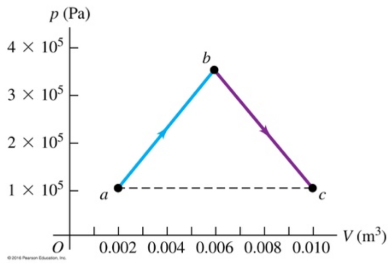
兩式相等得$x = 0.6$。$T_f = 360\text{ K} = 87℃$。

另解：$n = \frac{PV}{RT}$，可以得到$n_{H_2}\!:n_{O_2} = \frac{2.0V_0}{300}\!:\!\frac{1.0 \cdot 0.5V_0}{300} = 4\!:1$。

莫耳數不變：$\frac{3.0(0.2+x)V_0}{T_f}\!:\!\frac{1.0xV_0}{T_f} = 4\!:1$，得$x = 0.6$。

3. 考慮 0.25 莫耳可視為理想氣體的氦氣，狀態a, b, c的壓力分別是$p_a = p_c = 1.0 \times 10^5\text{Pa}, p_b = 3.6. \times 10^5\text{Pa}$，體積為$V_a = 2.0 \times 10^{-3}\text{m}^3, V_b = 6.0 \times 10^{-3}\text{m}^3, V_c = 1.0 \times 10^{-2}\text{m}^3$。如圖所示。

$R = 8.31$ J/mol·K.



A. 狀態a, b, c的溫度分別為多少K？(7)

B. 若是進行 PV 圖上水平直線過程ac，氦氣要吸熱Q_{ac}多少J?內能差$E_c - E_a$為多少J?(8)

C. 若是進行 PV 圖上斜行直線過程$a \rightarrow b \rightarrow c$，氦氣總吸熱$Q_{abc}$多少J？(10)

提示：$\Delta E_{\text{int}} = Q - W$. ΔE_{int}與過程無關。

解答：

A. 狀態a的溫度：$T_a = \frac{PV}{nR} = \frac{1.0\times10^5\times0.002}{0.25\times8.31} = 96\text{ K}$。$T_c = 5T_a = 480\text{ K}$。

狀態b的溫度$T_b = \frac{PV}{nR} = \frac{3.6\times10^5\times0.006}{0.25\times8.31} = 1039\text{ K}$。

B. 水平直線過程ac為等壓，$Q_{ac} = nc_P\Delta T = n\frac{5}{2}R\Delta T \sim 2000.0\text{ J}$

狀態c與狀態a的內能差$E_c - E_a = n\frac{3}{2}R\Delta T = 1200\text{ J}$。

C. 內能差不變：$Q_{ac} - W_{ac} = Q_{abc} - W_{abc}$。

$$W_{abc} - W_{ac} = \text{三角形面積} = 2.6\times10^5\times8.0\times10^{-3}/2 = 1040\text{ J}$$

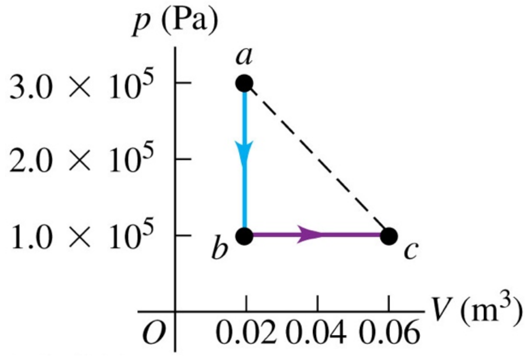
$$Q_{abc} = 2000 - 1040 = 3040\text{ J}$$

另解$E_c - E_a = Q_{abc} - W_{abc} = 1200\text{ J}$，

$$W_{abc} = \text{兩個梯形面積} = 2\times(3.6+1)\times10^5\times4.0\times10^{-3}/2 = 1840\text{ J}$$

$$Q_{abc} = E_c - E_a + W_{abc} = 3040\text{ J}$$

4. 考慮一缸2.0莫耳雙原子分子組成的氮氣。由狀態a膨脹至狀態c，而此過程在 PV 圖上，為由狀態a到狀態c的一條直線。兩個狀態的壓力與體積如圖所示。氮氣的定容比熱為 $c_V = 5/2\,R$。



A. 計算此氣體前後的溫度T_a, T_c分別是多少 K？(5)

B. 在這個過程中，氣體對外所作的功是多少 kJ？氣體的內能變化是多少 kJ？吸熱（或放熱）是多少 kJ？(10)

考慮同樣一缸氣體的另一過程，由狀態a先定容減壓至狀態b，再定壓膨脹至狀態c。

C. 狀態b的溫度T_b是多少 K？

D. 整個過程$a\rightarrow b\rightarrow c$，氣體的內能變化是多少 kJ？吸熱（或放熱）是多少 kJ？(10)

解答：

A. $T = \frac{PV}{nR}$，因此$T_a = \frac{3\times10^5\times0.02}{2\cdot8.31} = 361\text{K}$，$T_c = \frac{1\times10^5\times0.06}{2\cdot8.31} = 361\text{K}$，$T_a = T_c$。。

B. 所作的功即是直線下所包圍的面積：$W = 4\times10^5\times\frac{0.04}{2} = 8.0\text{kJ}$。。

內能變化：$\Delta E = nc_V\Delta T = 0$，吸熱：$Q = \Delta E + W = 8.0\text{ kJ}$ 。

C. $T_b = \frac{1\times 10^5 \times 0.02}{2\cdot 8.31} = 120\text{K}$，

D. 整個過程$a \to b \to c \;\; \Delta E = nc_V \Delta T = 0$

吸熱：$Q_{acb} = Q_{ac} + Q_{cb} = nc_p(T_c - T_b) + nc_V(T_b - T_a) = 2\frac{7}{2}R \cdot 241 - 2\frac{5}{2}R \cdot 241 =$ 4.0kJ。