近代物理期中考

Apr 2025

1. Consider a quantum simple harmonic oscillator:. The angular frequency is . The ground state satisfies:and its energy eigenvalue is *.*
2. The ket denotes energy eigenstates: . They are orthonormal: *.* What is the energy eigenvalue of ? (5)
3. Calculate the expectation values: and . (10)

You can use the following formula:

1. Calculate the matrix elements: and . (10)

You can use .

1. Do your results in B,C agree with the quantization condition: ?

Solution:

1. Eigenvalue:
2. Plug in the formulae of and :

Because and , only the crossing terms remain:

And

This agrees with the quantization condition:

On the other hand:

This also agree with the quantization condition:

1. We denote the normalized eigenstates of as .

For a quantum axially symmetric rotator, assume that its Hamiltonian can be written as

1. Consider the case . How many states are possible? What are the possible quantum number ? (10)
2. Assume that the rotator is in the state of . It decays into the state and emits one photon (there is a selection rule for this kind of process: What is the energy of the photon it emits? (10)

Solution:

1. There are five states, with .
2. . Plug this into the formula:

For eigenstates of , the energy eigenvalues are

The emitted photon energy equals .

1. Consider the electron in a hydrogen atom. We learned in class that the radial part of its stationary state wave function can be written as:

with a polynomial of degree (次的多項式). We also learn the angular part of the wave function, called spherical harmonics, has the form

Now for a fixed principal quantum number , consider the case , and .

1. Write down the radial function . You can keep an undetermined constant in your answer. ( can be determined by normalization condition). (10)
2. Radial probability density is defined so that is the probability of finding the electron distance from origin to be between and . We showed in class that . Find the radius where has a maximum. (10)
3. Write down . You can also keep an undetermined constant in your answer. The angular probability density peaks at a certain angle . Find . (5)

A diagram of a function

Description automatically generated

Solution:

1. since is a polynomials of order, a constant.
2. The maximum occurs at

Therefore . This is equation (8-50) at p139 in our textbook.

1. , since , again a constant. It peaks at the peak of , ie That is on the plane.
2. As in class, use the two eigenstates of the -component spin to form a basis of the spin state space.
3. In this basis, the -component of spin can be written as the . Find the eigenvalues and eigenvectors of . Please normalize your answer so that . (10)
4. What is the representing the -component of spin?

Calculate the expectation value: . (10)

1. In the state of , measure . What are the possible values you can get and what are the corresponding probabilities? (10)
2. In the state of , what is the probability that the measurement of yields the value of +?

In the state of , what is the probability that the measurement of yields the value of +? (10)

Hint: Expand the state using eigenstates and : . Like components of 3D vectors, . The expansion components are the amplitudes of measurement . The absolute value squared of the amplitudes are the probabilities.

Solution:

1. *,* 此式只有在行列式為零時有非零解：

, ,

如預期。

若，eigevalue is .

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1. In measure . The probability of yielding equal

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1. In the state of , the probability that the measurement of yields the value of + equals

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