近代物理期中考

1. As in class, use the eigenstates of the spin operator in direction as the basis. We design an experiment with the electron beam passing two Stern-Gerlach (SG) setups with different directions of variant magnetic field. We assign the direction of the electron beam as axis. The first SG is the typical one with z-direction variant magnetic field. Rotate the magnet in the second SG experiment around axis by an angle . Denote the direction of the magnetic field of this second SG as . The spin operator pointing in the direction of is . In both the first and the second SG, only spin-up electron is allowed to pass, ie: .



1. Find the eigenvector .of , expressed as a column vector . You can choose the coefficients to be real. Normalize the coefficients so that .
2. Calculate the probability for the electron that has passed through the first SG to pass the second SG? (25)
3. Let those electrons passing through the whole set (in the state of continue to fly in the direction and is introduced into an area with a **constant** magnetic field pointing in the direction. It passed the area with in a time duration . You can think about the time evolution of the electrons as if they are at rest. Then we measure . Calculate the expectation value of . You can use the notation to simplify your expression. Express you answer in (15)

 Hint: Energy of electron in constant magnetic field:

 . Time evolution factors: .

Solution:

1. *,* 此式只有在行列式為零時有非零解：, ,

如預期。

若，，。

1. The probability for to pass as equals:
2. In , the state evolves to . The expectation value of equals

1. Consider a Simple Harmonic Oscillator whose Hamiltonian can be written as: . The eigenstates can be written as with eigenvalues .

This SHO is perturbed by a small potential: . Treating this potential as a perturbation. To the leading order of , the real ground state can be written as a linear combination of the unperturbed energy eigenstates

Find the possible or and calculate the coefficients.

You can use the formula:

解答：

1. Consider an infinite potential as discussed in class, with boundaries at and , while containing a small step potential:

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The eigenfunction of an infinite box is known to be:

with energy eigenvalues: .

Calculate the correction in first order of to the ground state energy

解答：