習題四解答

1. Consider a Simple Harmonic Oscillator whose Hamiltonian can be written as: . The eigenstates can be written as with eigenvalues .

This SHO is perturbed by a small potential: . Treating this potential as a perturbation. To the leading order of , the real ground state can be written as a linear combination of the unperturbed energy eigenstates

Find the possible or and calculate the coefficients.

You can use the formula:

解答：

1. Consider an infinite potential as discussed in class, with boundaries at and , while containing a small step potential:

.

.

Chart

Description automatically generated

The eigenfunction of an infinite box is known to be:

with energy eigenvalues: .

Calculate the correction in first order of to the ground state energy

解答：

1. We will work out here an example of degenerate perturbation theory.

A Simple Harmonic Oscillator has a Hamiltonian as: . The eigenstates can be written as with eigenvalues . We can extend the idea to 2D and consider a 2D SHO with a Hamiltonian:

**For convenience, here we choose a unit system where** . Just like 3D electron gas we discussed in class, the motion in and the motion in are separable and the 2D SHO can be separated into two 1D SHO, one in and one in , with no interaction between them. The eigenstates could be written (as in 3D electron gas) as the products of an eigenstate of 1D SHO in and an eigenstate of 1D SHO in :

with energy eigenvalues:

The ground state is . And there are two lowest excited states

with the same energy and hence this is a two-fold degeneracy.

1. What is the energy eigenvalue of the lowest excited states?

Now introduce a perturbation potential:

To calculate the energy corrections for the two degenerate states using perturbation, we need the matrix elements of between . They turn out to be very simple. We can identify as and as in the class discussion.

You don’t need to do it but the detailed calculation is as follow:

1. Use the above matrix elements to form the key matrix . Calculate the 1st order energy corrections .

Hint:

A screenshot of a math equation

Description automatically generated

A close-up of a book

Description automatically generated

Solution: For the lowest excited states,

For the degenerate perturbation calculation:

. Assume that eigenvalue is .

Eigenvalues, which are the energy corrections, are