習題三

1. In this problem, **the comments are points I hope you will think about later after exam and there is no need to answer them.**

Consider a hydrogen atom. We learned in class that the radial function of its stationary state wave function can be written as:

$$R\_{nl}\left(r\right)\~e^{-\frac{r}{na\_{0}}}∙r^{l}∙H\left(\frac{2}{na\_{0}}r\right)$$

with $H\left(ρ\right)$ a polynomial of $n-l-1$ order. We also learn the angle-depending part $Y\_{lm}\left(θ,ϕ\right) $of the wave function, called spherical harmonics, has the form

$$sin^{\left|m\right|}θ∙\left(a polynomial in\cos(θ)of l-m order\right)$$

Now for a fixed principal quantum number $n$, consider the case $l = n - 1$, and $m = n - 1$.

1. Write down the radial function $R\_{nn-1}\left(r\right)$, up to normalization.
2. $R\_{nn-1}\left(r\right)$ has a maximum. Find the radius $r\_{0}$ where $R\_{nn-1}\left(r\_{0}\right)$ has a maximum. Comments: Observe that as $n$ increase, the peak becomes sharper. Hence the probability density concentrates on a certain radius $r\_{0}$.
3. Write down $Y\_{n-1n-1}\left(θ,ϕ\right)$ up to normalization. It also peaks at a certain angle $θ\_{0}$. Find this angle.

Comments: Observe that as $n$ increase, the peak becomes sharper. Hence the probability density concentrates on a plane.

 

$$r\_{0}$$

$$r\_{0}$$

1. Ch. 10 Solve 10-1.

For the eigenstate with the higher eigenvalue, measure $S\_{z}$, what are (or is) the possible values? What are (or is) the corresponding probabilities?

1. For, $\left|\left.α\right⟩\right.=\frac{1}{\sqrt{65}}\left(\begin{matrix}4\\7\end{matrix}\right)$，calculate the expectation value of $S\_{x}^{4}:\left⟨α\right⟩$.

Measure $S\_{x}$, what are (or is) the possible values? What are (or is) the corresponding probabilities?

 Hint: The expansion components 分量 of $\left|\left.α\right⟩\right.$ along $\left|\left.x,\uparrow \right⟩\right.$ equals $\left⟨α\right⟩$.

1. As in class, use the eigenstates $\left|z,\left.\uparrow \right⟩\right., \left|z,\left.\downright \right⟩\right.$ of the spin operator in $z $direction $S\_{z} $as the basis of the electron spin States. Consider the spin operator $S\_{n}$ pointing in the direction of $\hat{n}=\left(\sin(θ)\cos(ϕ),\sin(θ)\sin(ϕ),\cos(θ)\right)$ with $ϕ=0$. This spin operator can be studied in the following setup. We assign the direction of electron beam as $y$ axis. Rotate the magnet in a SG experiment around $y$ axis by an angle $θ$. The magnetic field of this second SG is along $\hat{n}$.



1. Derive from $S\_{n}=\vec{S}∙\hat{n}$ that $S\_{n}=\frac{ℏ}{2}\left(\begin{matrix}\cos(θ)&\sin(θ)\\\sin(θ)&-\cos(θ)\end{matrix}\right)$ .
2. Find the eigenvectors of $S\_{n}$, expressed as column vectors in the basis of $\left|z,\left.\uparrow \right⟩\right., \left|z,\left.\downright \right⟩\right.$: $\left|n,\left.\uparrow \right⟩\right.=\left(\begin{matrix}c\_{1}\\c\_{2}\end{matrix}\right), \left|n,\left.\downright \right⟩\right.=\left(\begin{matrix}d\_{1}\\d\_{2}\end{matrix}\right)$. You can choose the coefficient $c\_{1,2},d\_{1,2} $to be all real. Calculate the coefficients $c\_{1,2}$. Please normalize the coefficients so that $c\_{1}^{2}+c\_{2}^{2}=1$.
3. As in the setup above, place a typical SG with $z$ magnetic field and then a second SG with $\hat{n}$ magnetic field. In the first SG only spin-up electron is allowed to pass, ie: $\left|\left.z,\uparrow \right⟩=\right.\left(\begin{matrix}1\\0\end{matrix}\right)$. Calculate the probability for it to pass the second SG as $\left|n,\left.\uparrow \right⟩\right., $ie with $σ\_{n}=\frac{ℏ}{2}$ measured? It is a function of $θ$. Think a little bit when the probability would be the largest and the smallest and whether it makes sense. (25)