習題三

1. In this problem, **the comments are points I hope you will think about later after exam and there is no need to answer them.**

Consider a hydrogen atom. We learned in class that the radial function of its stationary state wave function can be written as:

with a polynomial of order. We also learn the angle-depending part of the wave function, called spherical harmonics, has the form

Now for a fixed principal quantum number , consider the case , and .

1. Write down the radial function , up to normalization.
2. has a maximum. Find the radius where has a maximum. Comments: Observe that as increase, the peak becomes sharper. Hence the probability density concentrates on a certain radius .
3. Write down up to normalization. It also peaks at a certain angle . Find this angle.

Comments: Observe that as increase, the peak becomes sharper. Hence the probability density concentrates on a plane.

A diagram of a function

Description automatically generated 

1. Ch. 10 Solve 10-1.

For the eigenstate with the higher eigenvalue, measure , what are (or is) the possible values? What are (or is) the corresponding probabilities?

1. For, ，calculate the expectation value of .

Measure , what are (or is) the possible values? What are (or is) the corresponding probabilities?

Hint: The expansion components 分量 of along equals .

1. As in class, use the eigenstates of the spin operator in direction as the basis of the electron spin States. Consider the spin operator pointing in the direction of with . This spin operator can be studied in the following setup. We assign the direction of electron beam as axis. Rotate the magnet in a SG experiment around axis by an angle . The magnetic field of this second SG is along .

A diagram of a beam

AI-generated content may be incorrect.

1. Derive from that .
2. Find the eigenvectors of , expressed as column vectors in the basis of : . You can choose the coefficient to be all real. Calculate the coefficients . Please normalize the coefficients so that .
3. As in the setup above, place a typical SG with magnetic field and then a second SG with magnetic field. In the first SG only spin-up electron is allowed to pass, ie: . Calculate the probability for it to pass the second SG as ie with measured? It is a function of . Think a little bit when the probability would be the largest and the smallest and whether it makes sense. (25)