習題三

1. Consider an infinite potential, with boundaries at $x=0$ and $x=a$: $V\left(x\right)=\infty , x>a, x<0$ and $V\left(x\right)=0, 0< x<a$.



$$a$$

$$V\left(x\right)$$

A particle is known to be localized inside the box, with an instantaneous 瞬間 wavefunction at $t=0 $as:

$$Ψ\left(x,0\right)=Cx\left(a-x\right) 0<x<a, $$

$$ =0 x<0,x>a$$

Calculate the constant $C$. Use your result to write down the probability density $P $at $x=\frac{a}{2}$ at $t=0$. (25)

1. Consider an electron moving from left $x=-\infty $ to right $x=\infty $ and is scattered by a step potential at $x=0$. The step potential is: $V=0, x<0$ and $V=V\_{0}, x>0$.



A wave packet formulation of this scattering can be approximated by considering the stationary energy eigenfunction of this step potential. The solution is:

$$ψ\_{E}=e^{ikx}+Re^{-ikx} x<0 k≡\sqrt{\frac{2mE}{ℏ^{2}}}$$

$$ψ\_{E}=Te^{iqx} x>0 q≡\sqrt{\frac{2m}{ℏ^{2}}\left(E-V\_{0}\right)}$$



Calculate the probability density as a function of $x$ in terms of $T,R,k$ at $x<0.$ It is a constant at $x>0$.



