近代物理期中考

Apr 2024

The Following two problems are about a quantum simple harmonic oscillator. The mass of the particle is . The angular frequency is 。

1. The ground state of this system can be described by a wavefunction as

You can use the constant in the following to simplify your calculation. We use the raising operator to generate the first excited state from the ground state:

We also know the raising operator is the linear combination of and :

Find the wavefunction for .

Draw a rough graph of the wavefunction versus .

Hint:

Solution:

It is a gaussian form at infinity and cross the x axis once at the origin as an odd function.



1. Calculate for the quantum simple harmonic oscillator. You can use the following formula:

And .

Hint: My first step is to show:

using . And we have calculated the right-hand side in class.

Solution:

Because , the above can be simplified:

Here we have used .

We also know that

The above is further reduced to

Using we can reduce the first term:

Using we can reduce the second term:

Hence

1. We denote the eigenstates of as (normalized), with

Consider the case .

1. A state is a combination of and ：. When I measure on , what are (or is) the possible values? What are (or is) the corresponding probabilities?
2. Now switch to the state of . When I measure on this state, what are (or is) the possible values? What are (or is) the corresponding probabilities?

Solution:

1. The measured values of could only be its eigenvalues. For , the eigenvalues are and . The probabilities are the square of the components of the expansion. In this case, *.* The components are *.* Hence the probabilities are for both and .
2. . Since is the eigenstate of , it is also the eigenstate of . Hence the measurement will give a certain values:

1. In this problem, **the comments are points I hope you will think about later after exam and there is no need to answer them.**

Consider a hydrogen atom. We learned in class that the radial function of its stationary state wave function can be written as:

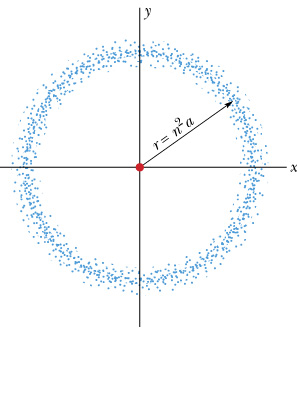
with a polynomial of order. We also learn the angle-depending part of the wave function, called spherical harmonics, has the form

Now for a fixed principal quantum number , consider the case , and .

1. Write down the radial function , up to normalization.
2. has a maximum. Find the radius where has a maximum. Comments: Observe that as increase, the peak becomes sharper. Hence the probability density concentrates on a certain radius .
3. Write down up to normalization. It also peaks at a certain angle . Find this angle.

Comments: Observe that as increase, the peak becomes sharper. Hence the probability density concentrates on a plane.

A diagram of a function

Description automatically generated 

Solution:

1. since is a polynomials of order, a constant.
2. The maximum occurs at . Therefore .
3. , since , again a constant. It peaks at the peak of , ie That is on the plane.