習題三

1. Prove that $\left[L\_{z},L\_{x}\right]=iℏL\_{y}$.

$$\left[L\_{z},L\_{x}\right]=\left[\left(xp\_{y}-yp\_{x}\right),\left(yp\_{z}-zp\_{y}\right)\right]=\left[xp\_{y},yp\_{z}\right]+\left[yp\_{x},zp\_{y}\right]$$

$$=\left[xp\_{y},y\right]p\_{z}+z\left[yp\_{x},p\_{y}\right]=x\left[p\_{y},y\right]p\_{z}+z\left[y,p\_{y}\right]p\_{x}=iℏ\left(-xp\_{z}+zp\_{x}\right)=iℏL\_{y}$$

1. Prove that $\left[L^{2},L\_{x}\right]=0$.

$$\left[L\_{x}^{2}+L\_{y}^{2}+L\_{z}^{2},L\_{x}\right]=\left[L\_{z}^{2},L\_{x}\right]+\left[L\_{y}^{2},\hat{L}\_{x}\right]$$

$$=L\_{z}\left[L\_{z},L\_{x}\right]+\left[L\_{z},L\_{x}\right]L\_{z}+L\_{y}\left[L\_{y},L\_{x}\right]+\left[L\_{y},L\_{x}\right]L\_{y}$$

$$=iℏL\_{z}L\_{y}+iℏL\_{y}L\_{z}-iℏL\_{y}L\_{z}-iℏL\_{z}L\_{y}=0$$

1. Prove that $\left[L\_{z},L\_{-} \right]=-ℏ∙L\_{-}$ and $L\_{\pm }\left.\left|a,m\right.\right⟩$ is the eigenstate of $L\_{z} $with eigenvalue $\left(m\pm 1\right)ℏ$.

$$\left[L\_{z},L\_{x}\right]=iℏL\_{y}$$

$$\left[L\_{z},L\_{y}\right]=-iℏL\_{x}$$

 第一式減第二式乘$i$：$\left[L\_{z},L\_{-} \right]=-ℏ∙L\_{-}$

$$L\_{z}L\_{-}\left.\left|a,m\right.\right⟩=\left(L\_{-}L\_{z}-ℏL\_{+}\right)\left.\left|a,m\right.\right⟩$$

$$=mℏ∙L\_{-}\left.\left|a,m\right.\right⟩-ℏL\_{+}\left.\left|a,m\right.\right⟩=\left(m-1\right)ℏ∙L\_{+}\left.\left|a,m\right.\right⟩$$











