習題二

1.

$V_s = \left(\frac{h}{e}\right)\frac{f-\phi}{e}$. Choosing two points on the graph, one has $\left(\frac{h}{e}\right)(4\times10^{14}\text{ Hz})-\frac{\phi}{e}=0$ and $\left(\frac{h}{e}\right)(8\times10^{14}\text{ Hz})-1.7\text{ eV}$. Combining these two expressions one obtains:

(a) $\phi = 1.6\text{ eV}$

(b) $\frac{h}{e} = 4.0\times10^{-15}\text{ Vs}$

(c) For cut-off wavelength, $\lambda_c = \frac{hc}{\phi} = \frac{1\,240\text{ eV nm}}{1.6\text{eV}} = 775\text{ nm}$.

(d) Accepted $\frac{h}{e} = 4.14\times10^{-15}\text{ Vs}$, about a 3% difference.

15. With $V(r) = V_0\,(r/a)^k$, the equation describing circular motion is

$$m\frac{v^2}{r} = \left|\frac{dV}{dr}\right| = \frac{1}{r}kV_0\left(\frac{r}{a}\right)^k$$

so that

$$v = \sqrt{\frac{kV_0}{m}}\left(\frac{r}{k}\right)^{k/2}$$

The angular momentum quantization condition $mvr = n\hbar$ reads

$$\sqrt{ma^2kV_0}\left(\frac{r}{a}\right)^{\frac{k+2}{2}} = n\hbar$$

We may use the result of this and the previous equation to calculate

$$E = \frac{1}{2}mv^2 + V_0\left(\frac{r}{a}\right)^k = (\frac{1}{2}k+1)V_0\left(\frac{r}{a}\right)^k = (\frac{1}{2}k+1)V_0\left[\frac{n^2\hbar^2}{ma^2kV_0}\right]^{\frac{k}{k+2}}$$

In the limit of $k >> 1$, we get

$$E \to \frac{1}{2}(kV_0)^{\frac{2}{k+2}}\left[\frac{\hbar^2}{ma^2}\right]^{\frac{k}{k+2}}(n^2)^{\frac{k}{k+2}} \to \frac{\hbar^2}{2ma^2}n^2$$

Note that V_0 drops out of the result. This makes sense if one looks at a picture of the potential in the limit of large k. For $r < a$ the potential is effectively zero. For $r > a$ it is effectively infinite, simulating a box with infinite walls. The presence of V_0 is there to provide something with the dimensions of an energy. In the limit of the infinite box *with the quantum condition* there is no physical meaning to V_0 and the energy scale is provided by $\hbar^2/2ma^2$.