習題一

1.

Two events occur in an inertial system K as follows:

Event 1: $x_1 = a, \quad t_1 = 2a/c, \; y_1 = 0, \; z_1 = 0$

Event 2: $x_2 = 2a, \quad t_2 = 3a/2c, \; y_2 = 0, \; z_2 = 0$

In what frame K′ will these events appear to occur at the same time? Describe the motion of system K′.
Is there a frame K′ in which the two events described in Problem 13 occur at the same place? Explain.

提示：

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

解答

From the Lorentz transformations $\Delta t' = \gamma\left[\Delta t - v\Delta x / c^2\right]$. But $\Delta t' = 0$ in this case, so solving for v we find $v = c^2 \Delta t / \Delta x$. Inserting the values $\Delta t = t_2 - t_1 = -a/2c$ and $\Delta x = x_2 - x_1 = a$, we find $v = \dfrac{c^2(-a/2c)}{a} = -c/2$. We conclude that the frame K′ travels at a speed $c/2$ in the $-x$-direction. Note that there is no motion in the transverse direction.

Try setting $\Delta x' = 0 = \gamma(\Delta x - v\Delta t)$. Thus $0 = \Delta x - v\Delta t = a + va/2c$. Solving for v we find $v = -2c$, which is impossible. There is no such frame K′.

2.

The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is N_0, the number at time t is given by $N = N_0 e^{-t/\tau}$ where τ is the mean lifetime, equal to 2.2 μs. Suppose the muons move at a speed of $0.95c$ and there are 5.0×10^4 muons at $t = 0$. (a) What is the observed lifetime of the muons? (b) How many muons remain after traveling a distance of 3.0 km?

提示：τ是移動的 muon 的$\Delta t'$， $N = N_0 e^{-t/\tau}$

解答：

(a) $\tau = \gamma \tau' = \left[1-(0.95)^2\right]^{-1/2}(2.2\ \mu s) = 7.05\ \mu s$

(b) $\Delta t' = \frac{d}{0.95c} = \frac{3\times 10^3\ \text{m}}{0.95c} = 1.05\times 10^{-5}$ s, therefore,

$$N = N_0 \exp\left(-\frac{\Delta t}{\tau}\right) = \left(5\times 10^4 \text{ muons}\right)\exp(-1.487) \approx 1.128\times 10^4 \text{ muons}.$$

3.

Show that the energy–momentum relationship given by $E^2 = p^2c^2 + (mc^2)^2$ follows from the expressions $E = \gamma mc^2$ and $p = \gamma mu$.

解答：

$E = \gamma mc^2$, $p = \gamma mu$; $E^2 = \left(\gamma mc^2\right)^2$; $p^2 = (\gamma mu)^2$;

$$E^2 - p^2c^2 = \left(\gamma mc^2\right)^2 - (\gamma mu)^2 c^2 = \gamma^2\left\{\left(mc^2\right)^2 - (mc)^2u^2\right\}$$

$$= \left(mc^2\right)^2\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)^{-1} = \left(mc^2\right)^2 \text{ Q.E.D.}$$

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

4.

An unstable particle having a mass of 3.34×10^{-27} kg is initially at rest. The particle decays into two fragments that fly off with velocities of $0.987c$ and $-0.868c$. Find the rest masses of the fragments.

提示：設質量為m_1, m_2，寫下能量守恆與動量守恆！

解答：

In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_l c^2}{\sqrt{1-v_l^2/c^2}} + \frac{m_h c^2}{\sqrt{1-v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987c)(6.22) = (m_h)(0.868c)(2.01)$$
$$m_l = \frac{(m_h)(0.868c)(2.01)}{(0.987)(6.22)} = 0.284m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains $3.34 \times 10^{-27}\ \text{kg} = 6.22m_l + 2.01m_h$ which in turn gives

$3.34 \times 10^{-27}\ \text{kg} = (6.22)(0.284)m_l + 2.01m_h$ or $m_h = \frac{3.34 \times 10^{-27}\ \text{kg}}{3.78} = 8.84 \times 10^{-28}\ \text{kg}$ and

$m_l = (0.284)m_h = 2.51 \times 10^{-28}\ \text{kg}$.