近代物理期末考 June 2024

考卷領取時間：June 7 (Friday) 14:00 A213

1. Assume that is the unit vector pointing in the direction . The spin operator of an electron along the direction can be written as
2. Find the eigenvector with the positive eigenvalue. Please normalize it to .

Hint: Since you know is the spin operator, its eigenvalues must be . You can use this fact to save time in finding eigenvectors.

1. When the electron is in the state , measure the component spin . What are the possible results you would get? Find the corresponding probabilities.
2. Given the component spin , calculate the expectation value of in the state :.

Solution:

For the eigenvector of eigenvalue :

We find , hence .

Measuring , we will get the eigenvalues of : . can be written in terms of the two eigenvectors of . The probabilities are: and .

The expectation value of in the state of cab be calculated using matrix multiplication:

.

1. An infinite box consists of a potential as follows:

The energy eigenfunction in this infinite box is known to be:

with energy eigenvalues: .

Now add a perturbation of a small step potential:

.

.

The full potential now is shown in the following diagram:

Chart

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Calculate the correction in the first order of to the energy eigenvalue of the th eigenstate

解答：

1. We will work out here an example of degenerate perturbation theory.

A Simple Harmonic Oscillator has a Hamiltonian as: . The eigenstates can be written as with eigenvalues . We can extend the idea to 2D and consider a 2D SHO with a Hamiltonian:

**For convenience, here we choose a unit system where** . Just like 3D electron gas we discussed in class, the motion in and the motion in are separable and the 2D SHO can be separated into two 1D SHO, one in and one in , with no interaction between them. The eigenstates could be written (as in 3D electron gas) as the products of an eigenstate of 1D SHO in and an eigenstate of 1D SHO in :

with energy eigenvalues:

The ground state is . And there are two lowest excited states

with the same energy and hence this is a two-fold degeneracy.

1. What is the energy eigenvalue of the lowest excited states?

Now introduce a perturbation potential:

To calculate the energy corrections for the two degenerate states using perturbation, we need the matrix elements of between . They turn out to be very simple. We can identify as and as in the class discussion.

You don’t need to do it but the detailed calculation is as follow:

1. Use the above matrix elements to form the key matrix . Calculate the 1st order energy corrections .

Hint:

A screenshot of a math equation

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A close-up of a book

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Solution: For the lowest excited states,

For the degenerate perturbation calculation:

. Assume that eigenvalue is .

Eigenvalues, which are the energy corrections, are

1. Which of the following statements are true? (5 points for each correct answer)
2. When the energy bands are all totally occupied, the solid is a conductor.
3. The conductivity of semiconductors grows as temperature becomes larger.
4. When a semiconductor is added impurities of atoms with 5 valance electrons, it becomes an n-type semiconductor, and the current carriers are electric holes.
5. In 3D electron gas, the electron energy has an upper limit, . The thermal and conductivity properties of the electron gas are dominated by electron with energy near .

Solution: B,D

1. Consider an infinite box with boundaries at and . The energy eigenstates and eigenvalues are described in problem 2. Put two electrons in the box. The wavefunction of the ground state of the two-electron system, denoted by the two quantum number can be written as:

This overall antisymmetric wavefunction consists of a symmetric space part (of two identical wavefunction) and an antisymmetric spin part.

1. Consider the first excited state. What is the energy eigenvalue?
2. Write down the wavefunction of the **four** first excited state, using the above notation.
3. What is the energy eigenvalue of the second excited states?

Solution:

1. The first excited state has one electron in and one in . The energy is the sum:
2. The overall wavefunctions need to be antisymmetric. Therefore, it could consist of a symmetric space part and an antisymmetric spin part,

or a antisymmetric space part and a symmetric spin part,

1. The second excited state has both electron in (. The energy is the sum: