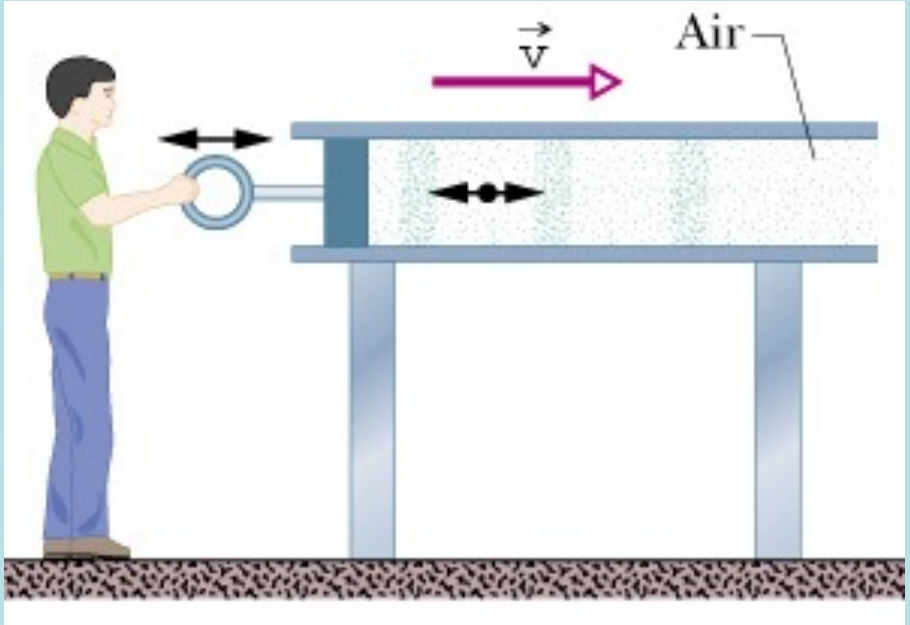
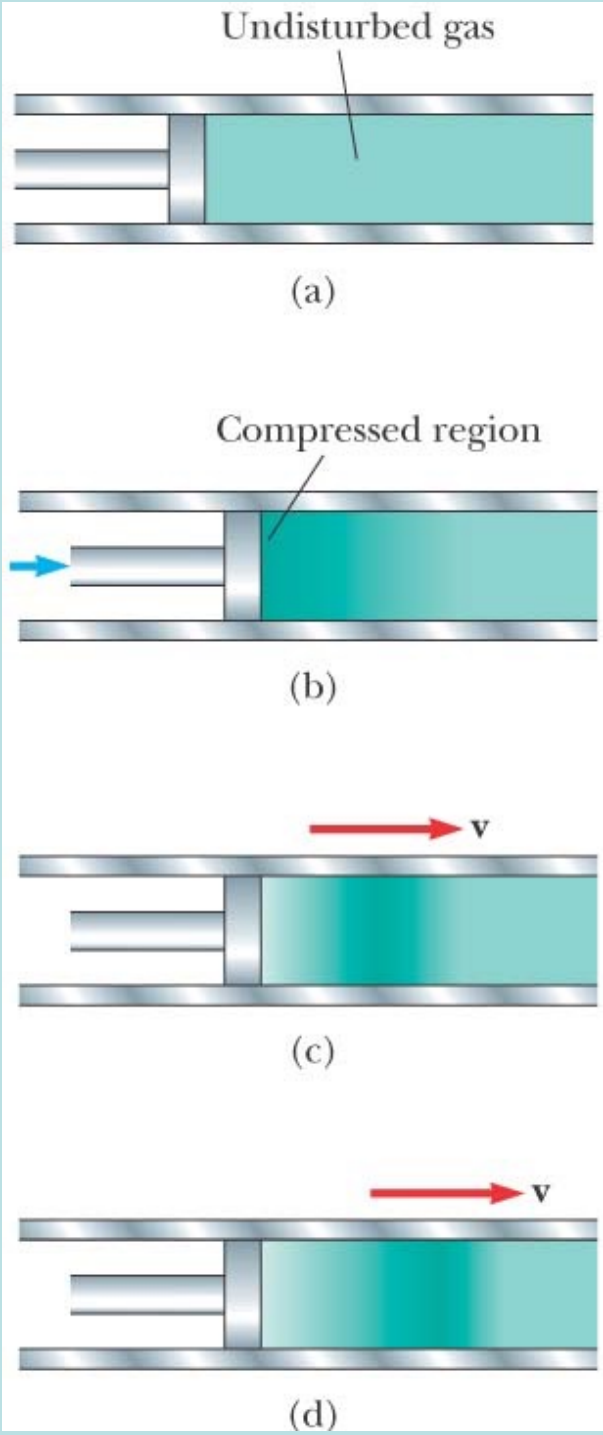
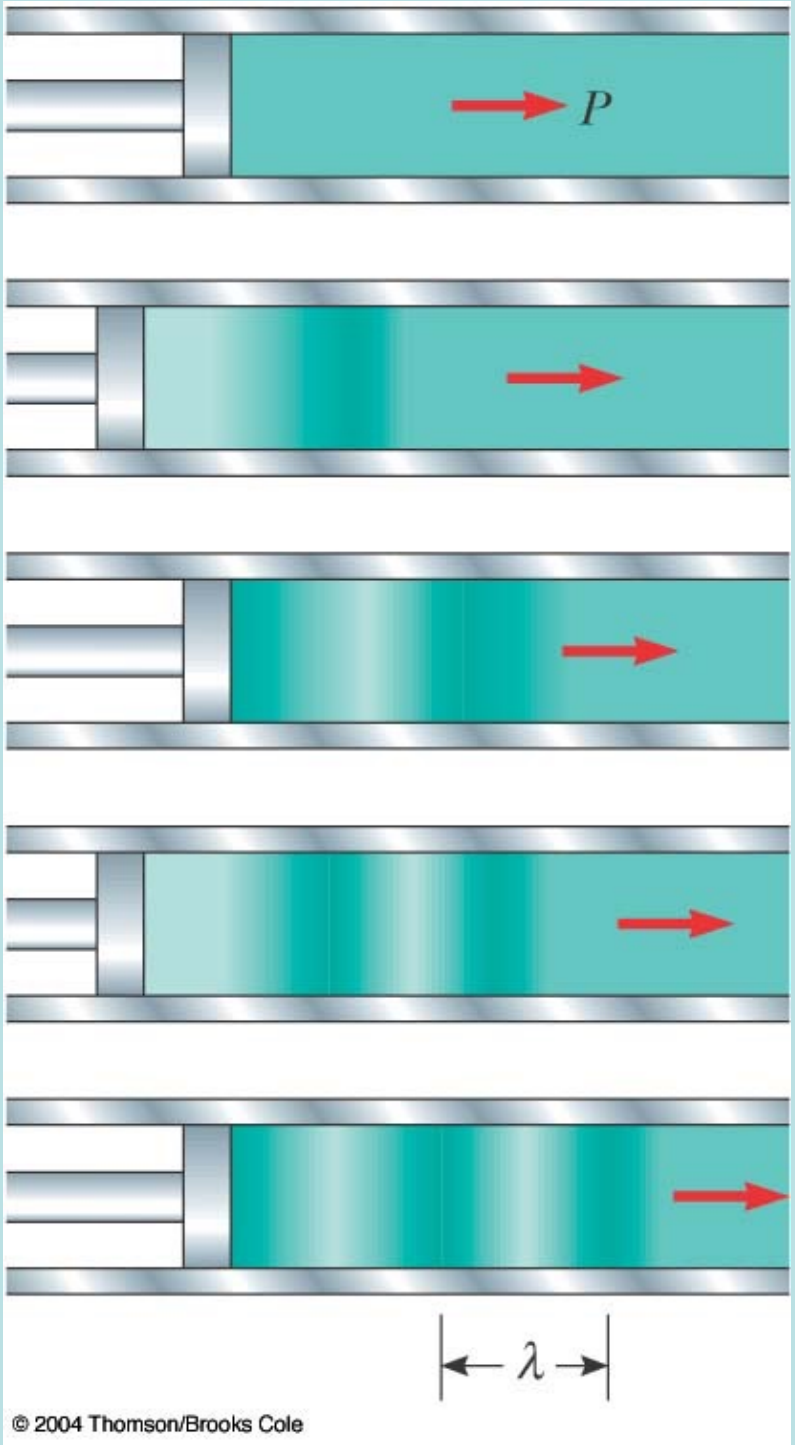
ROCK FESTIVAL

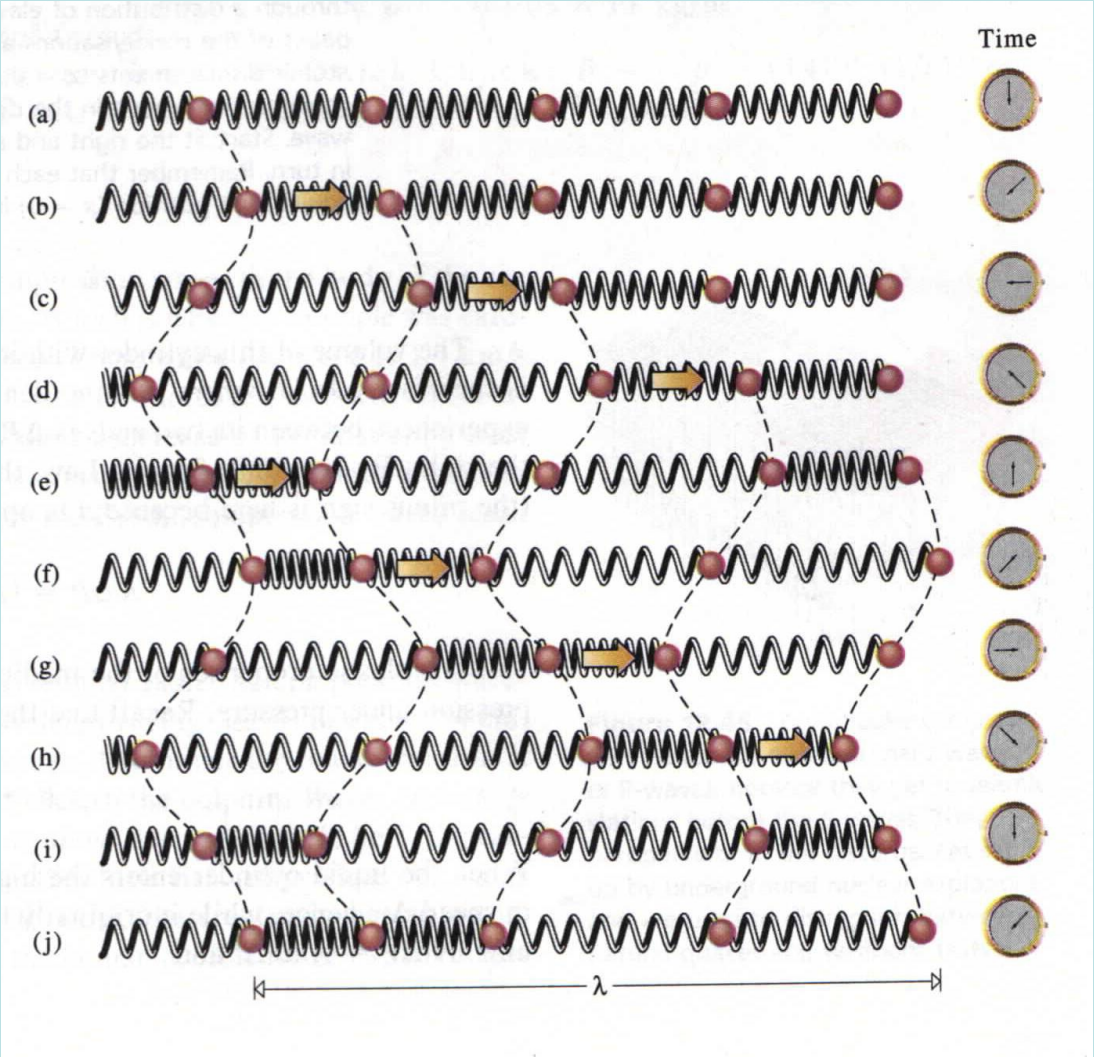
空氣柱中的聲波

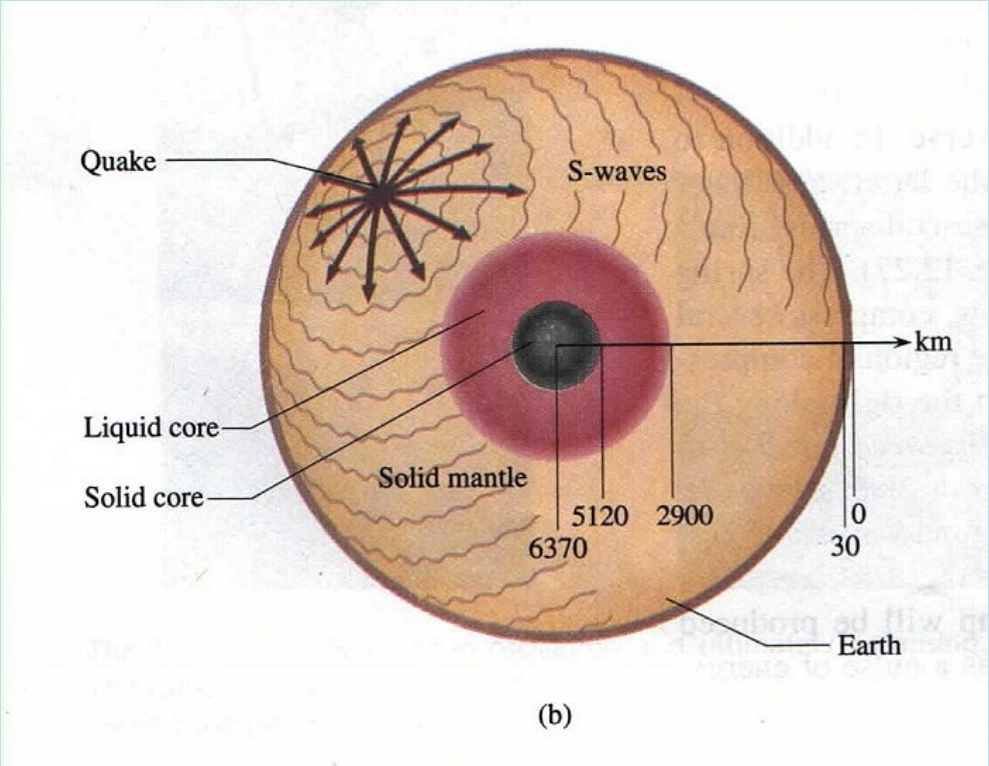


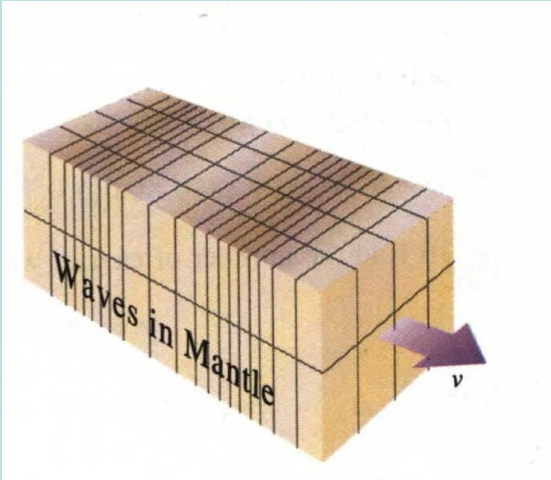

Undisturbed gas
(a)
Compressed region
(b)
v
(c)
v
(d)

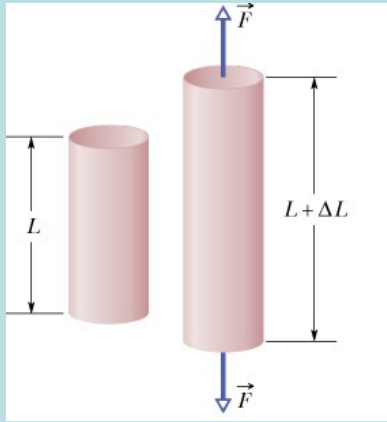

P
λ
© 2004 Thomson/Brooks Cole

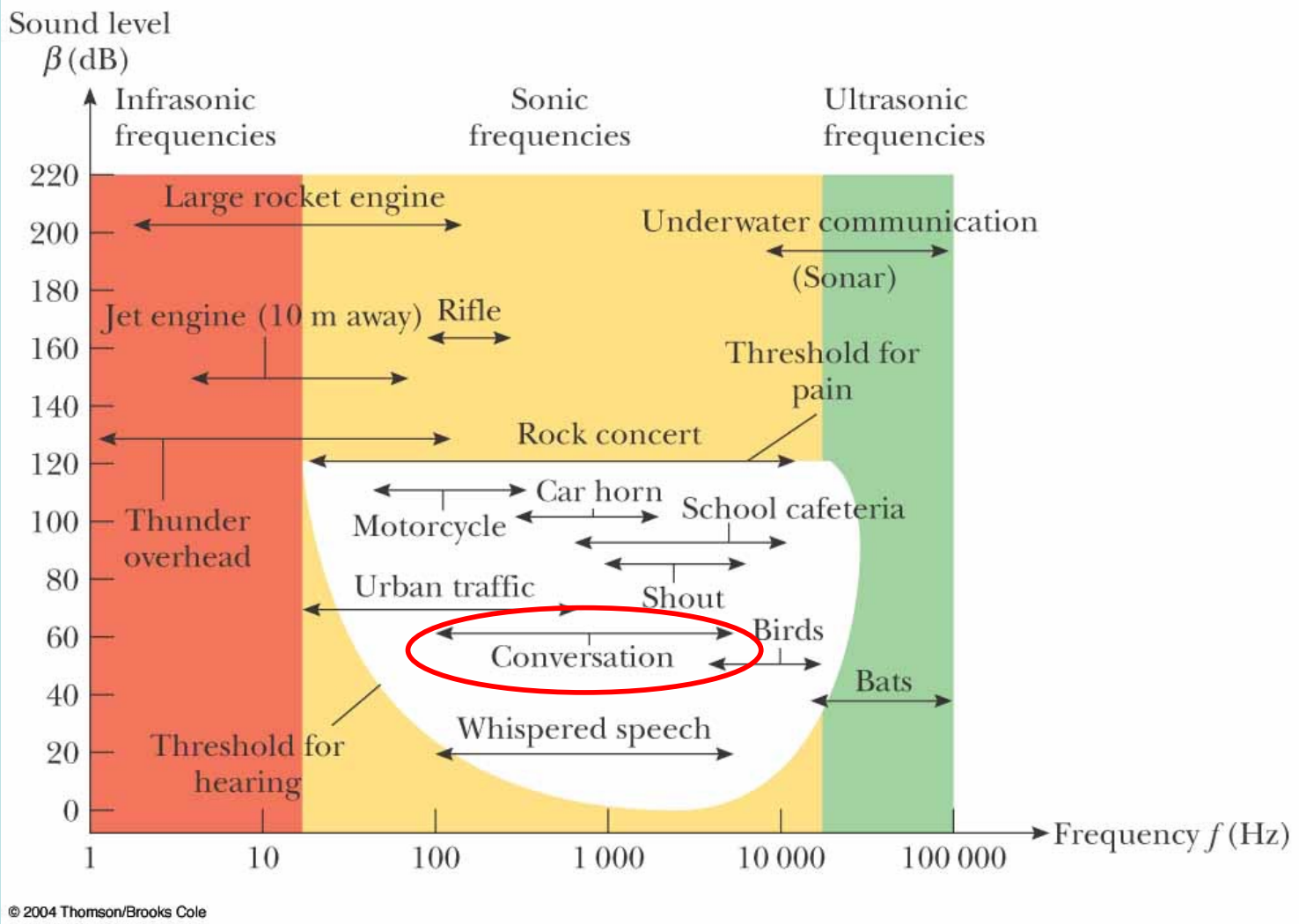
縱波　彈簧




Quake
S-waves
km
Liquid core
Solid core
Solid mantle
5120
2900
0
6370
30
Earth
(b)


Waves in Mantle
v


$\vec{F}$
L
$L + \Delta L$
$\vec{F}$

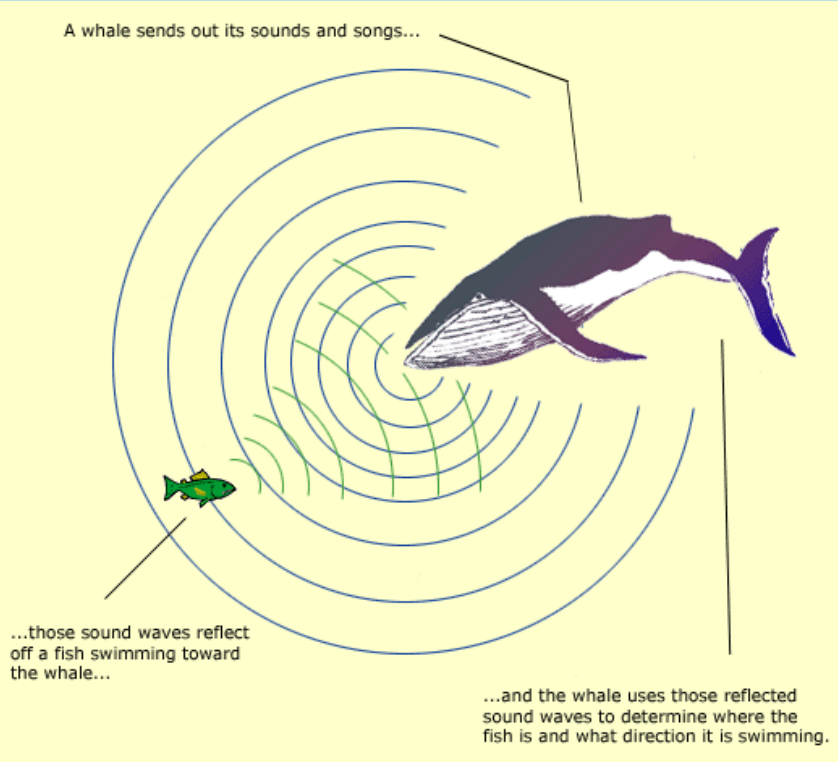


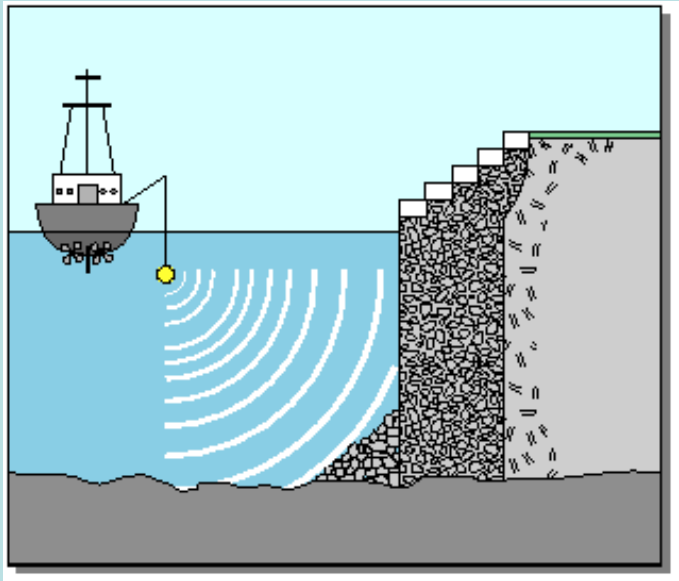
聲音的感覺與頻率非常相關！

聽的到的聲波頻率遠大於弦波： $f \approx 100 \rightarrow 10{,}000\text{Hz}$

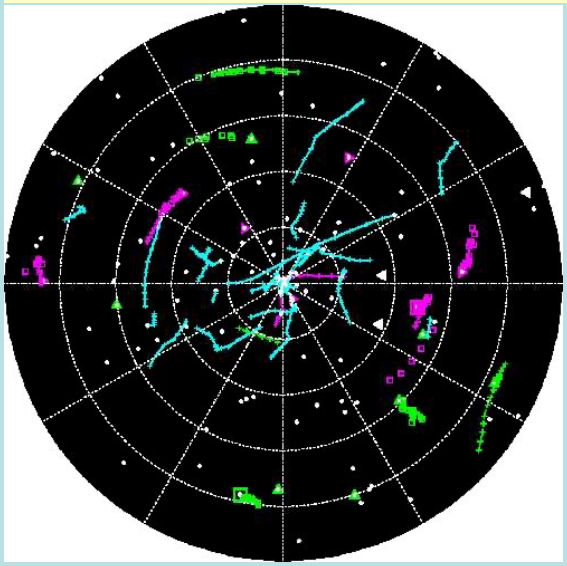
一般聲波可以視為不同頻率正弦波的疊加，討論可以正弦波為主。

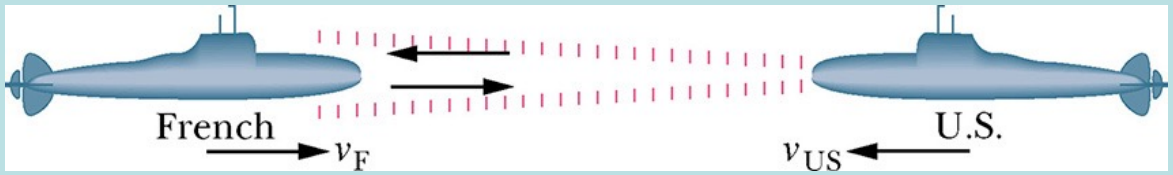
Sonar **SO**und **NA**vigation and **R**anging



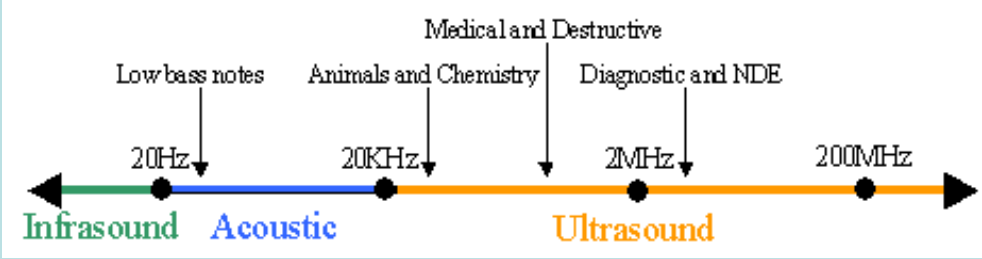


$$f \approx 10{,}000 \rightarrow 100{,}000\text{Hz}$$

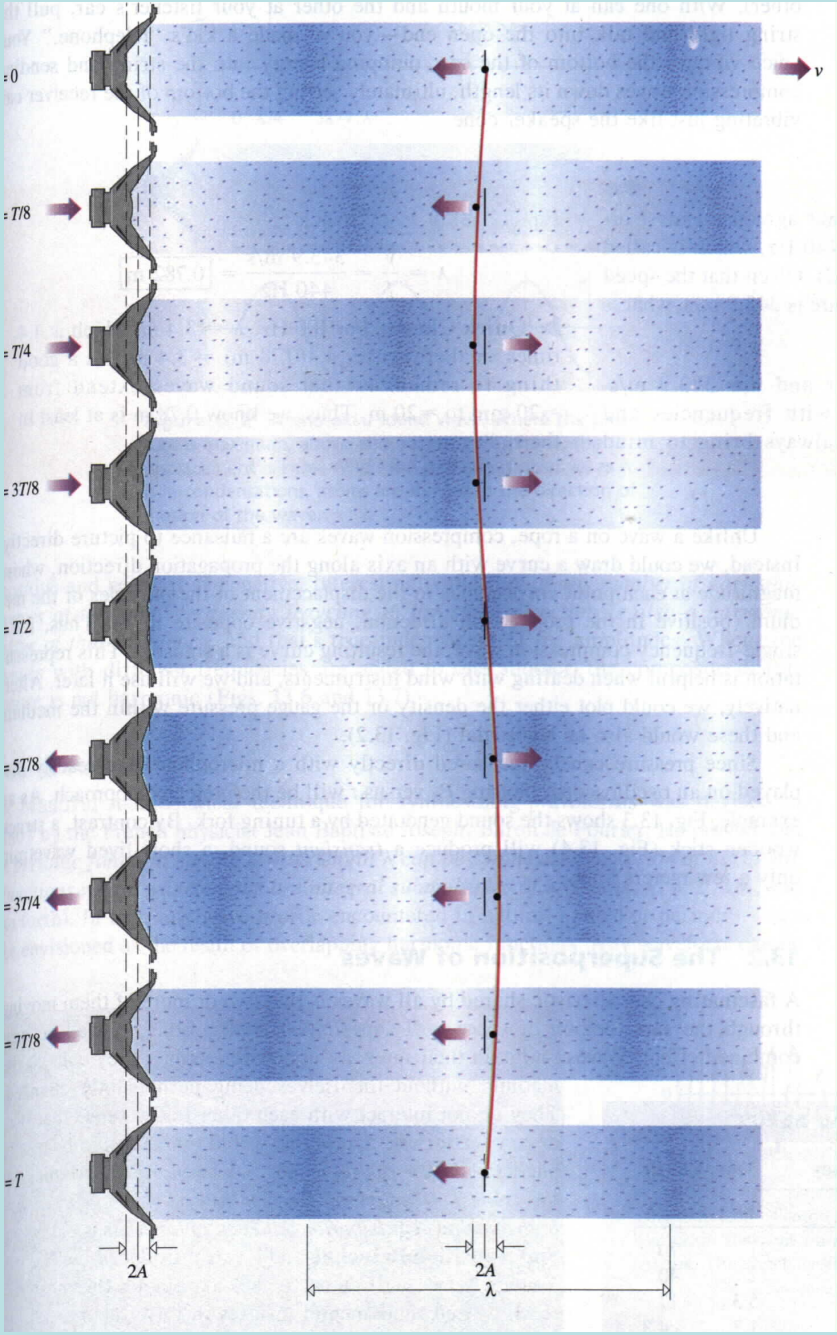


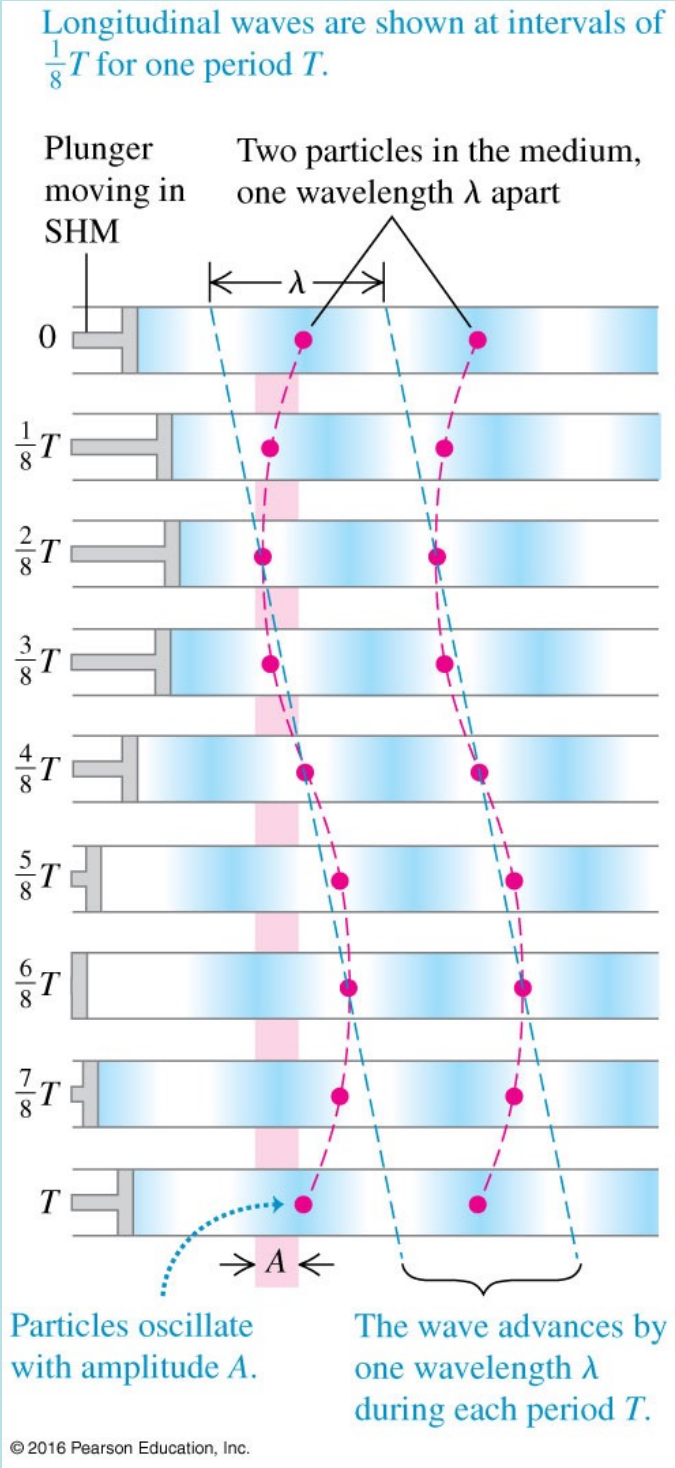


Ultra Sound


Medical and Destructive
Low bass notes
Animals and Chemistry
Diagnostic and NDE
20Hz
20KHz
2MHz
200MHz
Infrasound
Acoustic
Ultrasound

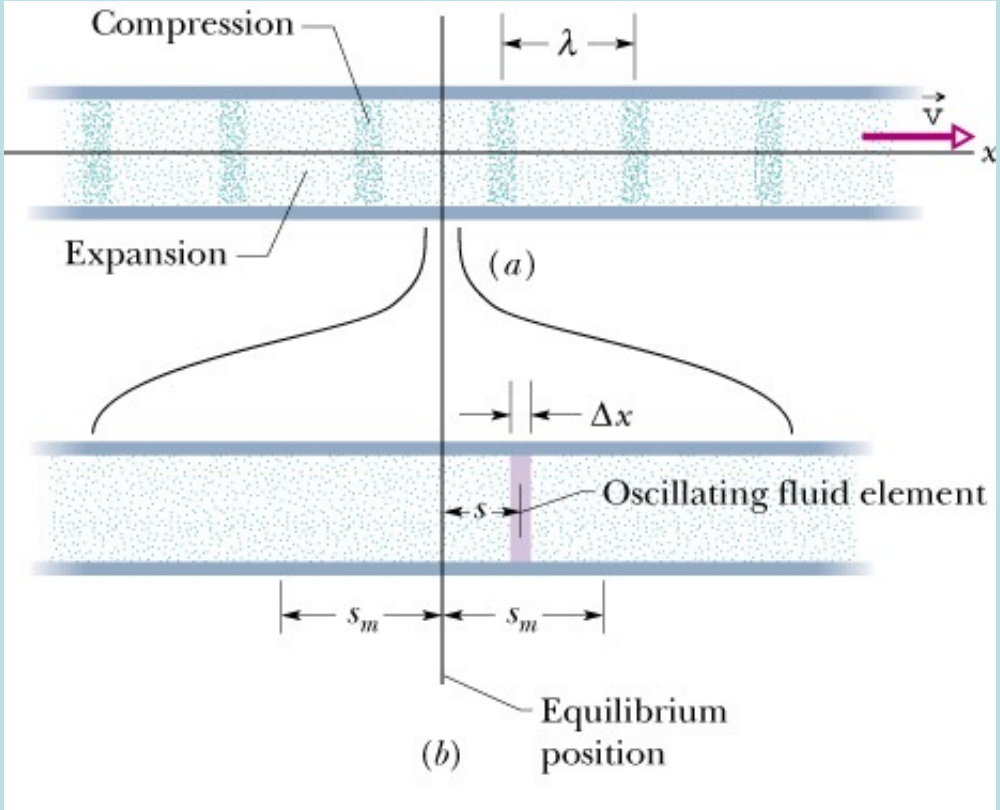
CHOI LUCIA
CPMC
07-12-2004
LC12/6/70
RAB 4-8P/Obstetric
11.8cm
14Hz
10:09:11
Fet.A.CRI
Har-low
Pwr 0
Gn 0
C9 / M7
E0
MI 1.2
CRI





縱波：介質粒子的震動方向與波的方向一致。

聲波的波函數

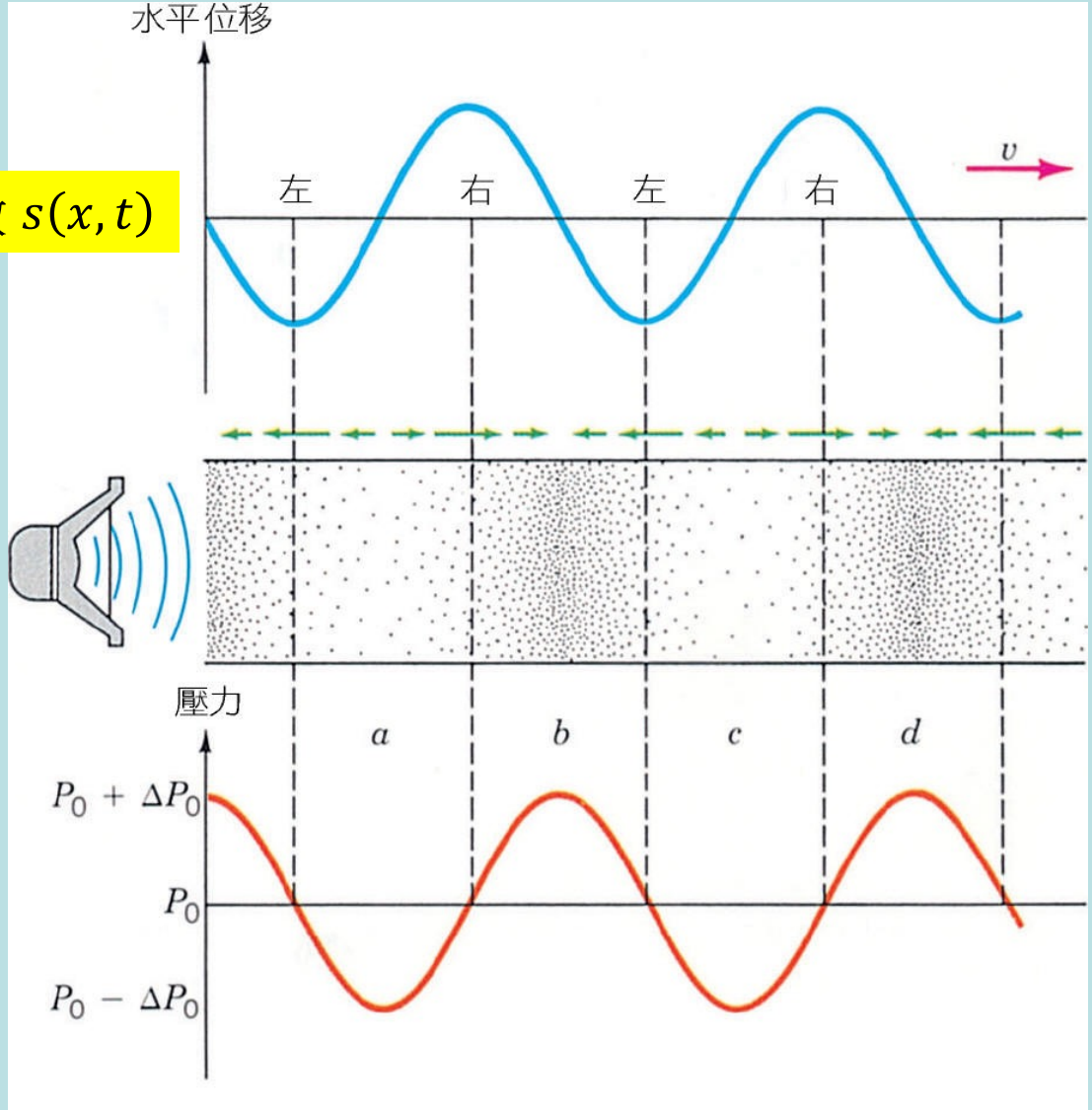


波函數 $s(x,t)$ 為以x為平衡點的氣體分子，在時間 t 時的位移

因此，時間為t時，此分子的位置為$x+s(x,t)$。

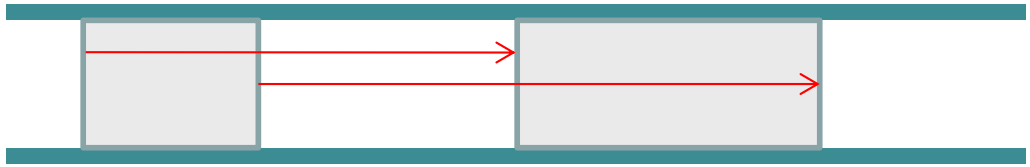
波函數可以是水平位移，直接但難測，以壓力表示時測量較為容易！

波函數 $s(x,t)$

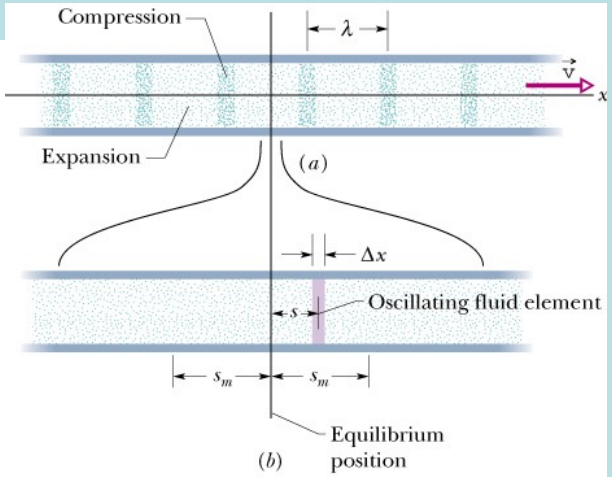


波函數 $\Delta P(x,t)$

s與ΔP兩個波函數是相關的！

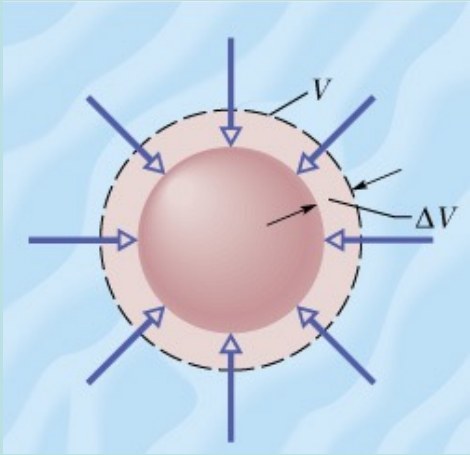


x $\quad x+\Delta x \quad x+s(x,t) \quad x+\Delta x+s(x+\Delta x\,,t)$



此塊氣體的體積變化等於：

$$\Delta V = A\cdot[s(x+\Delta x\,,t)-s(x\,,t)] = A\cdot\Delta x\cdot\frac{\partial s}{\partial x} = V\frac{\partial s}{\partial x}$$



$$\Delta P = -B\cdot\frac{\Delta V}{V}$$

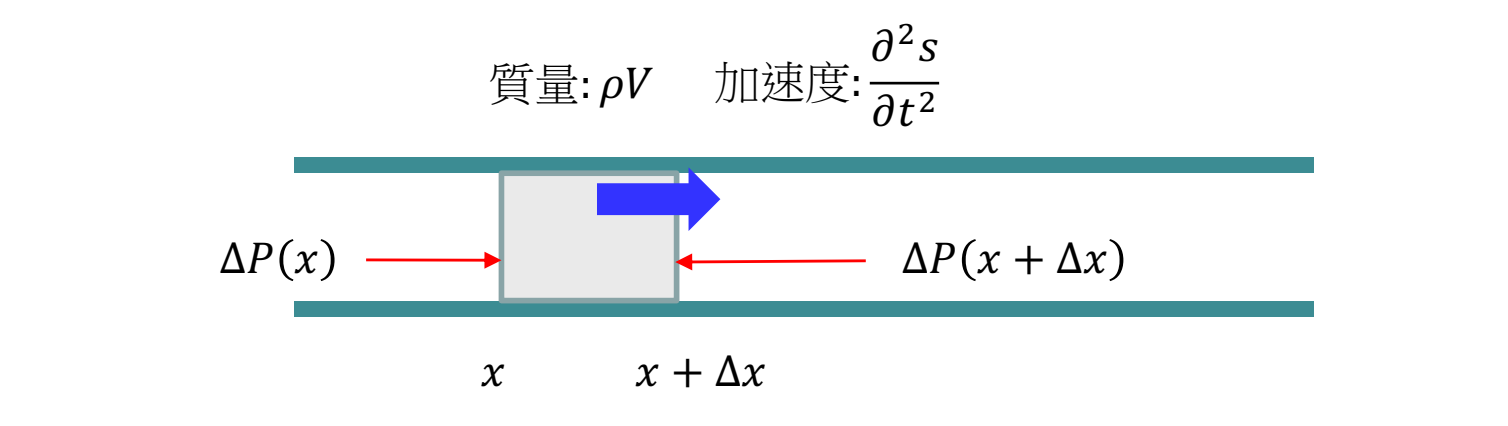
體積變化比例與壓力變化成正比！

B是Bulk Modulus

體積彈性係數

$$B = 1.4P$$

$$\Delta P = -B\frac{\partial s}{\partial x}$$



空氣柱中一小塊氣體的加速度等於$\frac{\partial^2 s}{\partial t^2}$，加速度正比於受力：

此塊氣體的受力來自兩邊的壓力差：

$$F = -A \cdot [\Delta P(x+\Delta x) - \Delta P(x)] = -A \cdot \Delta x \cdot \frac{\partial \Delta P}{\partial x} = -V \frac{\partial \Delta P}{\partial x}$$

力 = 質量 × 加速度 →

牛頓第二定律

$$\rho V \cdot \frac{\partial^2 s}{\partial t^2} = -V \frac{\partial \Delta P}{\partial x}$$

$$\Delta P = -B \frac{\partial s}{\partial x}$$

→

$$\rho \frac{\partial^2 s}{\partial t^2} = B \frac{\partial^2 s}{\partial x^2}$$

聲波的波方程式

$$\frac{\partial^2 s}{\partial x^2} = \frac{\rho}{B}\frac{\partial^2 s}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$

$$v = \sqrt{\frac{B}{\rho}}$$

聲速由彈性係數B決定。

$$\Delta P = -B \cdot \frac{\Delta V}{V}$$

將此式運用於很小的壓縮過程： $\Delta \rightarrow d$

$$dP = -B \cdot \frac{dV}{V}$$

$$B = -\frac{1}{V} \cdot \frac{dP}{dV}$$

若聲波傳送是等溫的壓縮過程：

$$P = nRT \cdot V^{-1}$$

$$B = -\frac{dP}{dV} \cdot V = nRTV^{-1} = P$$

$$v = \sqrt{\frac{P}{\rho}} \sim 288.1 \text{ m/s}$$

$$P = 1.01 \times 10^5 \text{Pa}$$
$$\rho = 1.21 \text{ kg/m}^3$$

比測量結果小！

The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

[a]At 0°C and 1 atm pressure, except where noted.
[b]At 20°C and 3.5% salinity.

$$v = \sqrt{\frac{B}{\rho}}$$

但聲波的傳播不是定溫而是絕熱過程，壓縮膨脹過程太快來不及傳熱。

$$P = c \cdot V^{-\gamma}$$

$$B = -\frac{dP}{dV} \cdot V = \gamma c V^{-\gamma} = \gamma P = 1.4P$$

$$B = 1.4P$$

$$P = 1.01 \times 10^5 \text{Pa}$$
$$\rho = 1.21 \text{ kg/m}^3$$

$$v = \sqrt{\frac{1.4P}{\rho}} \sim 341.8 \text{ m/s}$$

The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

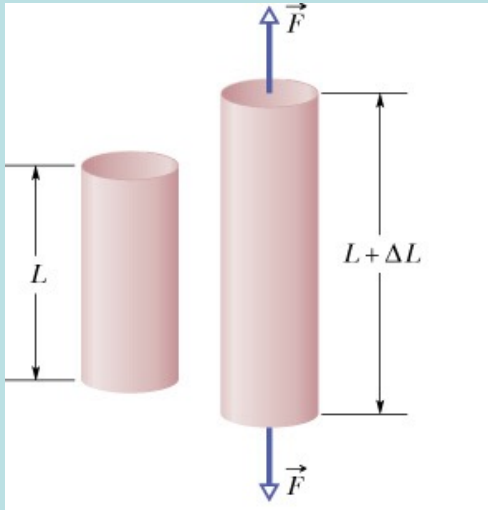
[a]At 0°C and 1 atm pressure, except where noted.
[b]At 20°C and 3.5% salinity.

固體介質中的聲速

$$v = \sqrt{\frac{B}{\rho}} \Rightarrow v = \sqrt{\frac{E}{\rho}}$$

$$E = \frac{F}{A}\frac{\Delta l}{l}$$

楊氏彈性係數



Some Elastic Properties of Selected Materials of Engineering Interest

Material	Density ρ (kg/m^3)	Young's Modulus E (10^9 N/m^2)	Ultimate Strength S_u (10^6 N/m^2)	Yield Strength S_y (10^6 N/m^2)
Steel[a]	7860	200	400	250
Aluminum	2710	70	110	95
Glass	2190	65	50[b]	—
Concrete[c]	2320	30	40[b]	—
Wood[d]	525	13	50[b]	—
Bone	1900	9[b]	170[b]	—
Polystyrene	1050	3	48	—

[a]Structural steel (ASTM-A36).
[b]In compression.
[c]High strength.
[d]Douglas fir.

The Speed of Sound[a]

Medium	Speed (m/s)
Gases	
Air (0°C)	331
Air (20°C)	343
Helium	965
Hydrogen	1284
Liquids	
Water (0°C)	1402
Water (20°C)	1482
Seawater[b]	1522
Solids	
Aluminum	6420
Steel	5941
Granite	6000

[a]At 0°C and 1 atm pressure, except where noted.
[b]At 20°C and 3.5% salinity.

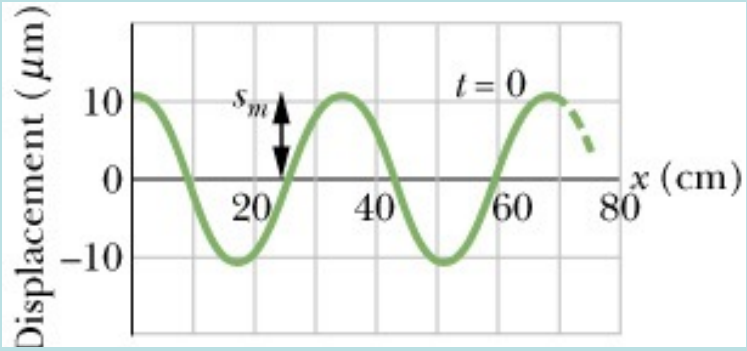
聲波的波函數s滿足與弦波一樣的波方程式，因此有一樣的解：

$$\frac{\partial^2 s}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$

$$s(x,t) = f(x - vt) + g(x + vt)$$

$$s(x,t) = s_m \cos(kx - \omega t)$$

正弦聲波



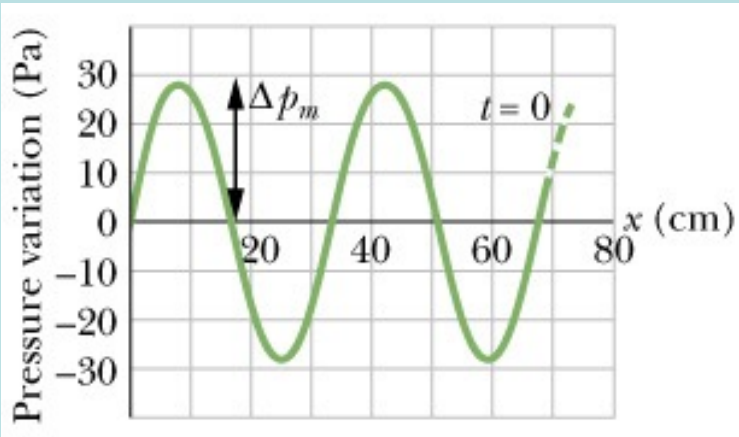
壓力波函數ΔP是s的偏微分，因此也滿足與弦波一樣的波方程式：

$$\Delta P = -B\frac{\partial s}{\partial x}$$

$$\frac{\partial^2 \Delta P}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Delta P}{\partial t^2}$$

$$\Delta P(x,t) = \Delta P_m \sin(kx - \omega t)$$

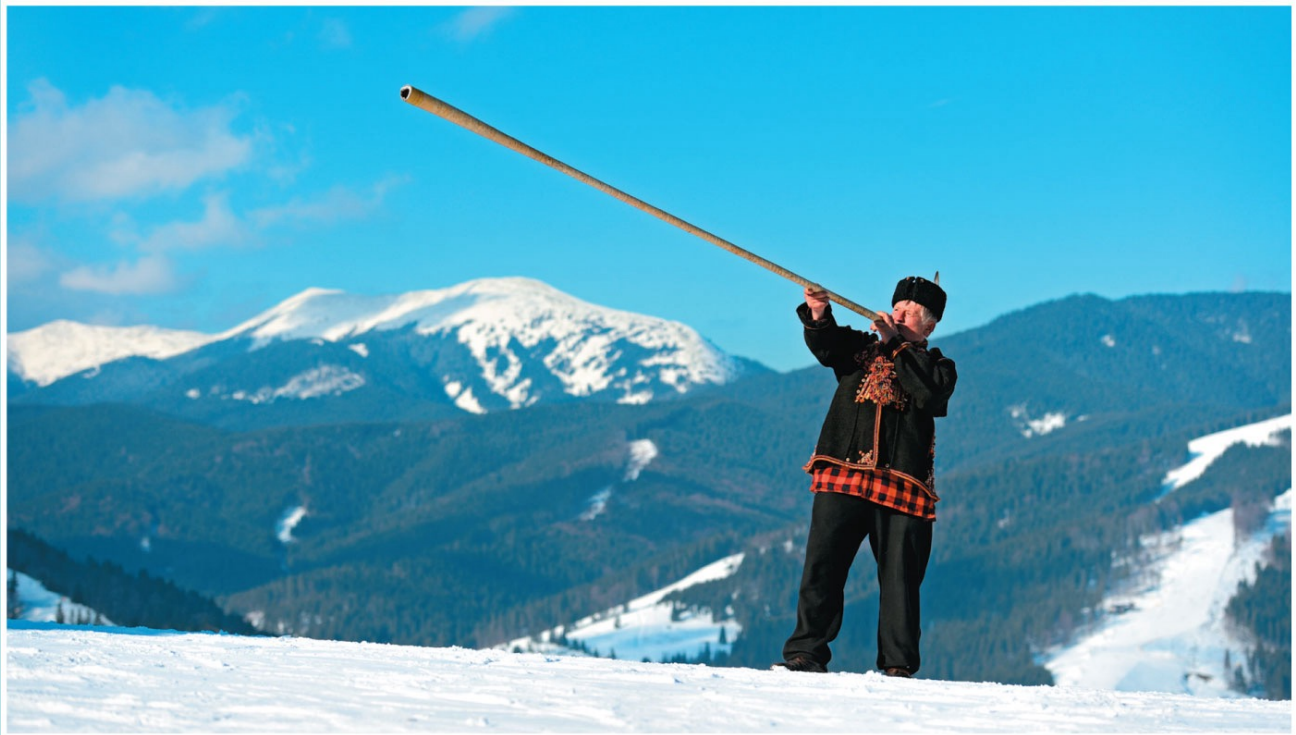
壓力與位移的相位差90°

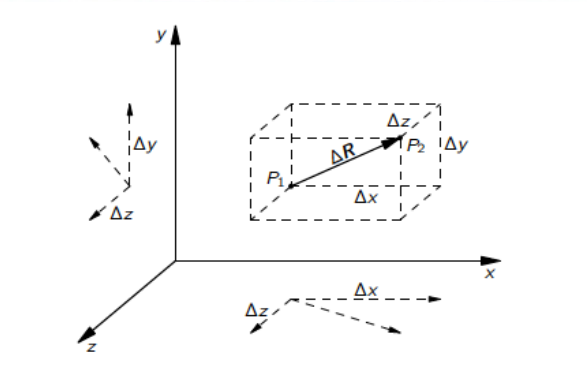


聲波可以是立體波，其波函數是三度空間座標的函數： $s(x, y, z, t)$

現在我們終於要正式進入立體世界！

在立體世界，所有物理量必須被歸類為純量、向量或是張量！





聲波可以是立體波，其波函數是三度空間座標的純量函數： $s(x,y,z,t)$

滿足三度空間波方程式：

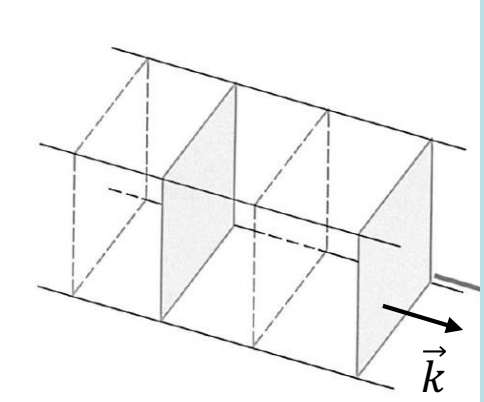
$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$

立體聲波的波源可以是一很大的平面，

如此波的傳播會沿垂直此面的方向，可定義一個角波數向量$\vec{k}$： $s_m\cdot\sin(\vec{k}\cdot\vec{r}-\omega t)$

對正弦波而言，以鄰近的波峰組成平面來圖示最方便，稱為波前。

如此波前為一個一個與波源面平行的平面，這樣的波稱為平面波。



$$s(\vec{r},t)=s_m\cdot\sin(\vec{k}\cdot\vec{r}-\omega t)=s_m\cdot\sin(k_x x+k_y y+k_z z-\omega t)$$

代入波方程式後，證實此解滿足波方程式。

而且可以得到角波數向量大小k與角頻率ω的關係，與一維相同。

$$k_x^2+k_y^2+k_z^2=k^2=\frac{1}{v^2}\omega^2$$

$$\omega=kv$$

$$k=\frac{2\pi}{\lambda}$$

波源也可能為一點波源，

如此因為球對稱，波前為一球面，稱為球面波。

如此波傳播的$\vec{k}$方向在各地都不一樣：垂直於當地波前。

因此就與$\hat{r}$同向， $\vec{k}\cdot\vec{r}\sim kr$

而由於球對稱的關係，波函數只會與r有關。因此猜測：

$$s(r,t)\propto\sin(kr-\omega t)$$

球面波的能量也就是強度會隨距離變大而減小：

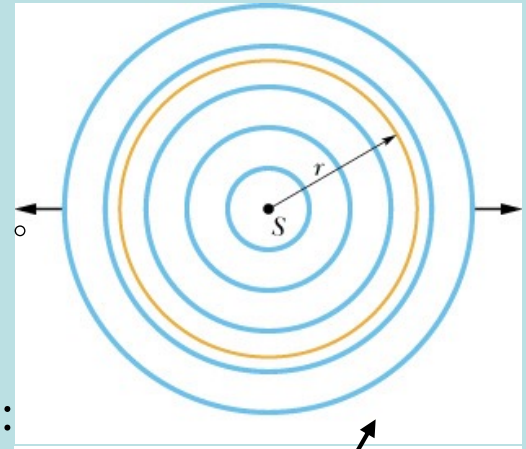
$$I=\frac{\text{Energy/Time}}{A}=\frac{P}{4\pi r^2}$$

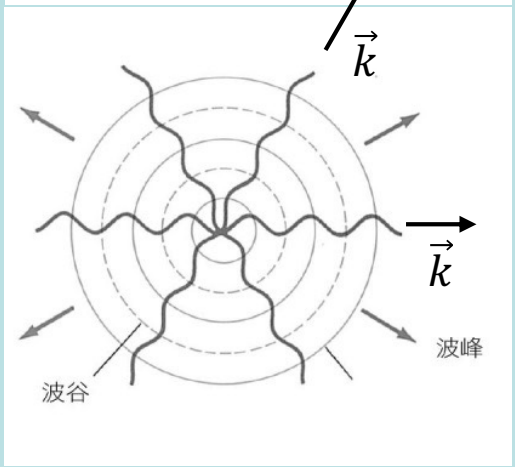
因此振幅不是常數，而是隨距離變大而減小。

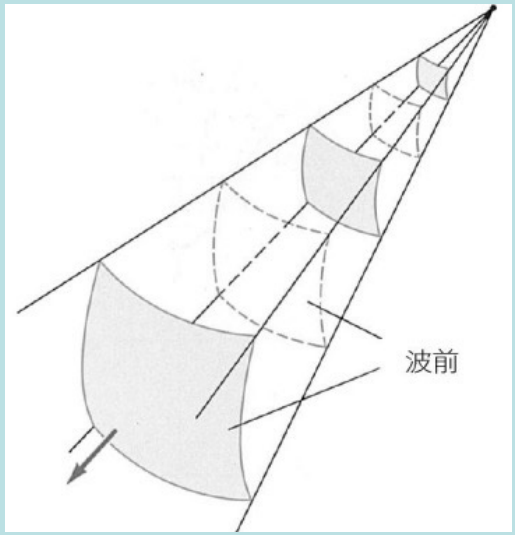
$$I\propto s_m^2 \qquad s_m\propto\frac{1}{r}$$

$$s(r,t)\propto\frac{1}{r}\cdot\sin(kr-\omega t)$$

球面波離波源甚遠時，可以以一平面波近似。





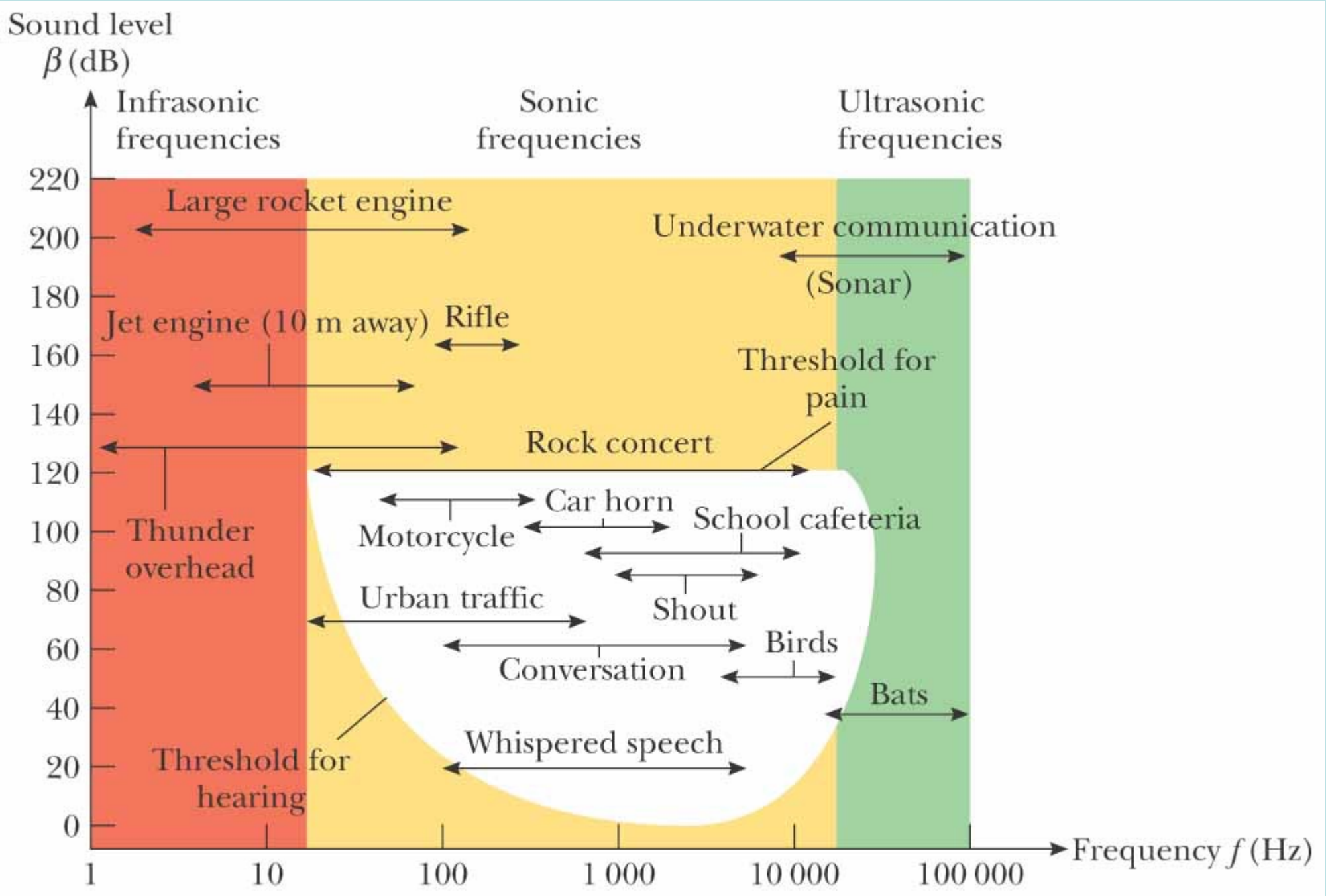


音量由波強度決定

Sound Level (Decibel)

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0}$$

Some Sound Levels (dB)	
Hearing threshold	0
Rustle of leaves	10
Conversation	60
Rock concert	110
Pain threshold	120
Jet engine	130



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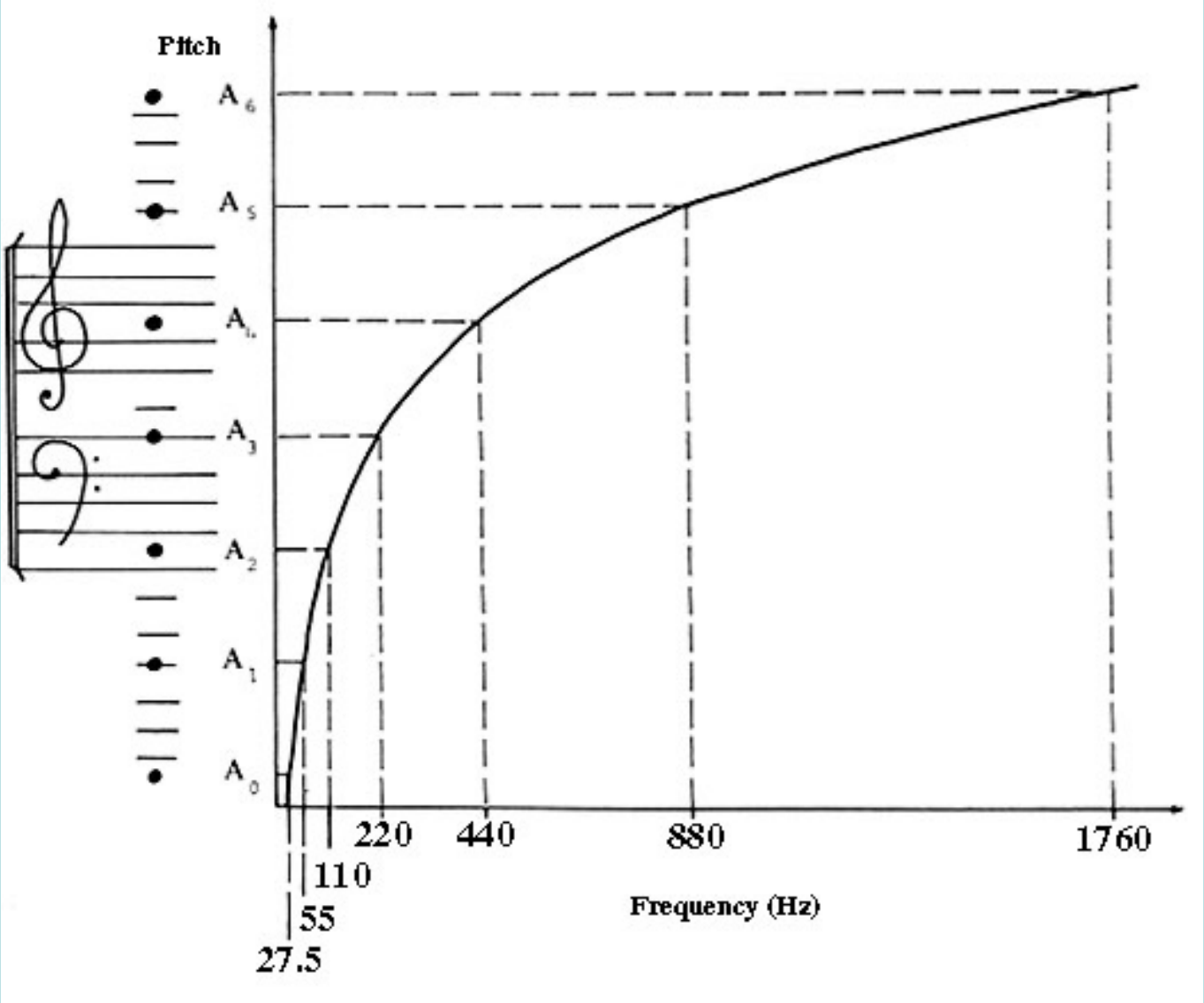
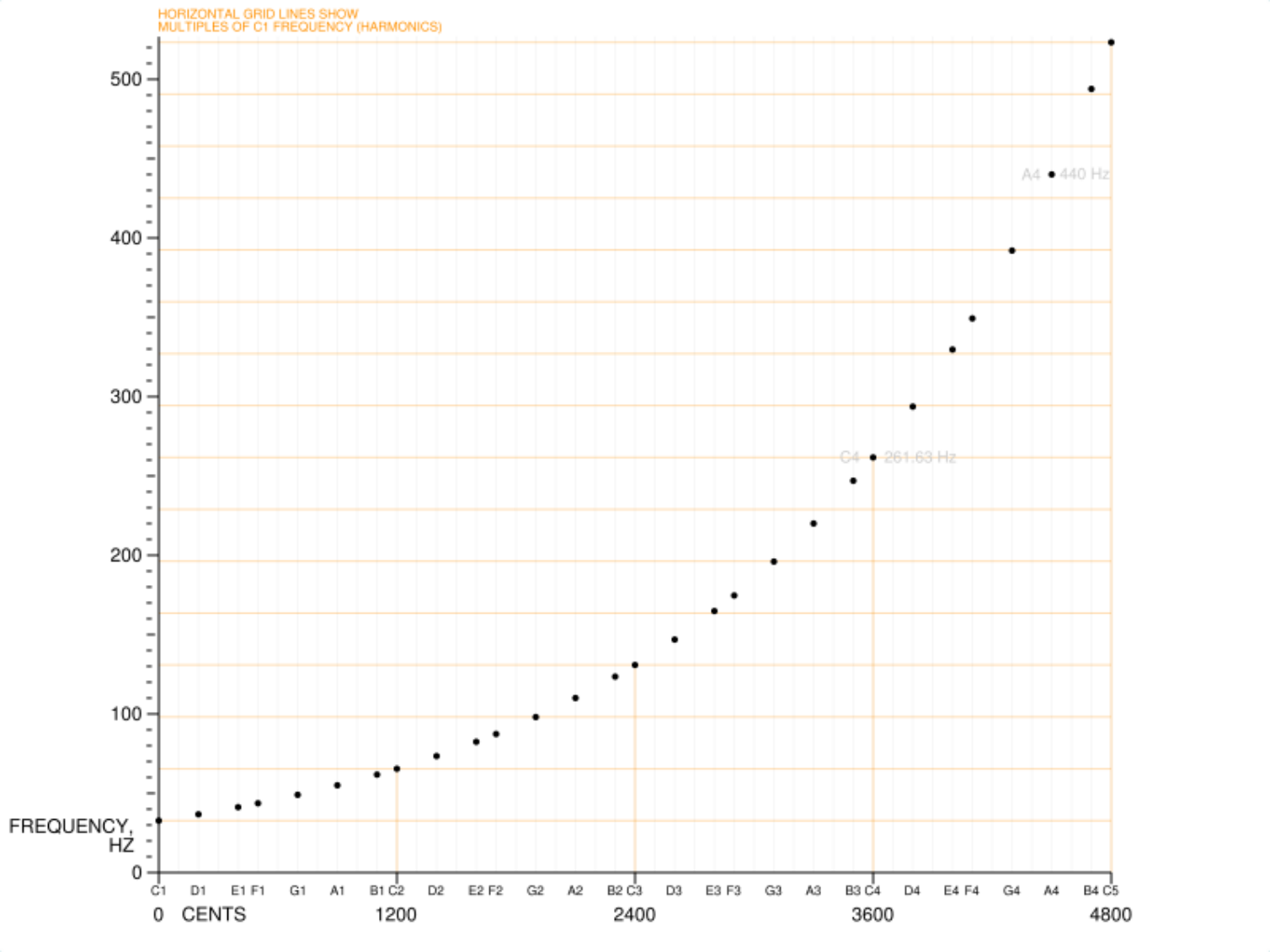
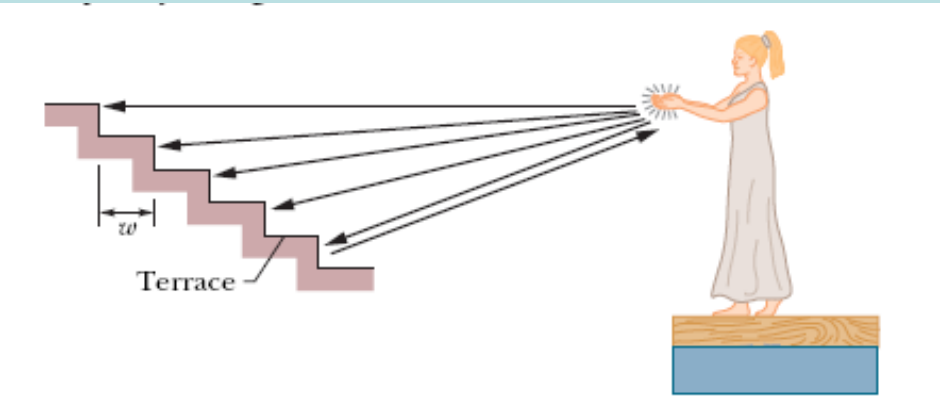
音高由頻率決定

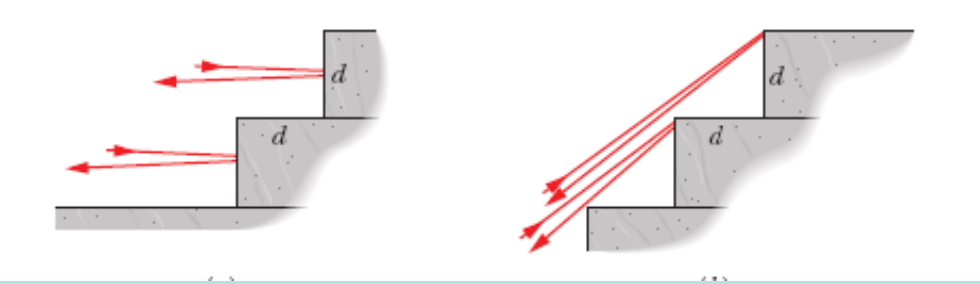

Pitch
A6
A5
A4
A3
A2
A1
A0
220
440
880
1760
110
55
27.5
Frequency (Hz)

音高由頻率決定


HORIZONTAL GRID LINES SHOW
MULTIPLES OF C1 FREQUENCY (HARMONICS)
A4 440 Hz
C4 261.63 Hz
FREQUENCY, HZ
0
100
200
300
400
500
C1 D1 E1 F1 G1 A1 B1 C2 D2 E2 F2 G2 A2 B2 C3 D3 E3 F3 G3 A3 B3 C4 D4 E4 F4 G4 A4 B4 C5
0 CENTS
1200
2400
3600
4800

音高由頻率決定





$$\Delta t_{\text{bot}} = \frac{L}{v} = \frac{2d}{v} \qquad (17\text{-}10)$$

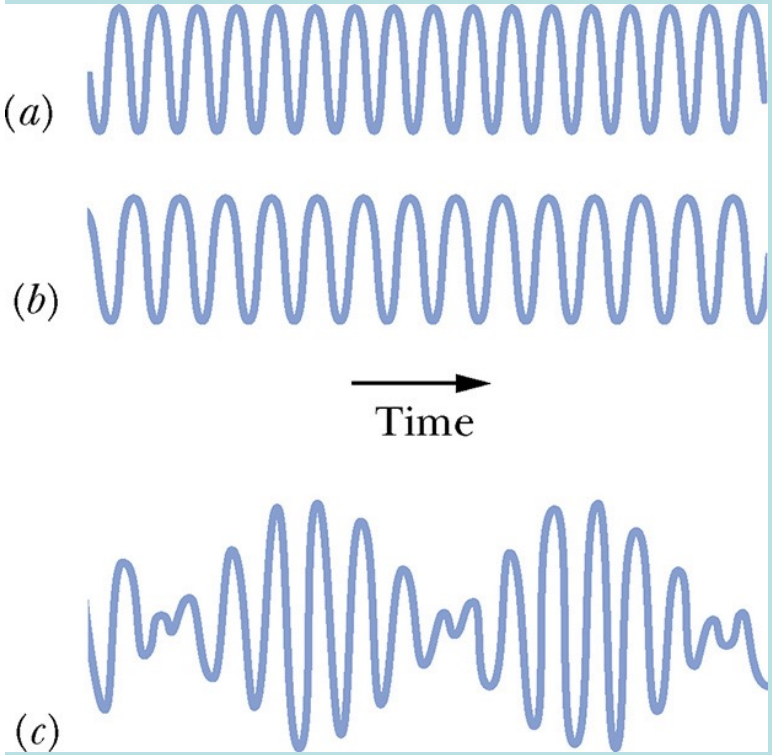
$$= \frac{2(0.263\ \text{m})}{343\ \text{m/s}} = 1.533 \times 10^{-3}\ \text{s}.$$

frequency f_{bot} at which the pulses arrive at the listener is

$$f_{\text{bot}} = \frac{1}{\Delta t_{\text{bot}}} \qquad (17\text{-}11)$$

$$= \frac{1}{1.533 \times 10^{-3}\ \text{s}} = 652\ \text{Hz}. \qquad \text{(Answer)}$$

Beat



$$s = s_1 + s_2 = s_m(\cos \omega_1 t + \cos \omega_2 t).$$

nometric identity (see Appendix E)

$$\cos \alpha + \cos \beta = 2 \cos[\tfrac{1}{2}(\alpha - \beta)] \cos[\tfrac{1}{2}(\alpha + \beta)]$$

te the resultant displacement as

$$s = 2s_m \cos[\tfrac{1}{2}(\omega_1 - \omega_2)t] \cos[\tfrac{1}{2}(\omega_1 + \omega_2)t].$$

$$\omega' = \tfrac{1}{2}(\omega_1 - \omega_2) \quad \text{and} \quad \omega = \tfrac{1}{2}(\omega_1 + \omega_2),$$

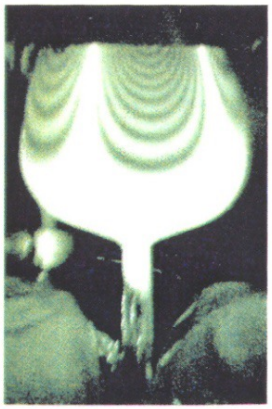
ite Eq. 17-43 as

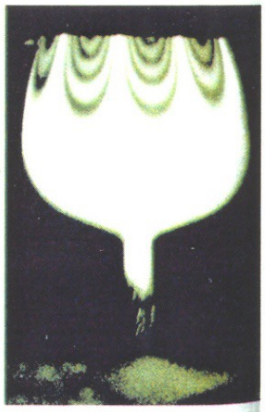
$$s(t) = [2s_m \cos \omega' t] \cos \omega t.$$

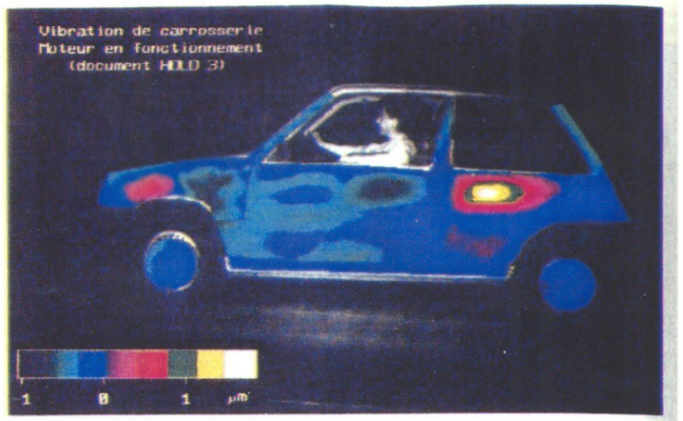
波的疊加式波函數疊加，因此會產生意想不到的結果

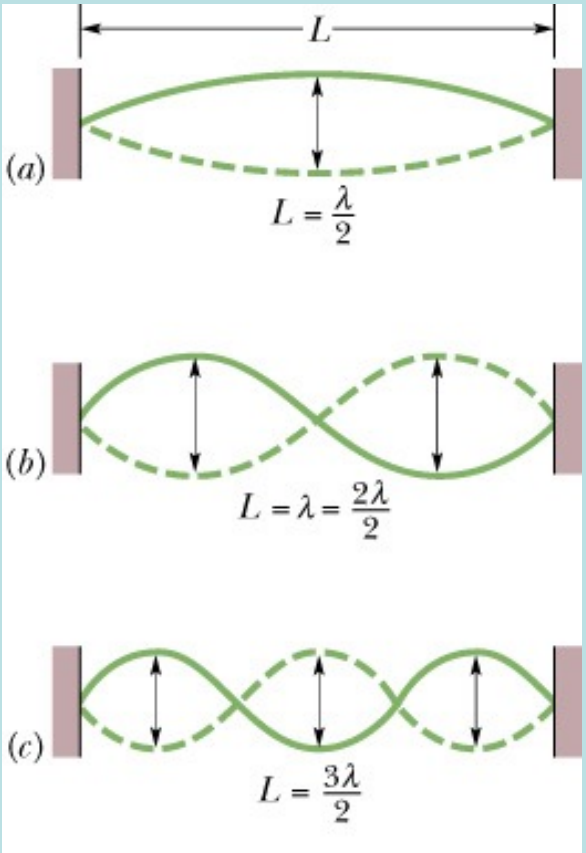
音色

樂器






Vibration de carrosserie
Moteur en fonctionnement
(document HOLD 3)
-1
0
1
µm



弦的駐波振盪造成音箱中空氣震動發出聲波。

弦的駐波頻率由弦長決定，因此手按不同位置可調整音高！

樂音以基頻為主

可是除了基頻，所有駐波態也都多多少少會被激發！

因此樂器的聲波除了基頻，會音樂器不同有不同泛音，這是音色不同的原因！

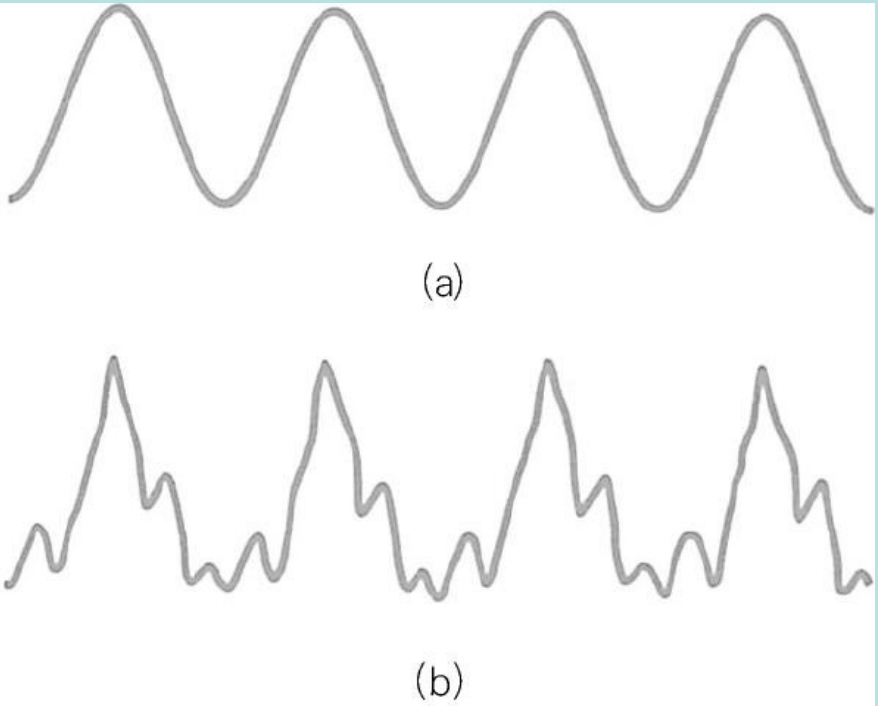
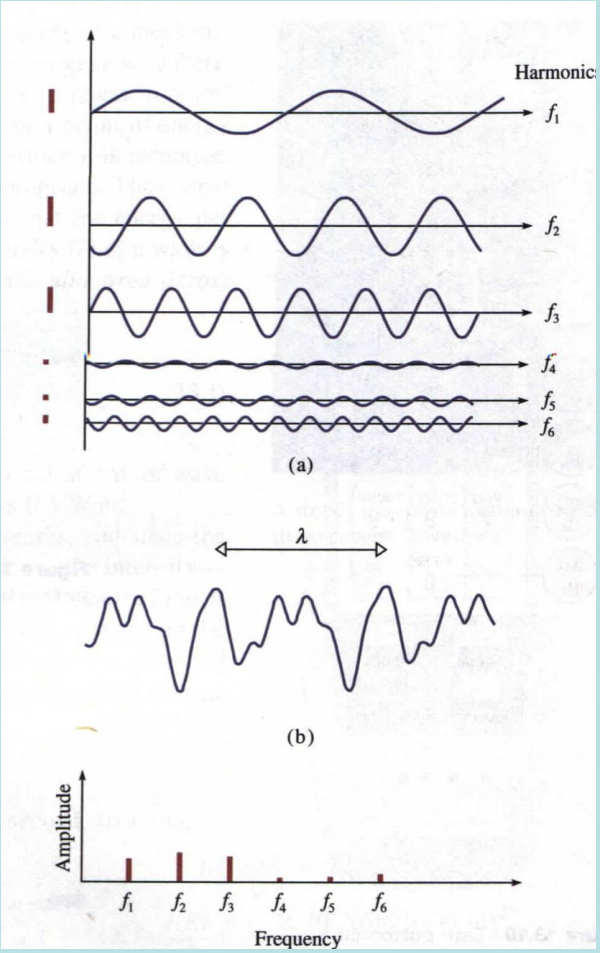


圖 17.15
(a) 音叉製造出壓力的正弦波形。
(b) 假想由樂器造出的壓力波形。



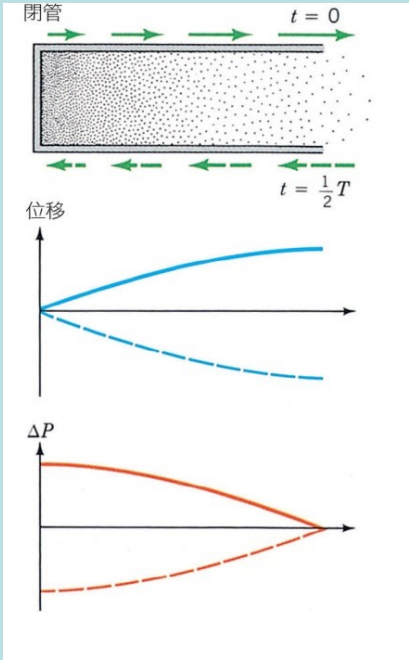


圖 17.4
閉管的基本模式。封閉端為一位移的節點，而且是壓力的腹點。實線說明在 $t = 0$ 的情形，而虛線則說明在 $t = T/2$ 之情形。

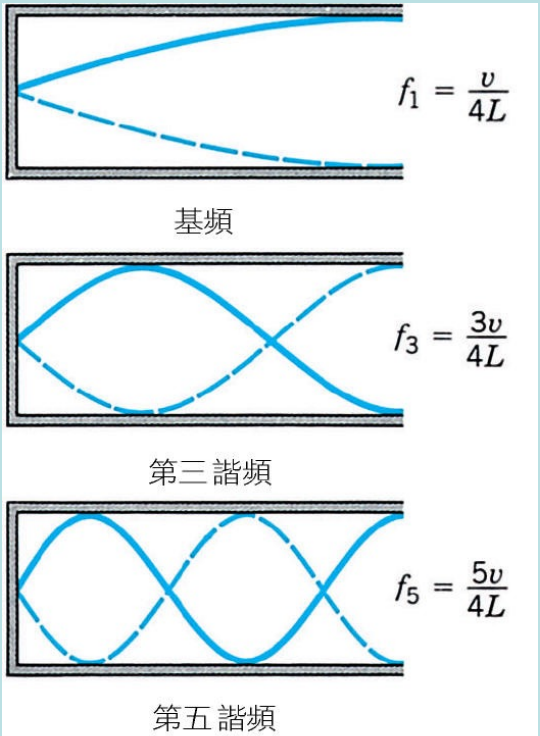
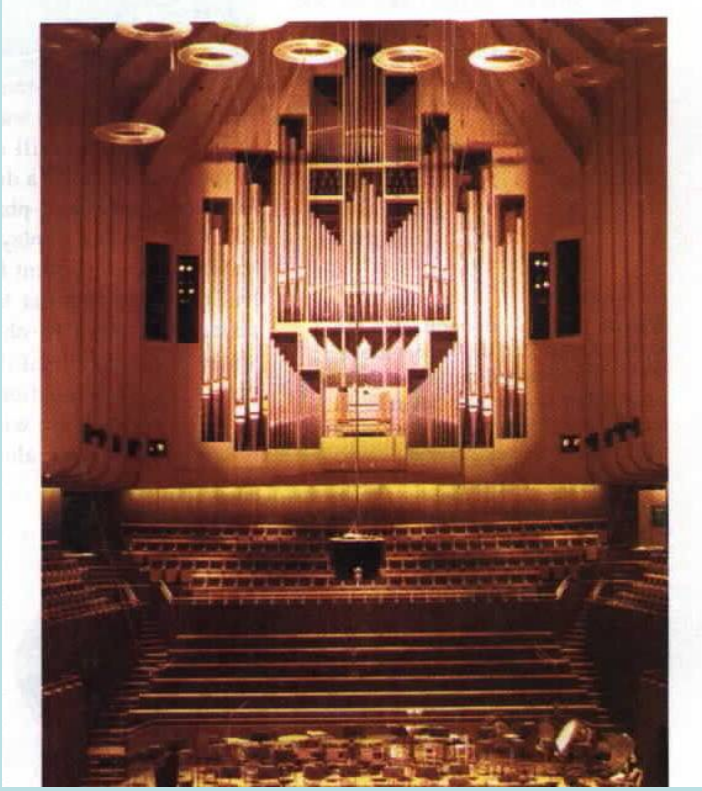
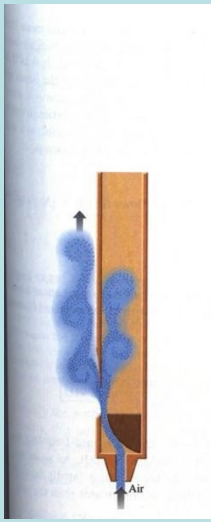


圖 17.5
閉管中前三種共振模。實線及虛線表示時差半週期的兩組縱波。





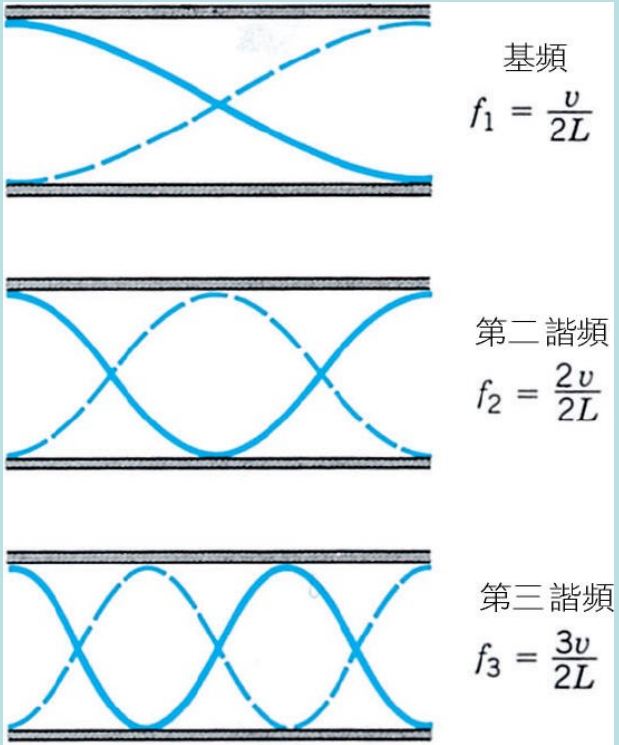
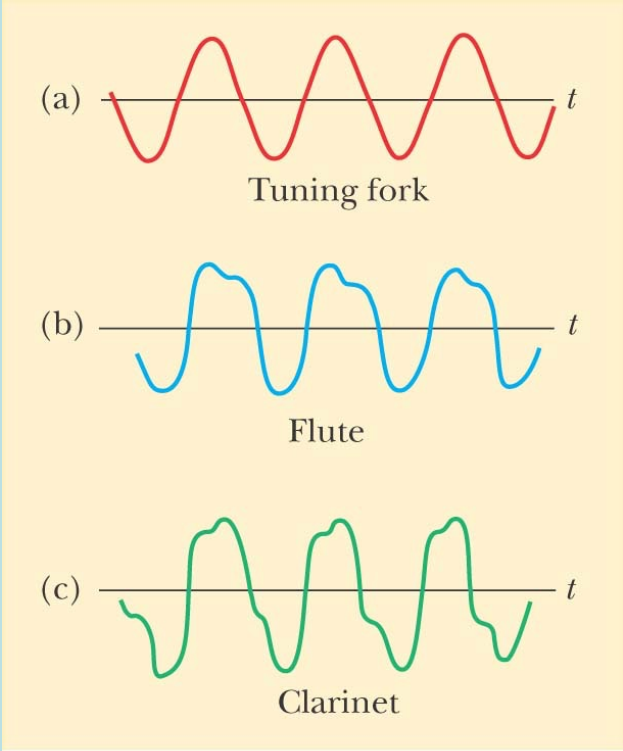


圖 17.6
開放管中前三種共振模。



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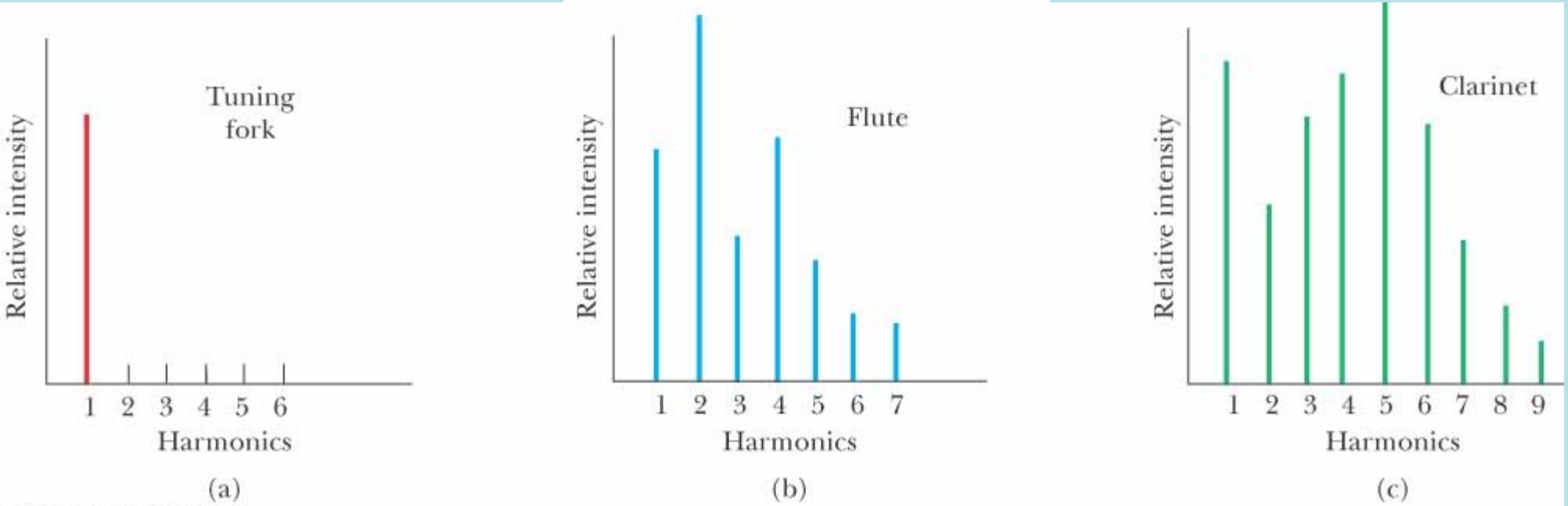

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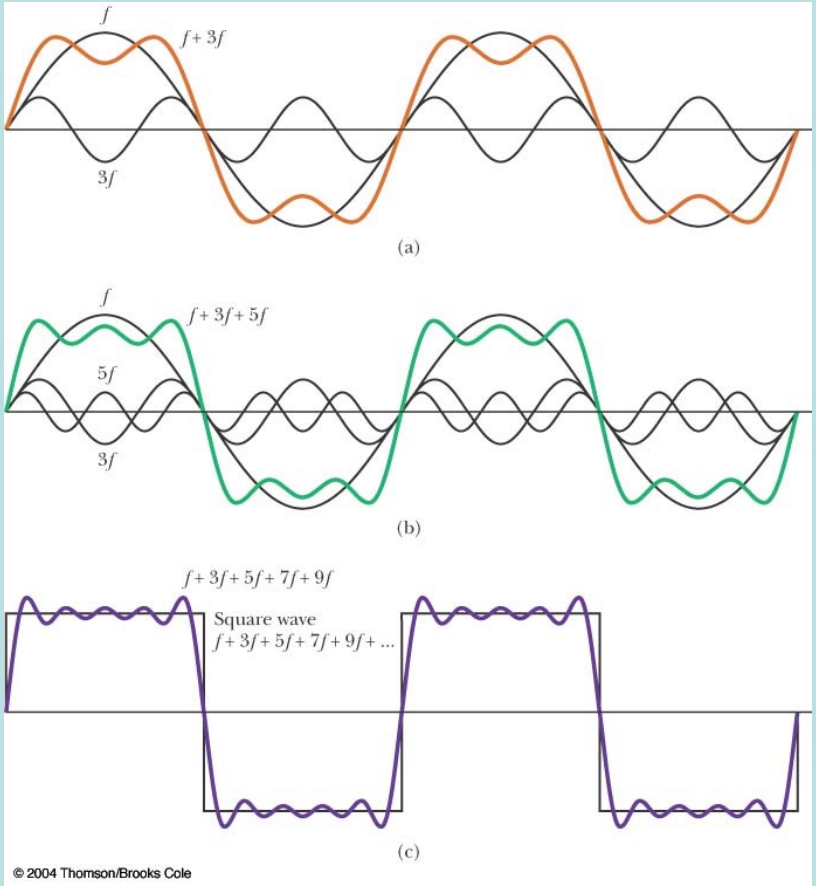
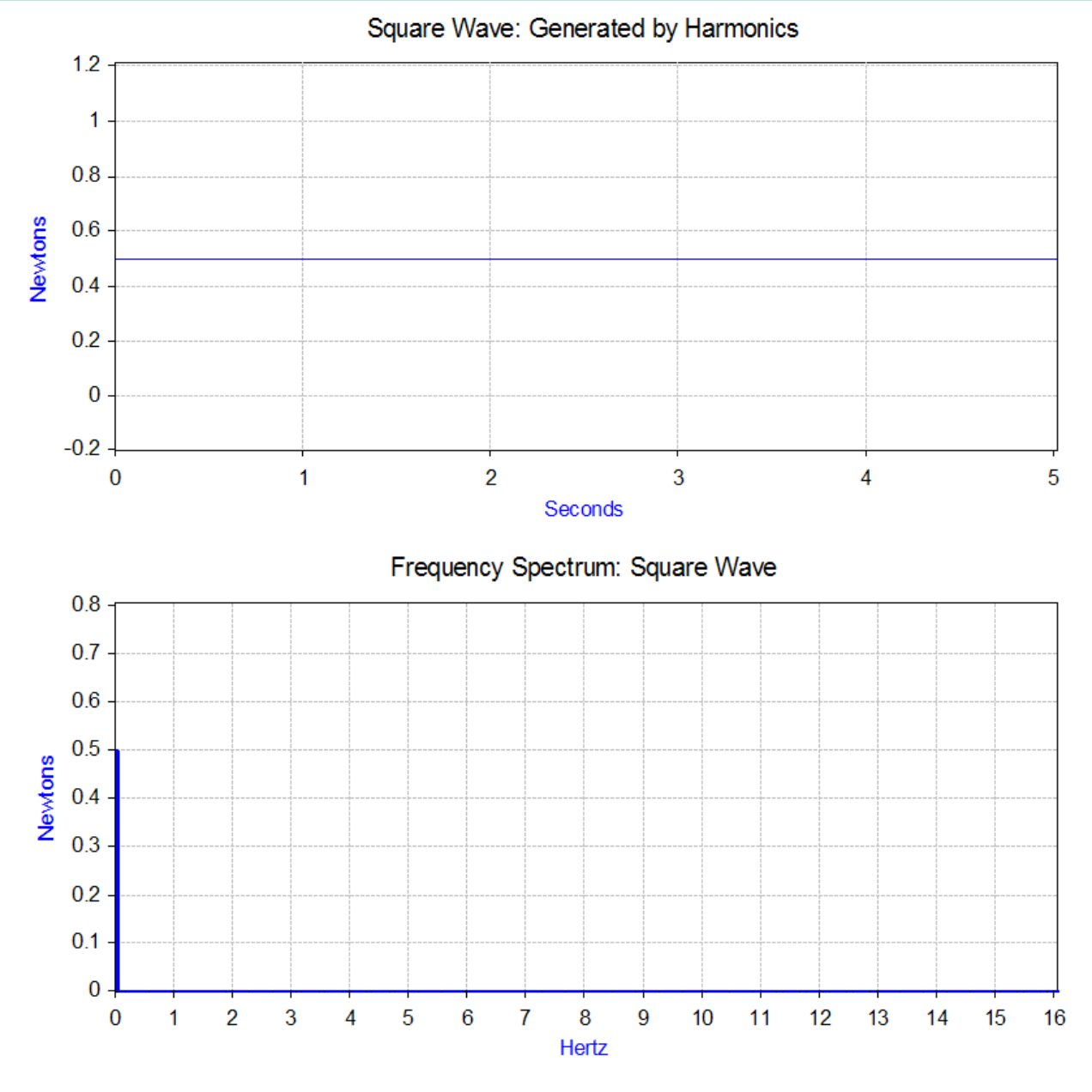
Fourier Series

任一週期函數可以寫成一系列正弦函數的疊加。

$$y(t) = \sum_{n=1}^{\infty} a_n \sin(n\omega_1 \cdot t)$$



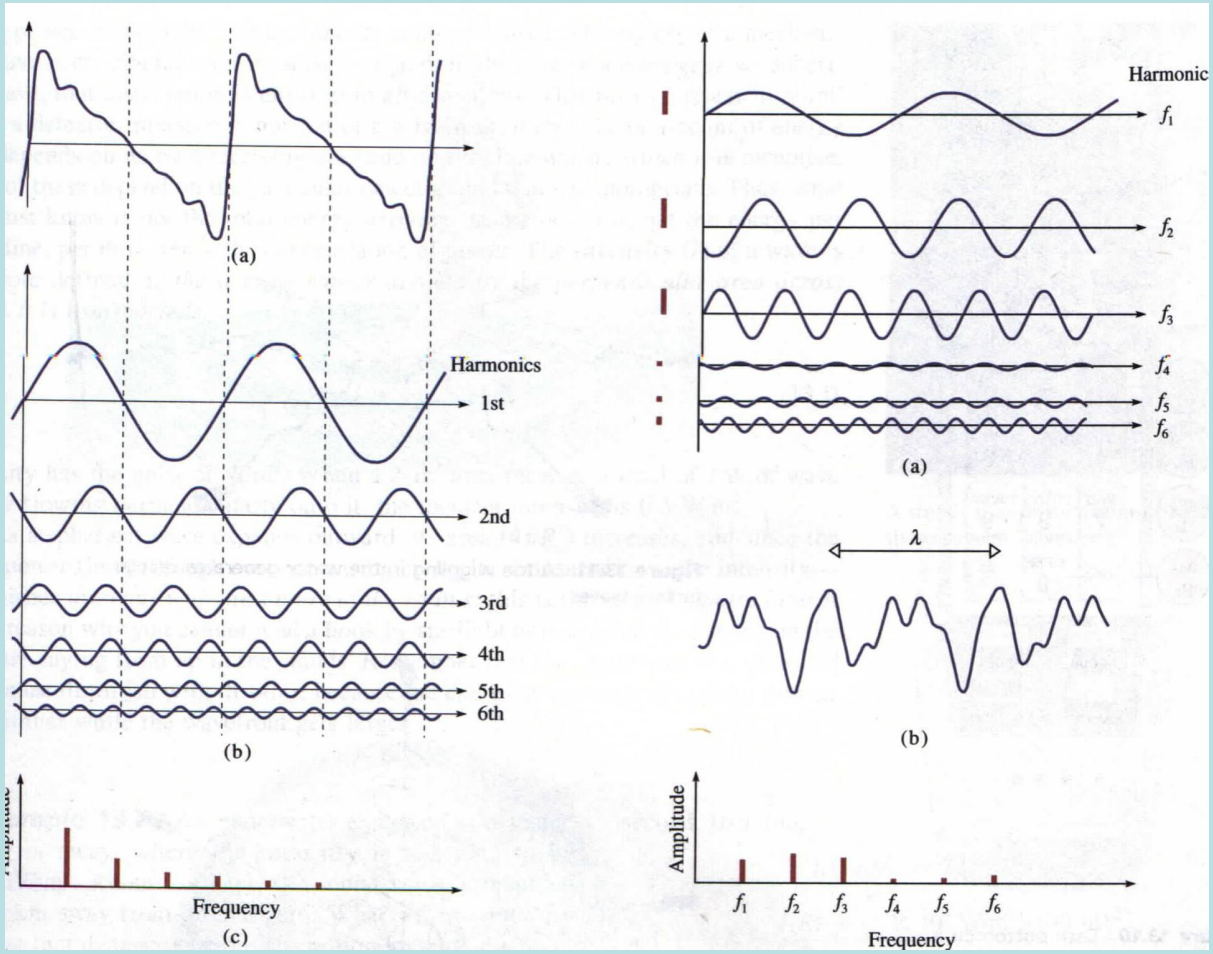

Square Wave: Generated by Harmonics
Newtons
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
Seconds
Frequency Spectrum: Square Wave
Newtons
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Hertz

所有週期震盪可以分解為簡諧震盪的疊加：

$$y(t) = \sum_{n=1}^{\infty} a_n \sin(n\omega_1 \cdot t)$$

任何週期波可以分解為正弦波的疊加！

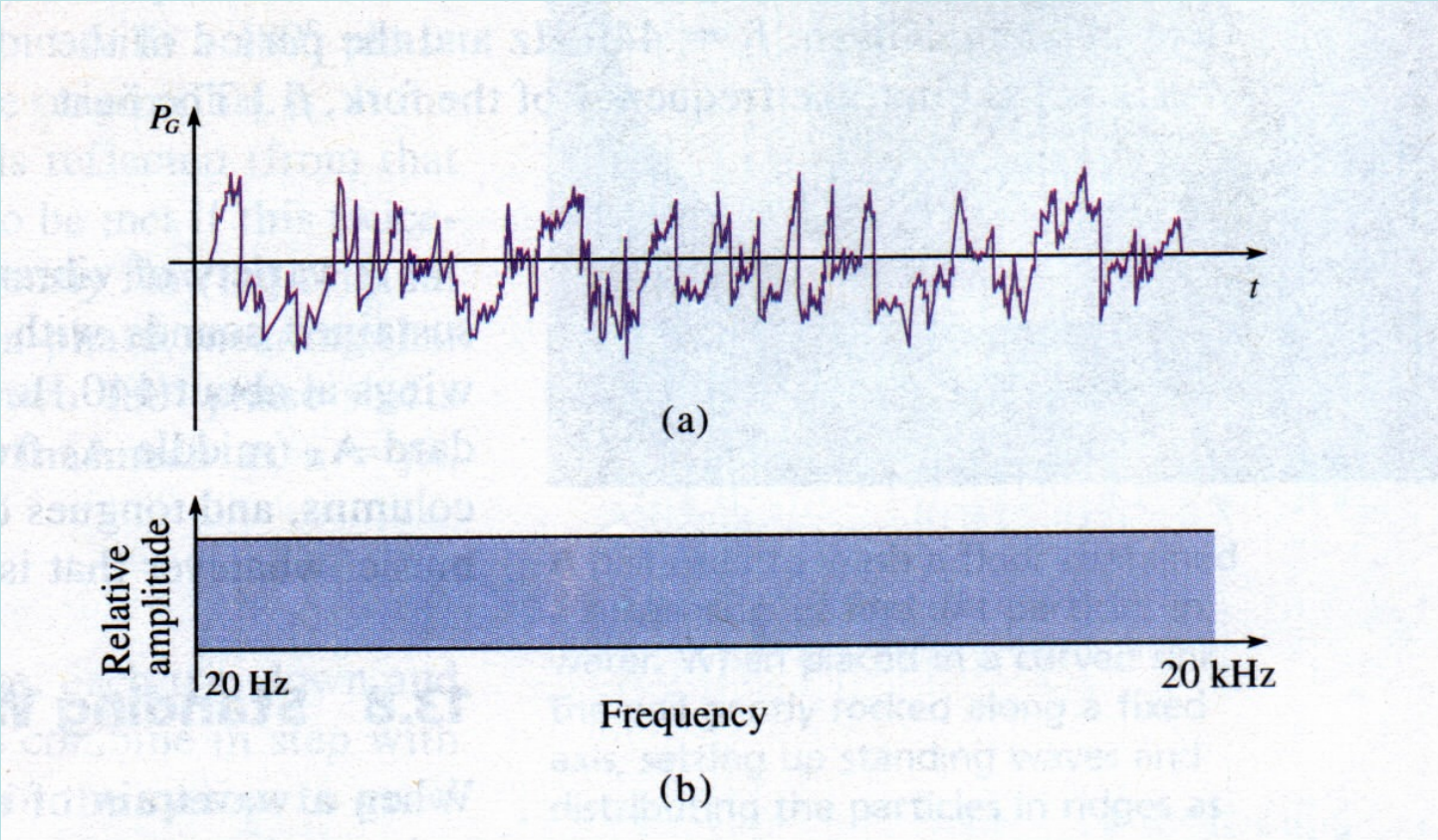
$$y = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\omega_1}{v} x - n\omega_1 \cdot t\right)$$



Fourier Analysis

整個波函數就與這個無限數列$\{a_n\}$完全對等！

White Noise

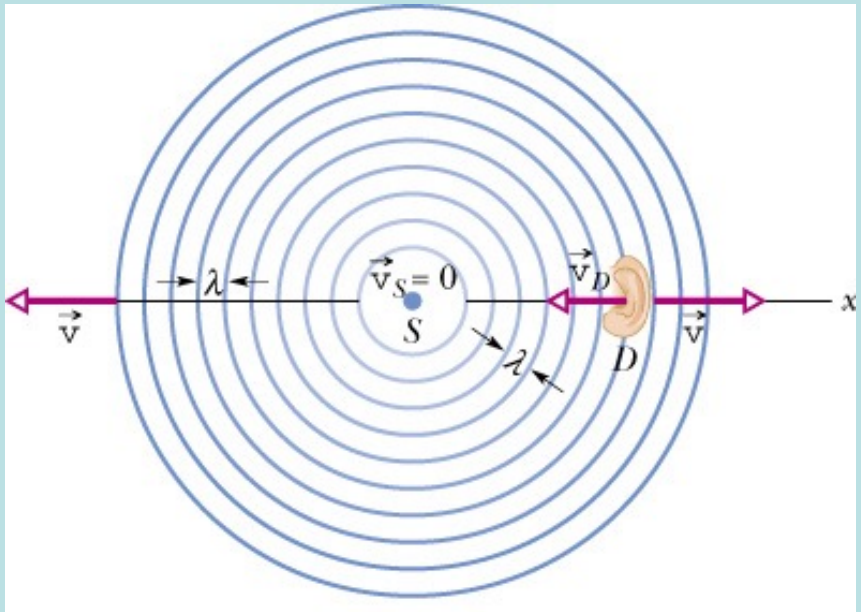


Doppler Effect

Detector moving

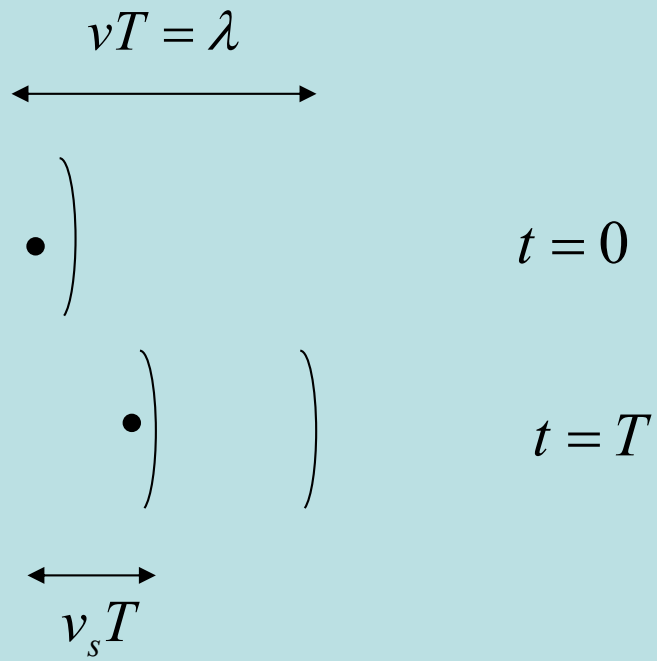
速度變化 $v \rightarrow v + v_D$

$$f = \frac{v}{\lambda} \rightarrow f' = \frac{v + v_D}{\lambda} = \frac{v + v_D}{v} \times \frac{v}{\lambda} = \frac{v + v_D}{v} f$$



$$f' = \frac{v \pm v_D}{v} f$$

Doppler Effect

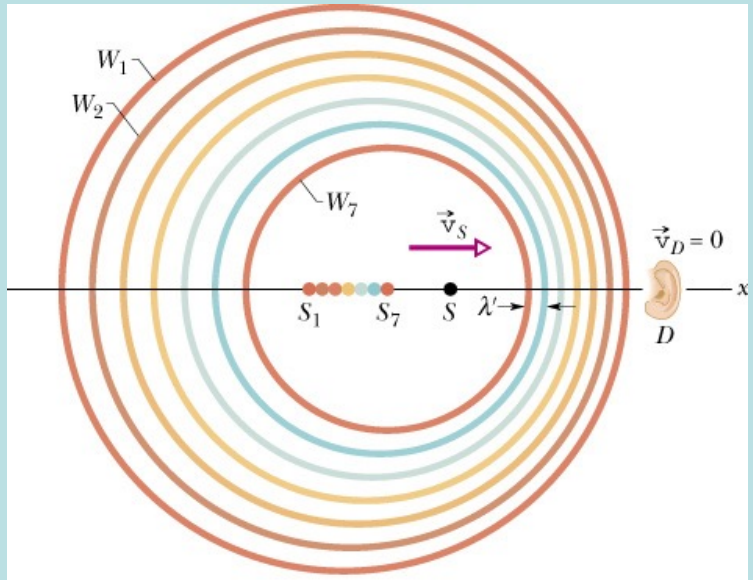


Source moving

波長變化 $\lambda = vT \rightarrow \lambda' = (vT - v_s T) = \frac{v - v_s}{v}\lambda$

$$f = \frac{v}{\lambda} \rightarrow f' = \frac{v}{\lambda'} = \frac{v}{v - v_s} \times \frac{v}{\lambda} = \frac{v}{v - v_s} \times f$$

$$f' = \frac{v}{v \mp v_s} \times f$$



Both Moving

$$f' = \frac{v \pm v_D}{v \mp v_s} \times f$$

$$v_{D,s} << v$$

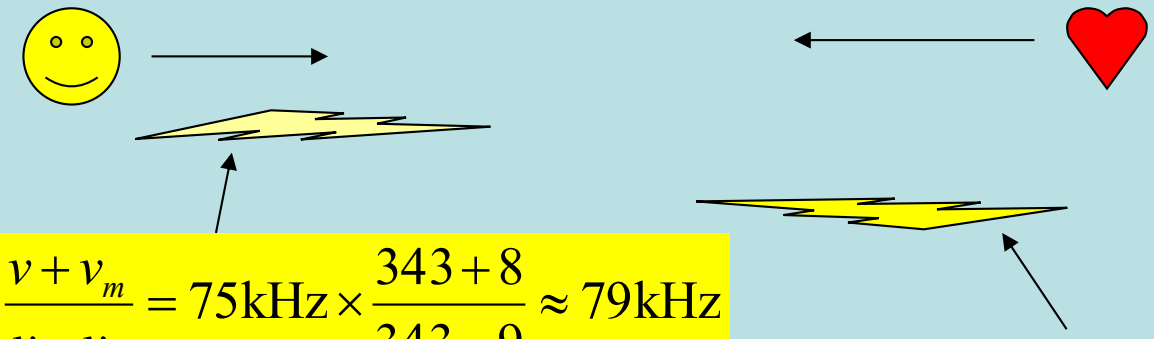
$$f' = \frac{v \pm v_D}{v \mp v_s} \times f = \frac{1 \pm \frac{v_D}{v}}{1 \mp \frac{v_s}{v}} \times f \approx \left(1 \pm \frac{v_D}{v} \pm \frac{v_s}{v}\right) \times f = f\left(1 \pm \frac{u}{v}\right)$$

u 是相對速度

此式與介質無關，對電磁波反而是正確的式子！

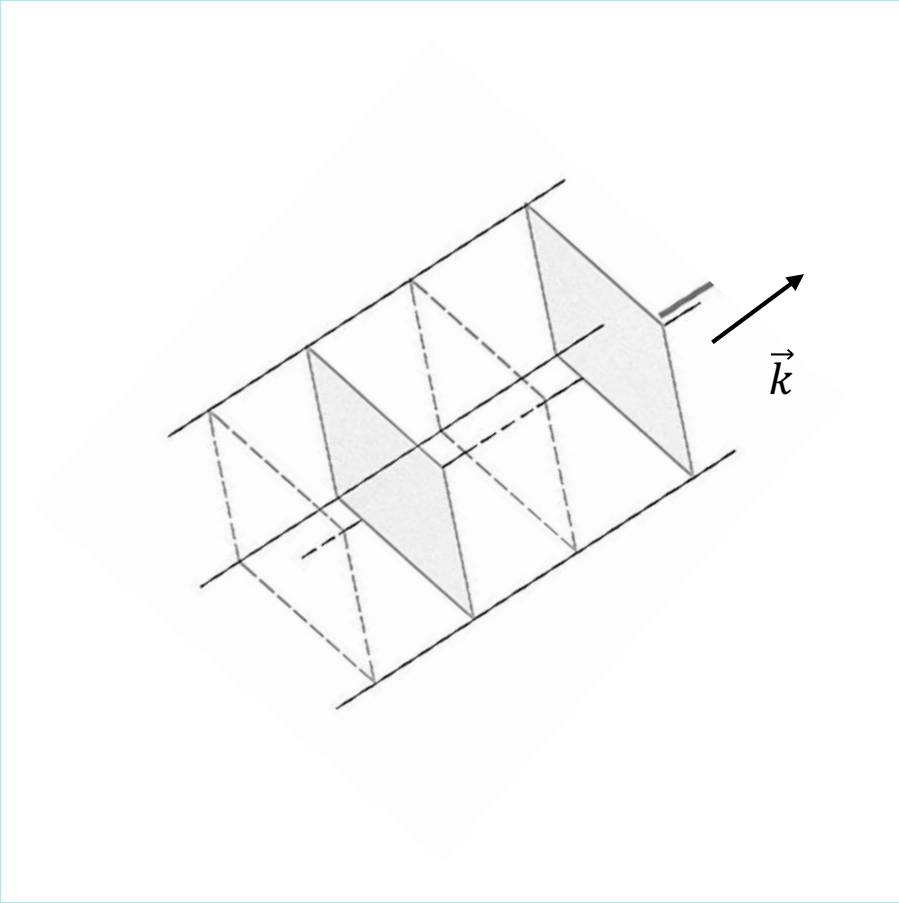
Bat

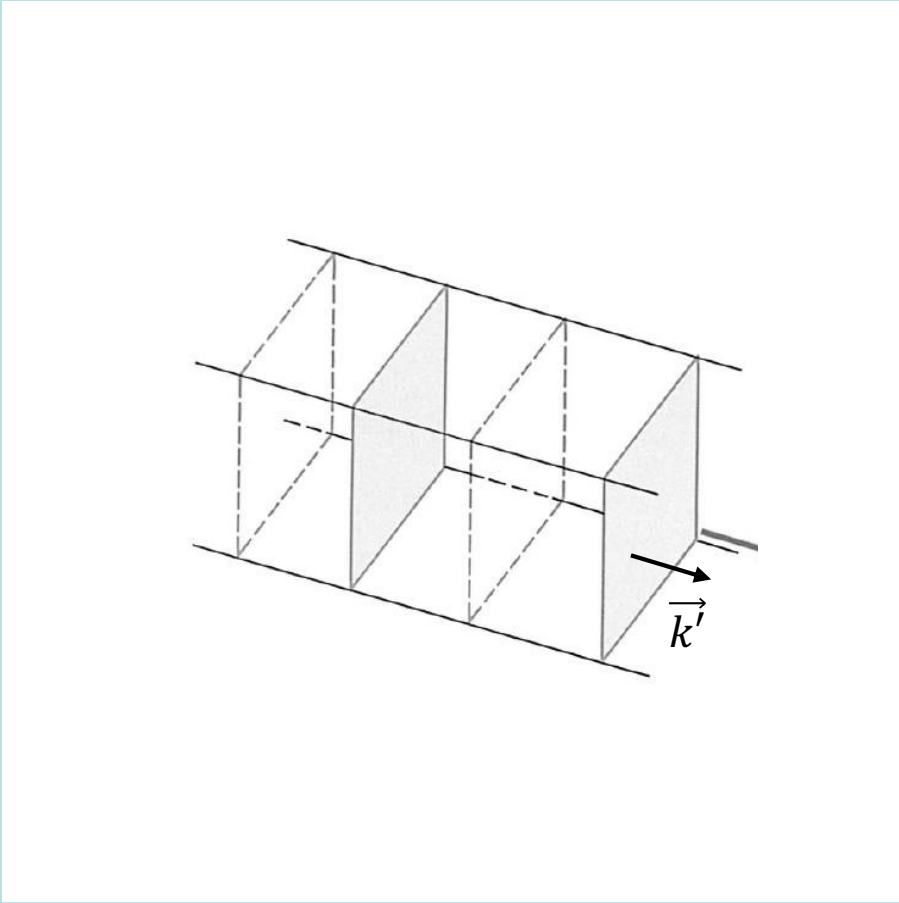
Bat



$$f_m = f_b \frac{v + v_m}{v - v_b} = 75\text{kHz} \times \frac{343 + 8}{343 - 9} \approx 79\text{kHz}$$

$$f'_b = f_m \frac{v + v_b}{v - v_m} = 79\text{kHz} \times \frac{343 + 9}{343 - 8} \approx 83\text{kHz}$$



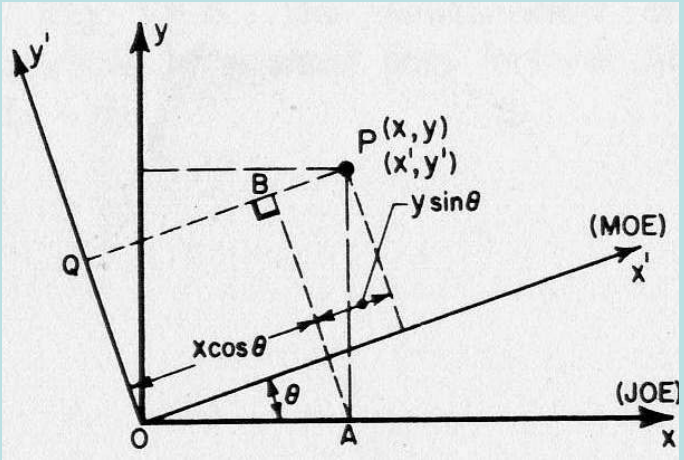


平面波性質與一維的波非常類似！這並不意外。

$$s(\vec{r},t) = s_m \cdot \sin\left(\vec{k}\cdot\vec{r} - \omega t\right)$$

如果重新選x'軸為沿$\vec{k}$的方向，那麼 $\vec{k}\cdot\vec{r} = kx'$

$s \to s_m \cdot \sin(kx' - \omega t)$ 在此新座標中，此平面波就是一維的波。



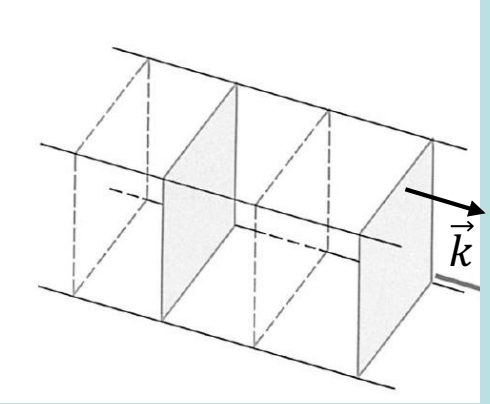
$$x'^2 + y'^2 + z'^2 = x^2 + y^2 + z^2$$

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{\partial^2 s}{\partial x'^2} + \frac{\partial^2 s}{\partial y'^2} + \frac{\partial^2 s}{\partial z'^2}$$

波方程式中波函數的二次微分和與向量內積非常類似！因此是一純量。

一個空間函數$U(\vec{r})$的空間微分和是不是一個向量？

$$\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) \overset{?}{\rightarrow} \text{a vector}$$



如果重新選x'軸為沿$\vec{k}$的方向，那麼 $\vec{k}\cdot\vec{r} = kx'$

$s \to s_m \cdot \sin(kx' - \omega t)$ 在此新座標中，此平面波就是一維的波。

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2} \quad \Rightarrow \quad \frac{\partial^2 s}{\partial x'^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2} = \frac{\partial^2 s}{\partial x'^2} + \frac{\partial^2 s}{\partial y'^2} + \frac{\partial^2 s}{\partial z'^2}$$

這個結果暗示波方程式在新座標與舊座標長得一模一樣！

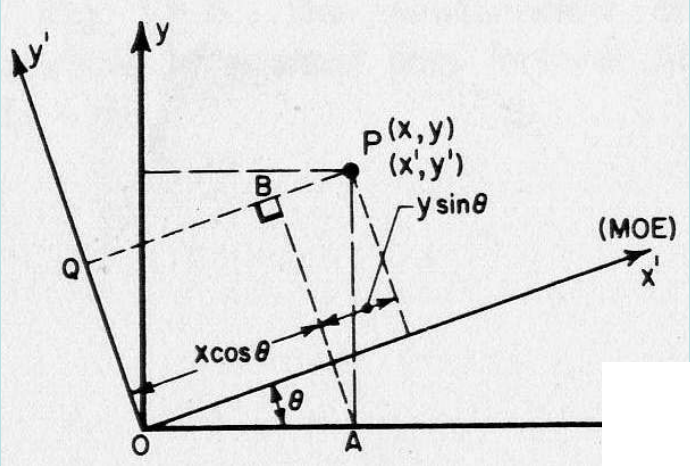
$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$ 是沒有方向性，與座標軸選擇無關！是一個純量！

介質空氣是均勻同向的，因此波方程式與方向無關是很自然的！

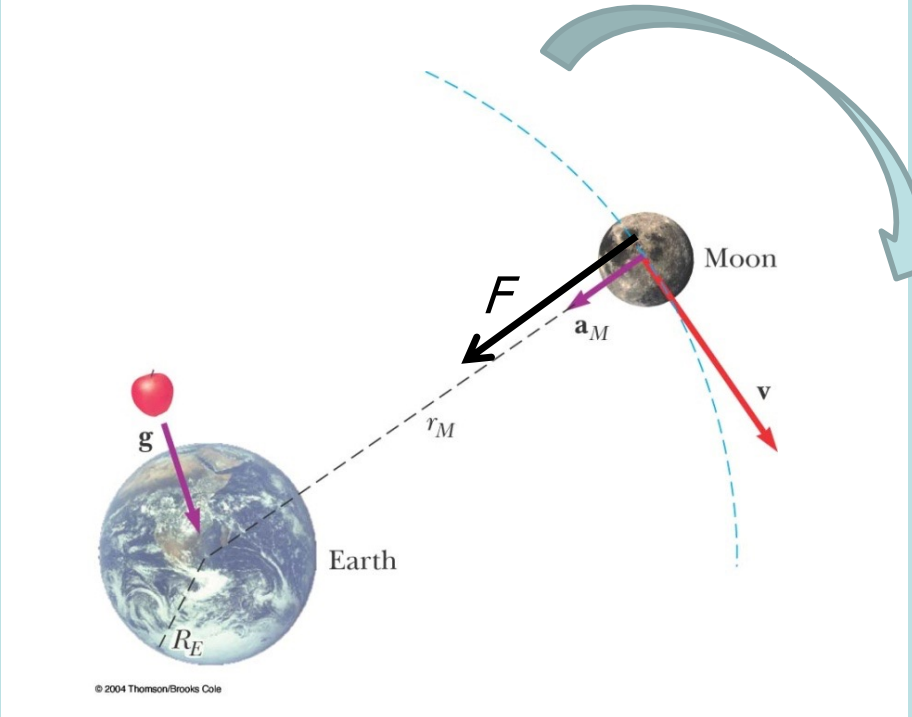
座標軸就可以自由選取！於是三維平面波就與一維波無兩樣。

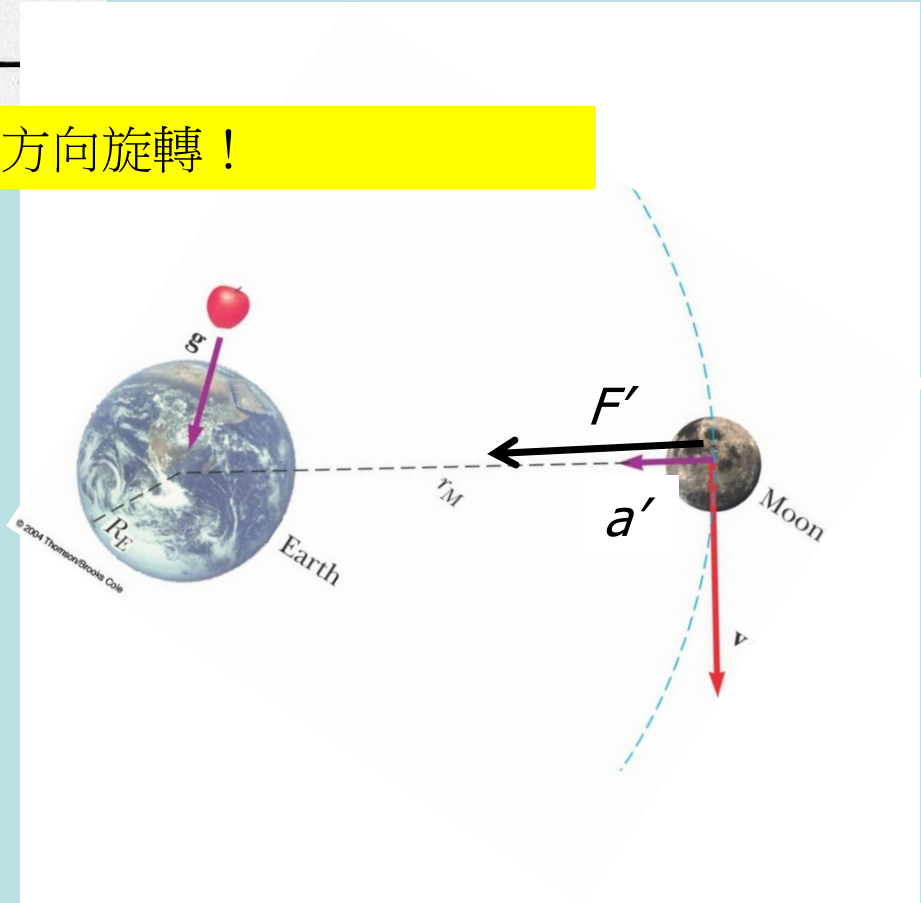
如果物理是與方向無關，座標軸就可以自由選取！

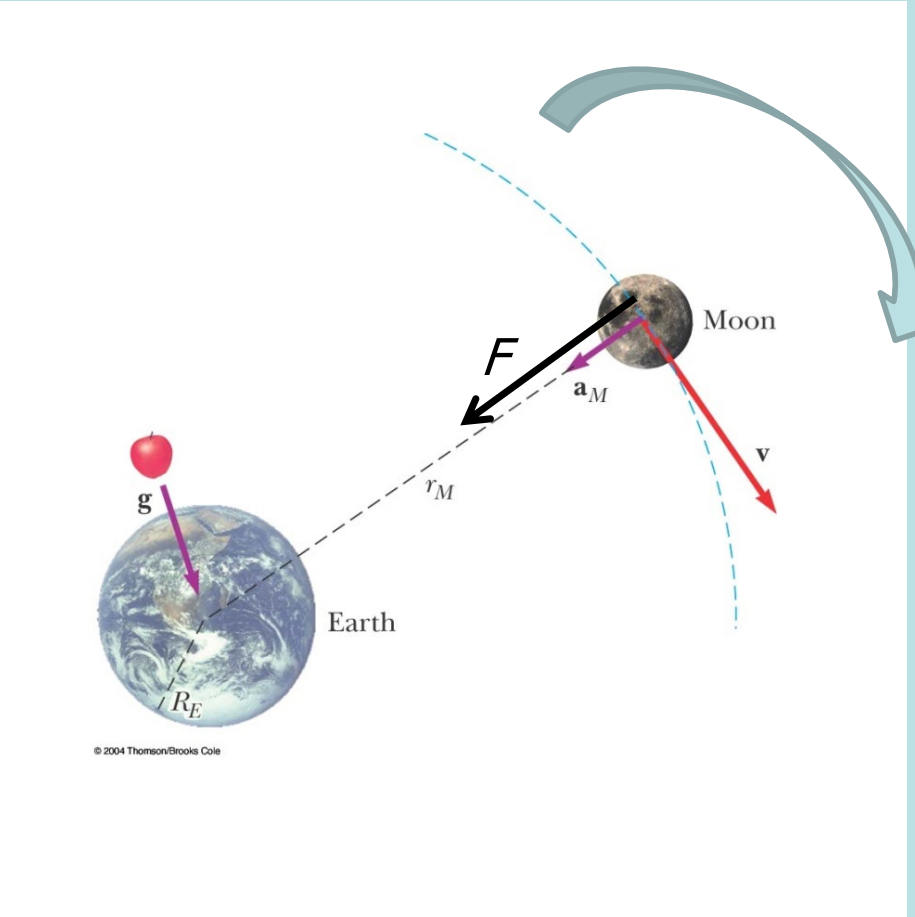
選擇新的座標軸後，位置座標的值會改變，稱為旋轉變換。

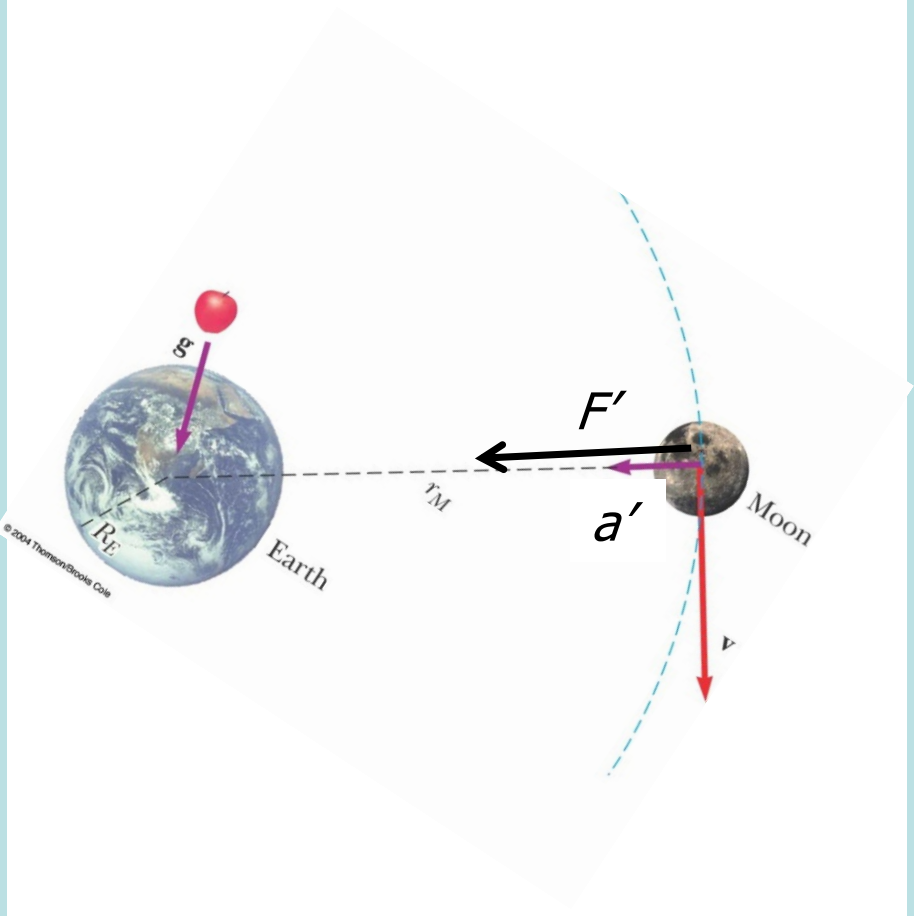


旋轉座標軸相當於將整個系統往反方向旋轉！









如果無法以物理方法分辨旋轉變換前後，旋轉前後的物理定律必須一樣！

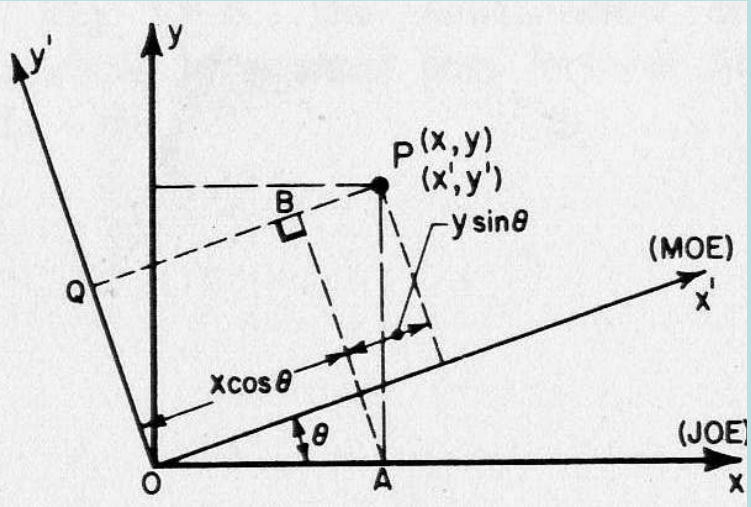
$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=\frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial x'^2}+\frac{\partial^2 s}{\partial y'^2}+\frac{\partial^2 s}{\partial z'^2}=\frac{\partial^2 s}{\partial t^2}$$

$$\vec{F}=m\vec{a}$$

$$\vec{F}'=m\vec{a}'$$

選擇新的座標軸後，位置座標的值會改變，稱為旋轉變換。



$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

所有的向量，分量在座標軸旋轉下都要滿足一樣的變換式：

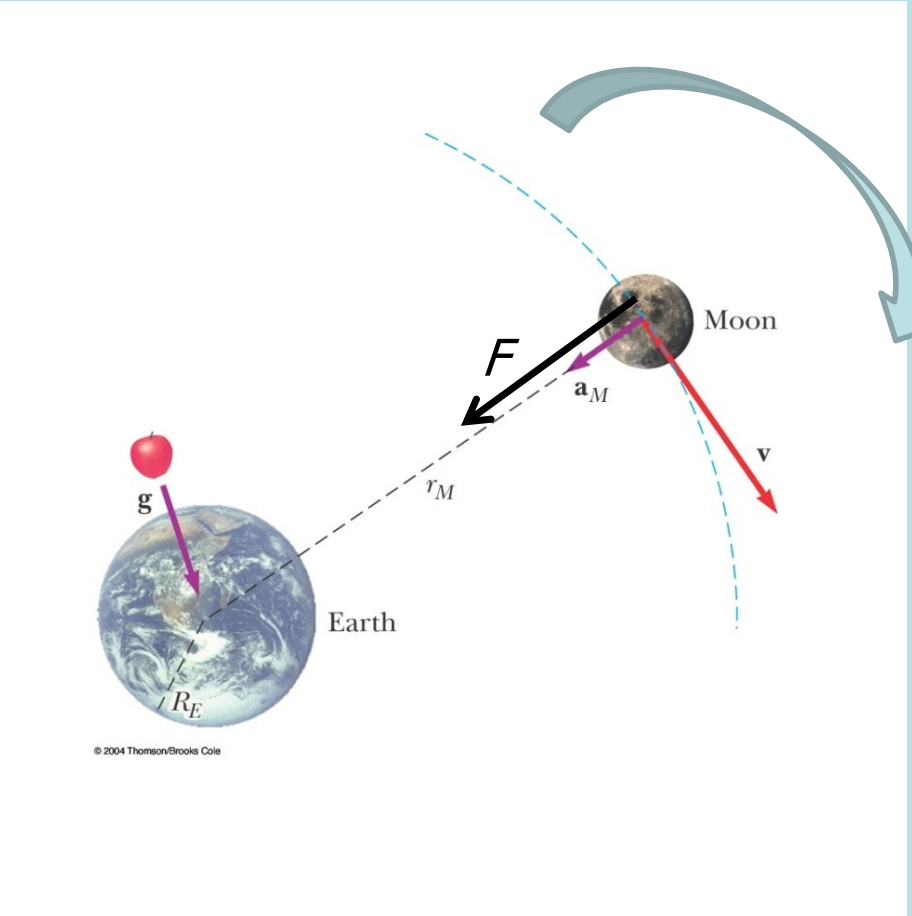
$$F_x' = F_x\cos\theta - F_y\sin\theta \qquad a_x' = a_x\cos\theta - a_y\sin\theta$$
$$F_y' = F_x\sin\theta + F_y\cos\theta \qquad a_y' = a_x\sin\theta + a_y\cos\theta$$

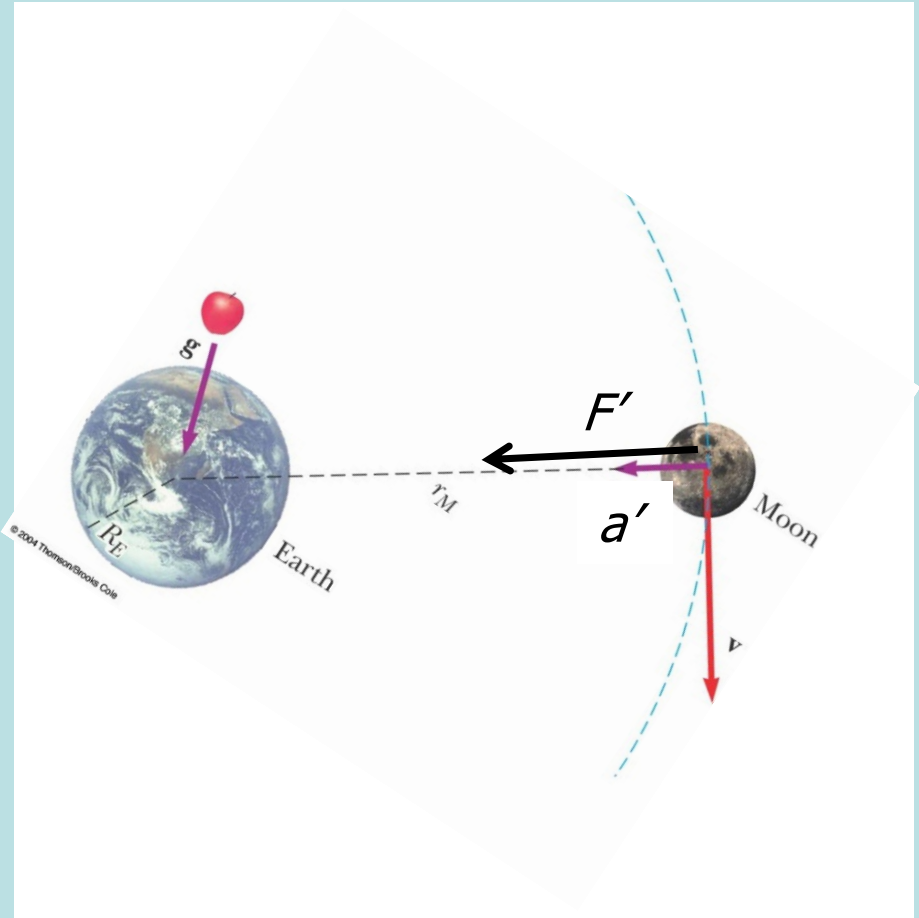
而純量是不變的：

$$x'^2 + y'^2 = x^2 + y^2$$

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{\partial^2 s}{\partial x'^2} + \frac{\partial^2 s}{\partial y'^2} + \frac{\partial^2 s}{\partial z'^2}$$

簡言之，在旋轉變化下如何變換，就定義了一個物理量究竟是純量向量或張量。





如果無法以物理方法分辨旋轉變換前後，旋轉前後的物理定律必須一樣！

$$\frac{\partial^2 s}{\partial x^2}+\frac{\partial^2 s}{\partial y^2}+\frac{\partial^2 s}{\partial z^2}=\frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial x'^2}+\frac{\partial^2 s}{\partial y'^2}+\frac{\partial^2 s}{\partial z'^2}=\frac{\partial^2 s}{\partial t^2}$$

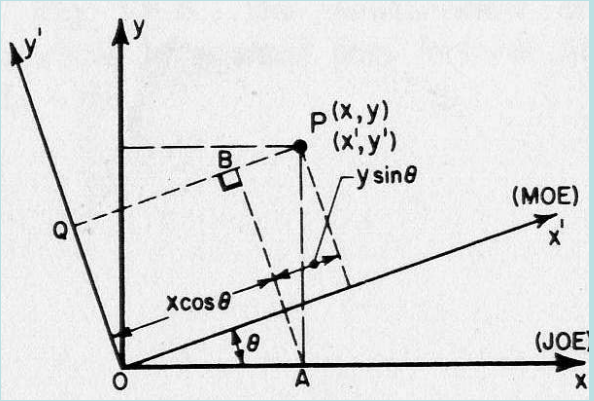
$$\vec{F}=m\vec{a}$$

$$\vec{F}'=m\vec{a}'$$

如何保證旋轉變換後，物理定律可以不變？

$$\vec{F} = m\vec{a} \quad \Longrightarrow \quad \vec{F}' \overset{?}{=} m\vec{a}'$$

注意等式兩邊都是向量。此定律可以寫成分量等式：



$$F_x = ma_x \qquad F_y = ma_y$$

$$x' = x\cos\theta - y\sin\theta,$$
$$y' = x\sin\theta + y\cos\theta$$

所有的向量分量在座標軸旋轉下滿足一樣的變換式：

$$F_x' = F_x\cos\theta - F_y\sin\theta \qquad a_x' = a_x\cos\theta - a_y\sin\theta$$
$$F_y' = F_x\sin\theta + F_y\cos\theta \qquad a_y' = a_x\sin\theta + a_y\cos\theta$$

注意分量等式的左右滿足同樣的轉換關係

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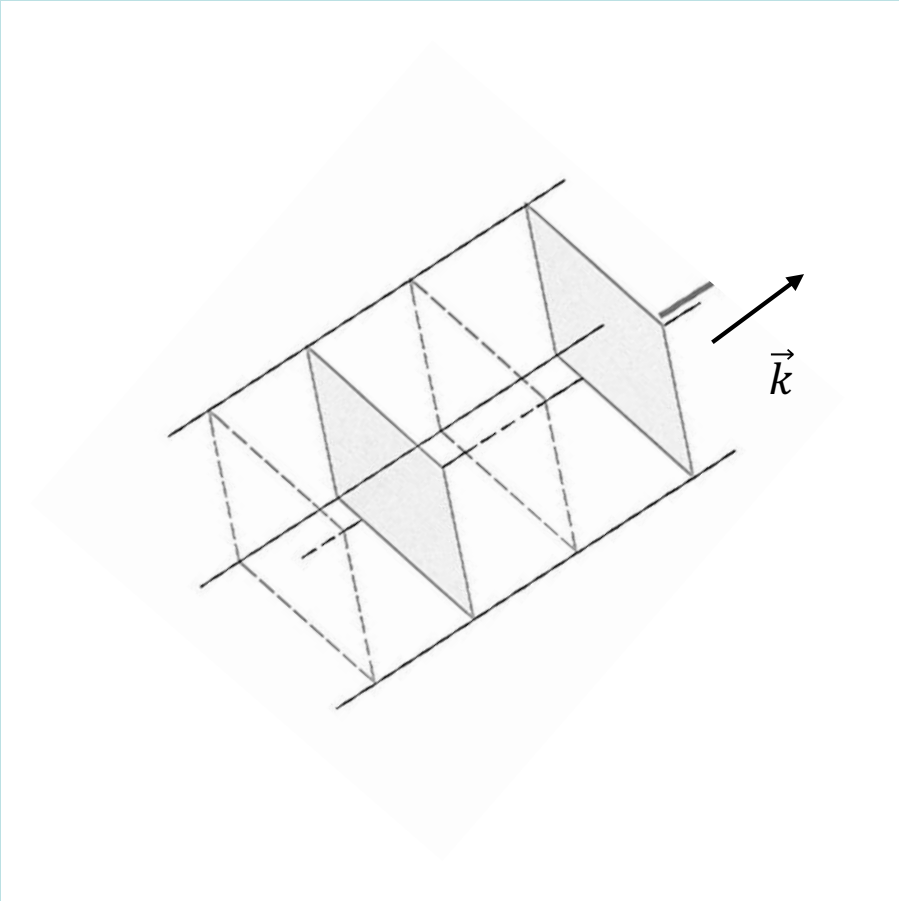
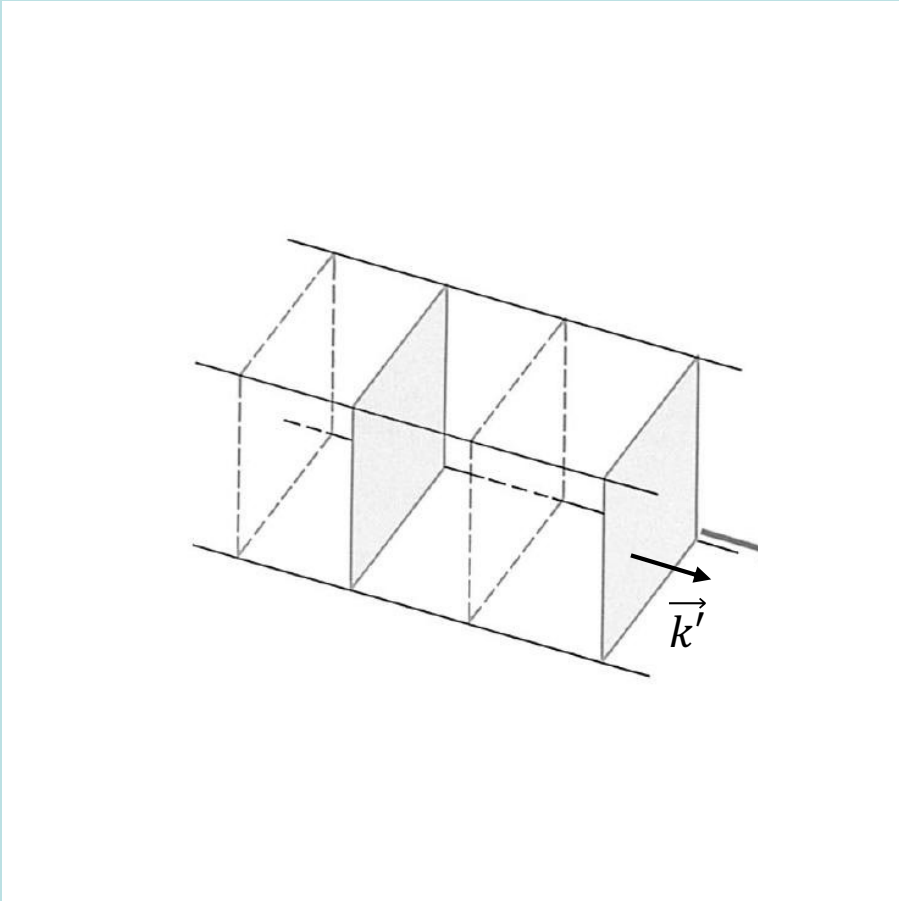
$$F_x = ma_x \quad F_y = ma_y \quad \Longrightarrow \quad F_x' = ma_x' \quad F_y' = ma_y'$$

$$\vec{F} = m\vec{a} \quad \Longrightarrow \quad \vec{F}' = m\vec{a}'$$

要求等式兩邊都是向量，就保證公式不隨座標軸變換而改變！





旋轉變換後，物理定律必須不變

$$\vec{F} = m\vec{a}$$

$$\vec{F}' = m\vec{a}'$$

要求物理定律等式兩邊都是純量、或向量、或張量，就保證公式在旋轉變換下不變

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial x'^2} + \frac{\partial^2 s}{\partial y'^2} + \frac{\partial^2 s}{\partial z'^2} = \frac{\partial^2 s}{\partial t^2}$$