Modern Physics Final

June 2023

1. Consider an infinite potential as discussed in class, with boundaries at and : and .



The energy eigenfunctions are known to be:

with eigenvalues: .

Put electrons into the potential. Assume we keep the system at .

1. Calculate the Fermi Energy . Remember electrons could be either spin up or down. (10)
2. Calculate the total energy . Note that it would be a function of . (10)
3. When the size is changed by a negative , does the total energy increase or decrease? Calculate the force of the system on whatever is changing the size , defined as . This is the equation of state of the system. (5)

Sol:

1. is the highest energy electrons will occupy. In 1D, there is only one quantum number and it rises with energy. Electron will occupy starting from to Note that a state could be occupied by two electrons spin up and down. Hence the energy of the state is .
2. The total energy would equal the sum of energies of states from to .
3. . It is more like the pressure of ideal gas than a restoring force of a spring.
4. Continue with the infinite potential well in problem 1. But now add only two electrons in the well. Denote the coordinates of the two electrons as . Use up and down arrows to denote the z component spins of the two electrons: for example, is the state with z-spin up for electron 1, z-spin down for electron 2.
5. What is the wavefunction of the ground state? You need to write down both the space part and the spin part. (10)
6. The first excited states contain two possibilities. The spin part could be symmetric, let’s call it spin-S, or antisymmetric, let’s call it spin-A. What is the wavefunction of the spin-A first excited state? There are 3 spin-S first excited states. What are the wavefunctions of the spin-S first excited states? (15)
7. Continue with the first excited states, assume the two electrons have a repulsive coulomb potential between them. Treat it as a perturbation. After considering the first order energy correction, which state S or A, will have lower energy? Why? Hint: In which states will the two electrons stay closer together? (5)

Sol:

1. The energy of eigenstates dependent on only the space part. The ground state would be for both electrons to be in , ie (1,1). The space part of the wavefunction is:

This space part wavefunction can only be symmetric. Hence to get a overall antisymmetric wavefunction, the spin part has to be antisymmetric. That could only be:

The whole wavefunction is hence:

1. The space part of the first excited state could be

or

Hence They could be symmetrized S or anti-symmetrized A.

Spin-A is antisymmetric in spin, and hence space part needs to be symmetric:

Spin-S is symmetric in spin and there are 3 of them. The space part needs to be anti-symmetric:

1. The first order energy correction is the expectation value of the perturbation.

The perturbation is the coulomb interaction. For A space part wavefunction, there is a smaller probability for the two electrons to have close coordinates. Hence the expectation value of the positive Coulomb interaction is smaller. Hence A states has lower energy.

1. 反應為一衰變，寫下原子核的質量數與質子數。此衰變發生後，因與質量接近，且遠大於氦原子核，可以近似假設衰變後鈾原子核是靜止的，計算粒子的動能（能量減去靜止能量）是多少。(20) 三個原子核的質量分別是，，，1 u

解答：。

能量守恆，因此，質量減少對應的能量都成為動能：。

1. 基本粒子的衰變是二體衰變：。如果是這樣，如上課所解釋的：那麼靜止的子所放出的緲子與微中子都有一個固定的能量。計算在子靜止的情況下，所放射出的微中子的能量。(25)

微中子可以視為無質量的粒子，答案可以表示，與上一題不同的是，在這個計算中，因為緲子與子質量有相當差距，緲子不能看成是靜止的。

提示：

代入真實的值：MeV，MeV，計算的能量是多少MeV。(20)

解答：因為子是靜止的，動量守恆給出：，。而因電子的質量可以忽略：。如此質子的能量也可以以電子能量表示：。根據能量守恆：，代入後兩邊平方：。MeV。