

習題一

1. Calculate $\frac{d}{dx}(1+2x)^3(8x^5+6x^7)^4$.

$$\begin{aligned}\text{Sol: By chain rule: } & (8x^5+6x^7)^4 \frac{d}{dx}(1+2x)^3 + (1+2x)^3 \frac{d}{dx}(8x^5+6x^7)^4 \\ &= (8x^5+6x^7)^4 3(1+2x)^2 \frac{d}{dx}(1+2x) + (1+2x)^3 4(8x^5+6x^7)^3 \frac{d}{dx}(8x^5+6x^7) \\ &= 6(8x^5+6x^7)^4(1+2x)^2 + (1+2x)^3 4(8x^5+6x^7)^3(40x^4+42x^6) \\ &= (8x^5+6x^7)^3(1+2x)^2[6(8x^5+6x^7) + 4(40x^4+42x^6)(1+2x)] \\ &= (8x^5+6x^7)^3(1+2x)^2[372x^7+168x^6+368x^5+160x^4]\end{aligned}$$

2. Calculate $\frac{d}{dx} \frac{1}{(1+2x)^3}$.

$$\text{Sol: } \frac{d}{dx} \frac{1}{(1+2x)^3} = -\frac{1}{(1+2x)^6} \frac{d}{dx}(1+2x)^3 = -\frac{3(1+2x)^2 \cdot 2}{(1+2x)^6} = -\frac{6}{(1+2x)^4}$$

3. Calculate $\frac{d}{dt}[6\cos(2t+3)]$ and $\frac{d^2}{dt^2}[6\cos(2t+3)]$.

$$\text{Sol: } \frac{d}{dt}[6\cos(2t+3)] = -6\sin(2t+3) \frac{d}{dt}(2t+3) = -12\sin(2t+3).$$

$$\begin{aligned}\frac{d^2}{dt^2}[6\cos(2t+3)] &= -\frac{d}{dt}[12\sin(2t+3)] = -12\cos(2t+3) \frac{d}{dt}(2t+3) = \\ &= -24\cos(2t+3).\end{aligned}$$

4. 考慮一個質量為0.34kg的粒子在二維 $x-y$ 平面上運動，已知它的 x, y 座標與時間的關係分別可以寫成： $x(t) = -15.0 + 2.0t + 4.0t^3$ ， $y(t) = 25.0 + 7.0t - 9.0t^2$ ，式中長度單位為 m，時間為 s。在時間為 $t = 0.7$ s時該粒子的速度與加速度，大小分別是多少？

解答：Differentiating $x(t) = -15.0 + 2.0t + 4.0t^3$ with respect to t , we get

$$v_x = \frac{dx}{dt} = 2.0 + 12.0t^2, a_x = \frac{dv_x}{dt} = 24.0t$$

Similarly, differentiating $y(t) = 25.0 + 7.0t - 9.0t^2$ twice with respect to t yields

$$v_y = \frac{dy}{dt} = 7.0 - 18.0t, a_y = \frac{dv_y}{dt} = -18.0$$

At $t = 0.7\text{s}$, we have $\vec{v} = (7.9, -5.6)$, $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = 9.7$.

$\vec{a} = (16.8, -18.0)$, $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2} = 24.6..$