Electromagnetic response of Weyl semimetal

A. Landau levels in magnetic field

B. Weyl orbit

C. Chiral anomaly

D. Chiral magnetic effect

E. Negative magnetoresistance

Weyl semimetal in magnetic field

$$\mathsf{H} = \chi v \boldsymbol{\sigma} \cdot \mathbf{p}. \qquad \chi = \pm$$

+B $\quad \vec{p} \rightarrow \vec{\pi} = \vec{p} + e\vec{A}$ $\quad$ Peierls substitution

$$\mathsf{H} = \chi v(\sigma_x \pi_x + \sigma_y \pi_y) + \chi v \sigma_z p_z,$$

$$[\pi_x, \pi_y] = \frac{\hbar e B}{i}.$$

The energy spectrum can be solved with creation and annihilation operator:

$$\begin{cases} a = \frac{1}{\sqrt{2\hbar e B}}(\pi_x - i\pi_y) \\ a^\dagger = \frac{1}{\sqrt{2\hbar e B}}(\pi_x + i\pi_y) \end{cases},$$

$$[a, a^\dagger] = 1.$$

$$\begin{aligned} \mathsf{H} &= \chi\hbar\omega(\sigma_+ a + \sigma_- a^\dagger) + \chi v \sigma_z \hbar k_z \\ &= \chi \begin{pmatrix} v\hbar k_z & \hbar\omega a \\ \hbar\omega a^\dagger & -v\hbar k_z \end{pmatrix}, \end{aligned} \qquad \hbar\omega \equiv v\sqrt{2\hbar e B}$$

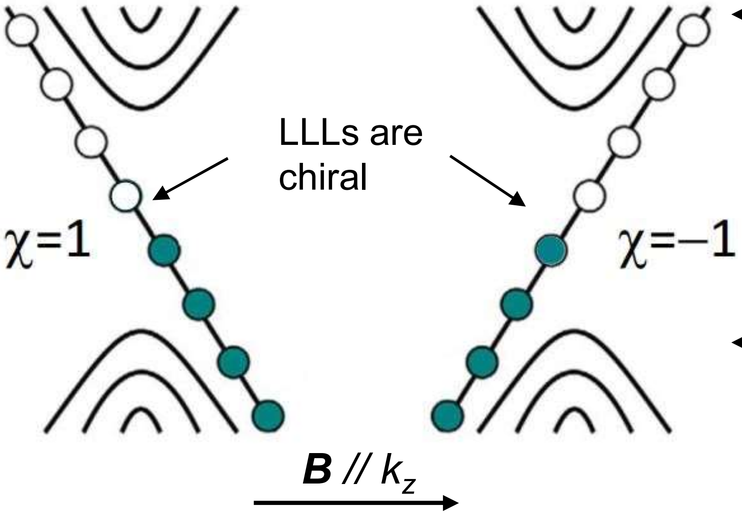
Solving eigenenergy

$$\mathrm{H}\Psi_n = \varepsilon_n \Psi_n,$$

$$\text{with } \Psi_n = u_n \begin{pmatrix} 1 \\ 0 \end{pmatrix} |n-1\rangle + v_n \begin{pmatrix} 0 \\ 1 \end{pmatrix} |n\rangle$$

$$\begin{cases} \varepsilon_0^\chi = -\chi v \hbar k_z. \\ \varepsilon_{n\pm}^\chi = \pm\chi\hbar\omega\sqrt{n+(vk_z/\omega)^2}, n \geq 1. \end{cases}$$

n=0: the LLL is chiral



- LLs disperse along ***B*** only

(electron cannot transport ⊥ ***B***)

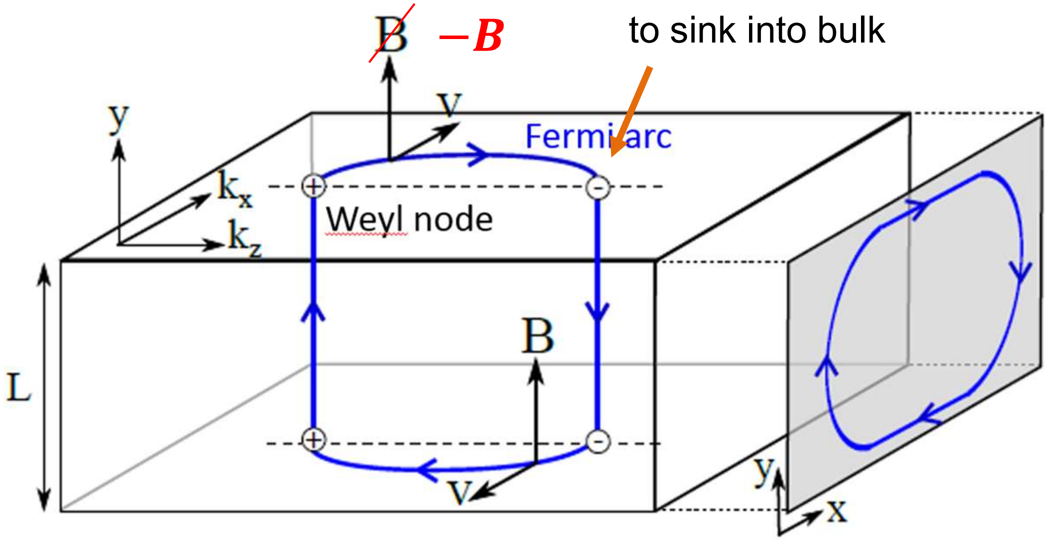
- Each LL is highly degenerate

(Kittel, chap 9)

degeneracy $D = \frac{\phi_{samp}}{\phi_0}$, $\phi_0 \equiv \frac{h}{e}$ Flux quantum

Weyl orbit

Coupling between SS and bulk state causes electrons to sink into bulk



$$T = 2t_{arc} + 2t_{bulk}.$$

$$\varepsilon_n = (n+\delta)\frac{h}{T}, \;\; n = 0, 1, 2, \cdots$$

➡ De Haas-van Alphen oscillation

Potter, Kimchi, and Vishnawath, Nat Comm 2014

For experiment, see e.g., Kar et al, Quantum Materials 8:8 (2023)

Dirac equation

$$\begin{pmatrix} \vec{\sigma}\cdot\vec{p} & -m \\ -m & -\vec{\sigma}\cdot\vec{p} \end{pmatrix}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix}\psi_+ \\ \psi_-\end{pmatrix}$$

(Chiral basis used. See, e.g., Itzykson and Zuber)

- Massless limit, decouples to two Weyl equations

$$\vec{\sigma}\cdot\vec{p}\psi_+ = i\frac{\partial\psi_+}{\partial t}$$

$$-\vec{\sigma}\cdot\vec{p}\psi_- = i\frac{\partial\psi_-}{\partial t}$$

- Vector and axial currents

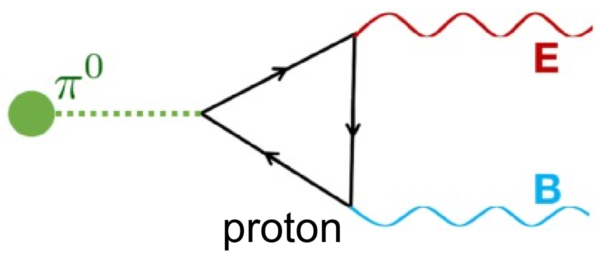
$$\vec{J} \equiv \vec{J}_+ + \vec{J}_- \qquad \vec{J}_A \equiv \vec{J}_+ - \vec{J}_-$$

- Two component are both conserved (because of the chiral symmetry when m=0).

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{J} = 0 \qquad \frac{\partial\rho_A}{\partial t} + \nabla\cdot\vec{J}_A = 0$$

- However, while studying the decay of a neutral pion into 2 photons, ABJ found that (1969)

$$\frac{\partial\rho_\pm}{\partial t} + \nabla\cdot\vec{J}_\pm = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B}$$

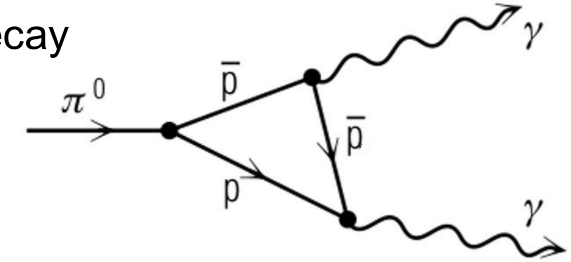


This shows that the (global) chiral symmetry is broken. This is called chiral anomaly or ABJ anomaly 手徵異常

Consequence of the anomaly

- Anomaly-induced neutral pion decay

$$\pi^0 \to \gamma\gamma$$



  - The decay rate does not depend on the mass of the fermion in the loop. Hence the effect survives even if $m_f \to 0$. We can therefore say that the decay is due to chiral anomaly.
  - Later calculation replaces the proton with u and d quarks and still has the same result (as long as quarks have 3 colors).

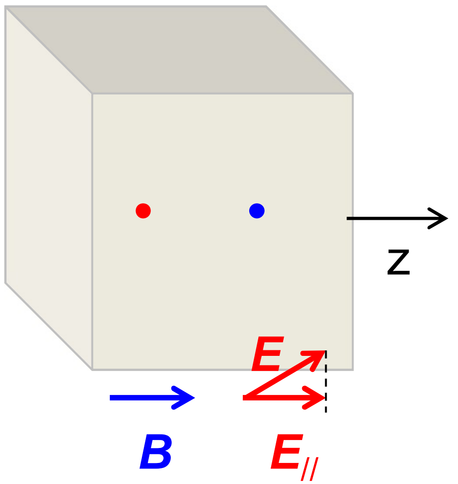
Decay amplitude $\propto (1^2 - 0^2) = 1 \;\Rightarrow\; \left(\left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2\right) N_c = \frac{N_c}{3}$

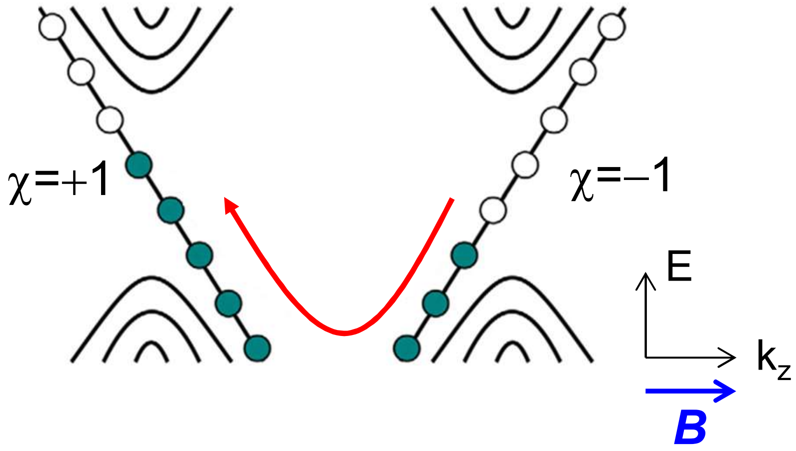
proton neutron → Up down

- A gauge field with local gauge symmetry spoiled by anomaly would not be renormalizable. Therefore, a valid gauge theory, such as the electro-weak theory, must be anomaly-free.

R. Percacci, Non-perturbative Quantum Field Theory

Chiral anomaly of lattice fermions (Nielsen and Ninomiya, Phys Lett B 1983)





Hosur and Qi, Comptes Rendus Phys 2013

1. LL degeneracy $\propto$ *B*
2. LLs disperse only along *B* !
3. 0-th Landau levels are chiral !
4. $E_{//}$ field drives the current

Current $\propto \quad (\vec{E}\cdot\hat{B})B = \vec{E}\cdot\vec{B}$

Chiral charge

$$\frac{d\rho_\pm}{dt} = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B}$$

(next page)

Same as the *chiral anomaly* in particle physics:

Covariant form

$$\partial_\mu j^\mu_\pm = \pm\frac{e^3}{h^2}\frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

Chern-Pontryagin density

- Difference: There is no lattice, no fermion doubling, and the Dirac sea has no bottom there

The simplest way to derive the chiral anomaly

- 1D charge transport

$$\begin{aligned}\frac{dQ_\chi^z}{dt} &= (-e)\chi\frac{\frac{\Delta k_z}{2\pi/L_z}}{\Delta t} \\ &= -e\chi\frac{\dot{k}_z}{2\pi/L_z}, \quad \hbar\dot{k}_z = -eE_z \\ &= e^2\chi\frac{E_\parallel L_z}{h}.\end{aligned}$$

- Degeneracy of a LL

$$D = \frac{\phi_{\rm tot}}{\phi_0} = \frac{A_{\rm samp}B}{h/e}$$

- 3D charge transport

$$\begin{aligned}\frac{dQ_\chi^{3D}}{dt} &= \frac{AB}{h/e}\frac{dQ_\chi^z}{dt} \\ &= \chi\frac{e^3}{h^2}AL_zBE_\parallel\end{aligned}$$

$$\frac{\partial\rho_\chi}{\partial t} = \chi\frac{e^3}{h^2}\mathbf{E}\cdot\mathbf{B}$$

Same result can be obtained without LLs
(Son and Spivak, PRB 2013)

Chiral anomaly and magnetoresistance $\rho(B)$

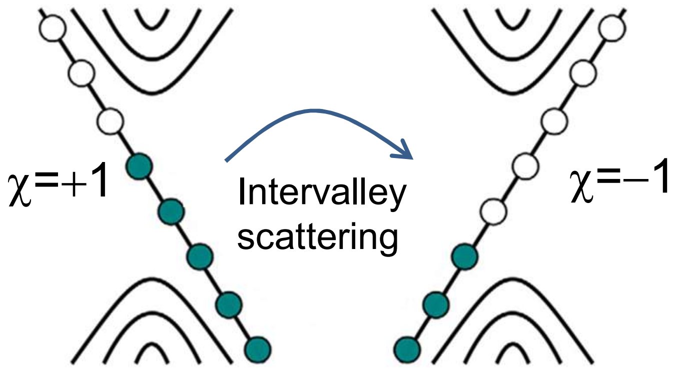
- Chiral anomaly (charge pumping between Weyl nodes)

$$\frac{d\rho_\pm}{dt} = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B} - \frac{\rho_\pm}{\tau_V}$$

Steady state $\rightarrow$ $\rho_\pm = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B}\,\tau_V$

Inter-valley relaxation time

$\rightarrow$ $\Delta\mu \equiv \mu_+ - \mu_- \propto \vec{E}\cdot\vec{B}\,\tau_V$



- Chiral magnetic effect (explained later)

$$\vec{j} = \frac{e^2}{h^2}\Delta\mu\vec{B}$$

- conductivity enhanced by B

$$\rightarrow \quad \vec{j} \propto \left(\vec{E}\cdot\vec{B}\right)\vec{B}\,\tau_V$$

Positive magneto-conductance, or **negative** magneto-resistance

max when $\vec{E}//\vec{B}$ $\quad \sigma(B) = \sigma_0 + \left(\frac{e^2 B}{2\pi}\right)^2 \frac{\tau_V}{g(\varepsilon_F)} \approx B^2$

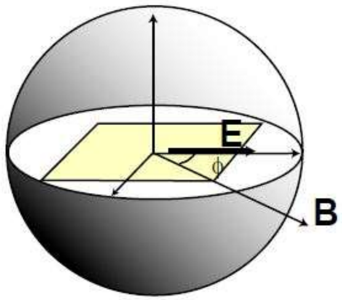
Fukushima, Kharzeev, and Warringa Phys. Rev. D 2008

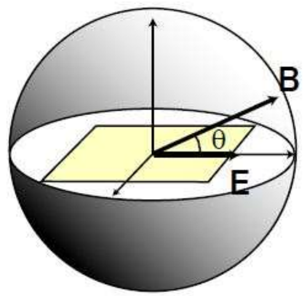
Li et al, Nature Phys 2016

Negative Magneto-Resistance (MR) in Weyl semimetal

- MR is usually positive (i.e. resistivity increases with B)
- An exception is a system with *weak localization*

  (also, a few other materials)



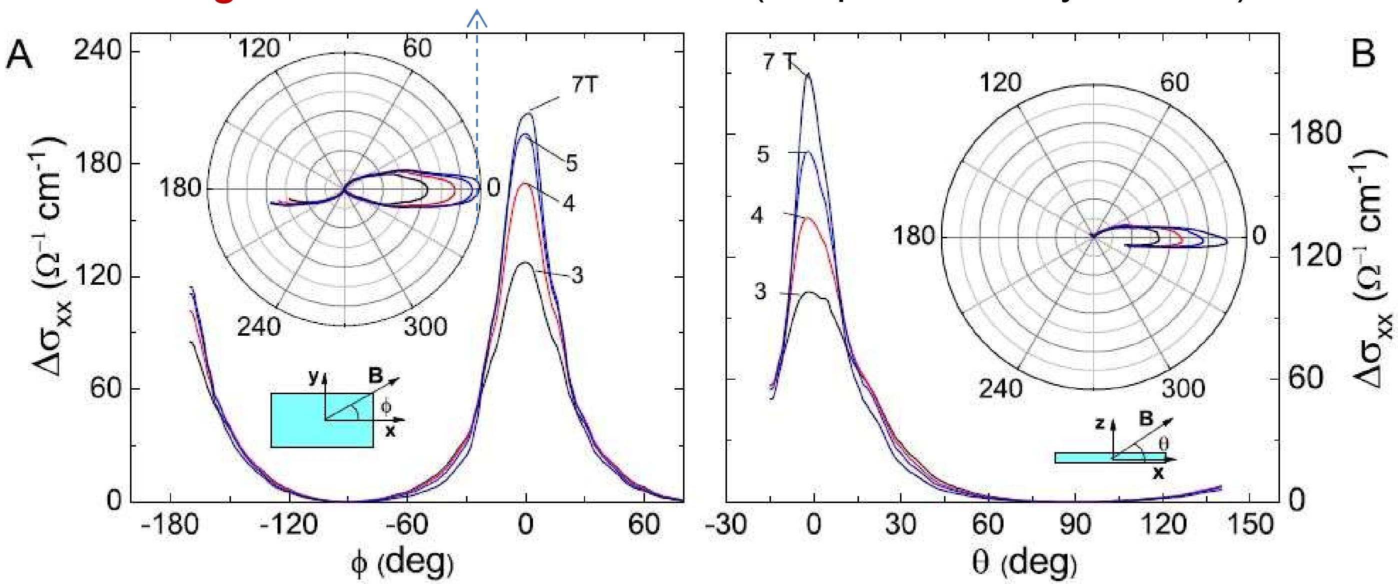


Xiong et al, 1503.08179

Xiong et al, Science express 2015

Na_3Bi
(Dirac node split by B field)

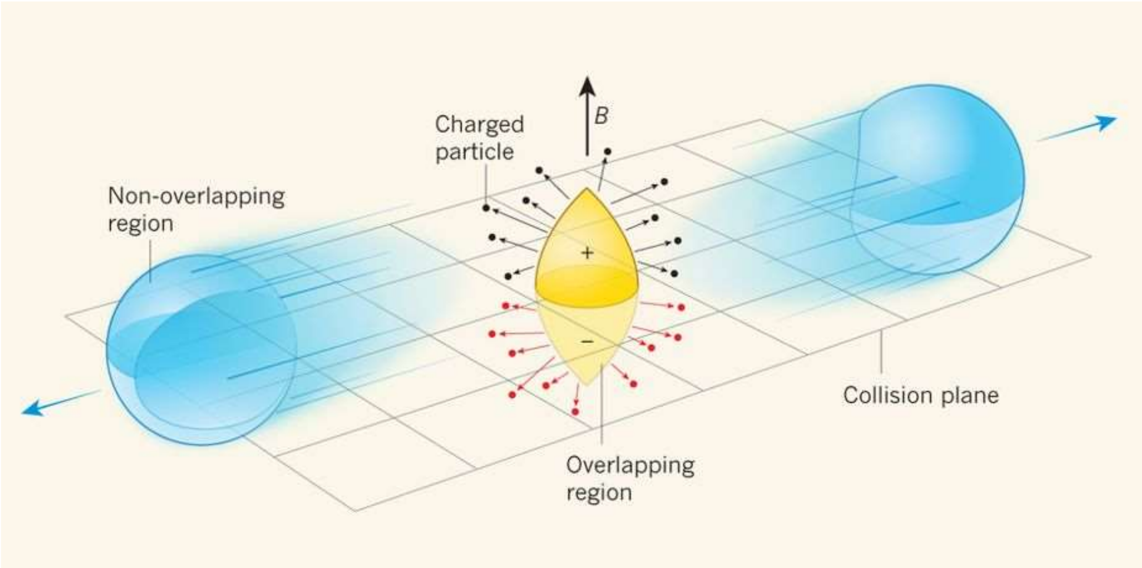
- Locking of the current max to B (not pinned to crystal axis)



Chiral magnetic effect (Vilenkin, Phys Rev D **22**, 3080, 1980)

$$\boxed{\vec{J}_{CME} = \alpha_B \vec{B}}$$

- Quark-gluon plasma in heavy ion collision (some evidence)
- Relativistic plasma in astrophysics
- Weyl semimetal (indirect evidence, through chiral anomaly)
- …



Figs from Dobrin Nature 2017, Chernodub arXiv 1002.1473, Vazifeh PRL 2013

https://www.bnl.gov/newsroom/news.php?a=119062

Kharzeev and Liao, Nat Rev Phys 2020

Symmetry in HE and CME

Hall effect

$$J_y = \sigma_H E_x$$

Space	–	–
Time	–	+

Chiral magnetic effect

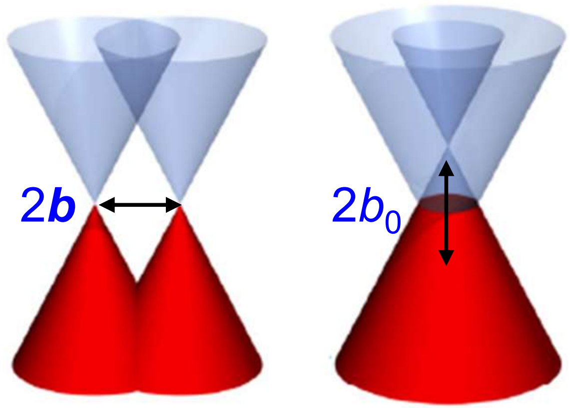
$$\boldsymbol{J} = \alpha_B \boldsymbol{B}$$

Space	–	+
Time	–	–

- $J_y = \sigma_H E_x$, needs to break TRS
- $\boldsymbol{J} = \alpha_B \boldsymbol{B}$, needs to break SIS (hard in high-energy experiment)

Chiral magnetic effect in WSM (Zyuzin and Burkov, PRB 2012)

$$H = \tau_z \vec{\sigma}\cdot\vec{k} + \vec{\sigma}\cdot\vec{b} + \tau_z b_0$$



➡ Dynamical axion in EM action
(derived with Fujikawa's path integral method)

$$S_\theta = \frac{1}{2\pi}\int dt d^3x\, \theta(\vec{r},t)\vec{E}\cdot\vec{B}\,\frac{e^2}{h}$$

$$\theta = 2(\vec{b}\cdot\vec{r} - b_0 t) \quad \text{(relativistic covariant)}$$

- Current density (recall TI SS)

$$\rightarrow \quad \vec{j} = \frac{1}{2\pi}\left(\nabla\theta\times\vec{E} + \partial_t\theta\,\vec{B}\right)\frac{e^2}{h}$$

$$= \left(\vec{b}\times\vec{E} - b_0\,\vec{B}\right)\frac{e^2}{\pi h}$$

AHE: $\vec{j}\perp\vec{E}$ — Chiral magnetic effect: $\vec{j}\parallel\vec{B}$

Consistent with previous Lecture

- "Relativistic covariance" requires AHE and CME to *both* exist. However, there is no such property in solids. So its existence is not guaranteed in WSM.

Note: This looks similar to the axion coupling for TI SS. But the current here is inside the bulk, not on surface.

However, an argument against CME for WSM

in equilibrium and in a static B field (Basar, Kharzeev, and Yee, PRB 2014)

$$\vec{J} = \alpha_B \vec{B}$$

- Work done by field on charge carriers

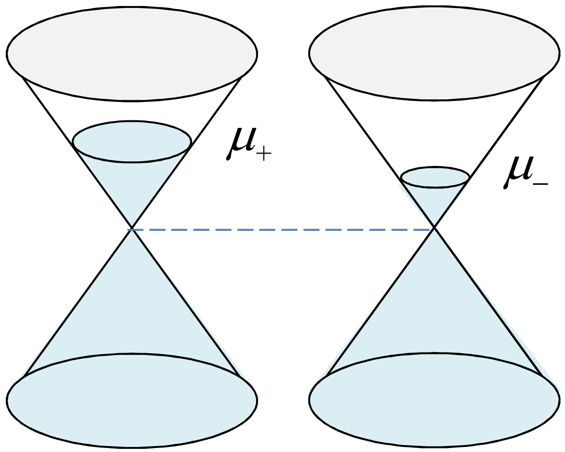
$$\frac{dP}{dt} = \vec{J} \cdot \vec{E} \sim \vec{E} \cdot \vec{B}$$ can be > 0 or < 0

Can extract energy out of equilibrium state!

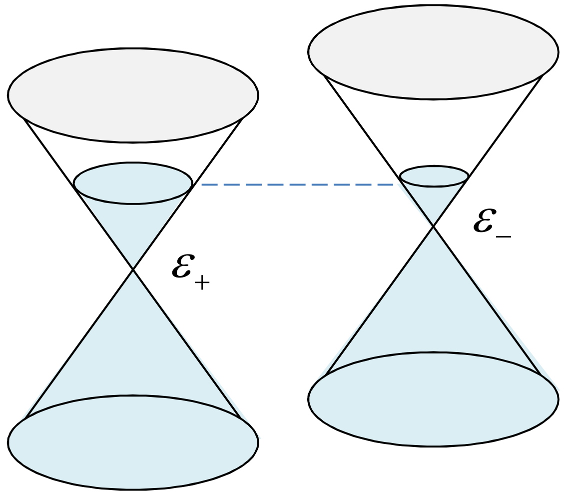
More detailed analysis (semiclassical, linear response)

also shows that static CME could not exist.

Non-equilibrium system



Non-static B field



- **Different chemical potentials**

$$\vec{J} = \frac{e^2}{h^2}\Delta\mu\vec{B} \quad \text{(non-equilibrium)}$$

Magnitude: $J \sim 0.01$ (A/mm^2)

if $\Delta\mu$=0.01 meV, B=0.1 T

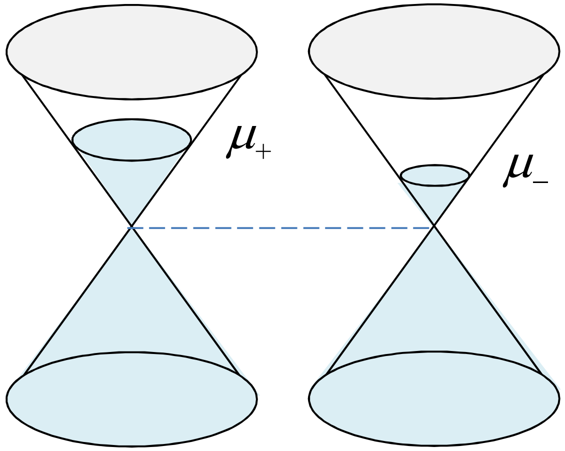
→ Negative MR mentioned earlier

Zhang et al, Nature Comm 2015

- **Same chemical potential**
  - Static *B* field: no current
  - Dynamic *B* field: can have CME current related to natural gyrotropic effect (no B field required) 旋光效應

Basar et al, Phys Rev B 2014
Yang and Chang, PRB 2015
Zhong et al, Phys Rev Lett 2016

An explanation for $\vec{J} = \frac{e^2}{h^2} \Delta\mu \vec{B}$



- An electric field moves electron charges Q from right to left. The displacement of charges costs an energy

$$dE = \frac{Q}{(-e)}(\mu_+ - \mu_-) > 0$$

- On the other hand, the rate of energy released (per unit volume) by electron relaxation should be

$$\begin{aligned} \frac{dE/V}{dt} &= \frac{1}{(-e)} \frac{\partial \rho}{\partial t} (\mu_+ - \mu_-) \\ &= -\Delta\mu \frac{e^2}{h^2} \mathbf{E} \cdot \mathbf{B} \qquad \Delta\mu \equiv \mu_+ - \mu_- \end{aligned}$$

Choose $\mathbf{E} \parallel \mathbf{B}$, and let $E \to 0$,

$$\Rightarrow \quad \mathbf{J} = -\Delta\mu \frac{e^2}{h^2} \mathbf{B}$$