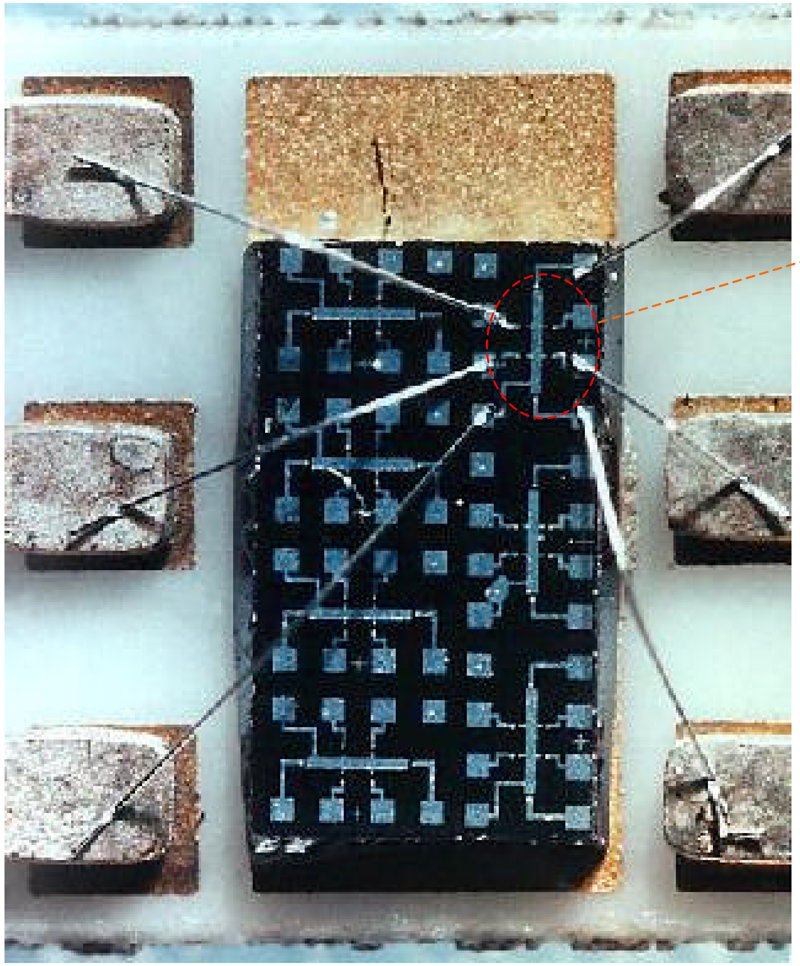
Quantum Hall effect in a nutshell

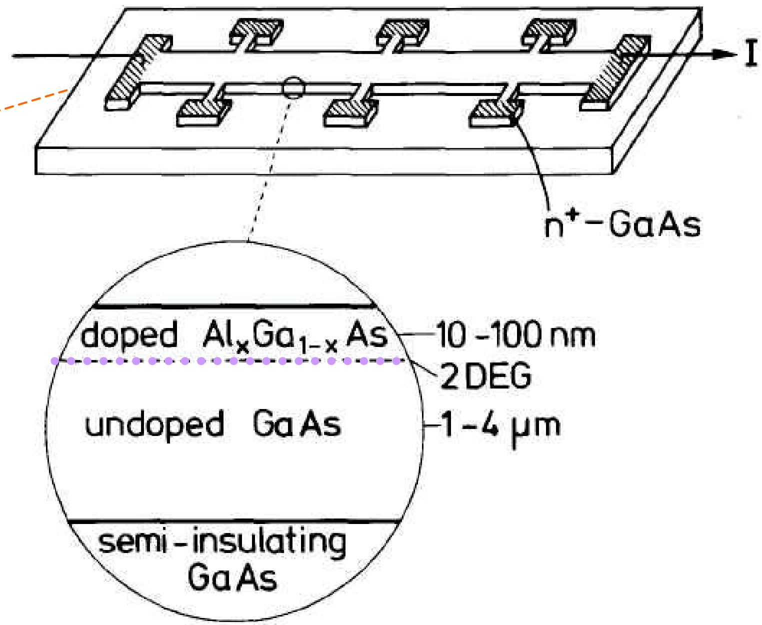
(Ref: Kittel p.564-577)

Dept of Phys

M.C. Chang

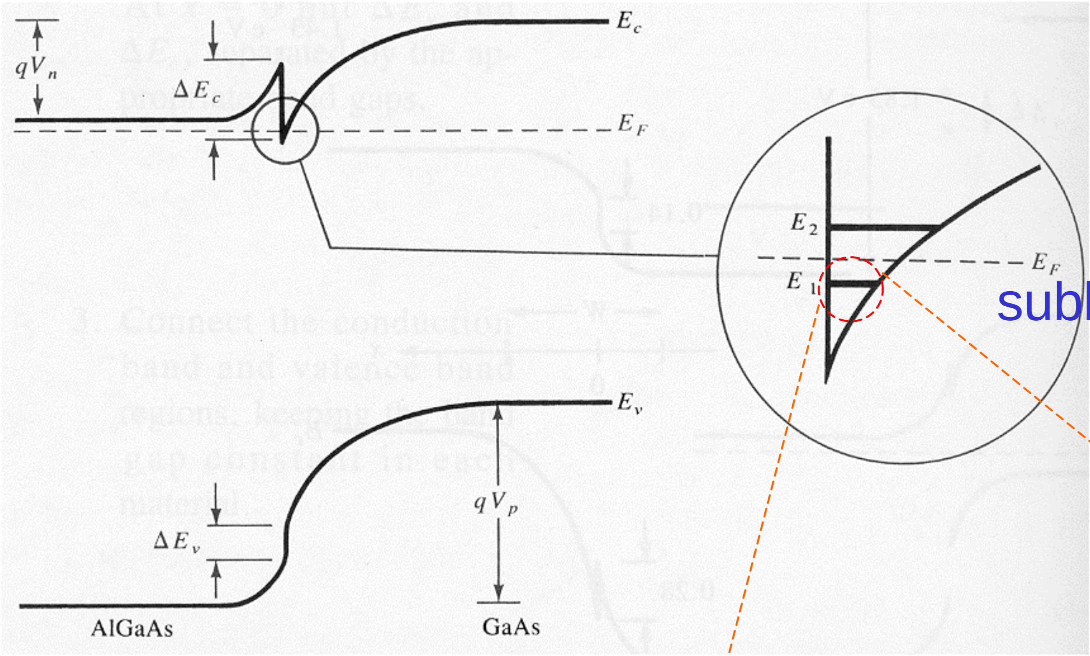
Measurement of Hall resistance





2-dim electron gas

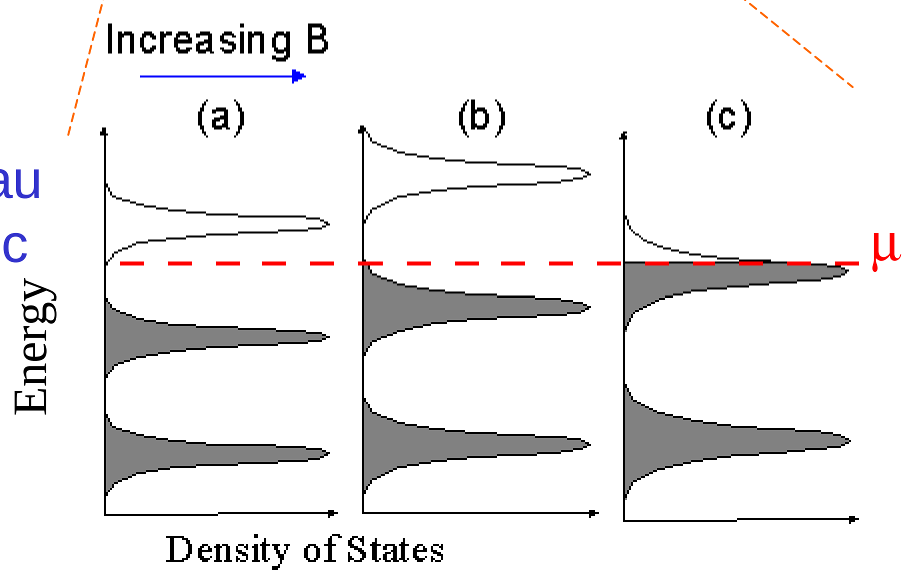
GaAs/AlGaAs heterojunction



Dynamics along z-direction is frozen in the ground state

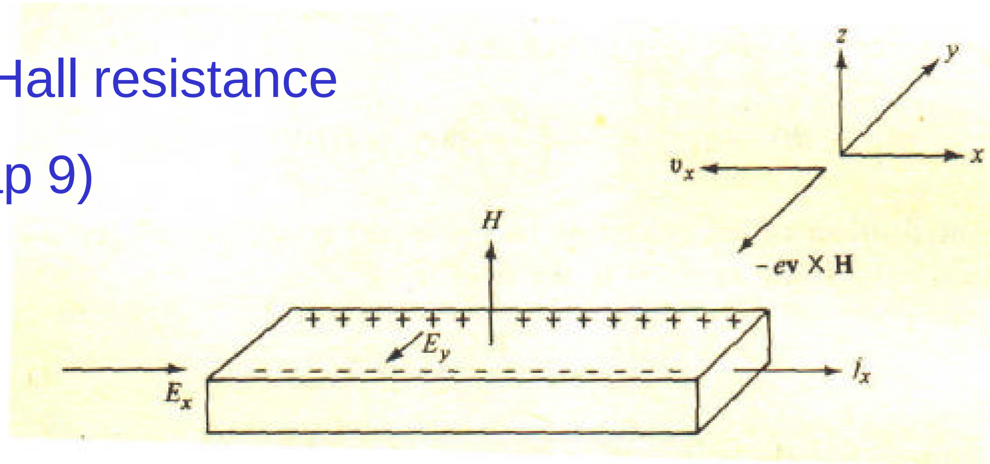
subband

(broadened) Landau levels in a magnetic field



Classical Hall resistance

(recall chap 9)

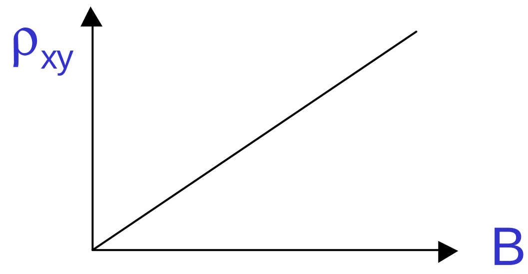


$$m_e \frac{d\vec{v}}{dt} = -e\vec{E} - e\vec{v} \times \vec{B} - m_e \frac{\vec{v}}{t}$$

$$\vec{B} = B\hat{z}; \quad d\vec{v}/dt = \vec{0} \text{ at steady state}$$

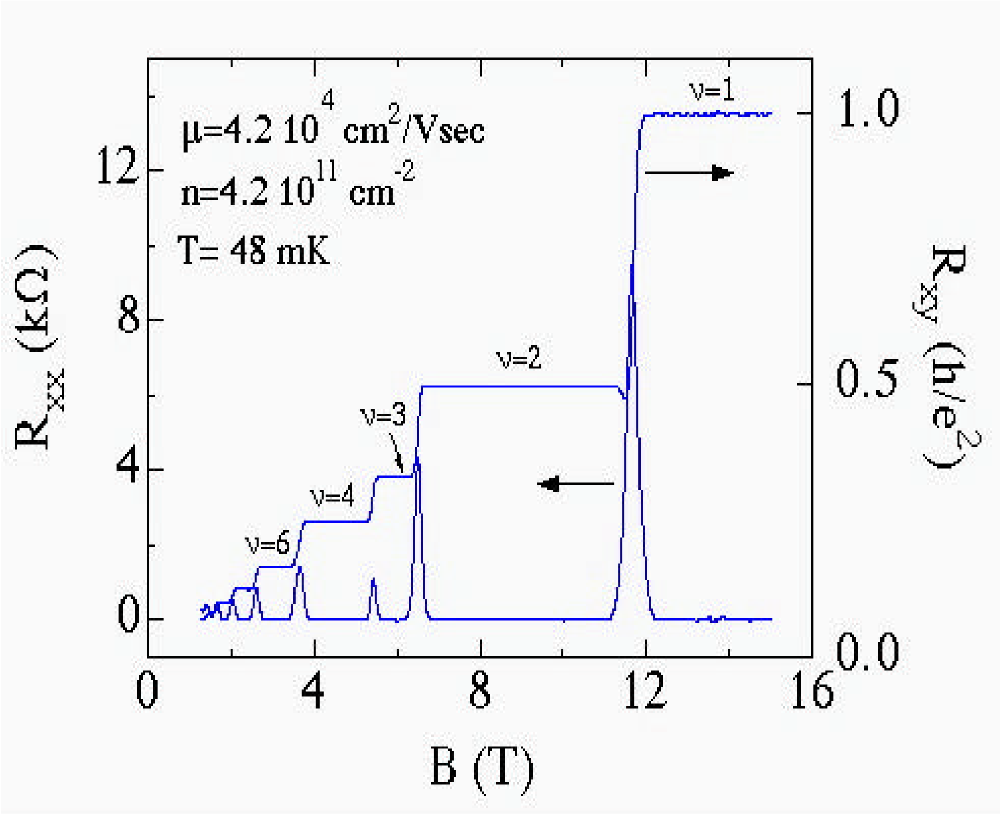
$$\Rightarrow \begin{pmatrix} m/t & eB \\ -eB & m/t \end{pmatrix} \begin{pmatrix} v_x \\ v_y \end{pmatrix} = -e \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \frac{m}{ne^2 t} & \frac{B}{ne} \\ -\frac{B}{ne} & \frac{m}{ne^2 t} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}, \quad \vec{j} = -en\vec{v}$$



$$\text{or} \quad \overleftrightarrow{s} = \overleftrightarrow{r}^{-1} = \begin{pmatrix} 0 & -\frac{ne}{B} \\ \frac{ne}{B} & 0 \end{pmatrix} \quad \text{as } r_0 \to 0$$

Quantum Hall resistance (von Klitzing, 1980)



1985

R_{xy} deviates from $(h/e^2)/n$ by less than 3 ppm on the very first report of the quantum Hall effect.

- This result is independent of the shape/size of sample.
- Different materials lead to the same effect (Si MOSFET, GaAs heterojunction…)

→ a very accurate way to measure α : $\alpha^{-1} = h/e^2c \approx 137.036$ (unit-indep.)

→ a very convenient resistance standard (later)

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

and

G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper
Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom
(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

experiment

$$\alpha^{-1}(\text{q. Hall}) = 137.035\,997\,9(32) \quad (0.024 \text{ ppm}),$$

$$\alpha^{-1}(\text{acJ}) = 137.035\,977\,0(77) \quad (0.056 \text{ ppm}),$$

$$\alpha^{-1}(h/m_n) = 137.036\,010\,82(524) \quad (0.039 \text{ ppm}).$$

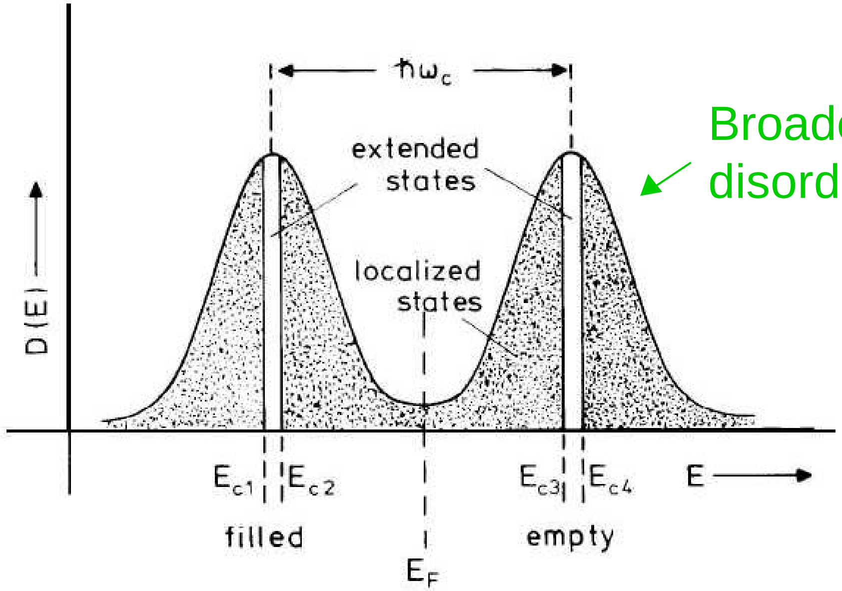
theory

$$\alpha^{-1}(a_e) = 137.035\,999\,44(57) \quad (0.0042 \text{ ppm}).$$

(Kinoshita, Phys. Rev. Lett. 1995)

Why there are plateaus?

- magnetic field -> Landau levels
- dirty interface -> electron localization -> plateaus



Broadened LL due to disorder

Quantum Hall effect requires

1. Two-dimensional electron gas
2. strong magnetic field (~ 10 Tesla)
3. low temperature (< 4 K)

Why R_H has to be exactly $(h/e^2)/n$?

- see Laughlin's argument next page

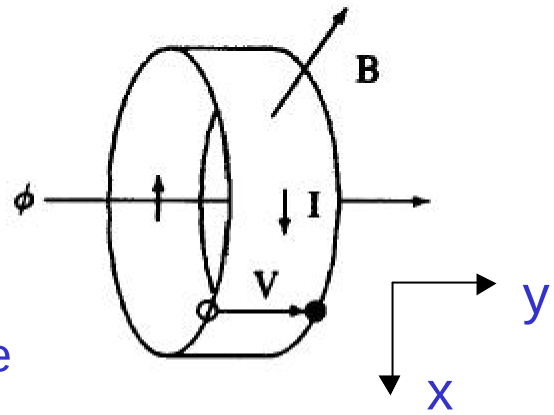
Quantization of Hall resistance (Laughlin, 1981)

$$H=\sum_i \frac{1}{2m}\left(\vec{p}_i+\frac{e}{c}\vec{A}(\vec{r}_i)\right)^2+V(\vec{r}_i)$$

if we let $\vec{A}\rightarrow\vec{A}+A_0\hat{x}$, then the current operator

$$j_x=\frac{-e}{m}\frac{1}{L_xL_y}\sum_i\left[\frac{\hbar}{i}\frac{\partial}{\partial y}+\frac{e}{c}A_x(\vec{r}_i)\right]=-\frac{c}{L_xL_y}\frac{\partial H}{\partial A_0}$$

A_0 has no physical meaning, unless the sample is wrapped into a loop. It is then related to a *fictitious* flux through the loop $f=A_0L_x$



Let us imagine such a system and solve

$$H_f|y_f>=E_f|y_f>$$

By the so-called Hellman-Feynman theorem, we have

$$<y_f|\frac{\partial H_f}{\partial f}|y_f>=\frac{\partial}{\partial f}<y_f|H_f|y_f>=\frac{\partial E_f}{\partial f}$$

$$\text{so}\quad j_x=-\frac{c}{L_y}\frac{\partial E_f}{\partial f}$$

- For localized states, no charge transfer whatever ϕ is.
- For extended states, integer charges may have transferred along y when ϕ is changed by one ϕ_0

$$j_x=-c\frac{n(-e)}{f_0}\frac{V_y}{L_y}=n\frac{e^2}{h}E_y$$

An accurate and stable resistance standard (1990)

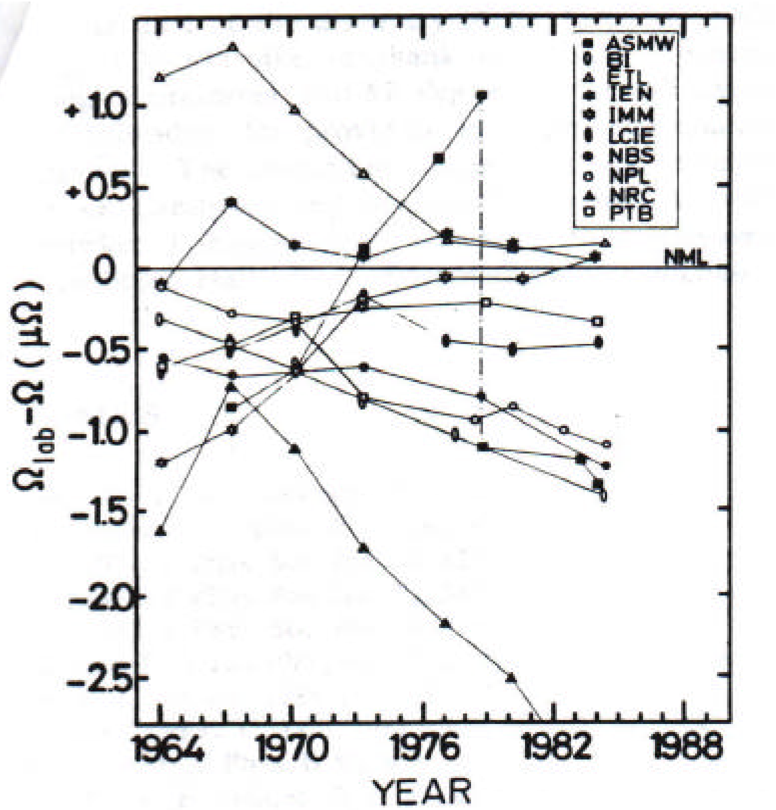


FIG. 26. Time dependence of the 1-Ω standard resistors maintained at the different national laboratories.

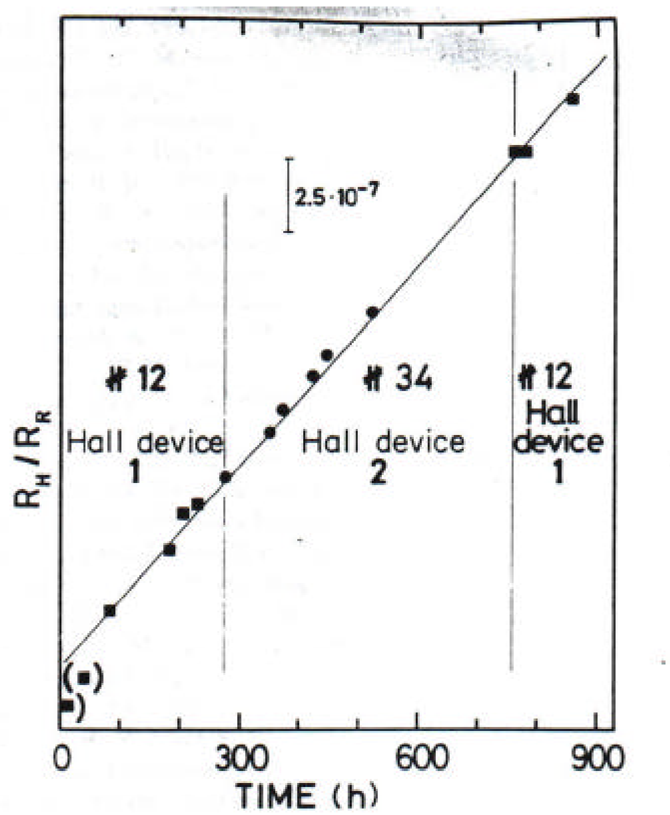
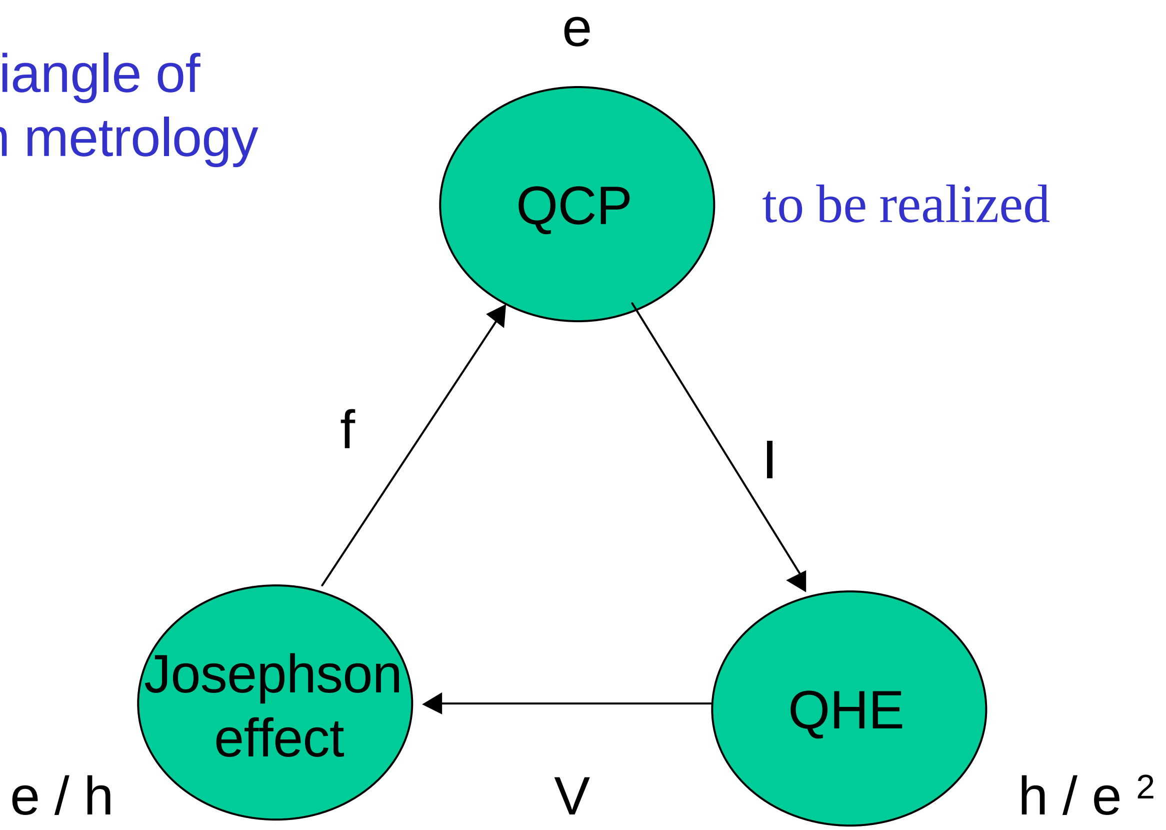


FIG. 27. Ratio R_H/R_R between the quantized Hall resistance R_H and a wire resistor R_R as a function of time. The result is time dependent but independent of the Hall device used in the experiment.

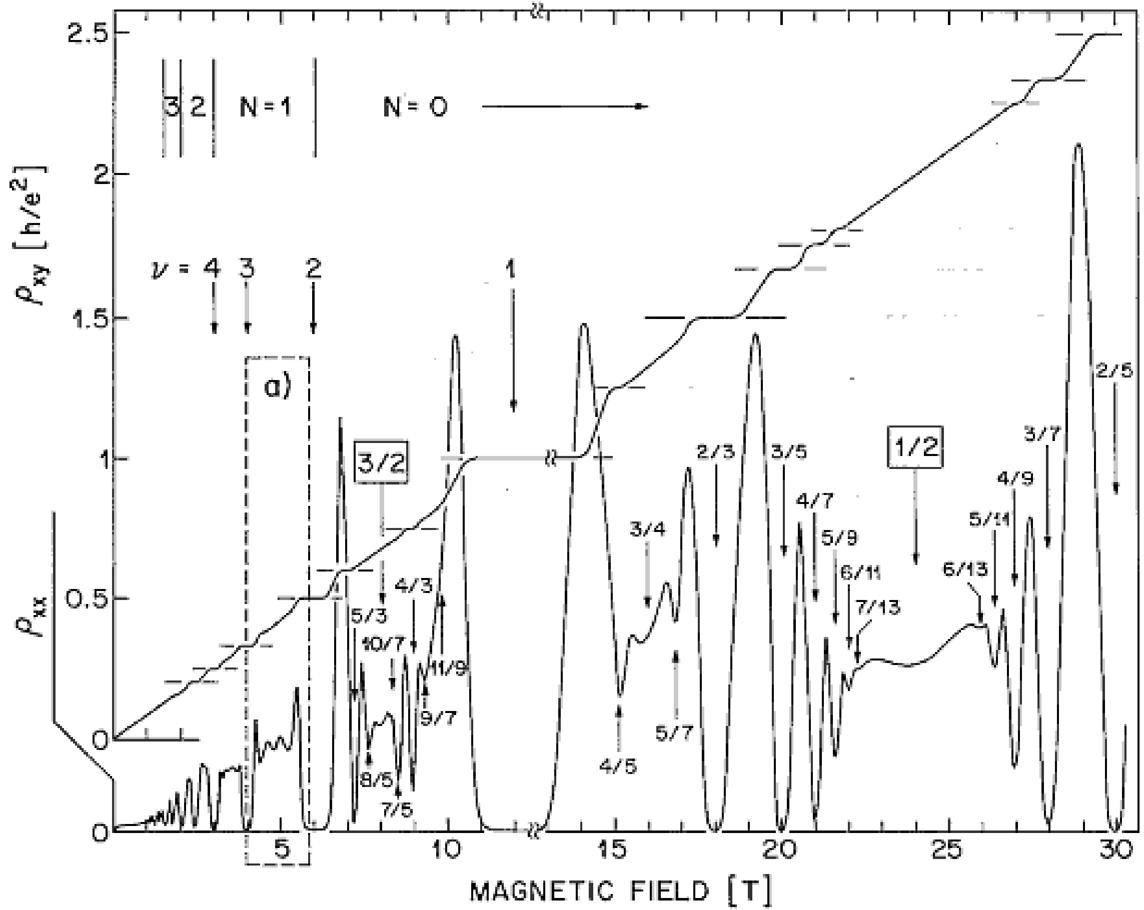
The triangle of
quantum metrology



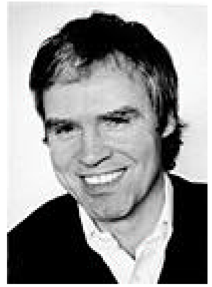
Josephson constant $K_J = 2e/h$	
Value	483 597.879 x 10^9 Hz V^{-1}
Standard uncertainty	0.041 x 10^9 Hz V^{-1}
Relative standard uncertainty	8.5 x 10^{-8}
Concise form	483 597.879(41) x 10^9 Hz V^{-1}

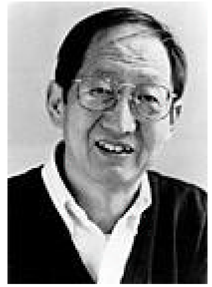
von Klitzing constant $R_K = h/e^2 = \mu_0 c/2\alpha$	
Value	25 812.807 449 Ω
Standard uncertainty	0.000 086 Ω
Relative standard uncertainty	3.3 x 10^{-9}
Concise form	25 812.807 449(86) Ω

Fractional quantum Hall effect
(Stormer, Tsui, Gossard, 1982)



GaAs/AlGaAs epitaxial heterojunction at 150 mK.





- superfluid analogy
- anyon excitation
- skyrmion excitation
- composite fermion
- stripe phase
- Josephson-like tunneling
- …

as rich as superconductor

1998