Chap 13

Nuclear interactions and applications

- Nuclear Reactions
- Reaction Kinematics
- Fission
- Fission Reactors
- Fusion

A short timeline on nuclear physics (after 1925)

- 1919 First artificial nuclear reaction by Rutherford:

$$\alpha + {}^{14}_{7}\mathrm{N} \rightarrow p + {}^{17}_{8}\mathrm{O}$$ ~ **Modern alchemy**

- 1932 Discovery of neutron (Chadwick)

  Discovery of deuterium (Urey)

  Discovery of positron (Anderson)
- 1932 Invention of particle accelerator (Cockroft and Walton) $p + {}^{7}\mathrm{Li} \rightarrow 2\alpha$
- 1933 Theory of beta decay (Fermi)
- 1934 Discovery of artificial radioactivity (Curie and Joliot) α
- 1938 Discovery of nuclear fission (Meitner) n
- 1939 Nuclear energy production in stars (Bethe et al)

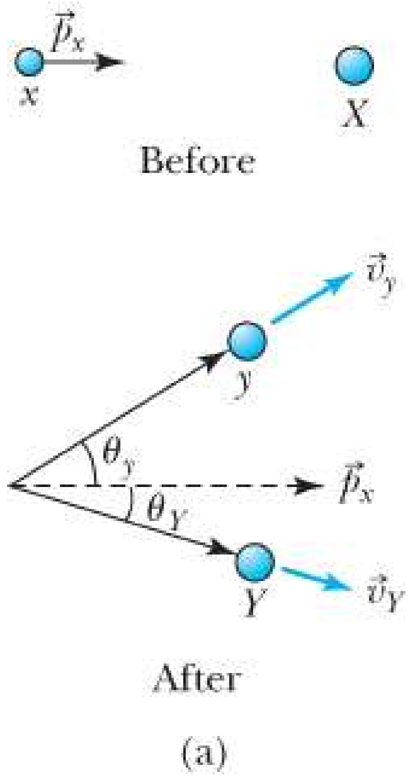
WWII

- 1942 The first nuclear reactor (Fermi et al)
- 1945 The first test of atomic bomb (Oppenheimer et al)
- 1952 The first test of hydrogen bomb (nuclear fusion, Teller et al)
- 1954 The first nuclear power plant (in Russia)

Nuclear reaction

- Consider the reaction: $x + X \rightarrow y + Y$. For a target X at rest, conservation of energy gives

$$M_x c^2 + K_x + M_X c^2 = M_y c^2 + K_y + M_Y c^2 + K_Y$$



Q-value (~ disintegration energy): 蜕變能

$$Q = M_x c^2 + M_X c^2 - \left(M_y c^2 + M_Y c^2\right) = K_y + K_Y - K_x$$

The difference between kinetic energies equals the difference between mass energies.

- Elastic collisions have Q = 0.
- When Q > 0, energy is released.

  When Q < 0, kinetic energy is converted to mass.

- In general a reaction $x + X \rightarrow y + Y$ can be rewritten as $X(x, y)Y$

For example, $\alpha + {}^{14}_{7}\mathrm{N} \rightarrow p + {}^{17}_{8}\mathrm{O}$ ⟺ ^{14}N(α,p)^{17}O

Ex 13.3: Calculate the ground state Q value for the reaction $^{14}N(\alpha, p)^{17}O$ in which Rutherford first observed a nuclear reaction. The kinetic energy of the α particles was 7.7 MeV. What was the sum of the kinetic energies of the exit channel?

$$M(^4He) = 4.002603\ u \qquad M(^1H) = 1.007825\ u$$

$$M(^{14}N) = 14.003074\ u \qquad M(^{17}O) = 16.999132\ u$$

Solution

$$\frac{Q}{c^2} = M(^4He) + M(^{14}N) - [M(^1H) + M(^{17}O)]$$

$$= -0.001280\ u$$

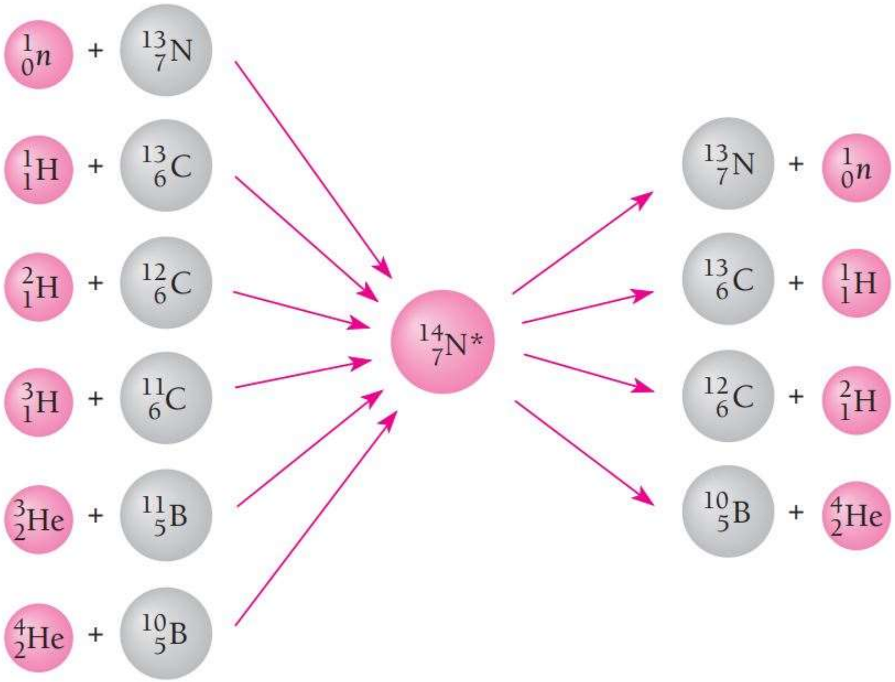
$$= (-0.001280\ c^2 \cdot u)\left(\frac{931.5\ MeV}{c^2 \cdot u}\right) = -1.192\ MeV$$

$$K(p) + K(^{17}O) = Q + K(\alpha)$$

$$= -1.192\ MeV + 7.7\ MeV = 6.5\ MeV$$

Compound nucleus 複核

- A composite of a (low energy) projectile (< 10 MeV) and a target nuclei, usually in a high state of excitation.
- The compound nucleus would decay into possible exit channels according to rules consistent with the conservation laws.



- The reaction often occurs near some specific projectile energy (resonance energy). The compound nucleus is also referred to as a nuclear resonance.

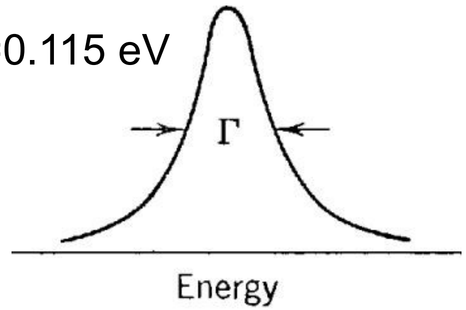
Resonance state: 共振態 A peak in cross sections of scattering experiments.

It can be considered as a particle with very short life.

- The neutron-capture cross section 中子捕獲截面 of $^{113}_{48}\mathrm{Cd}$ with respect to neutron energy:

$$^{113}\mathrm{Cd}(n, \gamma)^{114}\mathrm{Cd}$$

Half width Γ=0.115 eV



1 barn=100 fm^2
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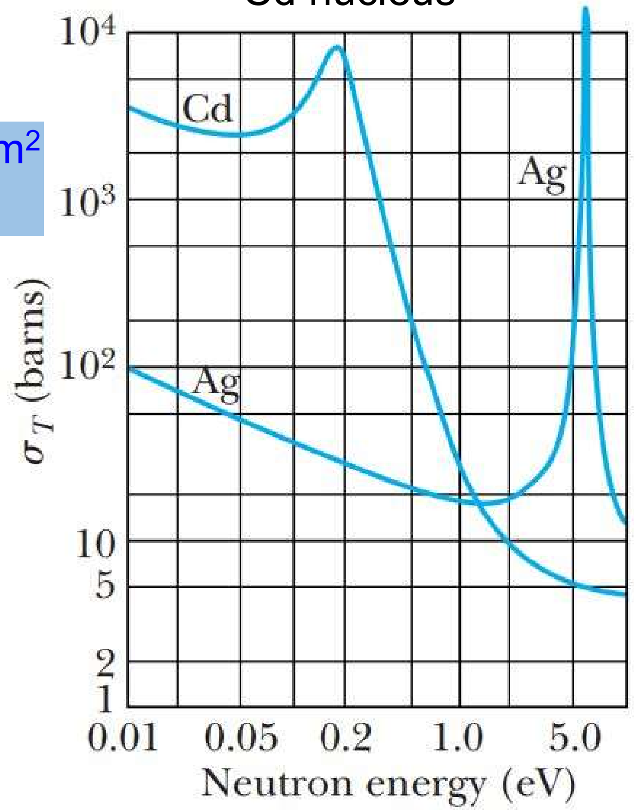
Excited state of ^{114}Cd nucleus



- Lifetime of resonance state: $\tau = \dfrac{\hbar}{\Gamma}$

$$\tau = \frac{1.054 \times 10^{-34}\,\mathrm{J \cdot s}}{(0.115\ \mathrm{eV})(1.60 \times 10^{-19}\ \mathrm{J/eV})} = 5.73 \times 1^{-15}\ \mathrm{s}$$

- Although ^{113}Cd constitutes only 12% of natural cadmium, its capture cross sections for slow neutrons are so great that it is used in control rods for nuclear reactors.

人工放射性

Artificial radioactivity (Curie and Joliot 1934)

- 1934, Curie and Joliot bombarded aluminum foil with alpha particles. The radioactivity persists after bombardment. This is the first **artificial radioactivity**.

$$ {}^{27}_{13}\mathrm{Al} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{30}_{15}\mathrm{P} + {}^{1}_{0}\mathrm{n} $$

$$ {}^{30}_{15}\mathrm{P} \rightarrow {}^{30}_{14}\mathrm{Si} + {}^{0}_{-1}\beta \qquad t_{1/2} \simeq 3 \text{ min} $$

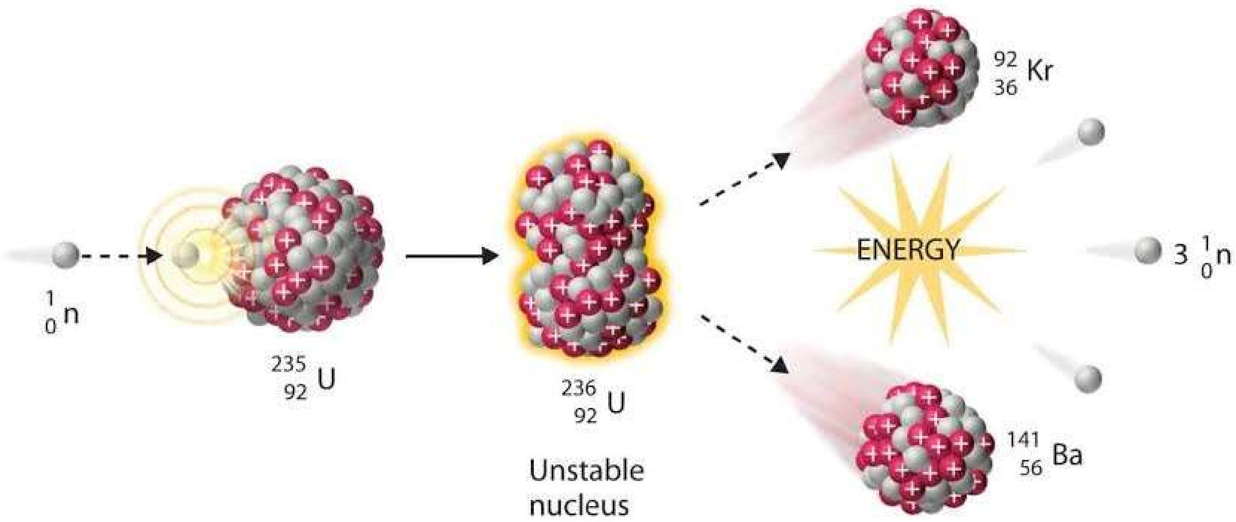
- It occurred to Fermi that neutrons would work better since there is no electric repulsion. 
- Neutrons are produced when alpha particles hit light isotopes of beryllium, carbon, or oxygen. Thus, a neutron source can be fabricated by mixing an alpha-emitter (radium, **polonium**) with these isotopes.
- Fermi *et al* started irradiating elements from light (H, Li…) to heavy, and finally irradiated the heaviest element then known: ^{238}U. But they're unable to fully understand the details of this reaction.
- Fermi also found that thermal neutrons are more efficient in nuclear reaction.

核分裂 **Nuclear fission** (Meitner 1938)

- 1938, Hahn and Strassman found Ba among the fragments of ^{235}U.

  This is soon correctly explained by Meitner and Frisch.

  (Frisch named it “fission”)



Note: Natural uranium has 99.27% ^{238}U and 0.72% ^{235}U.

Only the latter is fissionable.

Ex 13.5:

Calculate the ground state Q value of the induced fission reaction in Equation (13.14) if the neutron is thermal. A neutron is said to be *thermal* when it is in thermal equilibrium with its environment; it then has an average kinetic energy given by $\frac{3}{2}kT$.

$$n + {}^{235}_{92}\mathrm{U} \rightarrow {}^{236}_{92}\mathrm{U}^* \rightarrow {}^{99}_{40}\mathrm{Zr} + {}^{134}_{52}\mathrm{Te} + 3n$$

Solution

Because the kinetic energy of a thermal neutron is so small, its kinetic energy can be neglected.

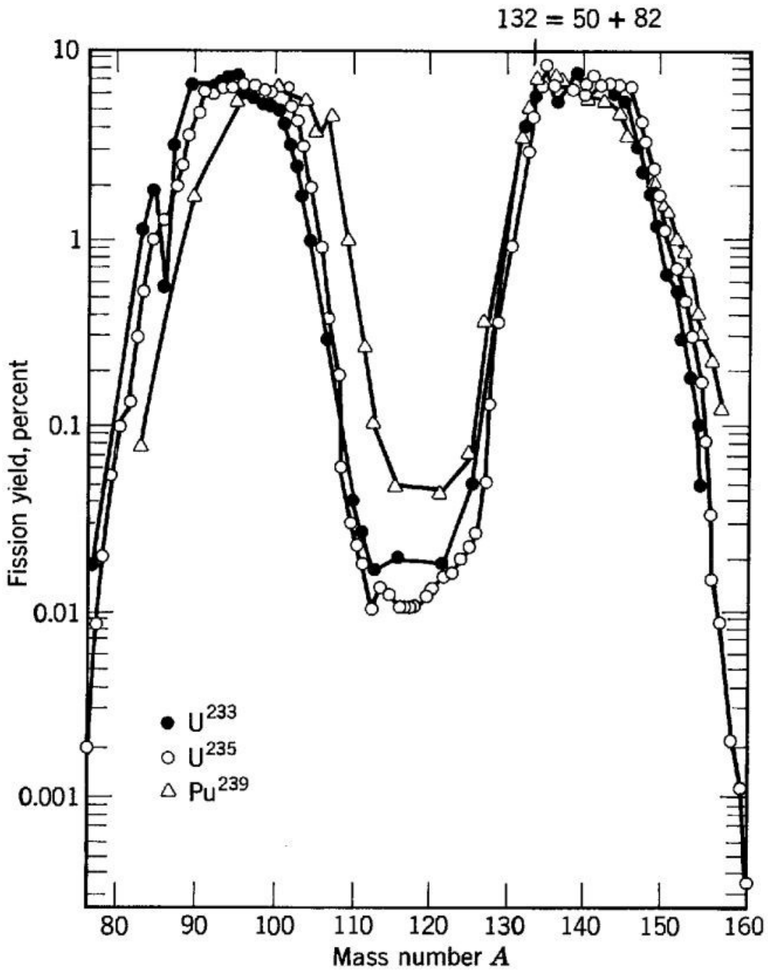
$$Q = \{M(^{235}\mathrm{U}) + m(n) - [M(^{99}\mathrm{Zr}) + M(^{134}\mathrm{Te}) + 3m(n)]\}c^2$$

$$= 0.1985\, c^2 \cdot \mathrm{u}$$

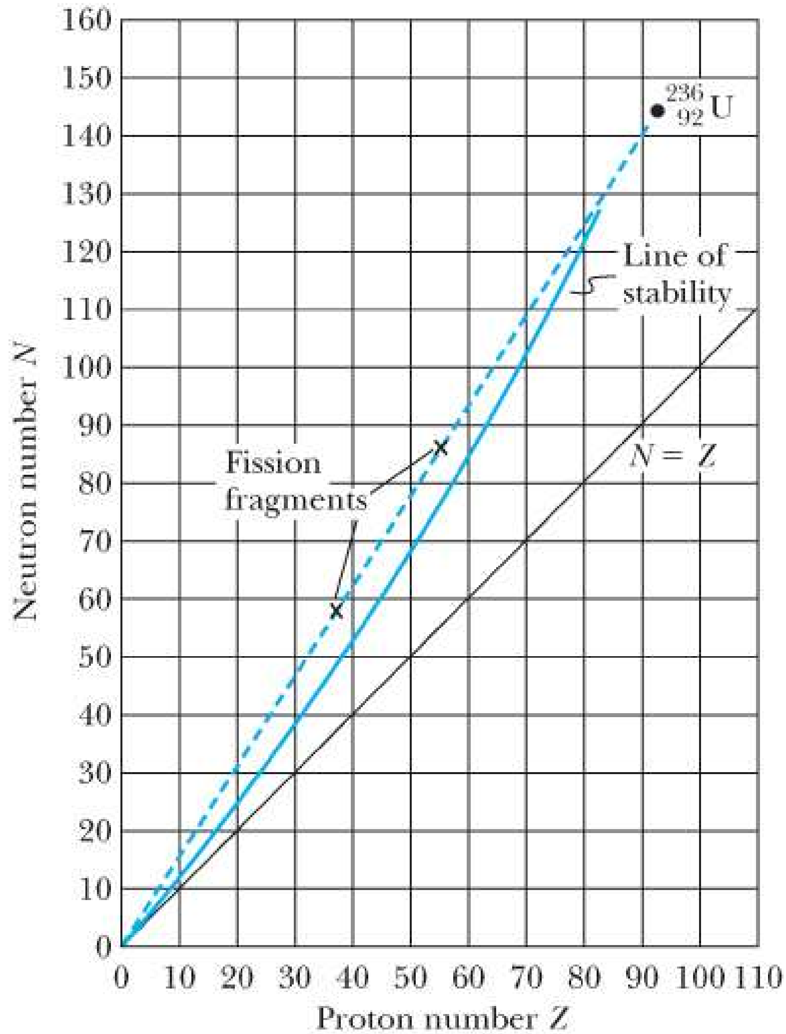
$$= 185\ \mathrm{MeV}$$

Even if the fission is induced by a thermal neutron of negligible kinetic energy on the nuclear scale, a tremendous amount of energy is released.

The distribution of mass numbers in the fragments

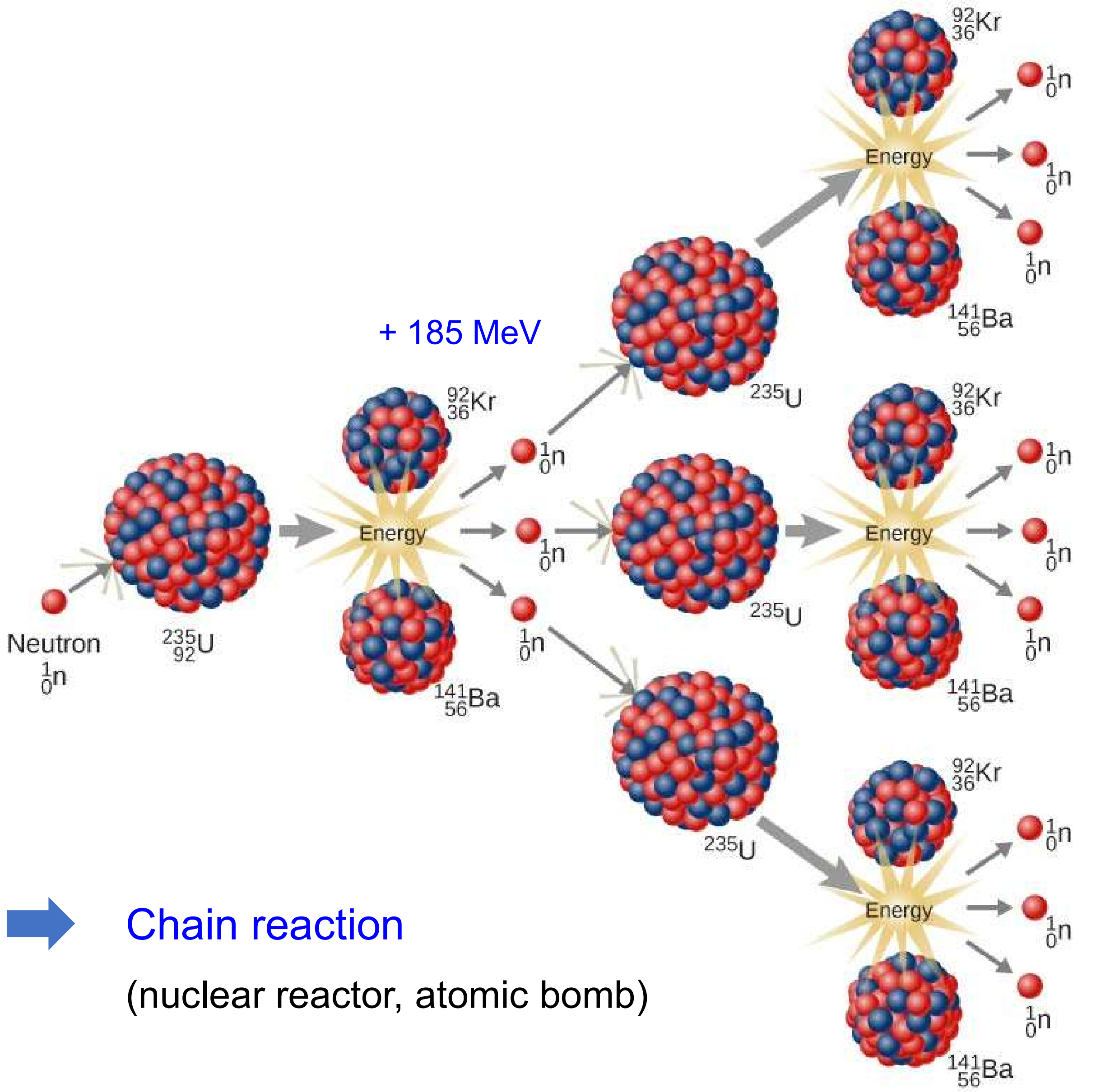


Usually 2 fragments are of unequal sizes



Because heavy nuclei have a greater neutron/proton ratio, the fragments contain an excess of neutrons. To reduce this excess, neutrons are emitted as soon as they are formed

1939, Fermi *et al*, 2 to 3 neutrons are emitted per fission of ^{235}U



Chain reaction

(nuclear reactor, atomic bomb)

Ex 13.7: It only takes a few microseconds for a neutron to be absorbed and cause another fission. Assume that 1.01 neutrons are captured on the average within 5 μs of each fission. Determine how many fissions will occur within 30 ms and the total energy produced.

Solution Within 30 ms the number of cycles of fission is 30 ms/5 μs = 6,000. The number of fissions is $(1.01)^{6,000} = 8 \times 10^{25}$. The total amount of energy produced is

$$\text{Energy} = (8 \times 10^{25}\ \text{fissions})\left(\frac{185\ \text{MeV}}{\text{fission}}\right)$$

$$= 10^{28}\ \text{MeV}$$

This is 10^{15} J, but the total world energy use in one year is about 10^{21} J. Fortunately, the process does not occur so quickly due to delayed neutrons, as we discuss next.

(not all neutrons are prompt, some are delayed by seconds and are emitted by daughter nuclides)

1 ton TNT ~ 4 x 10^9 J

Table 13.1 Energy Content of Fuels

Material	Amount	Energy (J)
Coal	1 kg	3×10^7
Oil	1 barrel (0.16 m^3)	6×10^9
Natural gas	1 ft^3 (0.028 m^3)	10^6
Wood	1 kg	10^7
Gasoline	1 gallon (0.0038 m^3)	10^8
Uranium (fission)	1 kg	10^{14}

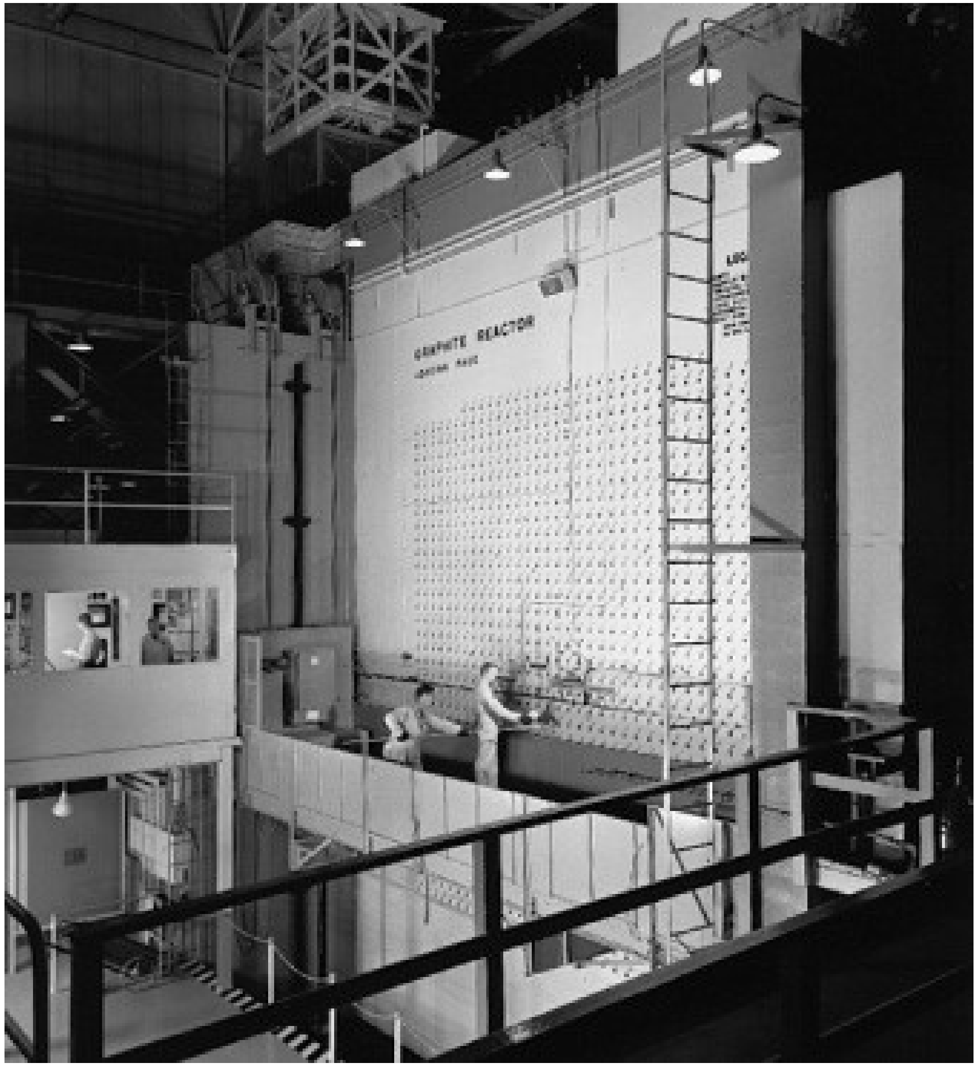
- **Table 13.2** Daily Fuel Requirements for 1000-MWe Power Plant

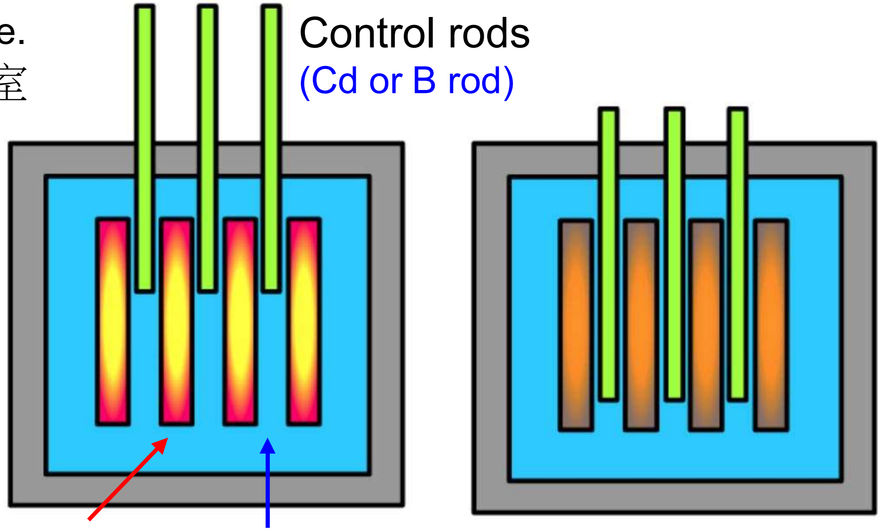
Material	Amount	
Coal	8×10^6 kg	(1 trainload/day)
Oil	40,000 barrels (6400 m^3)	(1 tanker/week)
Natural gas	2.5×10^8 ft^3 (7×10^6 m^3)	
Uranium	3 kg	

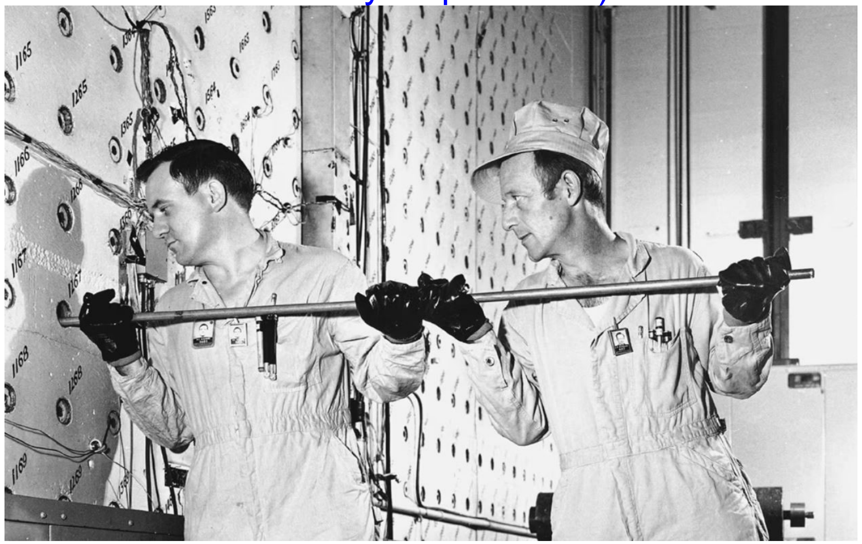
The first reactor that generated electricity (ORNL, 1948)

The plutonium for the Manhattan Project was produced here.

橡樹嶺國家實驗室

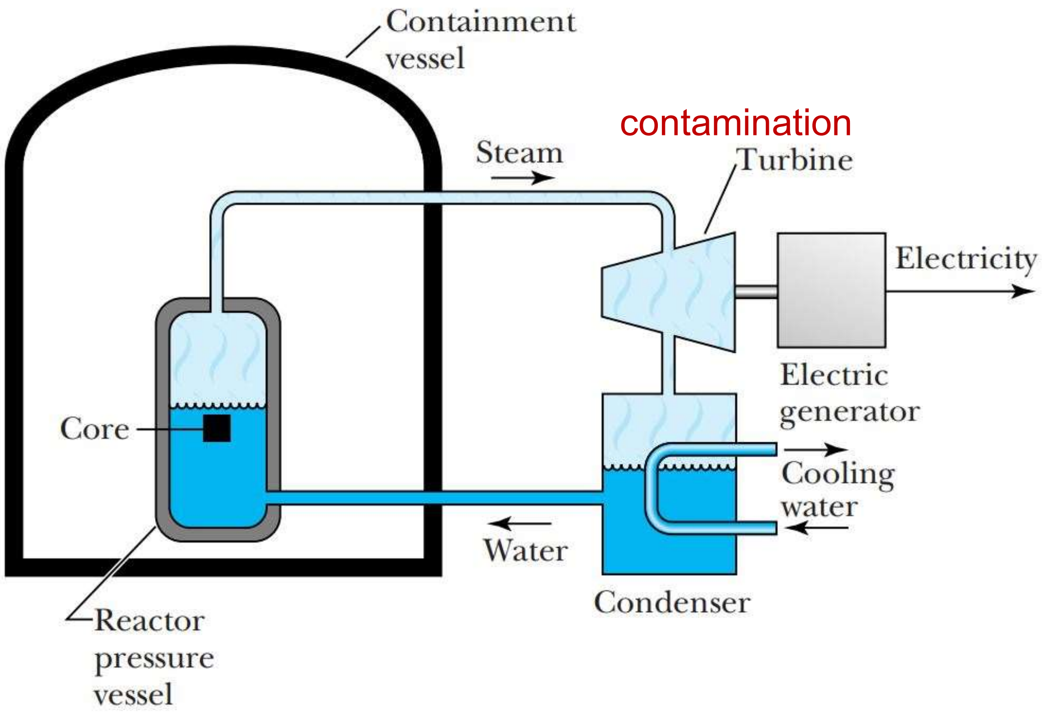


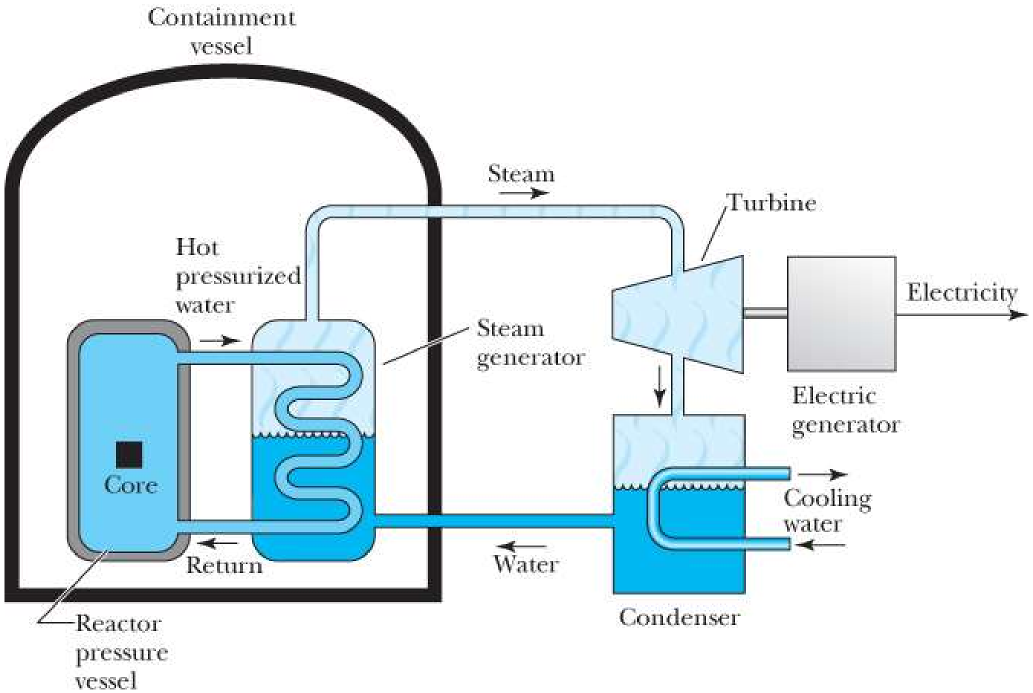




- Boiling water nuclear reactor

沸水式反應器





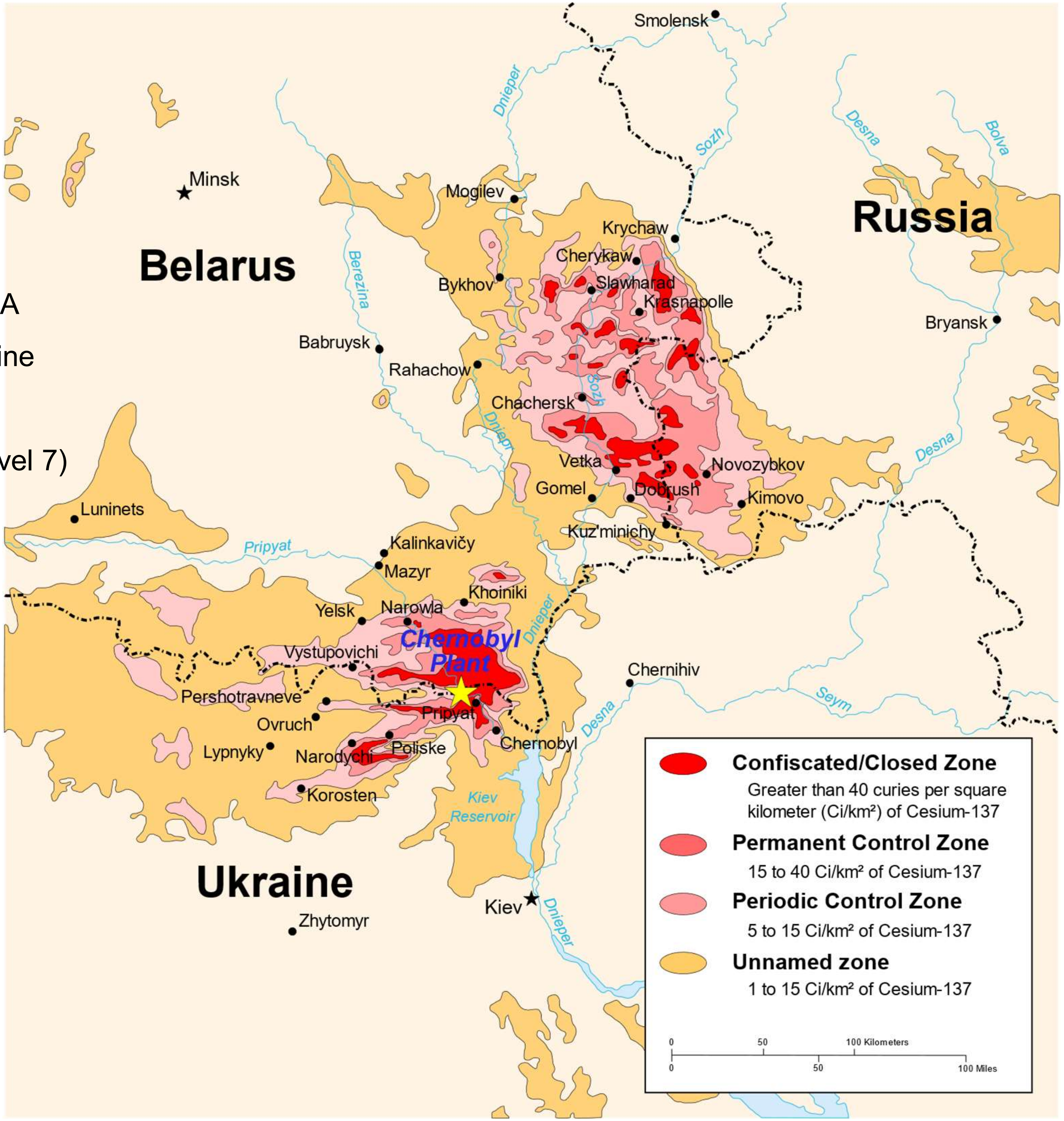
壓水式反應器

- Pressurized water nuclear reactor

Water at 155 atm to prevent boiling and avoid bubbling

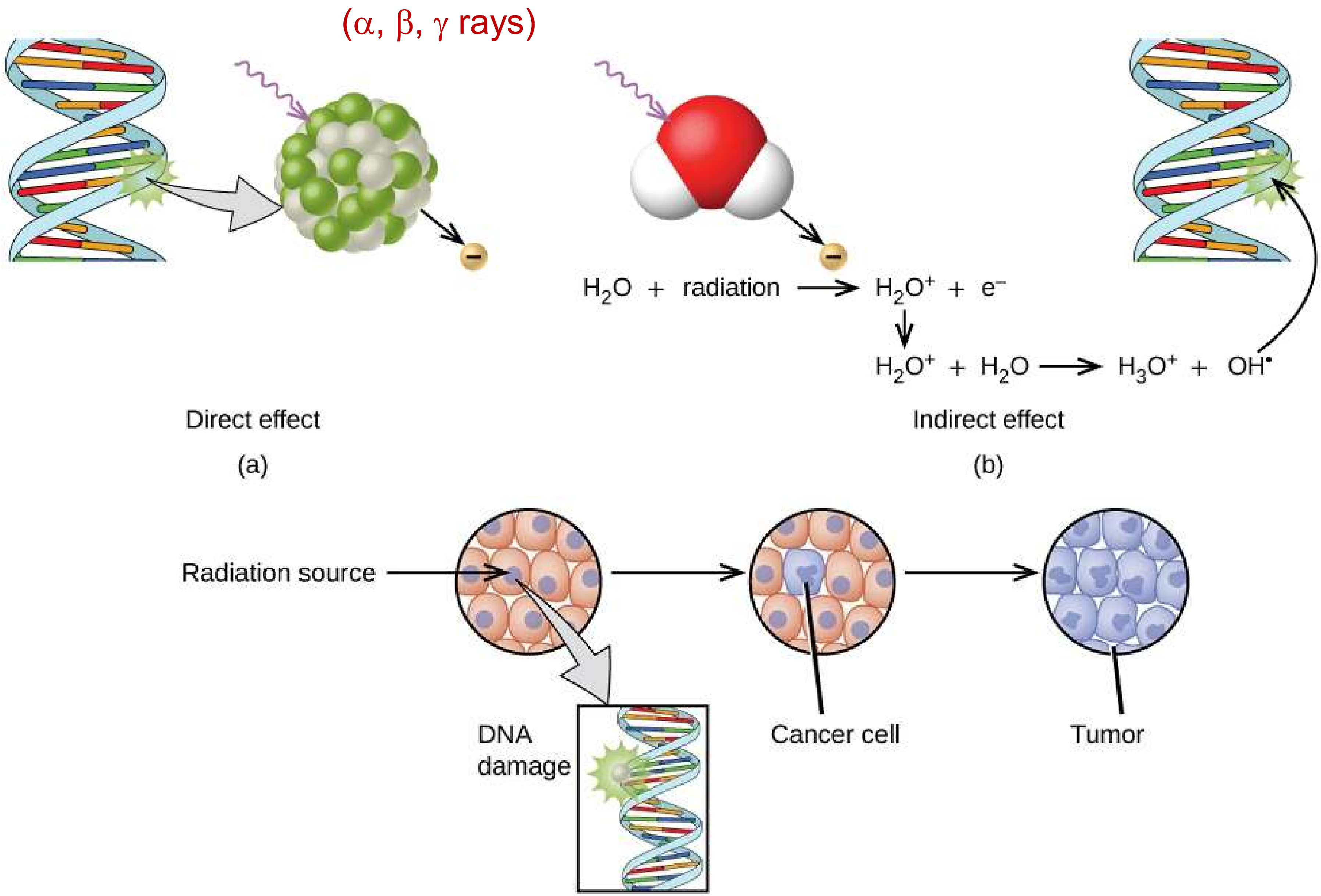
Three major accidents

- 1979 Three Mile Island, USA
- 1986/4/26 Chernobyl, Ukraine (level 7)
- 2011 Fukushima, Japan (level 7)



The Physics of the Disaster

Chernobyl — How It Happened



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Nuclear bomb

- ^{238}U underwent fission only if bombarded with neutrons of an energy greater than about 1 MeV, whereas slow neutrons could produce fission in ^{235}U (0.72% in natural uranium). In order to make a uranium bomb it Is necessary to separate them.
- On the other hand, plutonium (^{239}Pu) would undergo fission under slow neutron bombardment, and can be an alternative of ^{235}U.
- There were now two alternative ways of making an atomic bomb: the isotopic separation of uranium (by gaseous diffusion) or the formation of a sufficient quantity of plutonium (this requires a nuclear reactor using ^{238}U).

Critical mass:

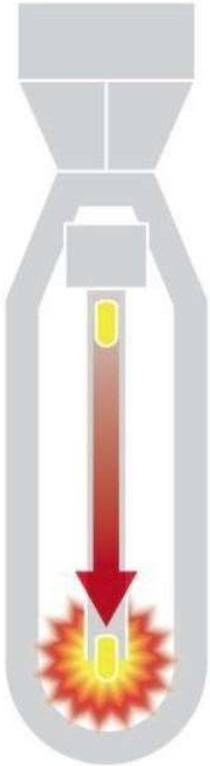
About 10 kgs of weapon's grade plutonium 239 or about 52 kgs of uranium 235.

2×10^{13} J would be released in 10^{-6} sec. (~10kT TNT)

https://en.wikipedia.org/wiki/Frisch%E2%80%93Peierls_memorandum

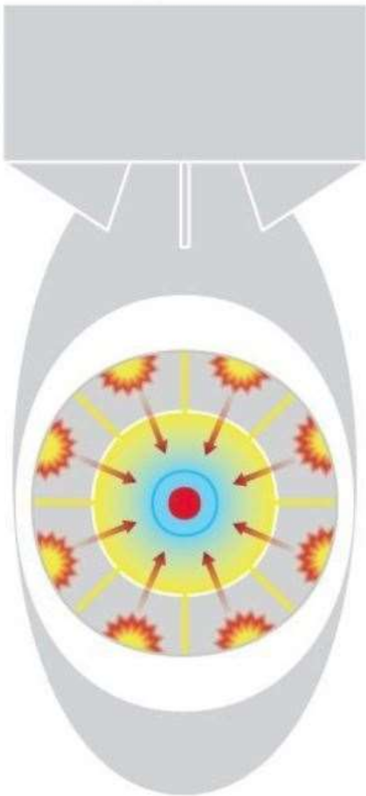
Manhattan project (Los Alamos, 1942-1946)

Uranium bomb
Gun type

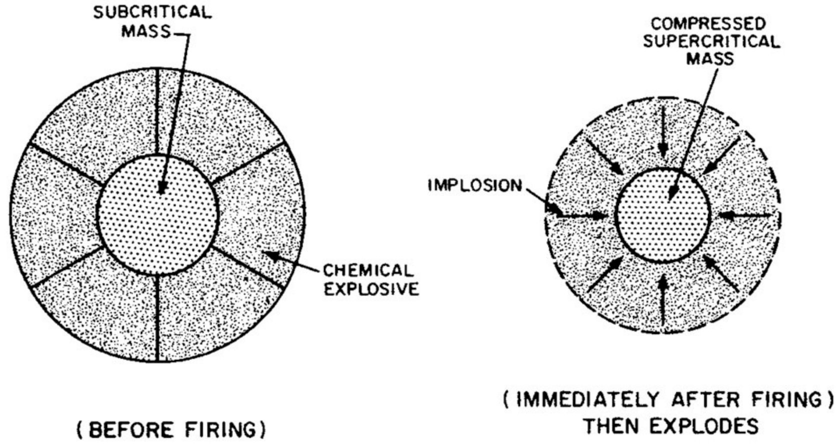


Hiroshima (60 kg 0f 80% U-235. 5% in Nuclear reactor)

Plutonium bomb
Implosion

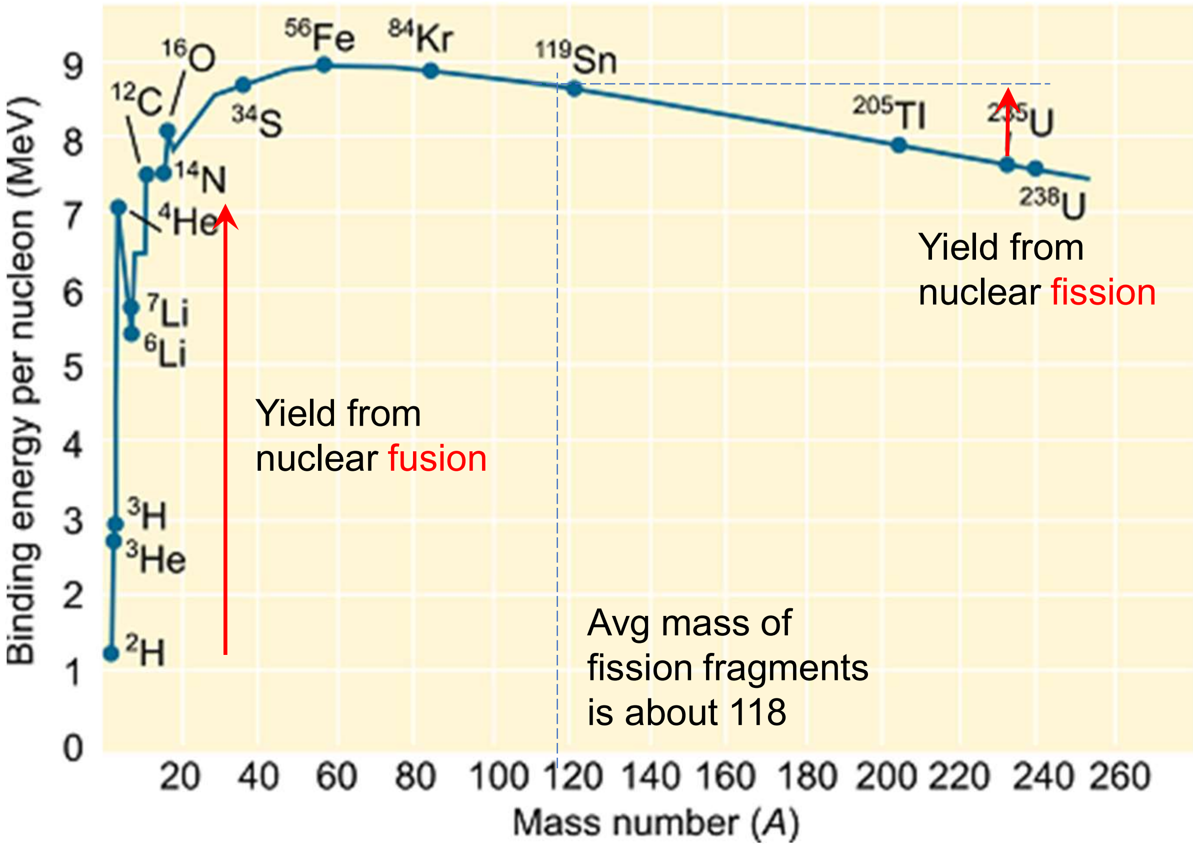


Trinity test
Nagasaki (6 kg of 96% Pu-239 + Implosion bomb)



From the movie Oppenheimer

Binding energy per nucleon



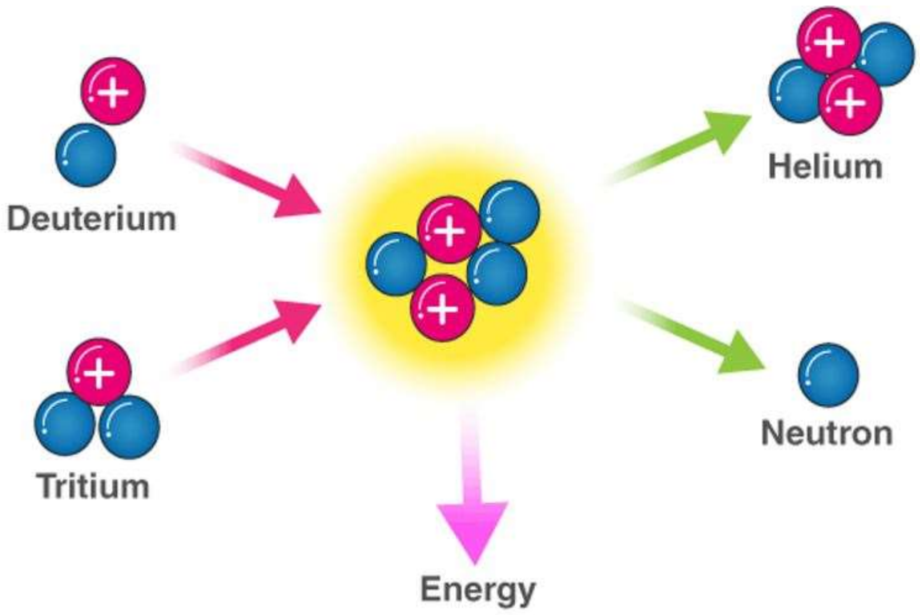
Nuclear fusion 核融合

Deuterium fusion, 2 examples:

$$\mathrm{D} + \mathrm{T} \rightarrow \mathrm{He}\ (3.52\ \mathrm{MeV}) + n\ (14.06\ \mathrm{MeV}) \qquad Q = 17.6\ \mathrm{MeV}$$

$$^{2}_{1}\mathrm{D} + ^{2}_{1}\mathrm{D} \rightarrow ^{3}_{1}\mathrm{T}\ (1.01\ \mathrm{MeV}) + p^{+}\ (3.02\ \mathrm{MeV}) \qquad 50\%$$

$$\rightarrow ^{3}_{2}\mathrm{He}\ (0.82\ \mathrm{MeV}) + n^{0}\ (2.45\ \mathrm{MeV}) \qquad 50\%$$



Nuclear fusion in stars

- "*Here on the earth, 150 million km from the sun, a surface 1 m^2 in area exposed to the vertical rays of the sun receives energy at a rate of about 1.4 kW. Adding up all the energy radiated by the sun per second gives the enormous total of 4×10^{26} W. And the sun has been emitting energy at this rate for billions of years. Where does it all come from*?" Beiser
- 1921, Eddington suggested hydrogen–helium fusion could be the primary source of stellar energy.

  Note: The sun consists of 70% hydrogen, 28% helium, and 2% of others
- Nuclear fusion required high temperature (~10^8 K) and density (10^{20} ions/m^3): At the sun's interior (1.5×10^7 K), the proton kinetic energy is about 1 keV, whereas the nuclear barrier for fusion is about 1 MeV (again, tunneling).

  (The core of the Sun is about 0.2 R, contains 34% of the Sun's mass.)
- Proton-proton chain and CNO cycle are the main nuclear reactions that can supply the energy in stars.

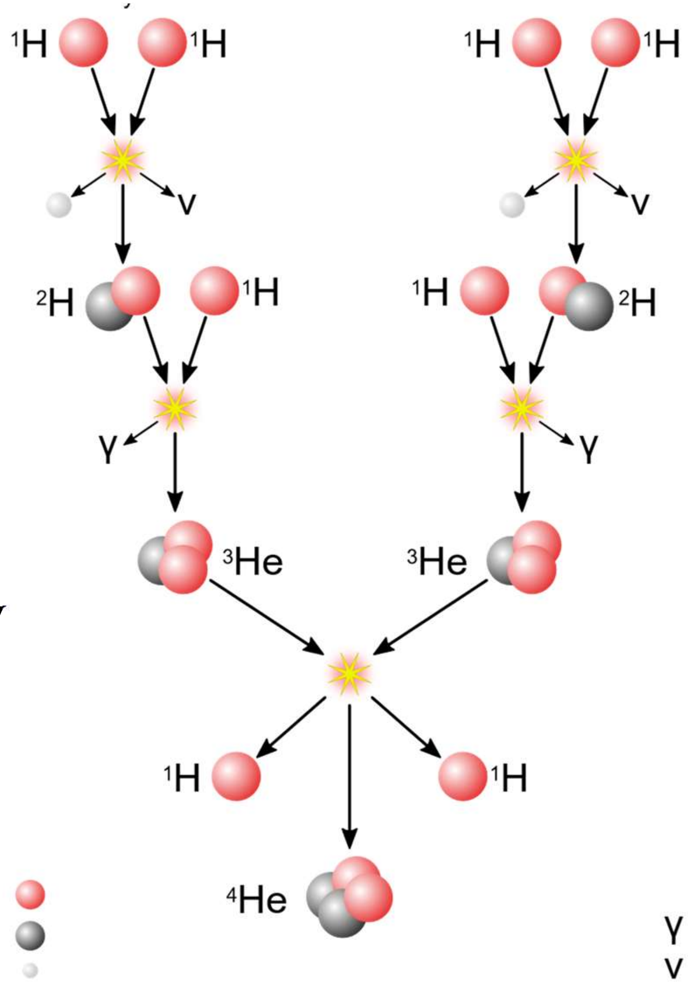
1. The **proton-proton chain** (Bethe 1939)

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + \beta^{+} + \nu \quad Q = 0.42\ \text{MeV}$$

$$^{2}H + {}^{1}H \rightarrow {}^{3}He + \gamma \quad Q = 5.49\ \text{MeV}$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H \quad Q = 12.86\ \text{MeV}$$

24.7 MeV (4×10^{-12} J) per He nucleus produced



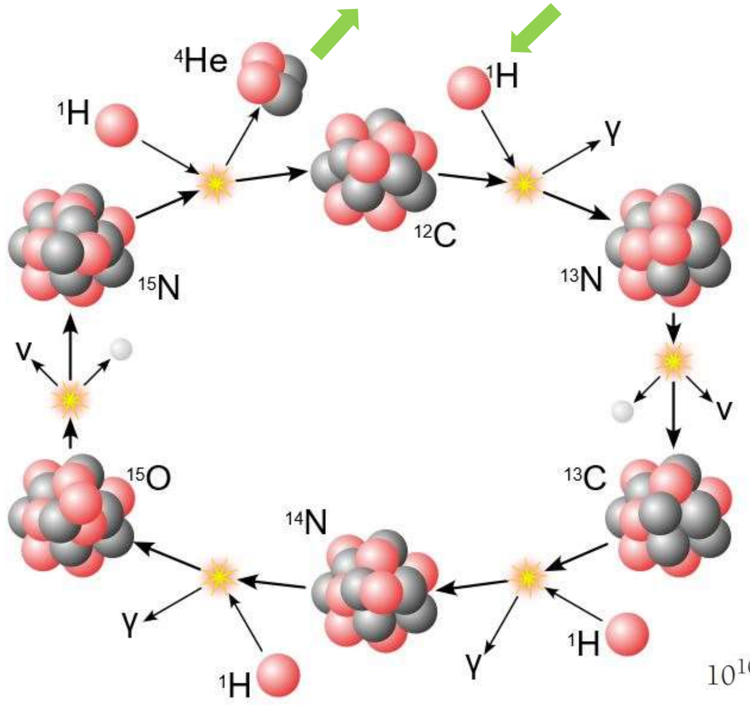
Note: one neutrino is produced every 2 protons

Q: The sun's power output of 4×10^{26} W means the sequence of reactions above must occur 10^{38} times per second. From the amount of hydrogen in the Sun, estimate how long can the sun keeps burning.

2. For stars 1.5 times heavier than the sun, the main reaction is

the **carbon cycle**, or **CNO cycle** (Bethe, Weizsäcker, 1937-1939)

https://astronomy.swin.edu.au/cosmos/C/CNO+cycle



Hydrogen in

$$^{12}C + {}^{1}H \longrightarrow {}^{13}N + \gamma$$

$$^{13}N \longrightarrow {}^{13}C + e^{+} + \nu$$

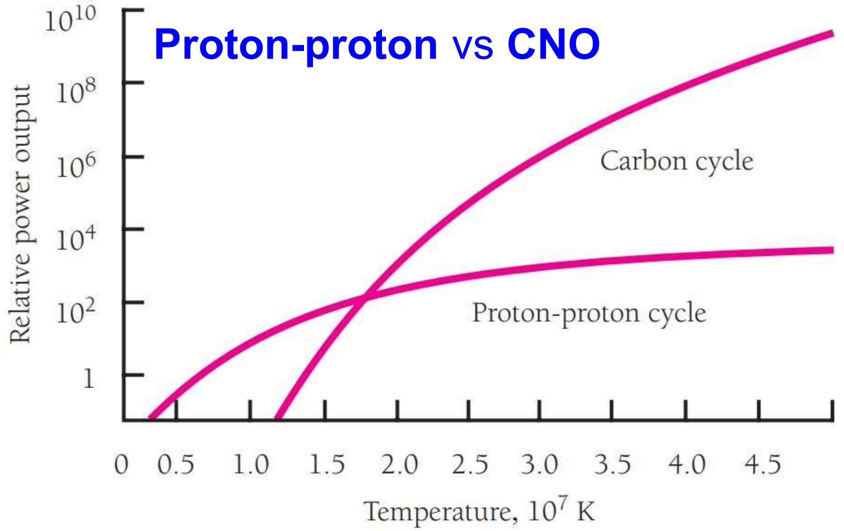
$$^{13}C + {}^{1}H \longrightarrow {}^{14}N + \gamma$$

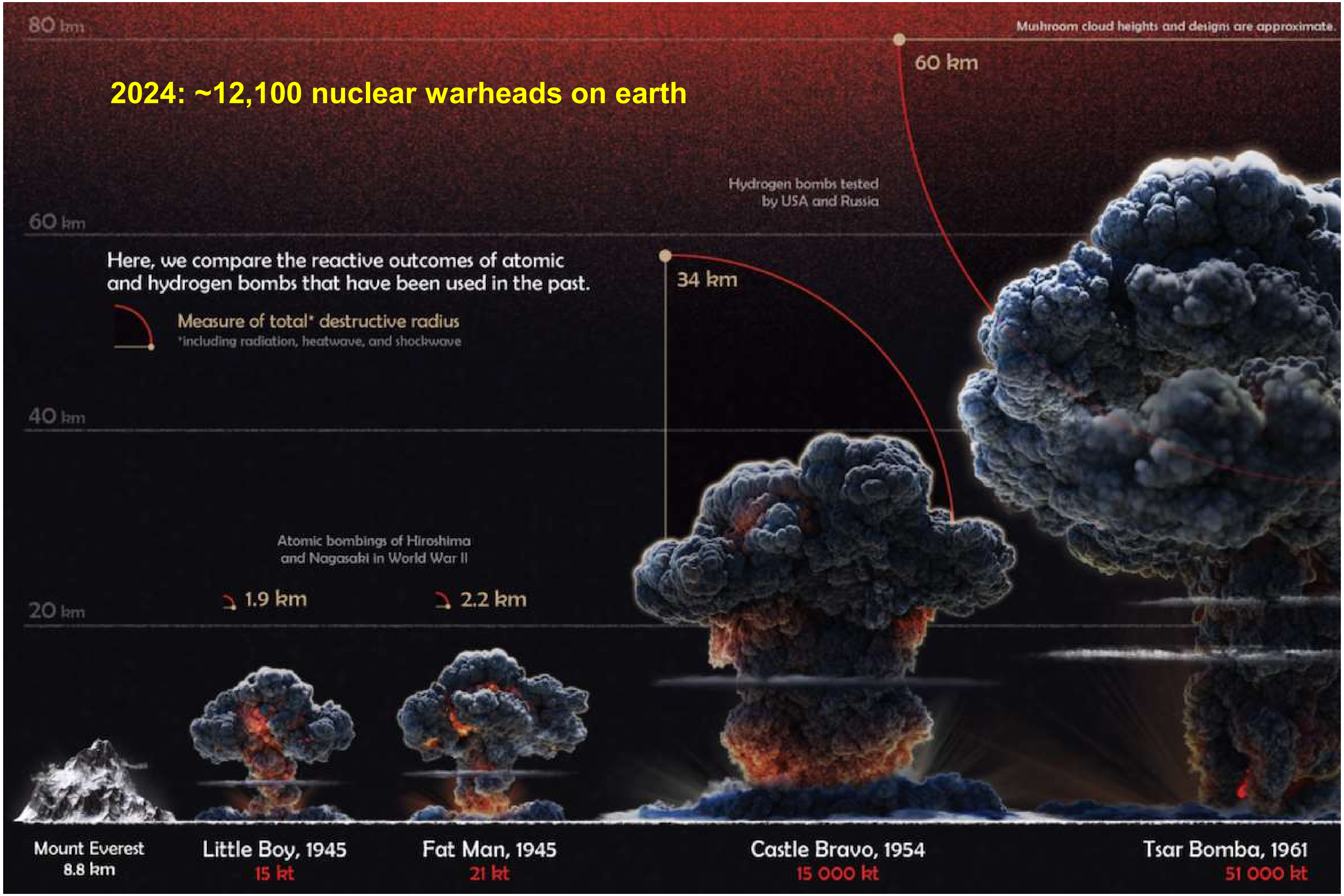
$$^{14}N + {}^{1}H \longrightarrow {}^{15}O + \gamma$$

$$^{15}O \longrightarrow {}^{15}N + e^{+} + \nu$$

$$^{15}N + {}^{1}H \longrightarrow {}^{12}C + {}^{4}He$$

Helium out





https://datainnovation.org/wp-content/uploads/2022/03/2022-Nuclear-Bombs.png

Artificially controlled thermonuclear fusion

$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow n + {}^{3}\mathrm{He} \quad Q = 3.3\,\mathrm{MeV}$$
$$^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow p + {}^{3}\mathrm{H} \quad Q = 4.0\,\mathrm{MeV}$$
$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow n + {}^{4}\mathrm{He} \quad Q = 17.6\,\mathrm{MeV}$$

Three main conditions are necessary for controlled nuclear fusion:

1) The temperature must be hot enough to overcome the Coulomb barrier and fuse their nuclei together. This requires a temperature of 100–200 million K.
2) The ions have to be confined together in close proximity to allow the ions to fuse. A suitable ion density is 2–3 × 10^{20} ions/m^3.
3) The ions must be held together in close proximity at high temperature long enough to avoid plasma cooling. A suitable time is 1–2 s.

Ex 13.9: Calculate the ignition temperature needed for the reaction

$$^{2}\mathrm{H} + {}^{3}\mathrm{H} \rightarrow n + {}^{4}\mathrm{He} \qquad Q = 17.6\ \mathrm{MeV}$$

Solution We need to calculate how much thermal energy is needed to overcome the Coulomb barrier. We will use 3 fm as the distance where the nuclear force first becomes effective.

The Coulomb potential energy that must be overcome is

$$V = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$= \frac{(9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C})^2}{3 \times 10^{-15}\ \mathrm{m}} = 7.7 \times 10^{-14}\ \mathrm{J}$$

The ignition temperature

$$T = \frac{2V}{3k} = 3.7 \times 10^9\ \mathrm{K}$$

- This is an overestimate. A more appropriate distance to use could be as great as 5 fm, which would result in a lower temperature.
- Also, the distribution of energies for plasma follows a statistical process. far out on the tail of the distribution, there are many particles with energies several times greater than 3/2kT. More accurate ignition temperature estimates for the D +T fusion reaction are in the range of 100–200 million K

ITER: International Thermonuclear Experimental Reactor

Giant international fusion project is in big trouble

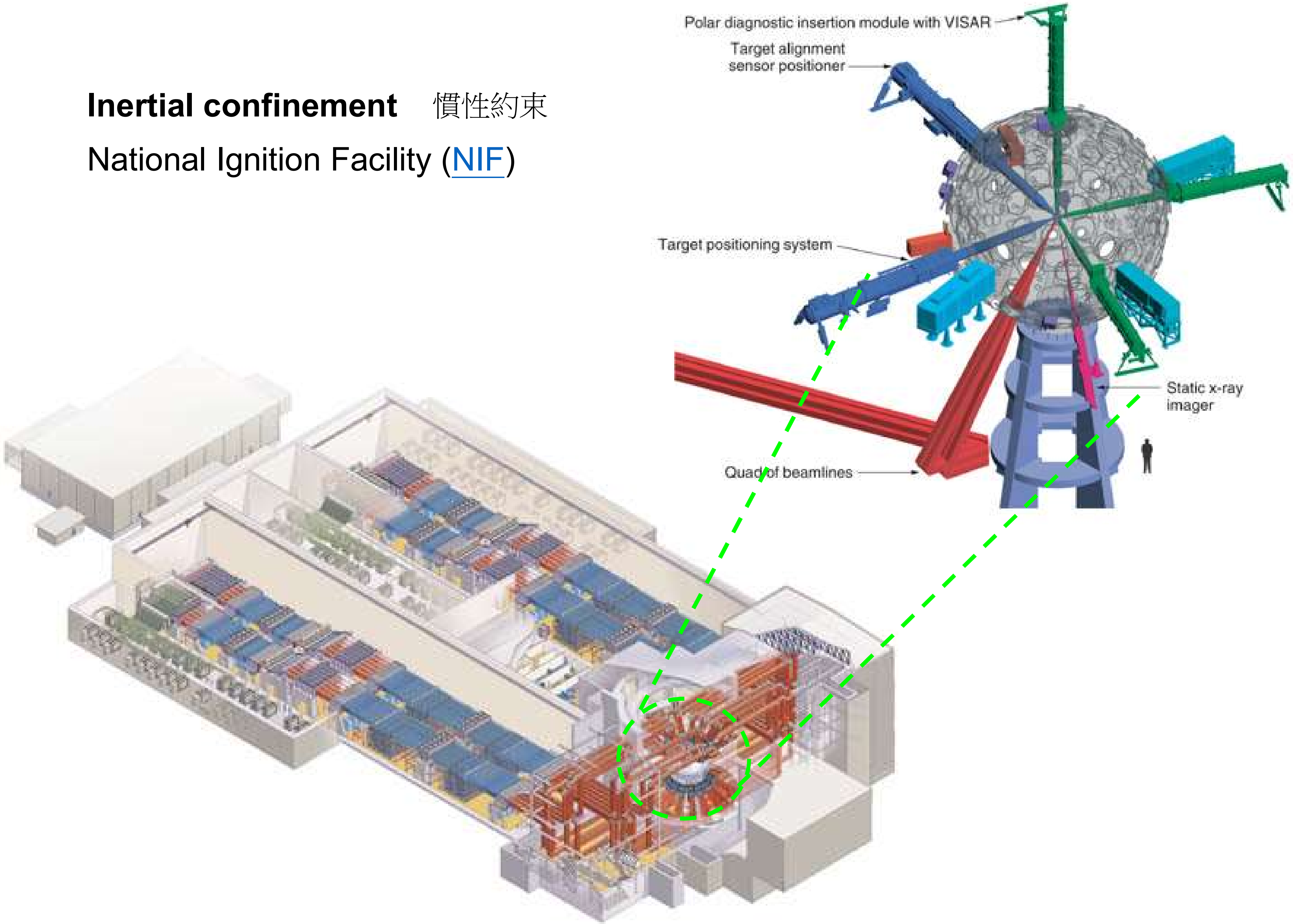
ITER operations delayed to 2034, with energy-producing reactions expected 5 years later

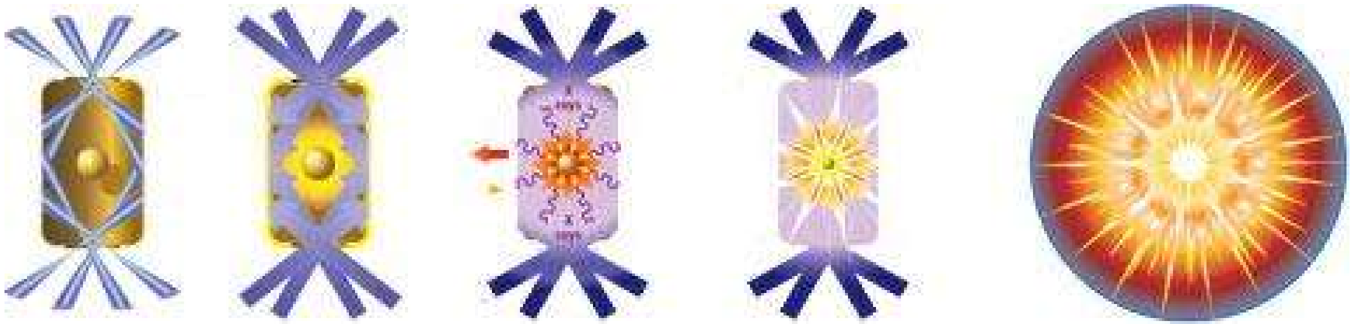
3 JUL 2024 · 4:30 AM ET · BY DANIEL CLERY

Science 2024

Inertial confinement 慣性約束

National Ignition Facility (NIF)



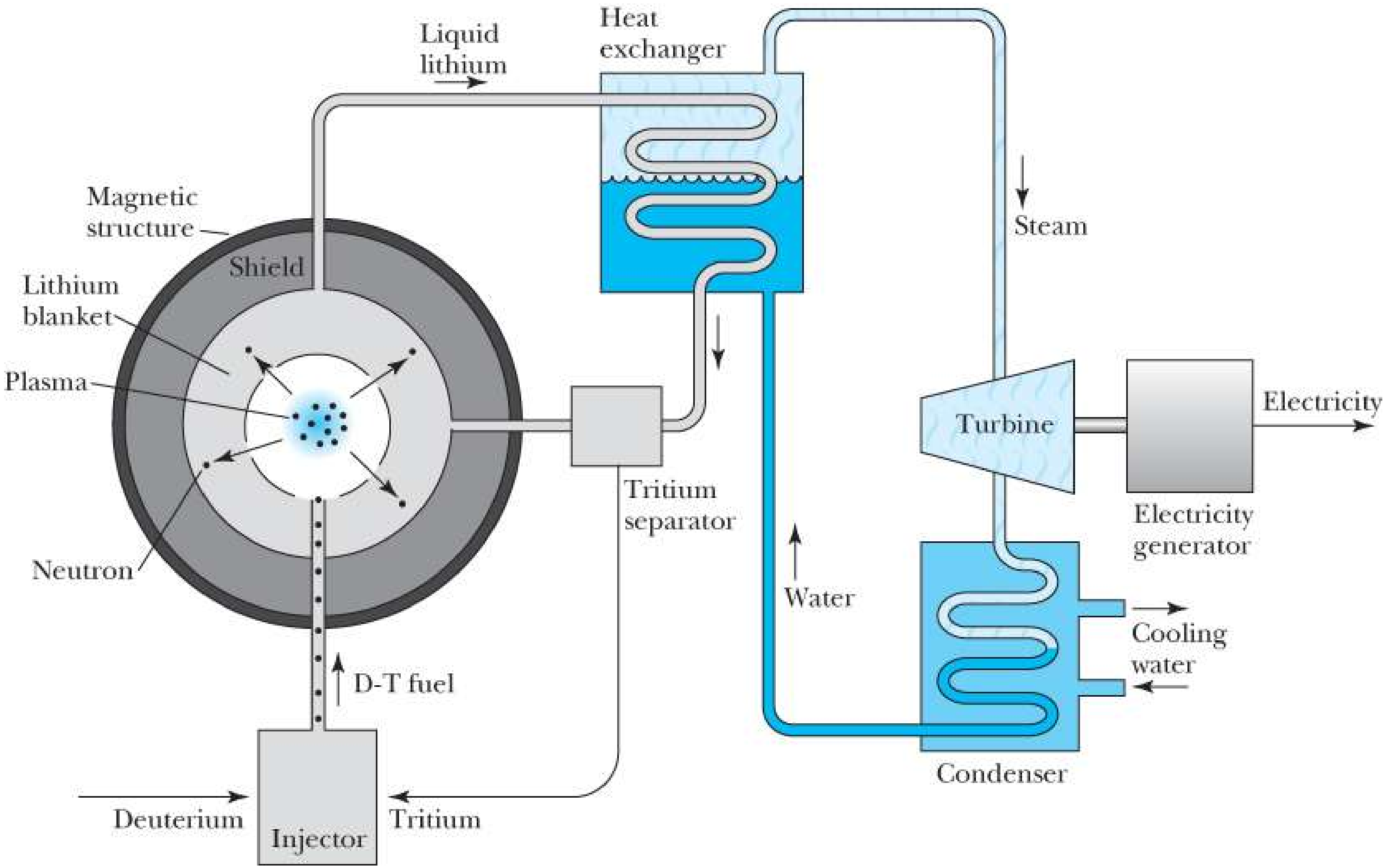


It's like a tiny hydrogen bomb.
Hotter than the center of the sun.

Aimed to create a single 500 terawatt (TW) peak flash of light from 192 lasers that reaches the target from numerous directions within a few picoseconds. Achieved the first instance of scientific breakeven controlled fusion in 2022 (Report from 60 minutes).

"*If ten 0.1-mg pellets are ignited every second, the average thermal output would be about 1 GW and could yield 300 MW or so of electric power, enough for a city of 175,000 people*." Beiser

Star Trek (2009) | Red Matter



"Humans are in a forever quest to find the most efficient way to boil water that spins something." a comment on youtube

One problem: T can only be produced by reactors. It’s current global reserve is estimated to be about 20 kgs only. ITER would need 300 gms/day to generate 800 MW (2% of France peak power consumption). Reactors can produce only about 100 gms/yr.

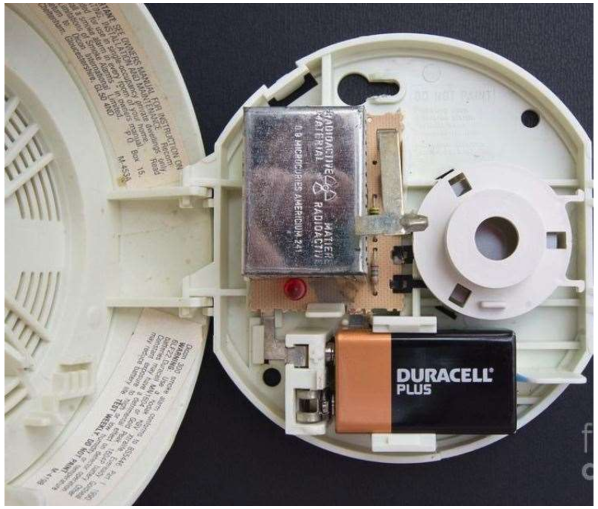
For more problems, see The Problem with Nuclear Fusion

(88 yrs half-life).

Some more (practical) applications of nuclear radioactivity

- Alpha-emitting radioactive sources have been used as power sources in heart pacemakers (replaced by Li battery).

- Smoke detectors use ^{241}Am sources of alpha particles as current generators. The scattering of the alpha particles by the smoke particles reduces the current flowing to a sensitive solid-state device, which results in an alarm.



- Spacecraft have been powered by radioisotope thermoelectric generators since the early 1960s. (Voyager 1, 470 W at launch from ^{238}Pu, half life 87.7 yrs)

Voyager 1: the most distant human-made object from Earth (~ 160 AU at 2024)

A pellet of $^{238}PuO_2$ as used in the RTG for the Cassini and Galileo missions