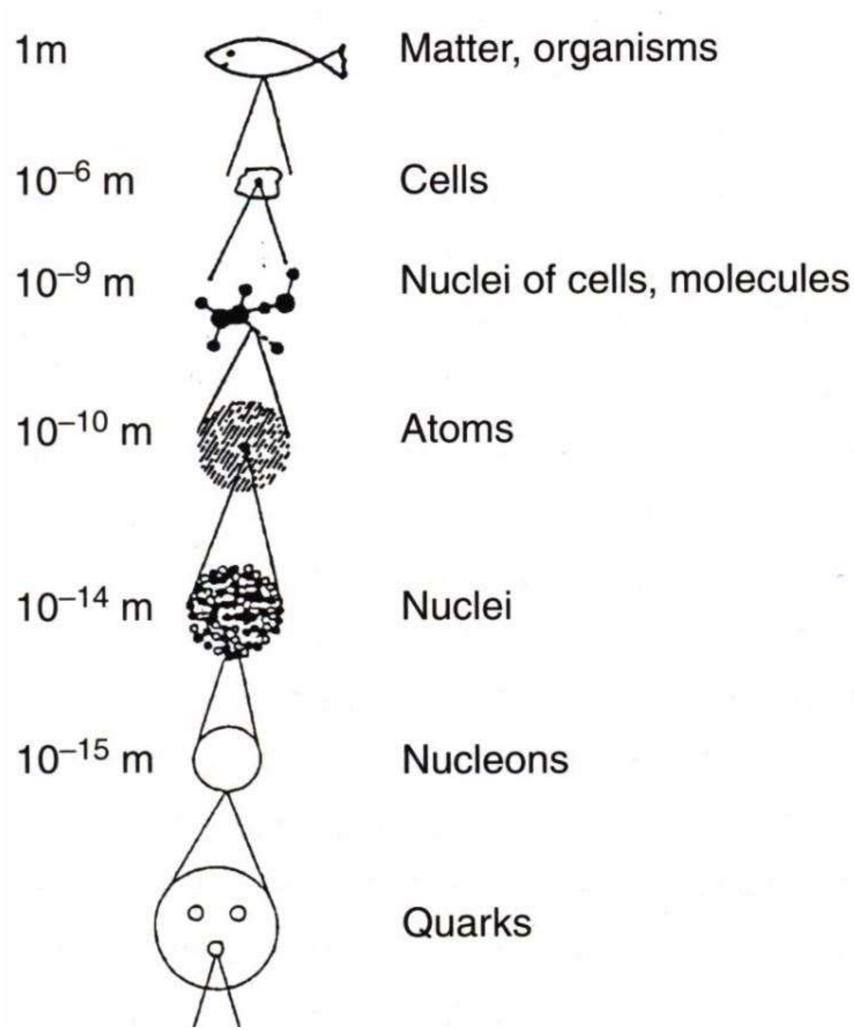
Chap 8
(atomic
physics)

Macro:
(Chaps 9, 10, 11)

Micro:
(Chaps 12, 13, 14)



Chap 12

The atomic nucleus

- Discovery neutron, deutrium
- Mass and radius of nucleus
- Magnetic moment, NMR
- Nucleus stability
- Nucleus models
- Radioactive decay
- Alpha, beta, and gamma decay

Rutherford proposed that the nuclei has two kinds of particles: protons and electrons. E.g.,, a ^{4}He nucleus has 4 protons and 2 electrons.

Problems of Rutherford's proposal:

1) Nuclear size

   Uncertainty principle: An electron confined to a box of nuclear dimensions have an energy larger than 20 MeV, whereas electrons emitted during beta decay have energies 2 or 3 MeV only.

2) Nuclear spin 氚

   If a deuteron (nucleus of a deuterium) consists of protons and electrons, the it must contain 2 protons and 1 electron. A nucleus composed of 3 fermions must have a half-integral spin. But according to the evidence from atomic spectroscopy, its spin is 1.

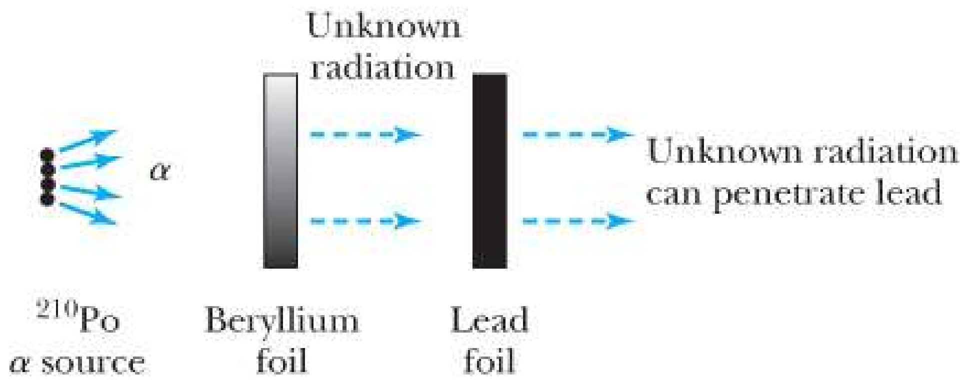
"*Despite these difficulties, the hypothesis of nuclear electrons was not universally abandoned until the discovery of the neutron in 1932.*"

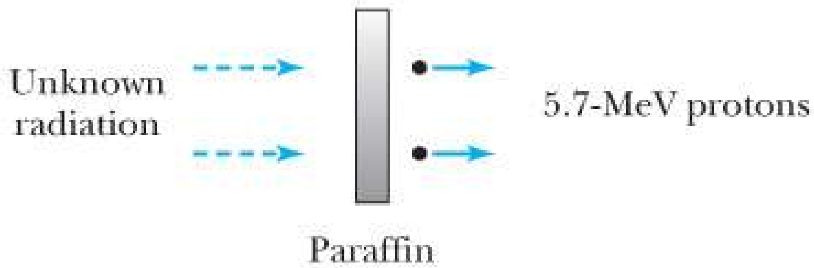
Beiser

Discovery of **neutron** (Chadwick 1932) 釙

- In 1930, Bothe and Becker used a polonium source that emitted α

鈹 particles to bombard a beryllium foil, the emitted ray (electrically neutral) could penetrate several centimeters of lead.





- Curie and Joliot passed this new ray through paraffin (which contains H). They

石蠟 found that protons with energies up to 5.7 MeV were ejected. They thought that it's due to γ rays. But this requires γ rays with very high energy (KeV ~ MeV).

- 1932, Chadwick studied the recoil velocities of different nuclei due to this new ray. He concluded that the neutral ray consists of particles with mass similar to proton’s.

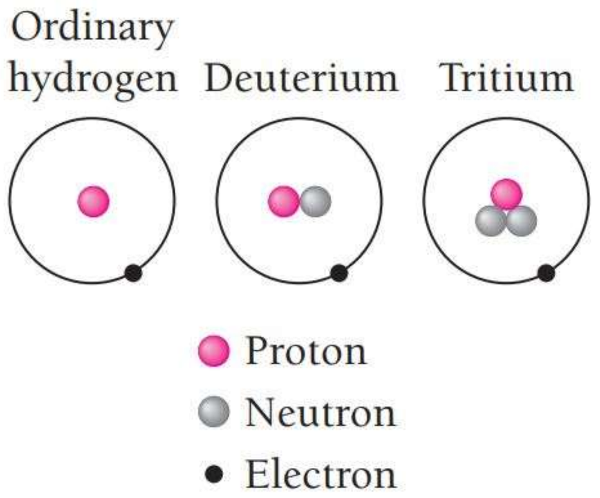
- 1932, Discovery of heavy hydrogen, or **deuterium,** by Urey.

  Compared to hydrogen, deuterium has a slight shift of spectral lines due to a different reduced mass. (there is little of this isotope ~ 0.015%). Ref: Chap 4

氘 • **Deuterium**: Has one proton and one neutron in its nucleus.

氚 • **Tritium**: Has one proton and two neutrons.

Tritium was first detected in 1934 by Rutherford *et al* after bombarding deuterium with deuterons.



同位素

- **Isotopes**: nuclei has the same Z, but different N's

The symbol of an atomic nucleus is $^{A}_{Z}X_{N}$,

where Z = atomic number (number of protons)

N = neutron number

A = mass number Z + N

原子質量單位

- **Atomic mass unit** *u* is defined so that the mass of a ^{12}C atom, the most abundant isotope of carbon, is exactly 12 u.

$1\ \text{u} = 1.66054 \times 10^{-27}\ \text{kg} = 931.49\ \text{MeV}/c^2$

Particle	Mass (kg)	Mass (u)		Mass (MeV/c^2)
Proton	1.6726×10^{-27}	1.007276	6p + 6n > ^{12}C	938.28
Neutron	1.6750×10^{-27}	1.008665		939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}		0.511
$^{1}_{1}$H atom	1.6736×10^{-27}	1.007825		938.79

Nucleus radius

- From the scattering of electrons off a nucleus, Rutherford estimated that the range of the nuclear force < 10^{-14} m.
- It is expected that

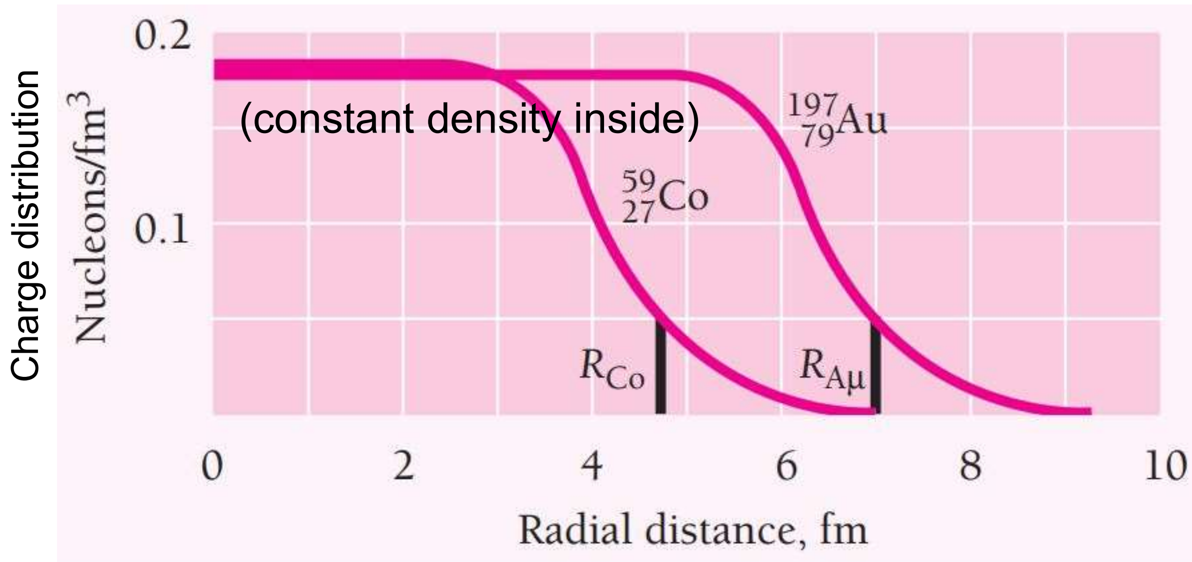
$$R = R_0 A^{1/3}, \qquad R_0 \simeq 1.2 \times 10^{-15}\ \text{m} = 1.2\ \text{fm}$$

1 fm = 10^{-15} m

For example, the radius of a ^{12}C nucleus is

$$R \simeq 1.2(12)^{1/3} \simeq 2.7\ \text{fm}$$

- Later it was found that range of nuclear force ≈ charge radius



Nucleus magnetic moment

- The magnetic moment of a nucleus is expected to be 1836 times smaller than electron's. But how do we determine them?

$$\boxed{\vec{\mu} = g_s \frac{-e}{2m} \vec{S}}$$

- Stern-Gerlach experiment on nucleus spin (1933)

  "*Pauli told Stern that if he enjoyed doing difficult experiments, he could do them, but it's a waste of time and effort because the result was already known* (g_s=2)."

  Segre

- From the experiment, it was found that

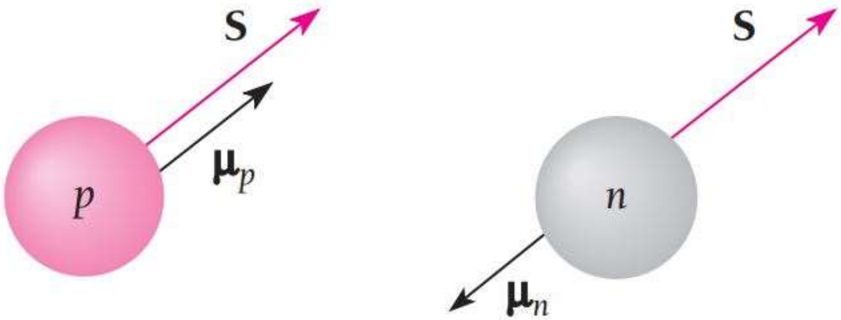
  The proton magnetic moment $\mu_p = 2.79\ \mu_N$. A surprise.

  The neutron magnetic moment $\mu_n = -1.91\ \mu_N$. Another surprise.

Nuclear magneton 核磁元

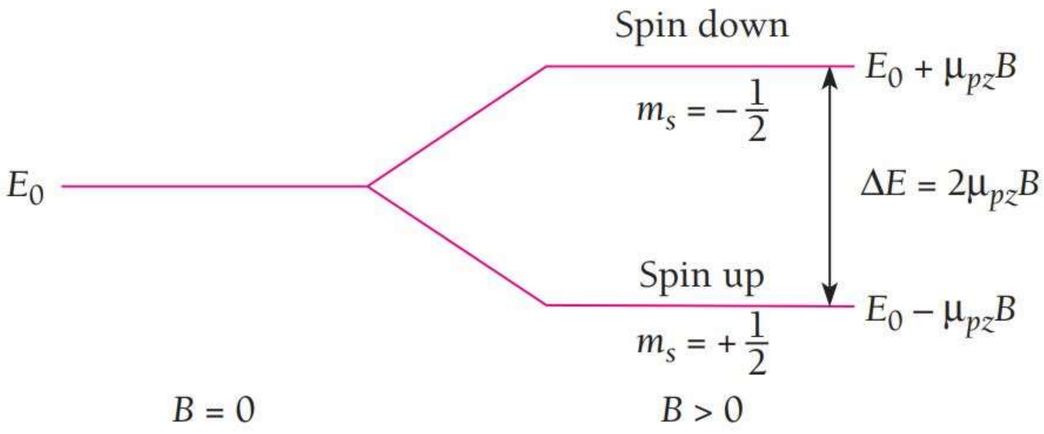
$$\mu_N = \frac{e\hbar}{2m_p} = 5.051 \times 10^{-27}\ \mathrm{J/T} = 3.152 \times 10^{-8}\ \mathrm{eV/T}$$

$$\boxed{\mu_B = \frac{e\hbar}{2m_e}}$$



The nonzero neutron magnetic moment implies that the neutron has negative and positive internal charge components at different radii.

Proton in magnetic field



(*a*) Find the energy difference between the spin-up and spin-down states of a proton in a magnetic field of $B = 1.000$ T (which is quite strong). (*b*) What is the Larmor frequency of a proton in this field? (resonant frequency)

Solution

(*a*) The energy difference is

$$\Delta E = 2\mu_{pz}B = (2)(2.793)(3.153 \times 10^{-8}\ \text{eV/T})(1.000\ \text{T}) = 1.761 \times 10^{-7}\ \text{eV}$$

If an electron rather than a proton were involved, ΔE would be considerably greater.
(*b*) The Larmor frequency of the proton in this field is

$$\nu_L = \frac{\Delta E}{h} = \frac{1.761 \times 10^{-7}\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 4.258 \times 10^{7}\ \text{Hz} = 42.58\ \text{MHz}$$

Beiser

Note: This is a good way to determine nuclear magnetic moments.

核磁共振

Nuclear magnetic resonance, NMR (Purcell, Bloch 1945)

NMR spectroscopy can be used to study the physical, chemical, and biological properties of matter (mostly through H or ^{13}C nuclei).

化學位移

- **Chemical shift**: different molecular electron distribution -> different local B field -> different NMR frequency
- **Medical NMR** (aka **MRI**, Lauterbur, Mansfield 1970s)
  - The hydrogen in water, fat … etc have different NMR frequencies and different relaxation times.
  - By changing the direction of B field gradient (1.5-3 T), we get an image that shows the proton density of H in a thin slice (3–4 mm) of the body.

For more, see a nice video: The Insane Engineering of MRI Machines

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- Radioactive decay
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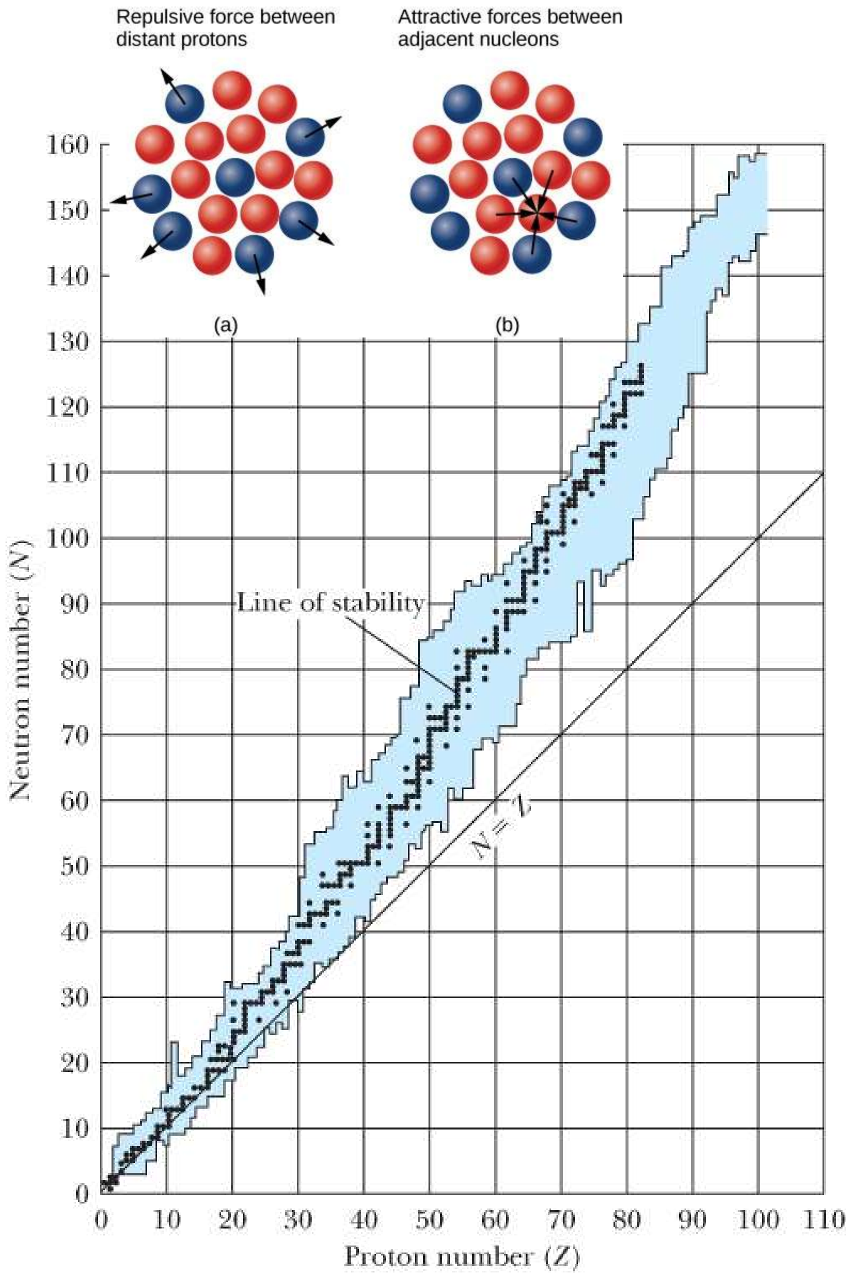
Nucleus stability

- For $Z \leq 20$, $Z \approx N$. For $Z \geq 20$, there is a preference for $N > Z$.
- The repulsion between protons becomes so great when $Z > 20$, so more neutrons is required for stability.
- Sixty percent of stable nuclides have both even *Z* and even *N*.
- Nearly all the others have either even *Z* and odd *N* or odd *Z* and even *N*, with the numbers about equal.
- Only five stable odd-odd nuclides are known:

$$^{2}_{1}\mathrm{H},\ ^{6}_{3}\mathrm{Li},\ ^{10}_{5}\mathrm{B},\ ^{14}_{7}\mathrm{N}, \text{and}\ ^{180}_{73}\mathrm{Ta}$$

- All nuclei with $Z>83$ and $A>209$ would decay spontaneously by emitting α particles



Binding energy of nucleus

Binding energy (>0)

- The mass of a deuteron is $m_d = m_p + m_n - B_d / c^2$

Mass of $^1_1$H atom	1.007825 u
+ mass of neutron	+1.008665 u
Expected mass of $^2_1$H atom	2.016490 u

But measured mass of $^2_1\mathrm{H} = 2.014102\ \mathrm{u}$

$$B_d / c^2 = 0.002388\ \mathrm{u}$$

u = 931.5 MeV/c^2

$$\Leftrightarrow 2.224\ \mathrm{MeV}$$

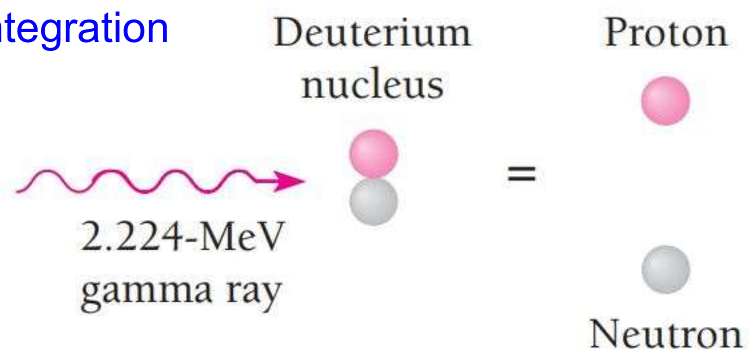
- Nuclear binding energies are very high. a typical binding energy is $8\mathrm{x}10^{11}$ kJ/kg.

  By contrast, the heat given off by burning gasoline is only $4.7\mathrm{x}10^{4}$ kJ/kg.

- Experimental confirmation by photo-disintegration

Use gamma rays to break a deuteron

$$\gamma + d \rightarrow n + p$$



The binding energy of a nucleus ${}^{A}_{Z}X$ against dissociation into any other possible combination of nucleons, for example nuclei R and S, is

$$B = [M(R) + M(S) - M({}^{A}_{Z}X)]c^2$$

Ex 12.5:

Show that the nuclide ^{8}Be has a positive binding energy but is unstable with respect to decay into two alpha particles.

Solution The binding energy of ^{8}Be is

$$B(^8\text{Be}) = [4m_n + 4M(^1\text{H}) - M(^8\text{Be})]c^2$$

$$= [4(1.008665\text{ u}) + 4(1.007825\text{ u}) - 8.005305\text{ u}]$$

$$\times\, c^2\left(\frac{931.5\text{ MeV}}{c^2\cdot\text{u}}\right) = 56.5\text{ MeV}$$

$$B(^8\text{Be} \to 2\alpha) = [2M(^4\text{He}) - M(^8\text{Be})]c^2$$

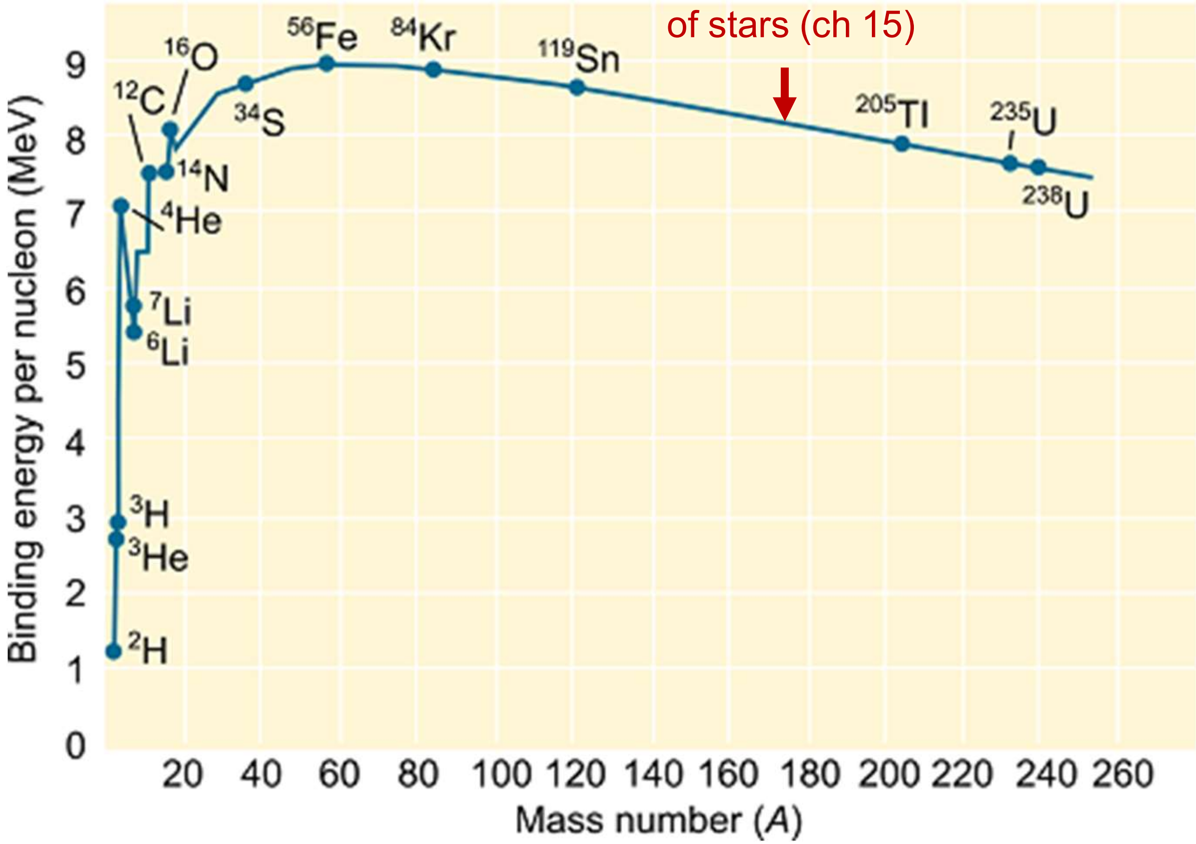
$$= [2(4.002603\text{ u}) - 8.005305\text{ u}]$$

$$\times\, c^2\left(\frac{931.5\text{ MeV}}{c^2\cdot\text{u}}\right) = -0.093\text{ MeV}$$

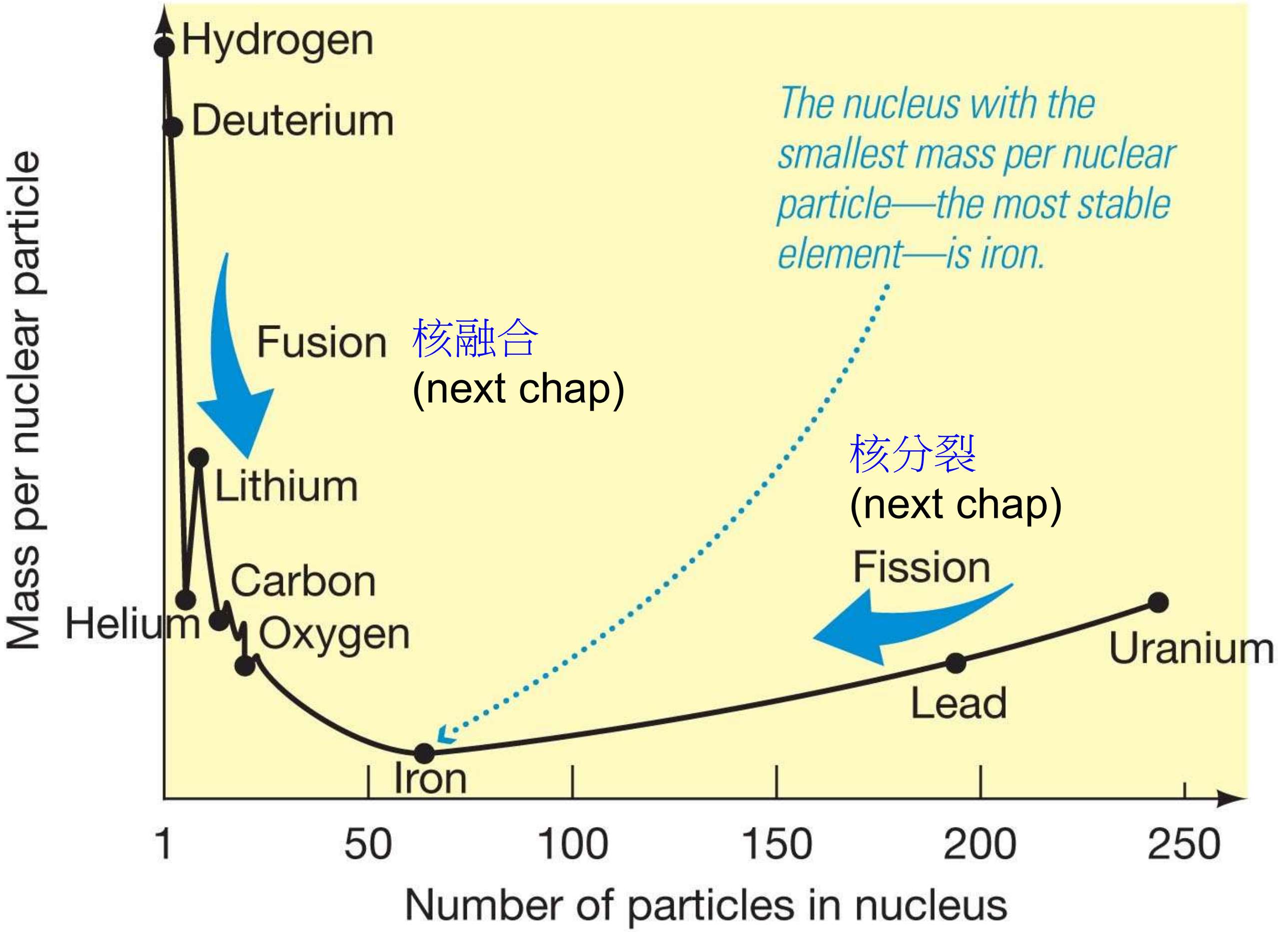
The instability of ^{8}Be is responsible for the fact that stars consist mostly of hydrogen and helium. Because of the instability of ^{8}Be, it is difficult for helium nuclei to join together to make heavier nuclei.

Binding energy per nucleon

Crucial for the death of stars (ch 15)



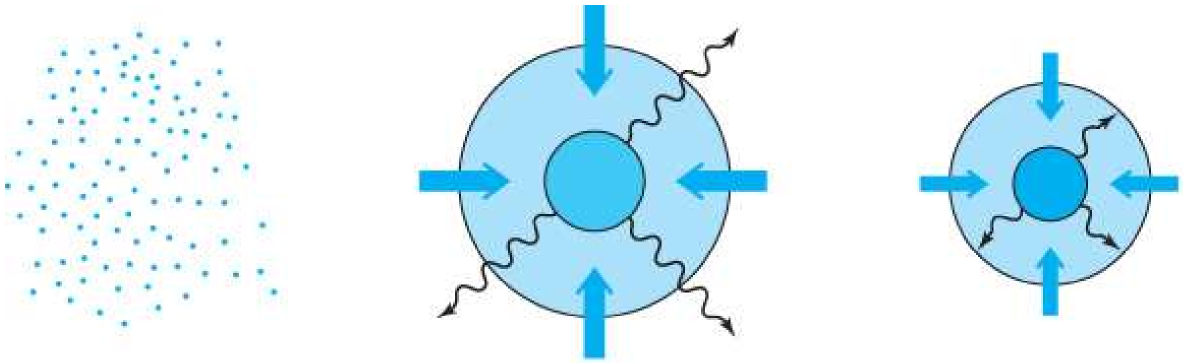
- The range is about 1.1 MeV (^{2}H) to 8.8 MeV (^{56}Fe).
- Note the saturation effect of nuclear force.
- Sharp peaks for the even-even nuclides ^{4}He, ^{12}C, and ^{16}O (tightly bound).


Hydrogen
Deuterium
Fusion
核融合
(next chap)
Lithium
Helium
Carbon
Oxygen
Iron
The nucleus with the smallest mass per nuclear particle—the most stable element—is iron.
核分裂
(next chap)
Fission
Uranium
Lead
Mass per nuclear particle
1
50
100
150
200
250
Number of particles in nucleus

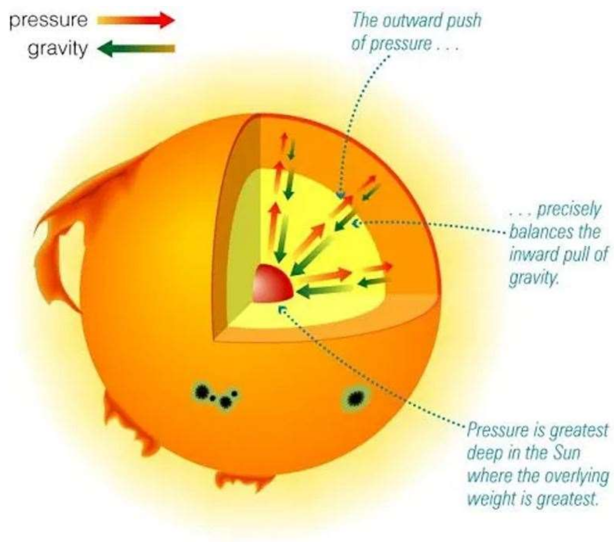
optional

Birth of stars

- As the universe cooled, gravitational forces attracted the matter into gaseous clouds, which formed the basis of stars.
- This process continued as the interior temperature and density of these clouds increased. Nuclear fusion began when the temperature reached 10^7 K.
- Initially, fusion created helium from the hydrogen nuclei.



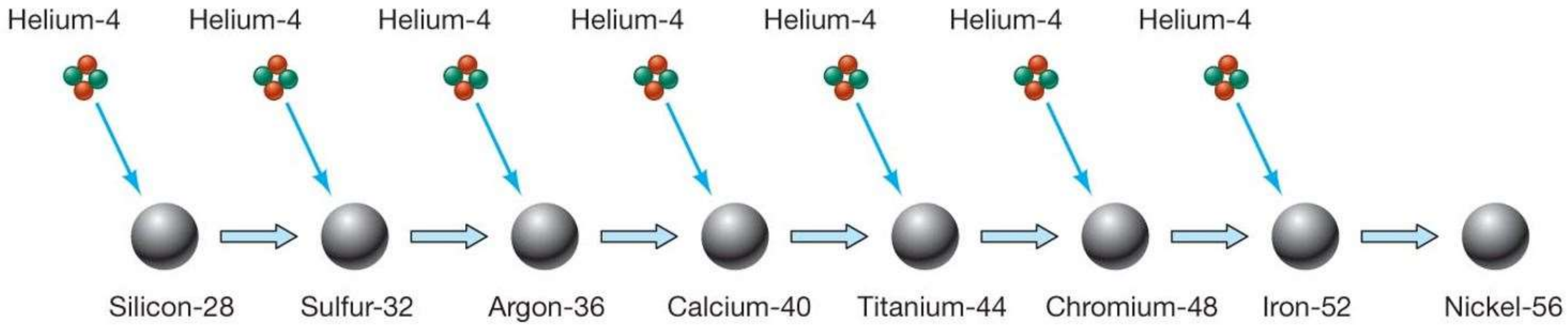
(a) (b) (c)



Sun, currently, H 70%, He 28%

optional

Evolution of stars



Temperature range for fusion processes

- 10^8K Helium “burning” → C (triple-α process), O 0.5 M
- 5 x 10^8K Carbon "burning" → O, Ne, Mg, Na 4 M
- 10^9 K Neon "burning" → O, Mg 8 M
- 1.5 x 10^9K Oxygen “burning” → Mg, Si, S, P > 8 M
- 2.7 x 10^9K Silicon "burning" → Fe and others (Ca, Ti, Cr, Ni...)

➡ Red giant (white dwarf), supernova (neutron star, black hole)

Our sun

Properties of the nuclear force (aka strong interaction)

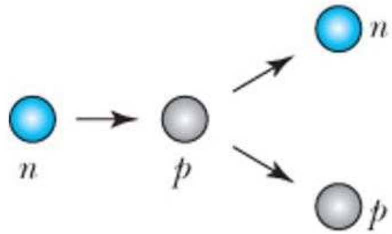
- Magnitude: The nuclear force holds nucleons together. If it was stronger by 1%, two protons could stick together without any neutrons needed, and the universe would be totally different (more next page).
- Range: Up to about 3 fm, the nuclear attraction between two protons is about 100 times stronger than the electric repulsion.

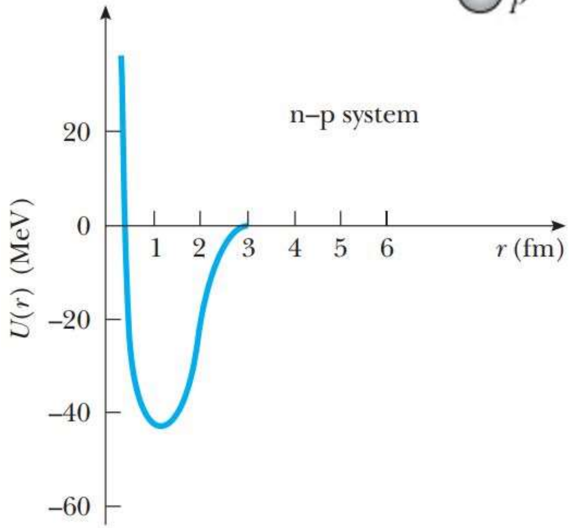
同位旋對稱

- Isospin symmetry: The nuclear force between two nucleons (n,p) is independent of their charges, $F_{pp} = F_{np} = F_{nn}$.

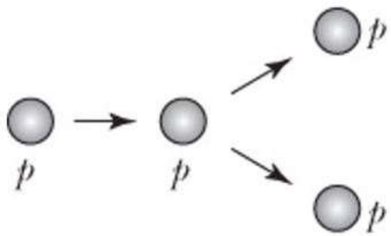
Nuclear potential

- **n-p**



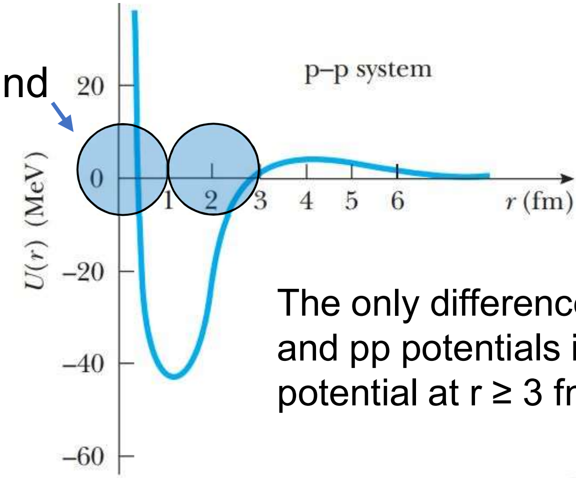


- **p-p**



$r_p \simeq 0.9$ fm

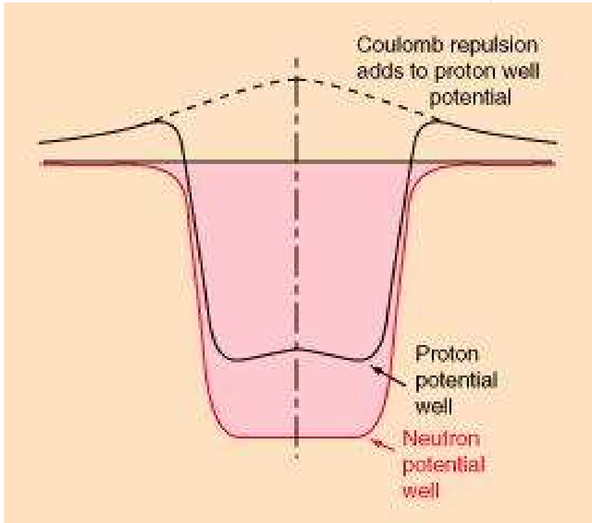
Fail to bind



The only difference between np and pp potentials is the Coulomb potential at r ≥ 3 fm.

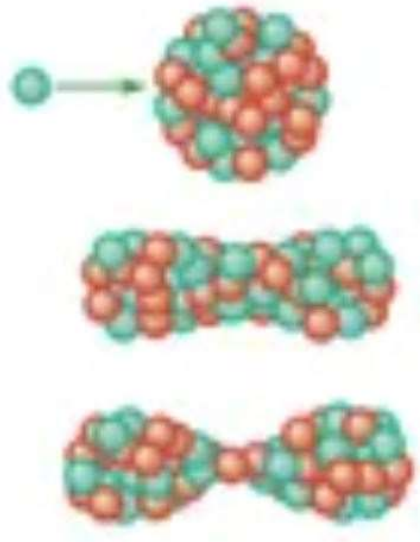
- **n-n**

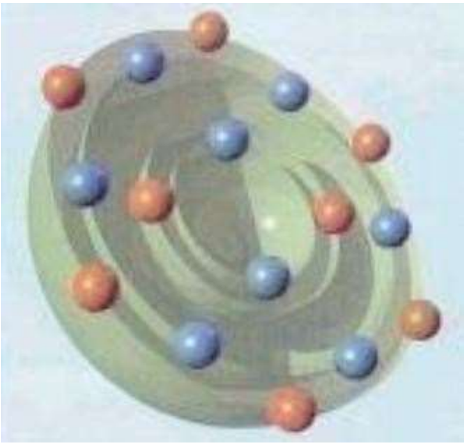
The n-n system is more difficult to study because free neutrons are not stable and could decay in about 15 minutes.



Models of nucleus

- The liquid drop model
- Fermi gas model
- The shell model
- …

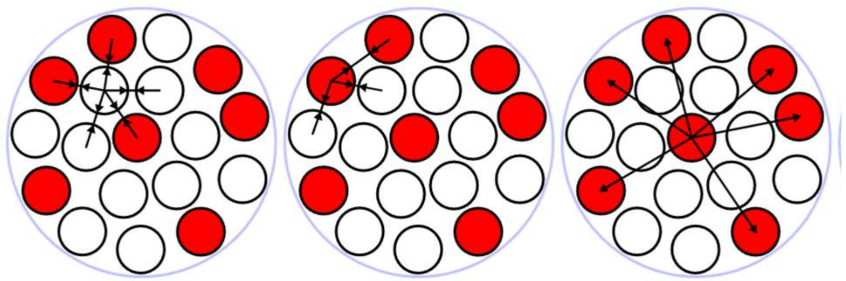




Liquid drop model (Gamow 1929, Weizsacker 1935)

Why not solid model?

A calculation shows that the vibrations of the nucleons about their average positions would be too great for the nucleus to be stable.



A semi-empirical formula for binding energy (positive):

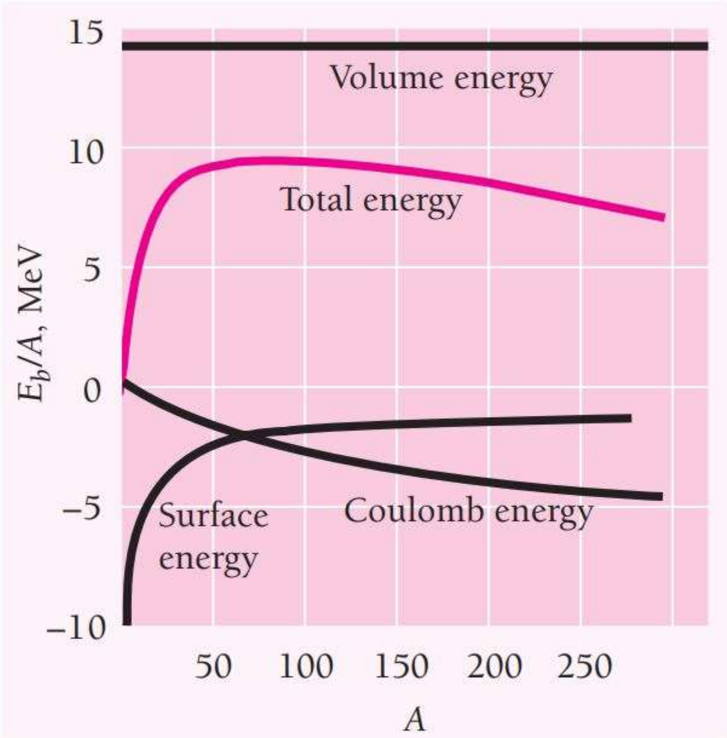
- Volume energy (A=mass number) $E_1 = a_1 A$
- Surface energy (less binding energy) $E_2 = -a_2 A^{2/3}$

  The number of nucleons on surface (with fewer bonds) is proportional to $A^{2/3}$

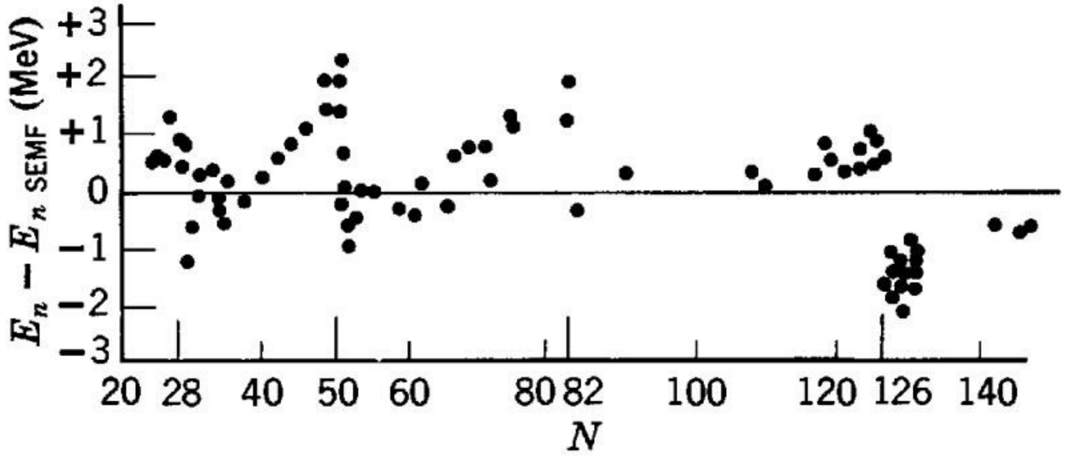
- Coulomb energy (less binding energy) $E_3 \simeq -\frac{Z(Z-1)}{2}\frac{e^2}{4\pi\varepsilon_0}\left\langle\frac{1}{r}\right\rangle = -a_3\frac{Z(Z-1)}{A^{1/3}}$

E_2 and E_3 are negative because they reduce nuclear stability.

$$\frac{B\left({}_{Z}^{A}X\right)}{A} \simeq 15.8 - 18.3\frac{1}{A^{1/3}} - 0.72\frac{Z(Z-1)}{A^{4/3}}$$



The binding energy of the last neutron (w.r.t the semiempirical formula with 5 parameters), as a function of the number of neutrons.



“The liquid drop model describes fairly accurately the masses of hundreds of nuclei in terms of only 5 parameters.” (3 here)

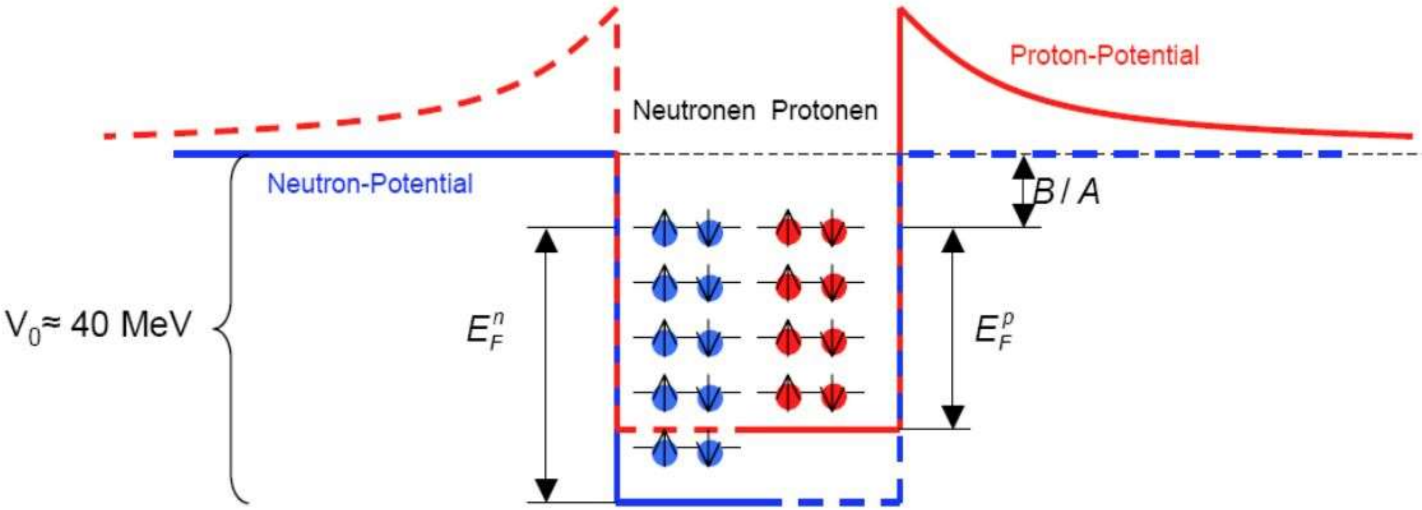
Eisberg and Resnick

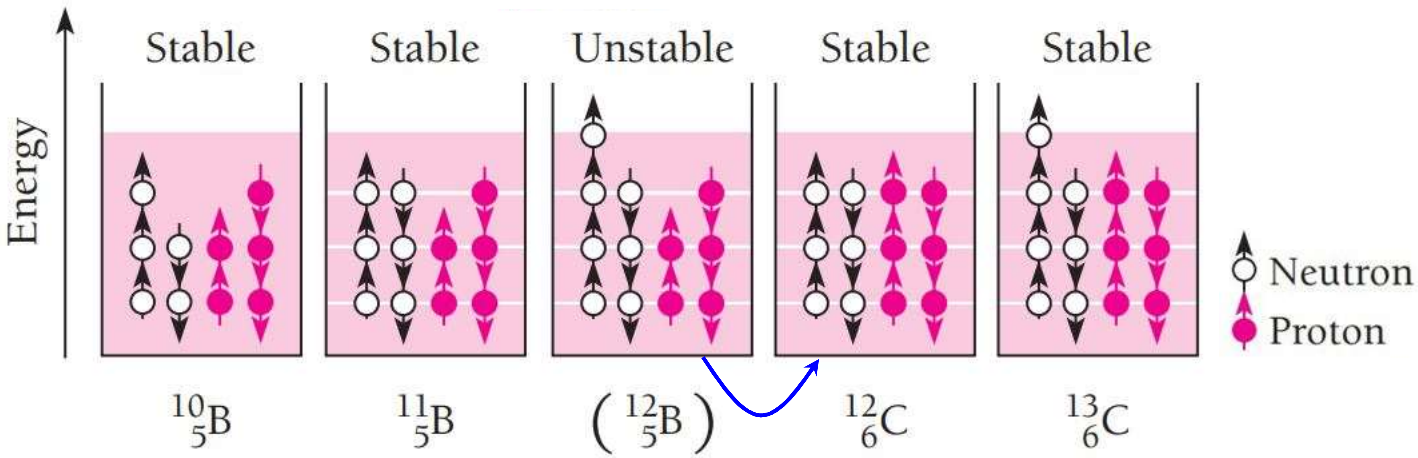
These data provide evidence for the magic numbers 2, 8, 20, 28, 50, 82, and 126, for neutrons. 魔數

The same magic numbers also for protons.

Fermi gas model, non-interacting (Fermi, Weisskopf)

Explains why even-even is preferred



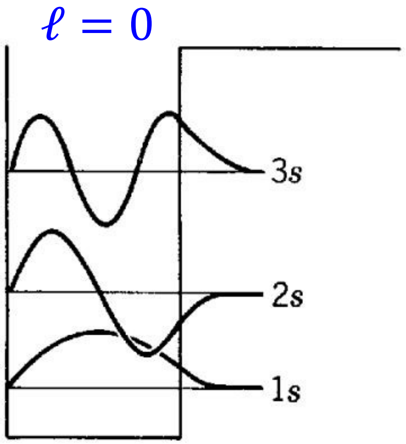


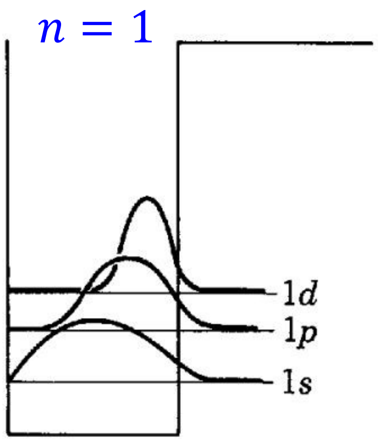
^{12}B nucleus changes by beta decay to
^{12}C nucleus in a fraction of a second.

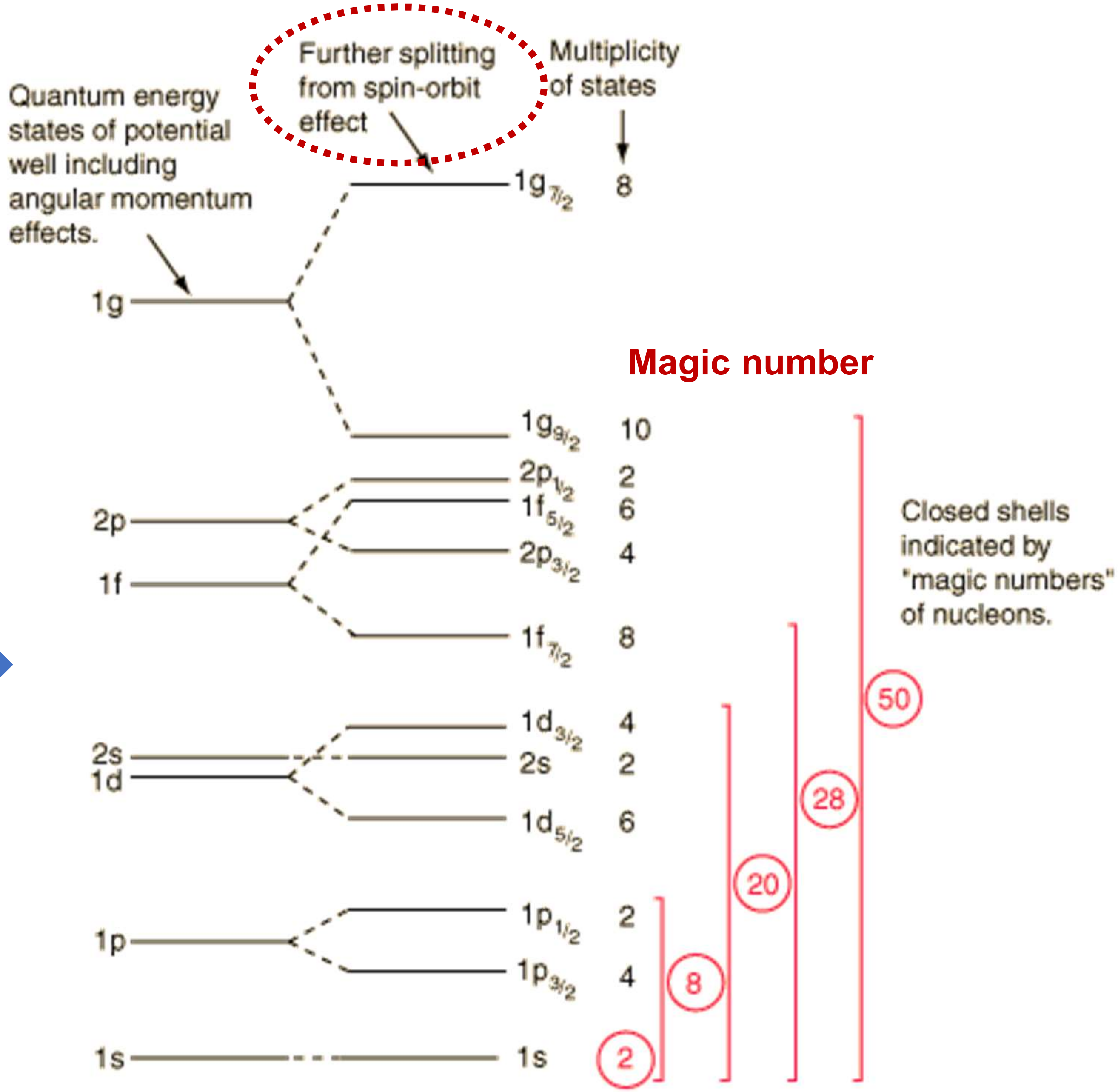
Nuclear shell model ~ atomic shell model

(Mayer, Jensen, 1949)

The 2nd female that got Nobel prize







Radioactive decay

- Decays rate

$$R = -\frac{dN}{dt} = \lambda N$$

λ: **decay constant**

吸收劑量	1 Gray (Gy) = 1 J/kg
等效劑量	1 Sievert (Sv) = 1 J/kg

貝克勒 **Becquerel** (Bq): 1 Bq = 1 decay / s

Curie (Ci): 3.7×10^{10} decays / s

- If $N(t)$ is the number of radioactive nuclei in a sample at time t, then

$$N(t) = N_0 e^{-\lambda t}$$

- Half-life $t_{1/2}$

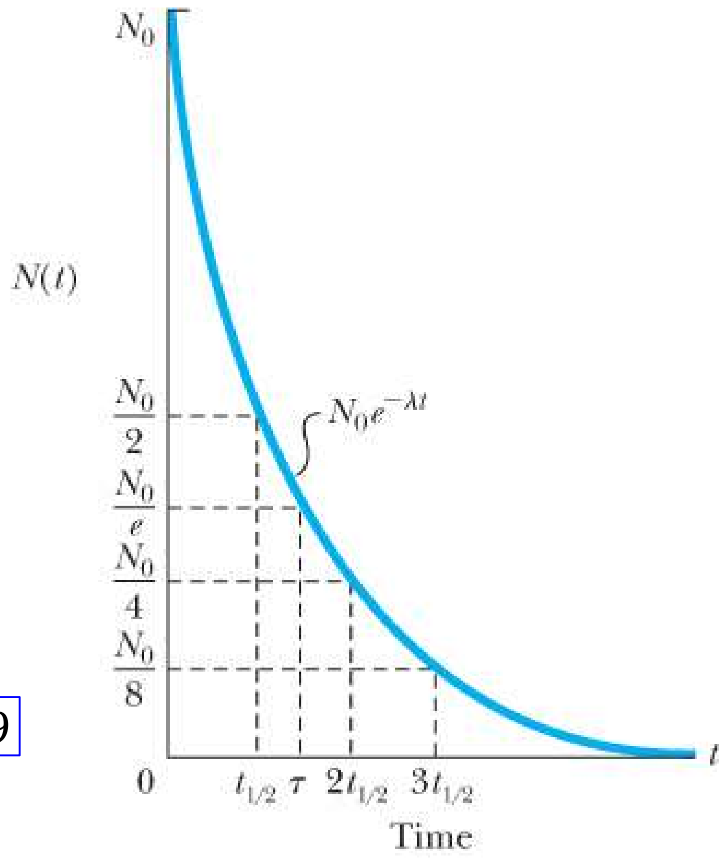
半衰期

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

The half-life is

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

$\boxed{\ln 2 = 0.69}$



Ex 12.10: A sample of ^{210}Po which α decays with $t_{1/2} = 138$ days is observed by a student to have 2000 disintegrations/s (2000 Bq).

(a) What is the activity in μCi for this source?

(b) What is the mass of the ^{210}Po sample?

Solution (a) We multiply the activity of 2000 decays/s by the factor that converts decays/s to Ci.

$$2000 \text{ decays/s}\left(\frac{1 \text{ Ci}}{3.7 \times 10^{10} \text{ decays/s}}\right) = 0.054 \times 10^{-6} \text{ Ci}$$

$$= 0.054 \text{ μCi}$$

(b) The number of radioactive nuclei.

$$N = \frac{R}{\lambda} = \frac{(R)(t_{1/2})}{\ln(2)} = 3.44 \times 10^{10} \text{ nuclei}$$

We use Avogadro's number to determine the mass from the number of atoms (nuclei).

$$\text{Mass} = 3.44 \times 10^{10} \text{ atoms} \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ atoms}} \frac{0.210 \text{ kg}}{1 \text{ mol}}$$

$$= 1.2 \times 10^{-14} \text{ kg}$$

Carbon dating 碳定年

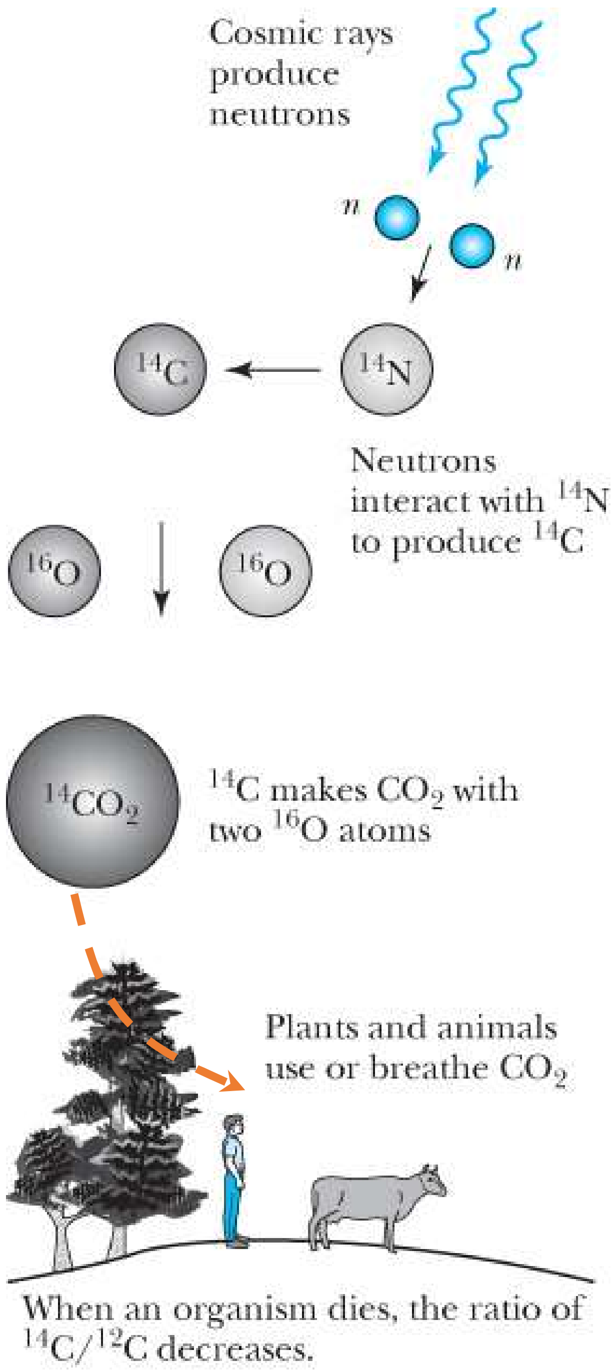
- Radioactive ^{14}C is produced in our atmosphere by the bombardment of ^{14}N by neutrons produced by cosmic rays.

$$n + {}^{14}\mathrm{N} \rightarrow {}^{14}\mathrm{C} + p$$

- When living organisms die, their intake of ^{14}C ceases, and the ratio of $^{14}C/^{12}C$ decreases as ^{14}C decays.

- The initial ratio of $^{14}C/^{12}C$ at the time of death was $R_0 = 1.2 \times 10^{-12}$

- Because the half-life of ^{14}C is 5730 years, it is convenient to use the $^{14}C/^{12}C$ ratio to determine the age of objects less than 45,000 years.

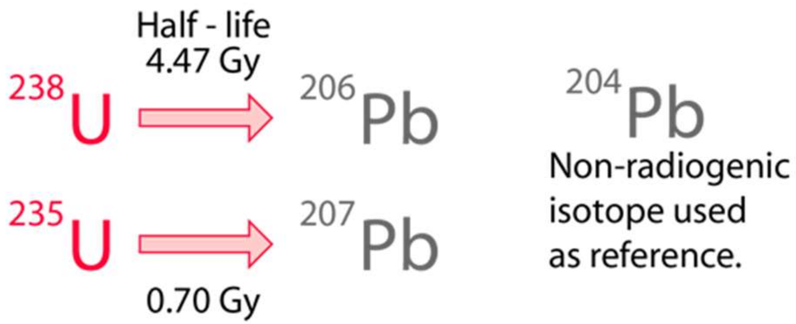


Shroud of Turin (Turin Cathedral, Italy)

In 1988, radiocarbon dating by three independent laboratories established that the shroud dates back to the Middle Ages, between the years 1260 and 1390.

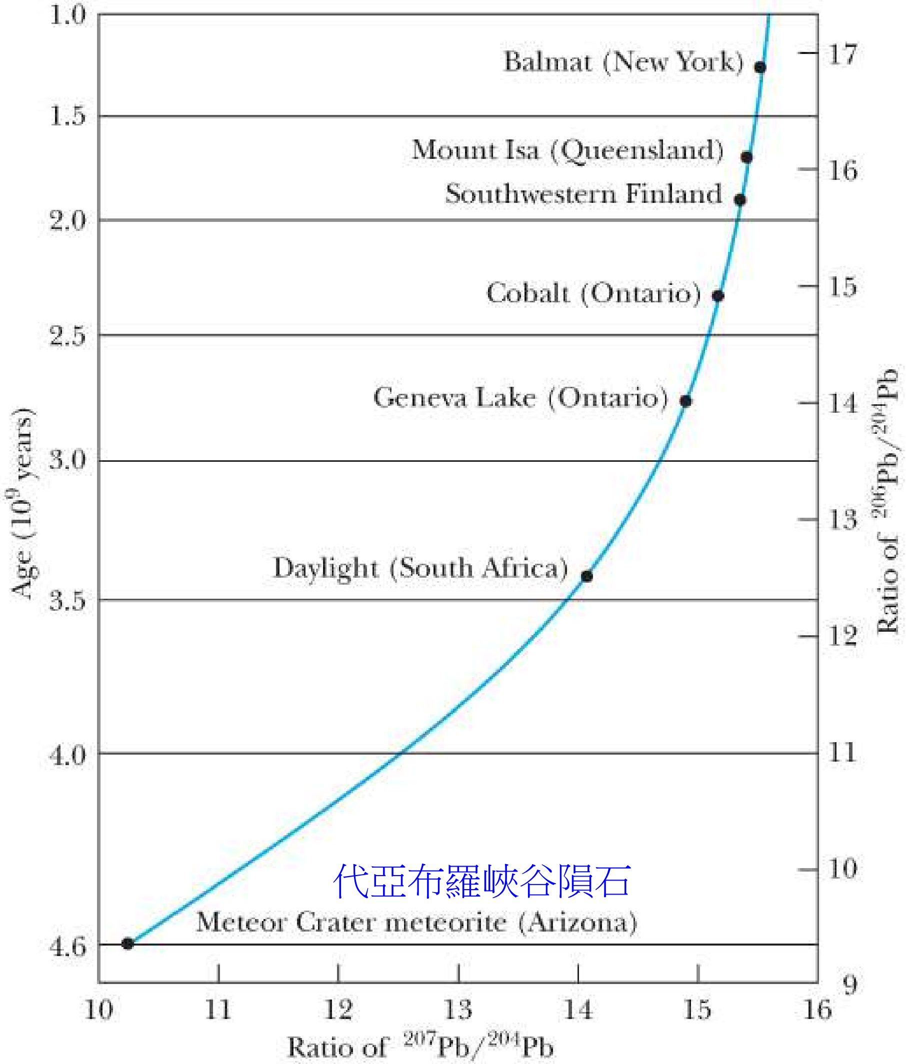
Lead-lead dating

- ^{204}Pb is stable, the other Pb isotopes are produced by U (or Th) decay:



- A plot of the abundance ratio of ^{206}Pb / ^{204}Pb versus ^{207}Pb / ^{204}Pb can be used to show that meteorites, leftovers from the formation of the solar system, are billions of years old.

The growth curve for lead ores from various deposits:



http://hyperphysics.phy-astr.gsu.edu/hbase/Nuclear/hhpb.html

Ex 12.19: A bone suspected to have originated during the period of the Roman emperors was found in Great Britain. Accelerator techniques gave its $^{14}C/^{12}C$ ratio as 1.10×10^{-12}. Is the bone old enough to have Roman origins?

AMS requires smaller sample sizes (~50 mg)

Strategy Remember that the initial ratio of $^{14}C/^{12}C$ at the time of death was $R_0 = 1.2 \times 10^{-12}$. We use the radioactive decay law to determine the time t that it will take for the ratio to decrease to 1.10×10^{-12}.

Solution The number of ^{14}C atoms decays as $e^{-\lambda t}$.

$$N(^{14}C) = N_0 e^{-\lambda t}$$

The ratio of ions is given by

$$R = \frac{N(^{14}C)}{N(^{12}C)} = \frac{N_0(^{14}C)e^{-\lambda t}}{N(^{12}C)} = R_0 e^{-\lambda t}$$

where R_0 is the original ratio. We can solve this equation for t.

$$e^{-\lambda t} = \frac{R}{R_0}$$

$$t = \frac{-\ln(R/R_0)}{\lambda} = -t_{1/2}\frac{\underline{\ln(R/R_0)}}{\ln(2)}$$

$$= -(5730 \text{ y})\left(\frac{\underline{-0.087}}{0.693}\right) = 720 \text{ y}$$

Ex 12.12: What is the alpha activity of a 10-kg sample of ^{235}U that is used in a nuclear reactor?

^{235}U has a half-life for emitting a particles of 7.04×10^8 y

Solution The number of ^{235}U atoms in a 10-kg sample is

$$N = M\frac{N_A}{M(^{235}\text{U})}$$

$$= (10\text{ kg})\left(\frac{10^3\text{ g}}{1\text{ kg}}\right)\left(\frac{6.02\times10^{23}\text{ atoms/mol}}{235\text{ g/mol}}\right)$$

$$= 2.56\times10^{25}\text{ atoms} = 2.56\times10^{25}\text{ nuclei}$$

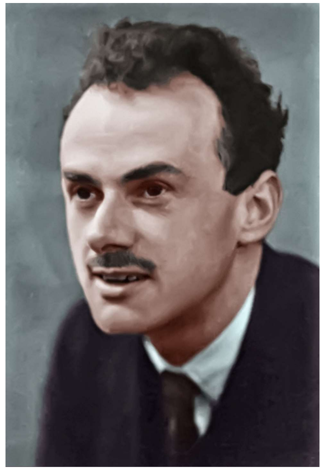
The activity is

$$R = \lambda N = \frac{\ln(2)\cdot N}{t_{1/2}}$$

$$= \frac{\ln(2)\cdot(2.56\times10^{25}\text{ nuclei})}{7.04\times10^8\text{ y}}$$

$$= 2.52\times10^{16}\text{ decays/y} = 8.0\times10^8\text{ Bq}$$

Theoretical discovery of **antiparticle** by Dirac (1929)



$$E = \frac{p^2}{2m}$$

→ Schrödinger eq. (1926)

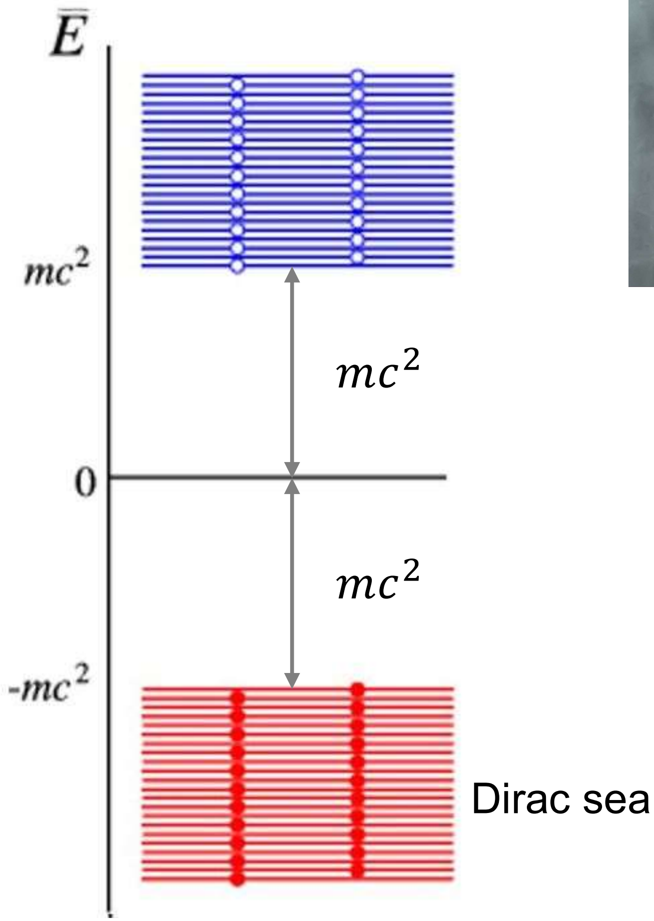
Quantum mechanics + Relativity

$$E^2 = p^2c^2 + m^2c^4$$

→ Klein-Gordon eq. (1926)
Spin-0 particle

$$E = \pm\sqrt{p^2c^2 + m^2c^4}$$

→ **Dirac eq.** (1928)
Spin-1/2 particle

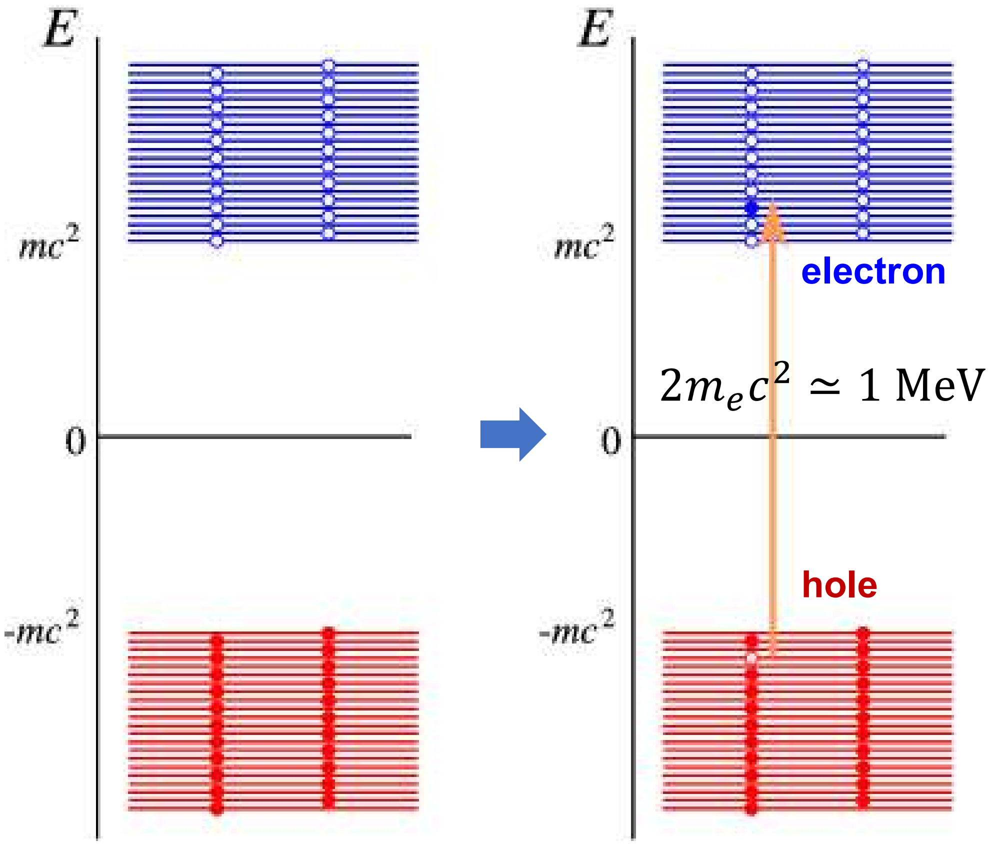


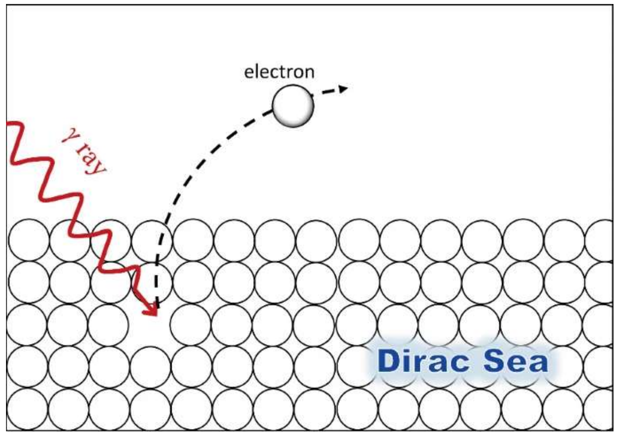
- The energy levels are found have two branches, one positive, one negative.
- To prevent electrons from falling to negative energy states, Dirac has to fill up the negative branch (this is the vacuum state).

https://oer.physics.manchester.ac.uk/NP/Notes/Notes/Notesse28.xht

Dirac sea: Filling negative energy states with electrons

- What happens if one lift an electron out of the Dirac sea?
  It creates a particle with positive charge in the sea.
- Dirac suggested that it is a proton, which was the only particle with positive charge known at that time. But its mass is wrong, so this became a puzzle.





A hole in Dirac sea has the same mass and spin as an electron, but with positive charge. It is called a positron.

正子

Antiparticles:

- Positron is the anti-particle of electron.
- All particles have antiparticles, except some neutral ones (e.g., photon, Z boson).
- An anti-particle has the same mass and lifetime as their associated particles. It has opposite electric charge (and some other quantum numbers).
- Particle and its anti-particle appear (and disappear) in pairs.

  E.g.,
$$\gamma \rightarrow e^{+} + e^{-}$$

- **Pair annihilation**: When a particle collide with its own anti-particle, they would annihilate each other and release energy $2mC^2$, e.g.,
$$e^{+} + e^{-} \rightarrow 2\gamma$$

  Q: why not just 1 γ?

Experimental discovery of positron (C.D. Anderson 1932)

The trajectory of a positron created from cosmic ray:

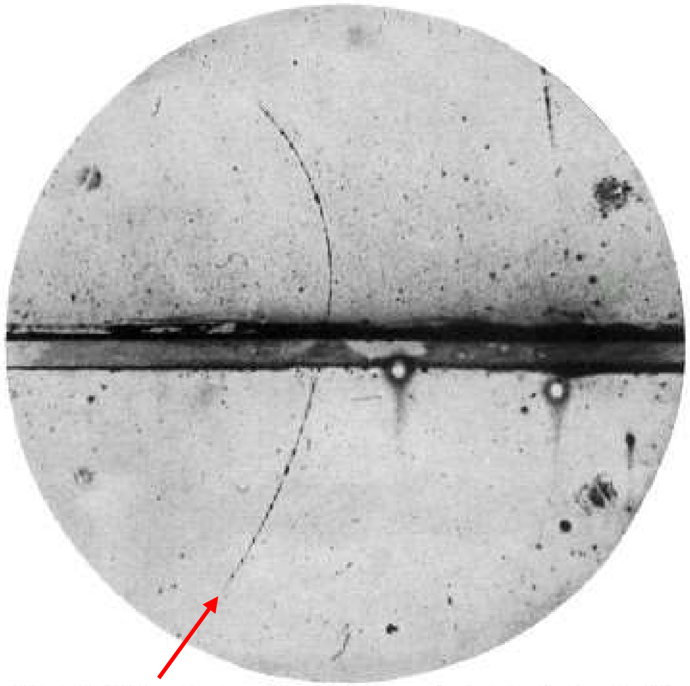
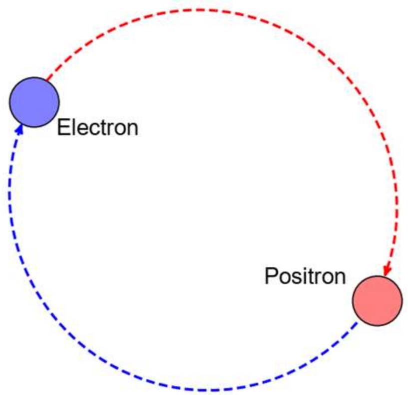


FIG. 1. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

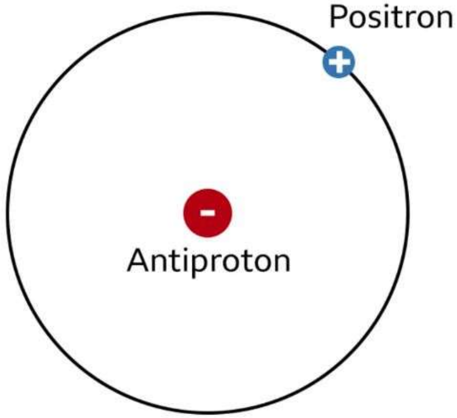
Note: Curie and Joliot also have seen positron trajectory in a cloud chamber, but they thought it's just an electron coming from above. (A missed opportunity for Nobel prize)

- Positronium (1951)　正負電子對

正正負電子對　Ortho-Positronium (same spin)
lifetime ~ 140 ns
仲正負電子對　Para-Positronium (opposite spins)
lifetime ~ 125 ps



Antihydrogen atom



- Anti-hydrogen (1995)
  Trapped for detailed study (2011)

- Antihyperhelium-4 (2024)
  2 antiprotons, 1 antineutron, and 1 antihyperon

➡ Anti-matter

➡ Anti-galaxy in the universe?

More on radioactive decay

Let the radioactive nucleus ${}^A_Z X$ be called the parent 母核

and have the mass $M({}^A_Z X)$. Two or more products can be produced in the decay. In the case of two products let the mass of the lighter one be M_y and the mass of the heavier one (normally called the *daughter*) be M_D. The conservation of energy is

子核

$$M({}^A_Z X) = M_D + M_y + Q/c^2$$

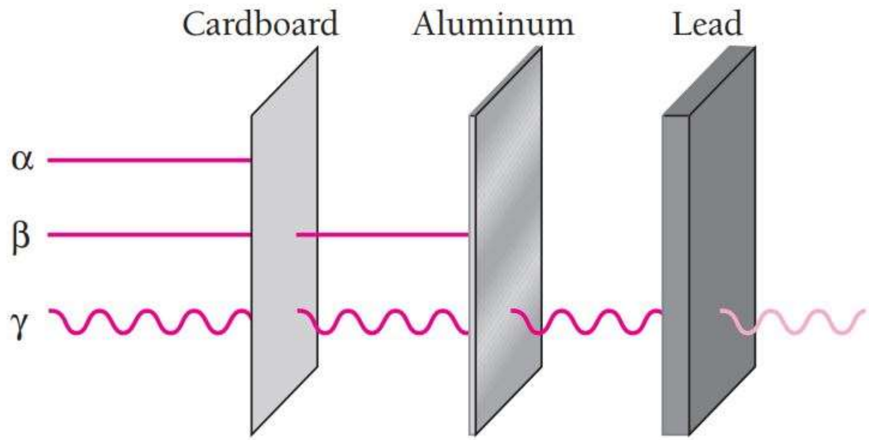
蛻變能

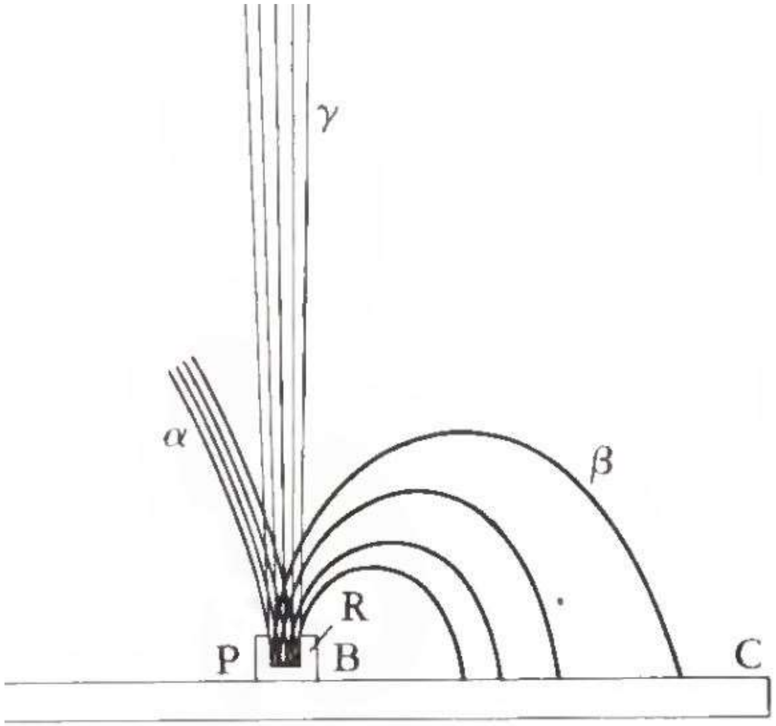
- The disintegration energy Q is the negative of the binding energy B.
- The binding energy normally refers to stable nuclei,

  whereas Q is normally used with unstable nuclei (>0).

Early discovery of Radioactive Decay

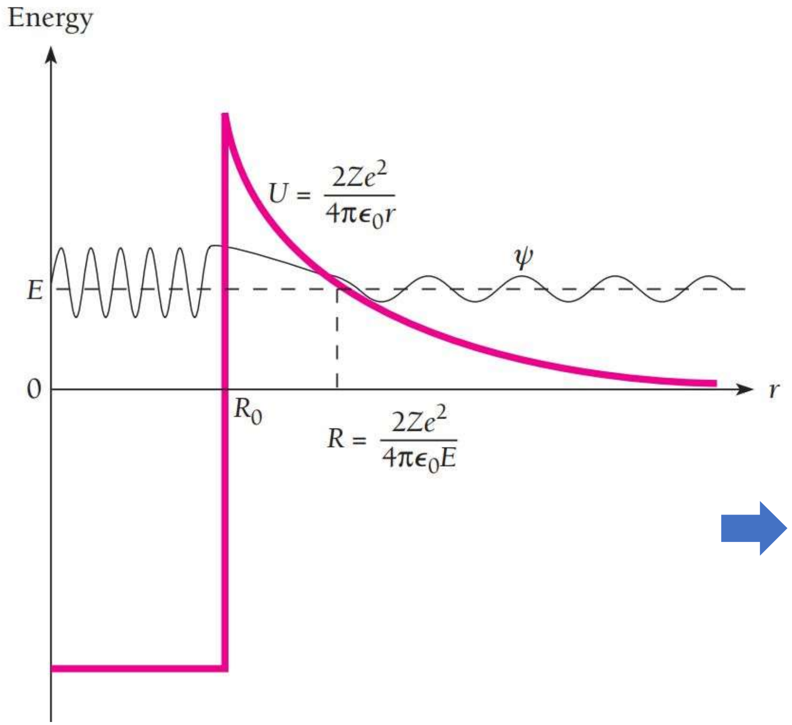
- 1896 Becquerel discovered radioactivity
- 1898 Rutherford realized that there are 2 kinds of radiation from uranium, he called them alpha and beta.
- 1900 Villard found gamma ray (~X-ray).
- 1904 Rutherford becomes convinced that alpha particles are helium ions





In the following, we study, α, β, χ decays in more details.

1. **Alpha decay** (1928, explained by Gamow)



- Many heavy nuclei can emit alpha particles, but their emission rates vary over a factor of 10^{13}, whereas their energies of alpha particles range only from 4 to 8 MeV.
- The alpha particles "tunnel" through the barrier (this is a probabilistic process).

The potential barrier (26.4 MeV for ^{212}Po) at the nuclear radius is higher than the energy of an alpha particle (8.78 MeV).

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}D + \alpha$$

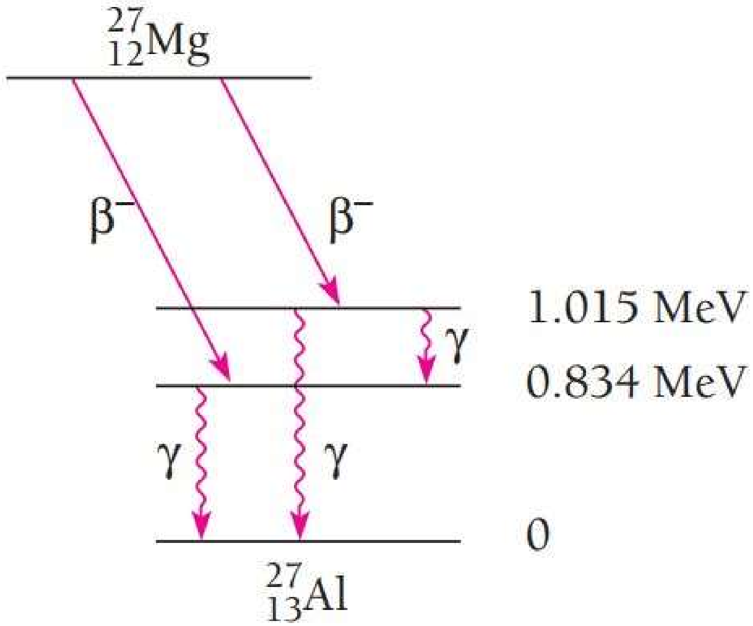
$$Q = [M(^{A}_{Z}X) - M(^{A-4}_{Z-2}D) - M(^{4}\mathrm{He})]c^2$$

If $Q > 0$, then alpha decay (^{4}He) is possible.

3. Gamma decay

Often follows α or β decay, an excited nucleus lowers its energy by emitting γ rays.

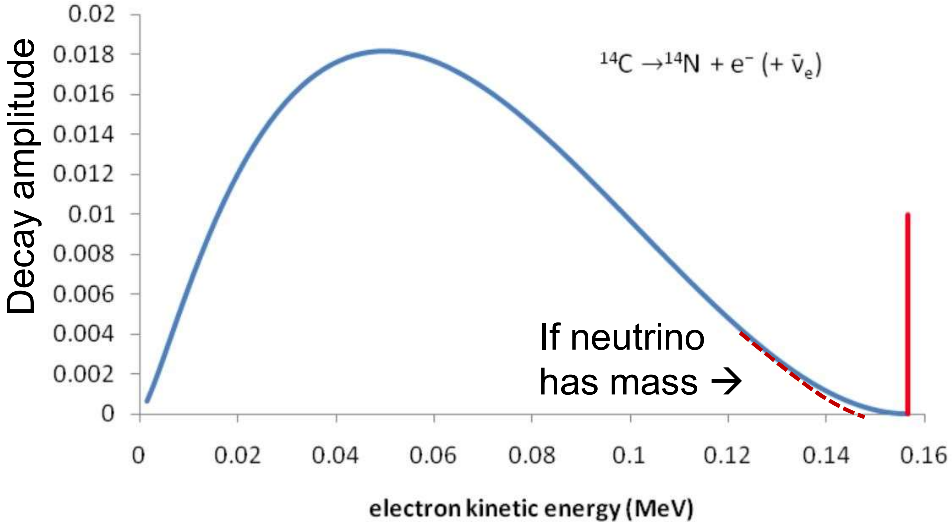
For example,



2. **Beta decay**

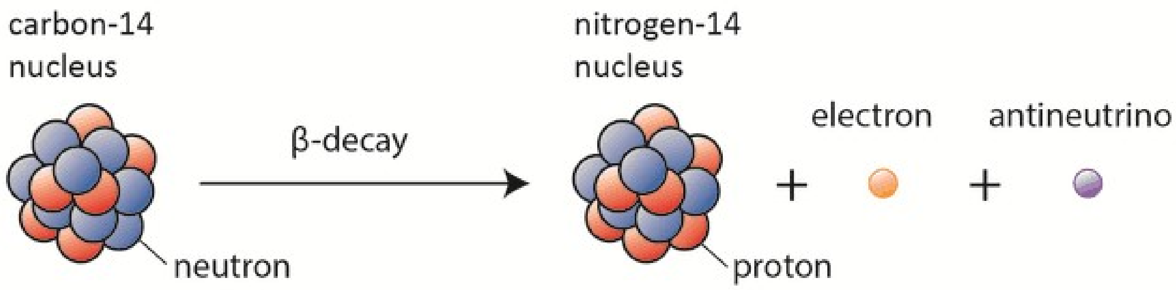
- A neutron can decay to a proton by beta decay, n → $p^+ + e^-$.
- Unstable nuclei may move closer to the line of stability by beta decay (n → p).
- There was a problem in neutron decay: spin ½ neutron cannot decay to two spin ½ particles, a proton and an electron.
- Also, the total energy of the decay does not seem to be conserved, (and no photon is observed).

Electron energy spectrum for beta decay of carbon-14. The red line marks the expected electron energy if only an electron were emitted.



- This puzzle leads Bohr to suggest that in the microscopic world, energy does not need to be conserved !

- 1934, Pauli suggested an unknown particle is produced in beta decay. From conservation laws, it has to have spin ½, charge 0, and carries away the missing energy. (later called **neutrino**) 微中子

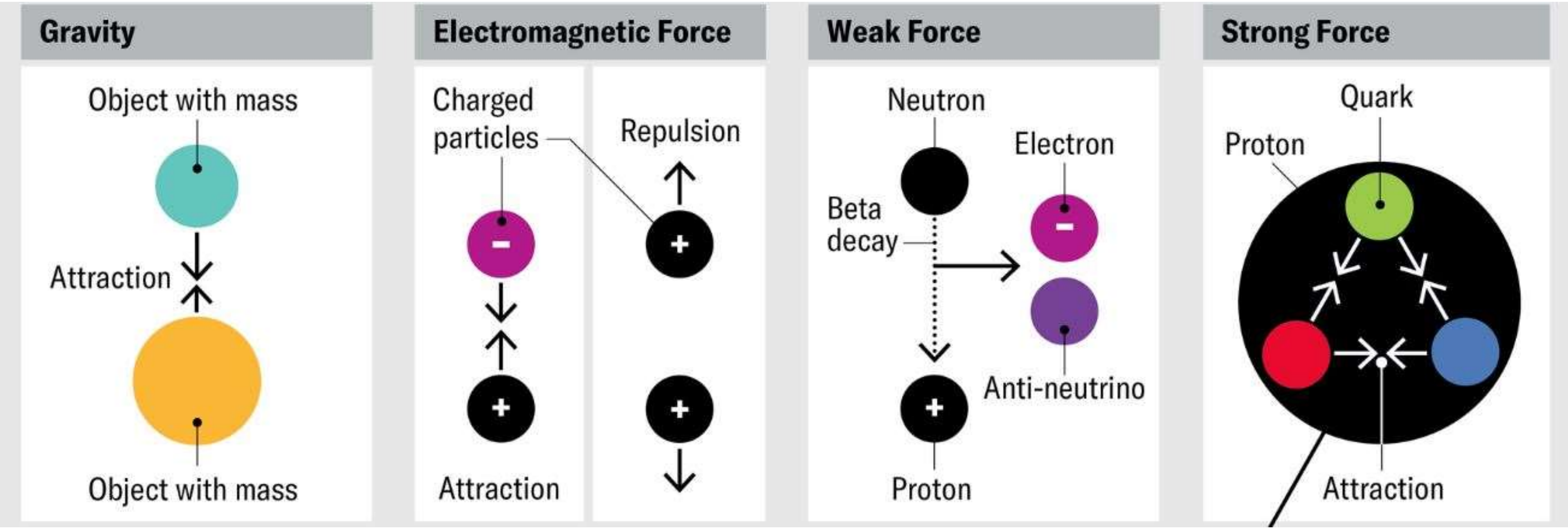


- The neutrino has no charge and almost no mass, hence it hardly interact with matters. In Pauli's words, It was, a “desperate remedy”. He said “*I have done a terrible thing, I have postulated a particle that cannot be detected*.” (At that time, it was considered a bad taste to introduce new particles).
- Neutrino can interact with others only with weak interaction, not EM nor strong interaction.
- One way to detect it: Inverse beta decay.

optional

4 fundamental Interactions (Chap 14)

Interaction	Relative Strength	Range	Mediating Particle
Gravitation	10^{-43}	∞	Graviton [hypothetical]
Electromagnetic	10^{-2}	∞	Photons
Weak	10^{-6}	10^{-18} m	$W^{\pm}$, Z bosons
Strong	1	10^{-15} m	Gluons 膠子

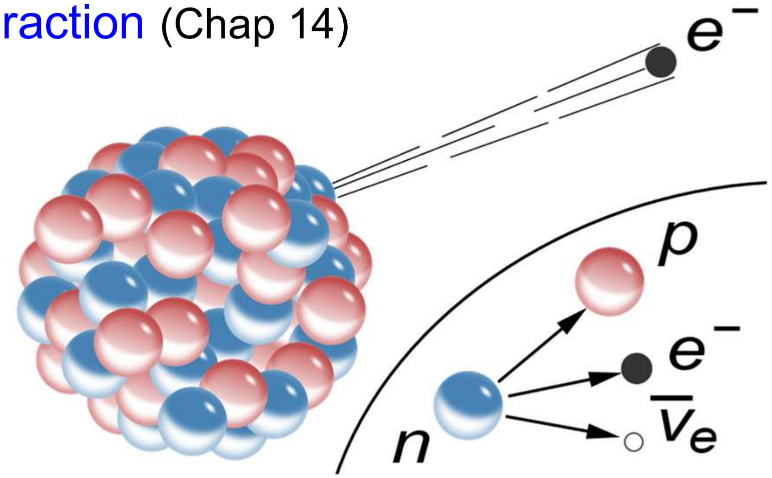


(Sci Am)

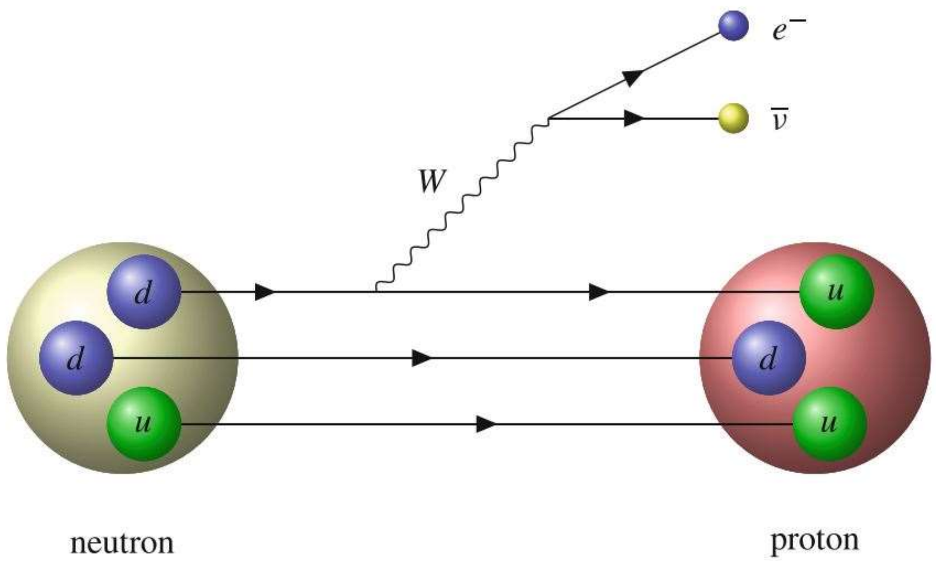
optional

Details of weak interaction (Chap 14)

- Beta decay



- Zooming in



The weak interaction is transmitted by 3 types of boson: $W^{\pm}, Z^0$

Salam, Weinberg (1964, 1967)

W^+ W^-	80.4 GeV
Z^0	91.2 GeV

~ 100 times the mass of a proton

→ Weak interaction very short-ranged

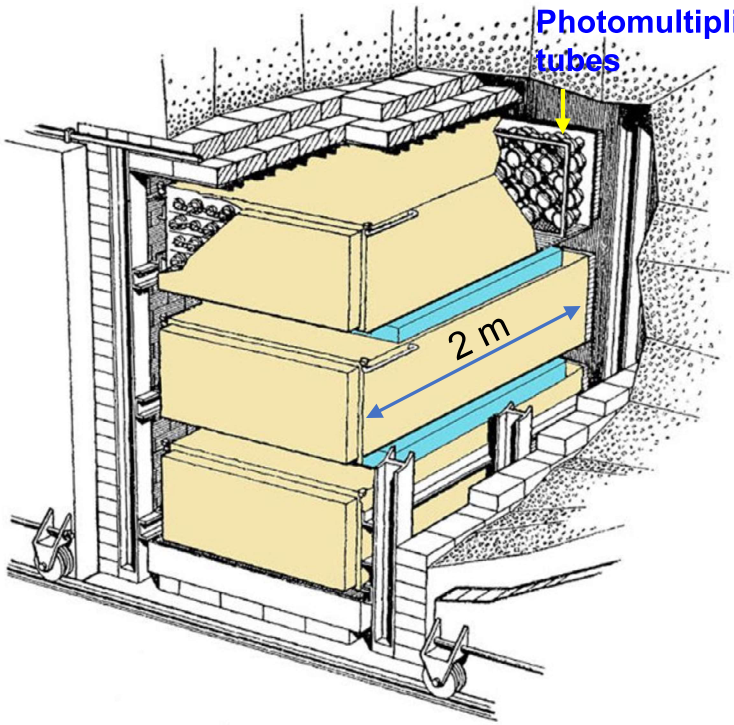
optional

Detecting neutrinos (Cowan and Reines, 1956)

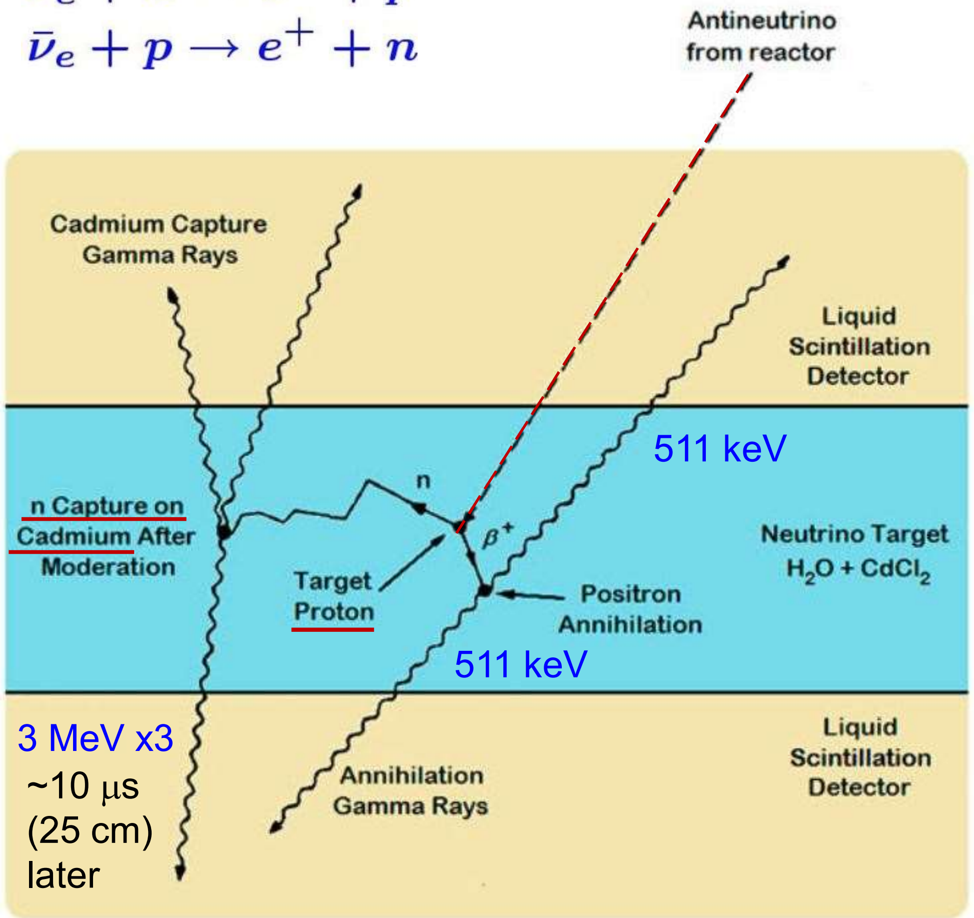
- Beta decay
- Inverse beta decay

$$n \rightarrow p + e^- + \bar{\nu}_e$$
$$p \rightarrow n + e^+ + \nu_e$$
$$\nu_e + n \rightarrow e^- + p$$
$$\bar{\nu}_e + p \rightarrow e^+ + n$$

Next to a nuclear reactor that produces 10^{12} to 10^{13}/cm^2 sec neutrinos.



Two target tanks (blue, $CdCl_2$ dissolved in water) between three scintillation detectors.



"delayed coincidence"

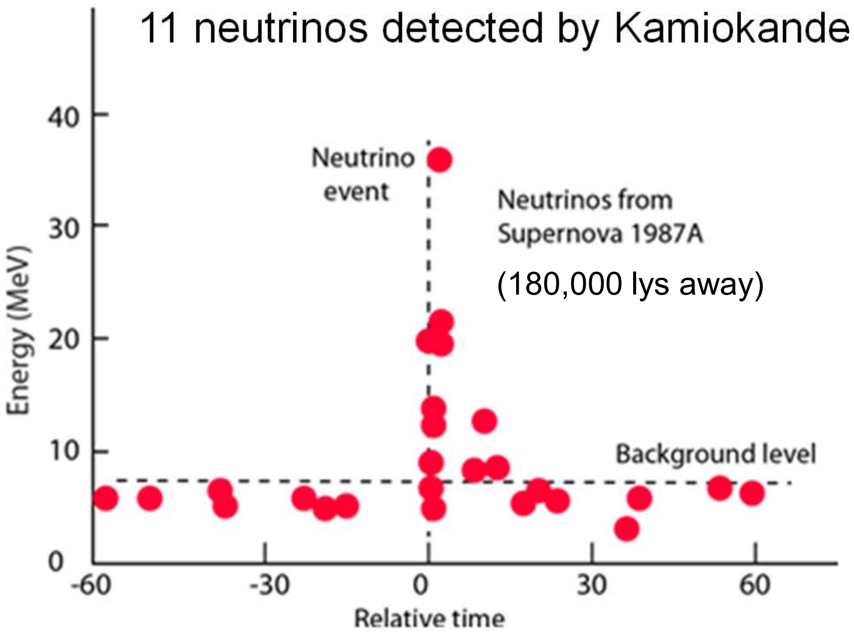
Link et al, Nuclear News Dec 2020

optional

Neutrino and supernova

- In the 80s, some experiments are designed to observe proton decay. They got null result (proton lifetime > 10^{33} yrs).
- However, on Feb 23 1987, 24 neutrino events were observed in neutrino detectors (Kamiokande and others), 3 hours before the light of SN 1987A hit us. (The brightest event since 1604)

11 neutrinos detected by Kamiokande II 神岡

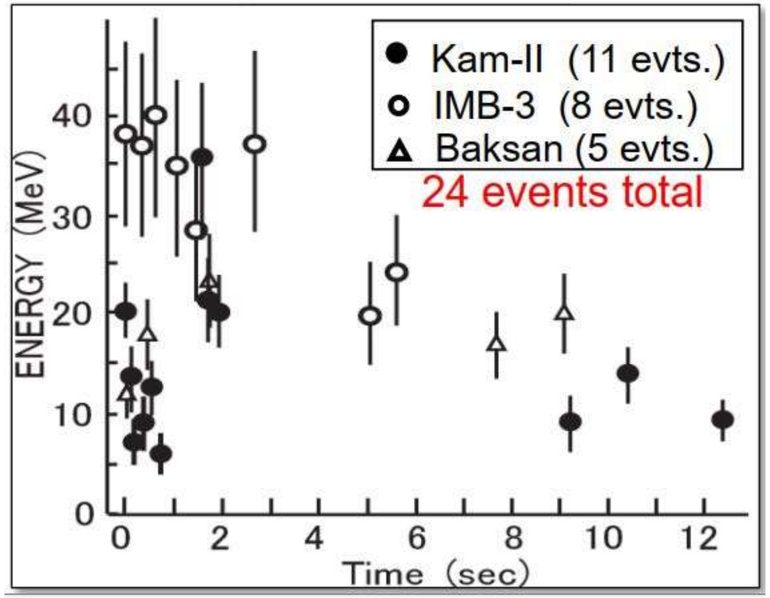


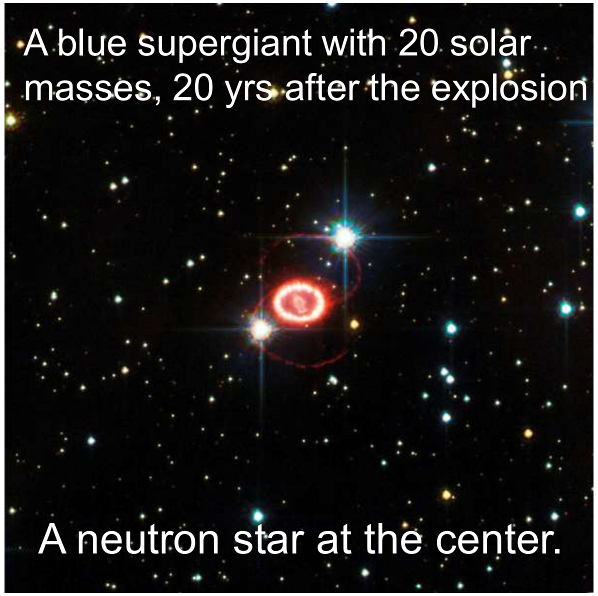
Note: Labs were equipped with automatic recording devices which also registered the time of events, so that, after the optical discovery of SN 1987A, scientists could go back over the records and discover the neutrino signal from it.

Death Of A Star - Documentary on supernova 1987a

optional

1987A neutrino events from 3 detectors worldwide





- 99% of the energy is carried away by neutrinos from the core of the star. 1% is carried away by light (when the shock wave hit the surface 3 hrs later).
- It is estimated that 10^{16} neutrinos went through the detector in 15 s. So the energy released by $\bar{\nu}_e$ was about $3\text{x}10^{46}$ J (~ 0.17 solar mass).
- The neutrinos arrived the earth within 15 s, after traveling 180,000 light years (from the Large Magellanic Cloud).

  → neutrino mass < 12 eV (cf: m_e=0.51 MeV).

β^- Decay

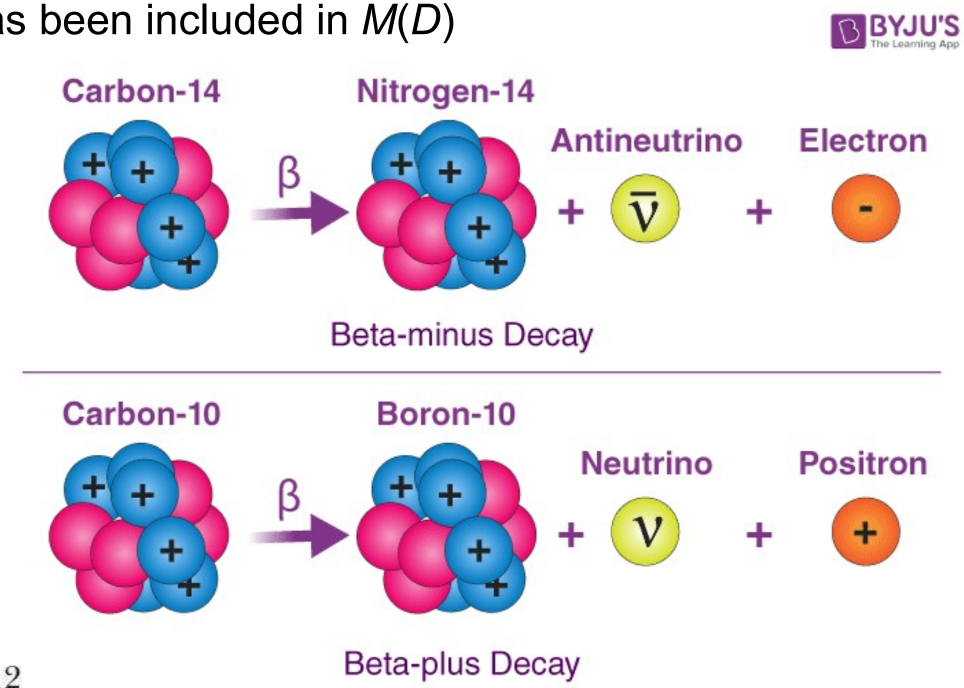
$$ {}^{A}_{Z}X \rightarrow {}_{Z+1}^{\;\;A}D + \beta^- + \bar{\nu} $$

$$ Q = [M({}^{A}_{Z}X) - M({}_{Z+1}^{\;\;A}D)]c^2 $$

Note: M is atomic mass, not nucleus mass. So the e-mass has been included in $M(D)$

e.g., the beta decay of a neutron of ^{14}C

$$ ^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \beta^- + \bar{\nu} \quad \beta^- \text{ decay} $$



β^+ Decay (positron emission, if there are too many protons)

$$ {}^{A}_{Z}X \rightarrow {}_{Z-1}^{\;\;A}D + \beta^+ + \nu $$

$$ Q = [M({}^{A}_{Z}X) - M({}_{Z-1}^{\;\;A}D) - 2m_e]c^2 $$

e.g.,

$$ ^{10}C \rightarrow {}^{10}B + \beta^+ + \nu \quad \beta^+ \text{ decay} $$

Ex 12.16: Find whether alpha decay or any of the beta decays are allowed for $^{226}_{89}\text{Ac}$.

錒

Solution

Alpha decay: $^{226}_{89}\text{Ac} \rightarrow {}^{222}_{87}\text{Fr} + \alpha$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{222}_{87}\text{Fr}) - M(^{4}\text{He})]c^2$$
$$= 5.54\text{ MeV} \qquad \text{Alpha decay is allowed}$$

β^- decay: $^{226}_{89}\text{Ac} \rightarrow {}^{226}_{90}\text{Th} + \beta^- + \bar{\nu}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{90}\text{Th})]c^2$$
$$= 1.12\text{ MeV} \qquad \beta^- \text{ decay is allowed}$$

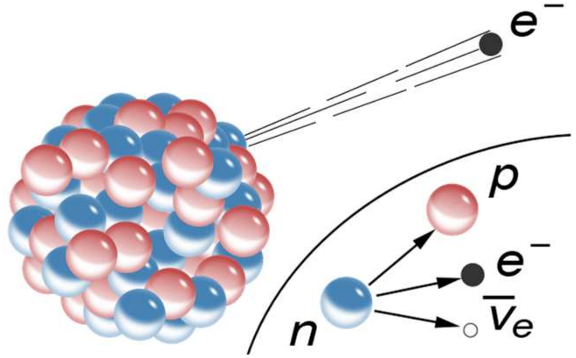
β^+ decay: $^{226}_{89}\text{Ac} \rightarrow {}^{226}_{88}\text{Ra} + \beta^+ + \nu$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra}) - 2m_e]c^2$$
$$= -0.38\text{ MeV} \qquad \beta^+ \text{ decay is not allowed}$$

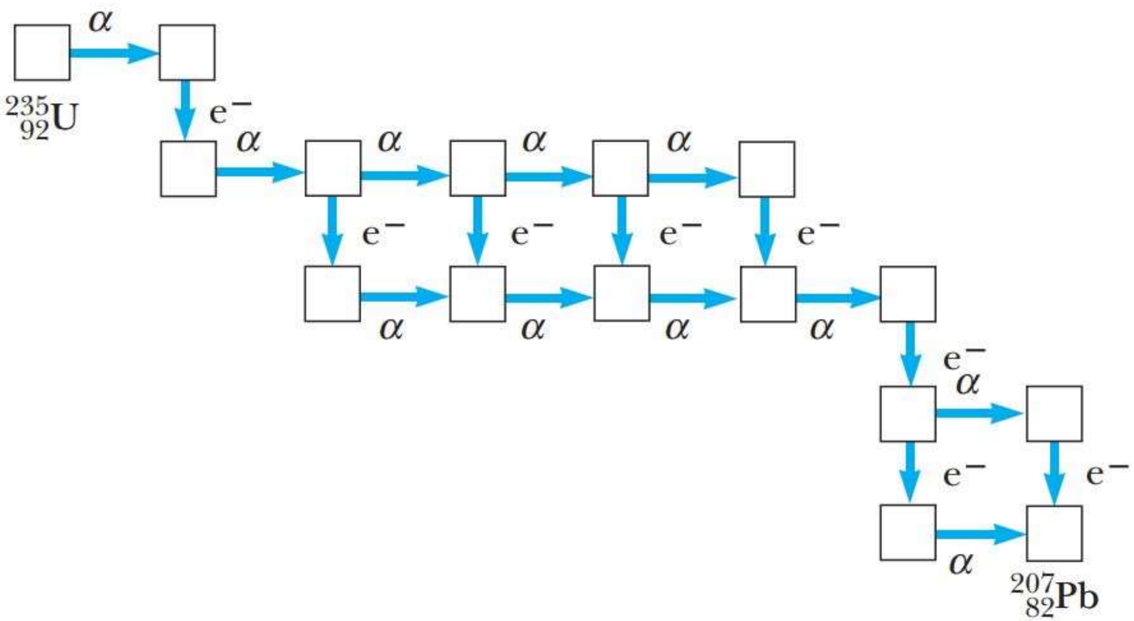
Experiment shows that alpha decay occurs only 0.006% of the time for ^{226}Ac, beta decay 83%, and electron capture 17%.

Summary of radioactive decay

Alpha decay	${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He$
Beta decay (e^-)	${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}X + e^- + \overline{\nu}$
Beta decay (e^+)	${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}X + e^+ + \nu$
Electron capture	${}^{A}_{Z}X + e^- \longrightarrow {}^{A}_{Z-1}X + \nu$
Gamma decay	${}^{A}_{Z}X^* \longrightarrow {}^{A}_{Z}X + \gamma$



For example,



Radioactive nuclides 核種

A particular nucleus of an atom with particular atomic number and atomic mass

- Heavy radioactive nuclides can change their mass number only by alpha decay ($^{A}X \rightarrow {}^{A-4}D$). They can change charge number Z by either alpha or beta decay.
- There are only four paths that heavy naturally radioactive nuclides may take as they decay:

Table 12.3 The Four Radioactive Series

Mass Numbers	Series Name	Parent	$t_{1/2}$ (y)	End Product
$4n$	Thorium	$^{232}_{90}\mathrm{Th}$	1.40×10^{10}	$^{208}_{82}\mathrm{Pb}$
$4n + 1$	Neptunium	$^{237}_{93}\mathrm{Np}$	2.14×10^{6}	$^{209}_{83}\mathrm{Pb}$
$4n + 2$	Uranium	$^{238}_{92}\mathrm{U}$	4.47×10^{9}	$^{206}_{82}\mathrm{Pb}$
$4n + 3$	Actinium	$^{235}_{92}\mathrm{U}$	7.04×10^{8}	$^{207}_{82}\mathrm{Pb}$

4n

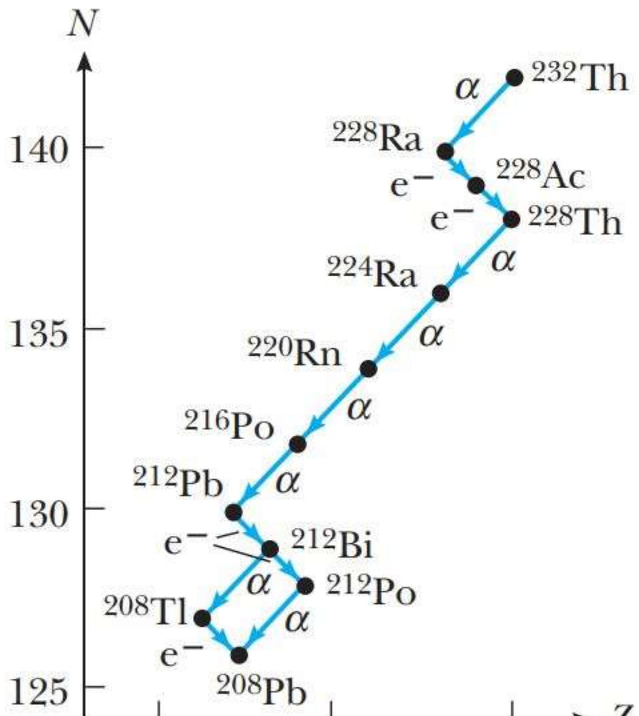


Figure 13.21 Successive decays for the ^{232}Th series.

4n+2