Chap 3

Experimental basis of quantum physics

- Discovery of X-ray
- Discovery of electron
- Line spectra (-> next chap)
- Blackbody radiation
- Photoelectric effect
- X-ray production (-> next chap)
- Compton effect
- Pair production

First, some history around 1900:

- The discovery of cathode ray (Plücker and Hittorf,1859)

Several things happened during 1895-1897

- Zeeman effect (1895) → electrons in atoms, more in Chap 7
- Röntgen's X-ray (1895).
- Electrons (J.J. Thomson,1897)
- Radioactivity (Becquerel,1897), more in Chap 12
- α, β, γ rays.
- Rutherford showed that α-ray is ionized helium atoms.
- He then used them to probe the atom structure, and discovered there is a nucleus in an atom, next Chap.

Inspires the discovery of

Cathode ray (1859, Plücker and Hittorf) 陰極射線

- Cathode rays casting a shadow of a Maltese cross (anode) in a Crookes tube, powered by a Ruhmkorff coil (a type of transformer that generates high voltage pulsefrom DC supply, a few *kV*).
- Need good vacuum.
- Cathode ray is wave or particle? Perrin showed that they are particles with negative charges in 1895 (Segre, ibid p.17).

https://www.britannica.com/science/range-particle-radiation

Discovery of X-ray (Röntgen, 1895)

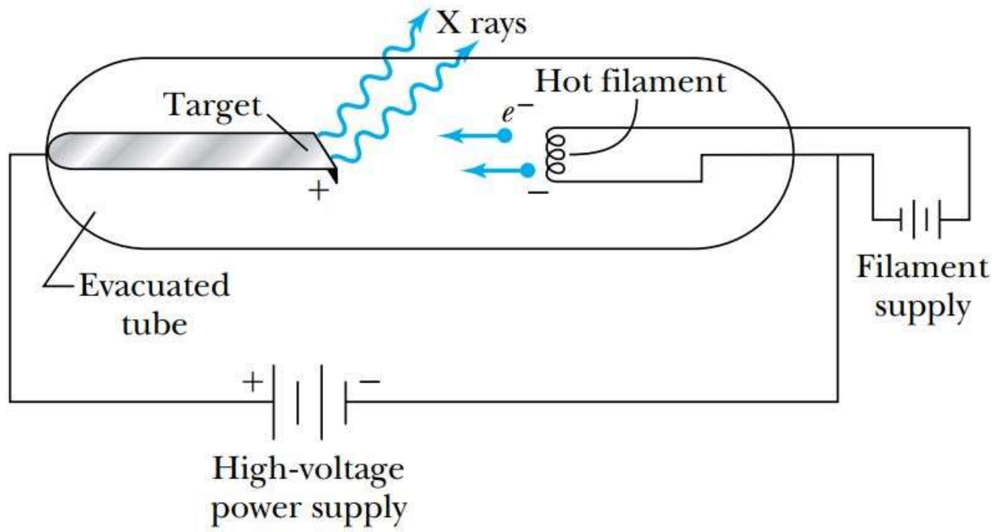
- Working with a completely shielded cathode tube, he noticed a glow coming from a nearby fluorescent screen, implying a ray emitted from the tube passing through the shielding. He dubbed this glow X-ray because of its unknown nature.

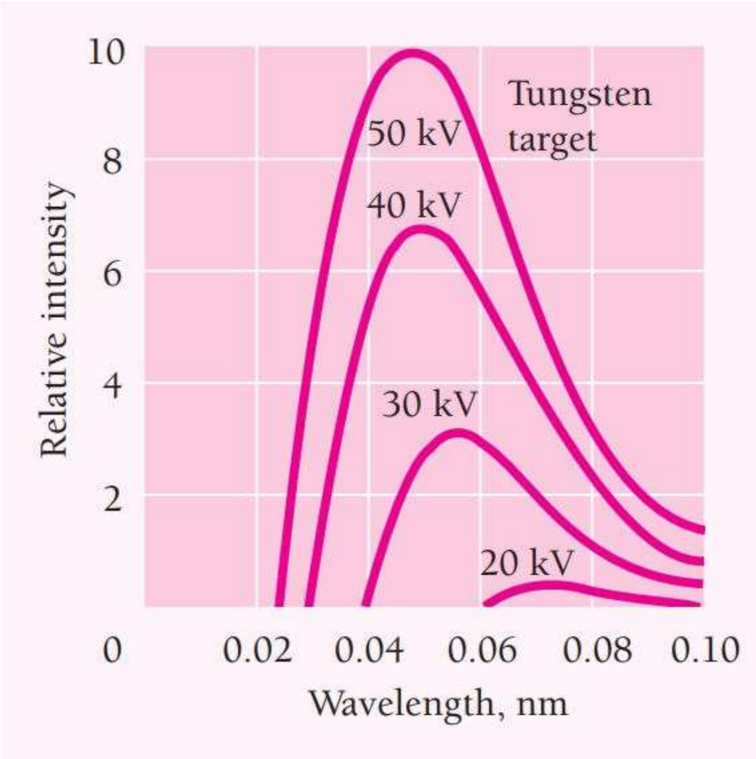
- He also found that opaque objects placed between the tube and the screen proved to be transparent to this new form of radiation. (www.britannica.com/science/X-ray)

- This first time people can see the inside of a body without cutting it open. This caused a sensation in academics and media.

How is the x-ray generated?

Braking radiation (bremsstrahlung)　制動輻射



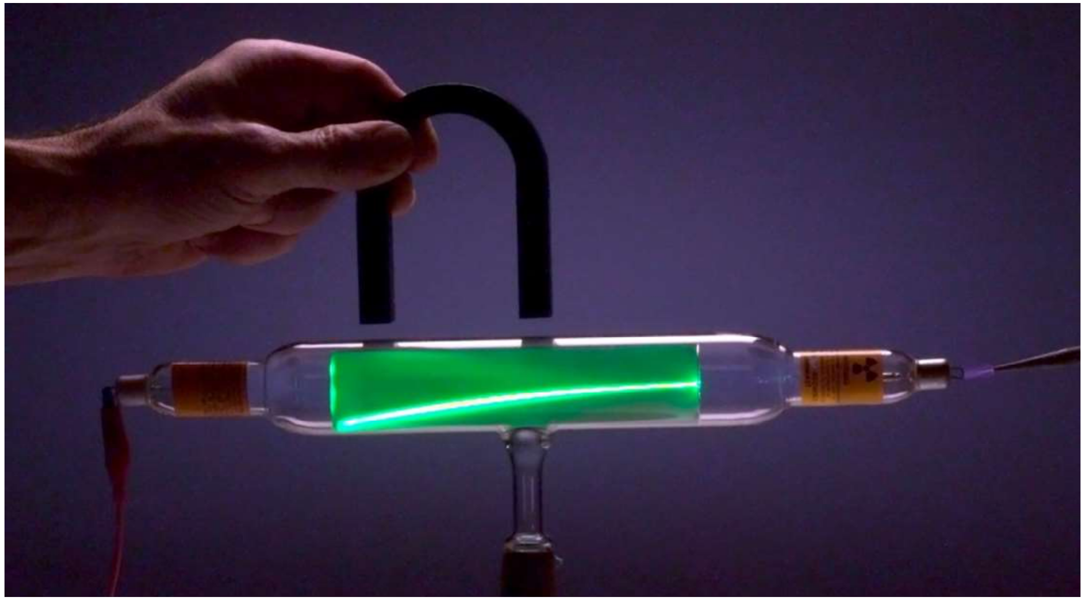


How do we know that X-ray is an EM wave?

1906, Barkla demonstrated the wave nature of X-rays by showing that they can be "polarized" by scattering from a solid (can also be explained with spinning particles).

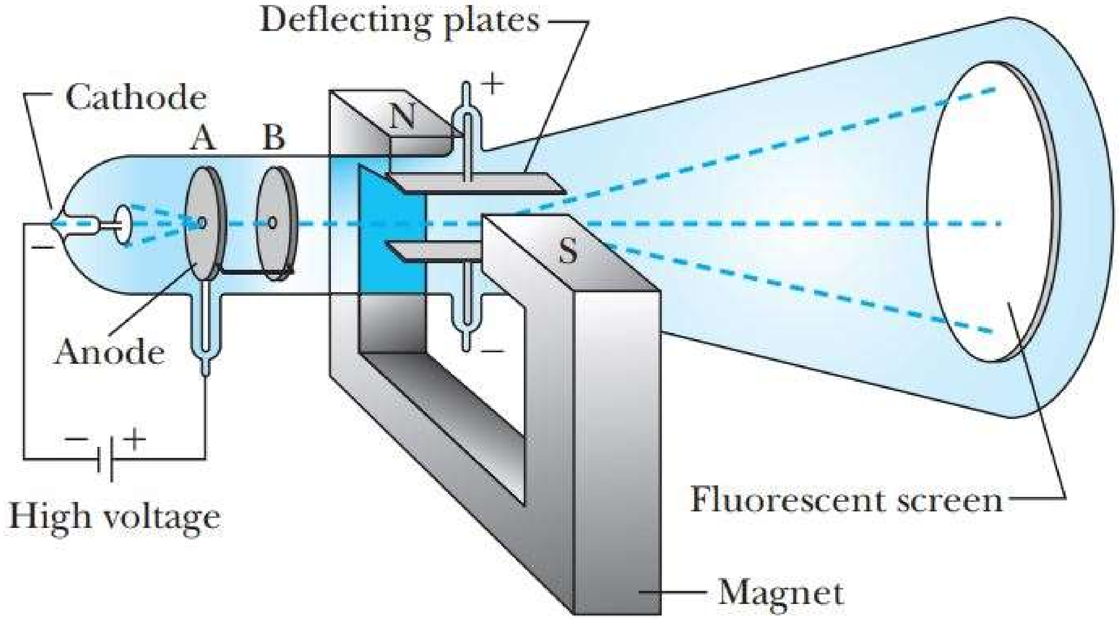
1912, von Laue's x-ray diffraction by crystals provided more evidence.

Discovery of electrons (Thomson, 1897)



Cathode ray is negatively charged, can be bent by a magnetic field and electric field. *It can also pass through a thin paper, indicating that it consists of something smaller than atoms.*

Determination of the *q*/*m* ratio (J.J. Thomson 1897)

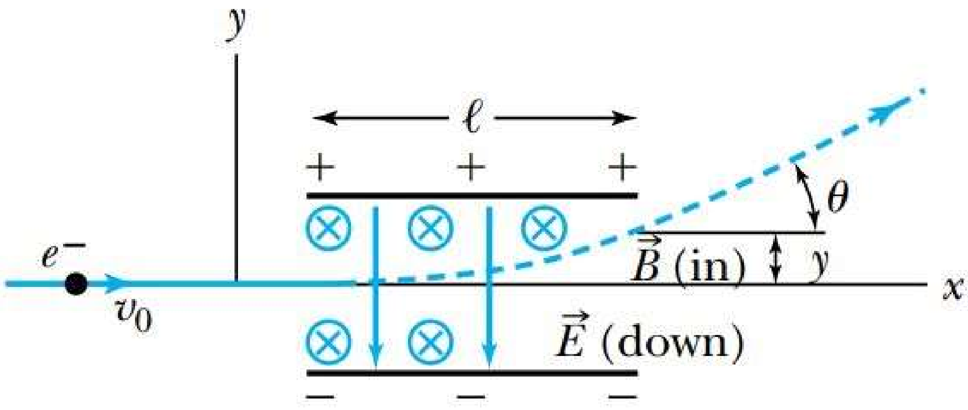


- Magnetic deflection has already been seen.
- Electric deflection is first observed by Thomson in 1897.

  Why so late?

  Again a good vacuum is required.

Determination of the *q*/*m* ratio



- Determine electron velocity by maintaining a horizontal beam

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = 0 \quad \Rightarrow \quad v_x = \frac{E}{B} = v_0$$

- Turn off B, measure the angle of deflection

$$\tan\theta = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{qE}{m}\frac{\ell}{v_0^2}$$

$$\Rightarrow \quad \frac{q}{m} = \frac{v_0^2 \tan\theta}{E\ell}$$

- Thomson also showed that this ratio is independent of the materials of nodes. So it is a universal component of matter. (Can still be explained with continuous charged "fluid".)

Ex 3.1: In an experiment similar to Thomson's, we use deflecting plates 5.0 cm in length with an electric field of 1.2×10^4 V/m. Without the magnetic field we find an angular deflection of 30°, and with a magnetic field of 8.8×10^{-4} T we find no deflection. What is the initial velocity of the electron and its q/m?

Solution We insert the values of E and B into Equation (3.4) to find

$$v_0 = \frac{E}{B} = \frac{1.2 \times 10^4 \text{ V/m}}{8.8 \times 10^{-4} \text{ T}} = 1.4 \times 10^7 \text{ m/s}$$

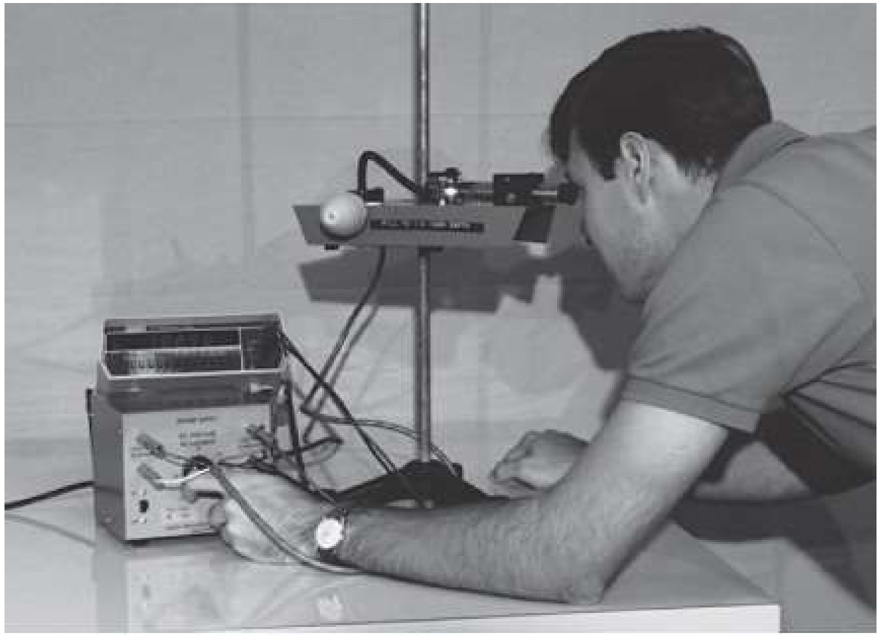
Because all our units for E and B are in the international system (SI), the value for v_0 is in meters/second. Equation (3.5) gives the following result for q/m:

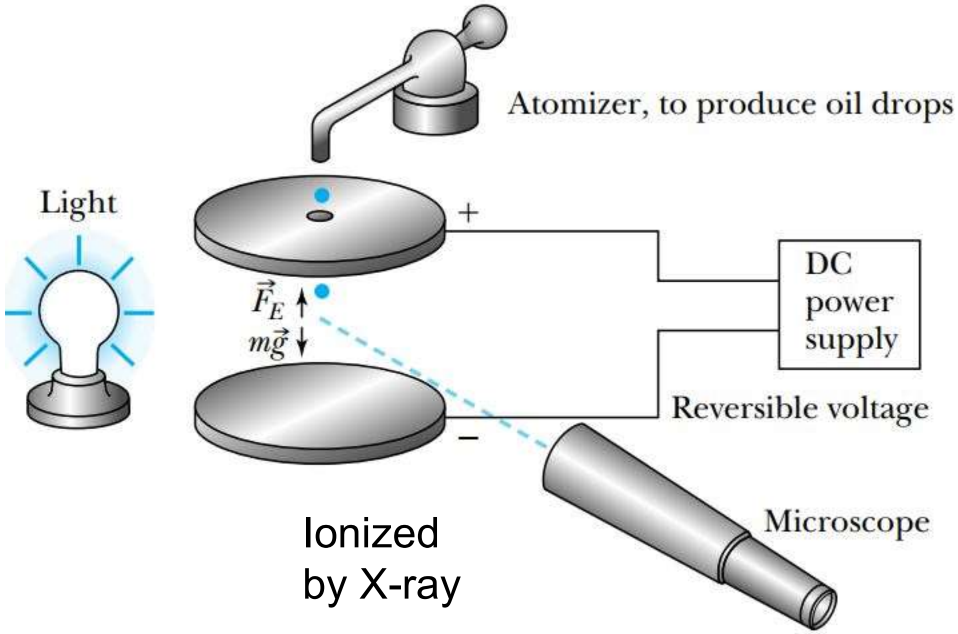
$$\frac{q}{m} = \frac{v_0^2 \tan\theta}{E\ell} = 1.8 \times 10^{11} \text{ C/kg}$$

Determination of electron charge (Millikan, 1909)

- First done by Wilson and J.J.Thomson with water vapor (1899).

  Got the order of magnitude of electron charge right.
- Later, Millikan's oil-drop experiment (1909)



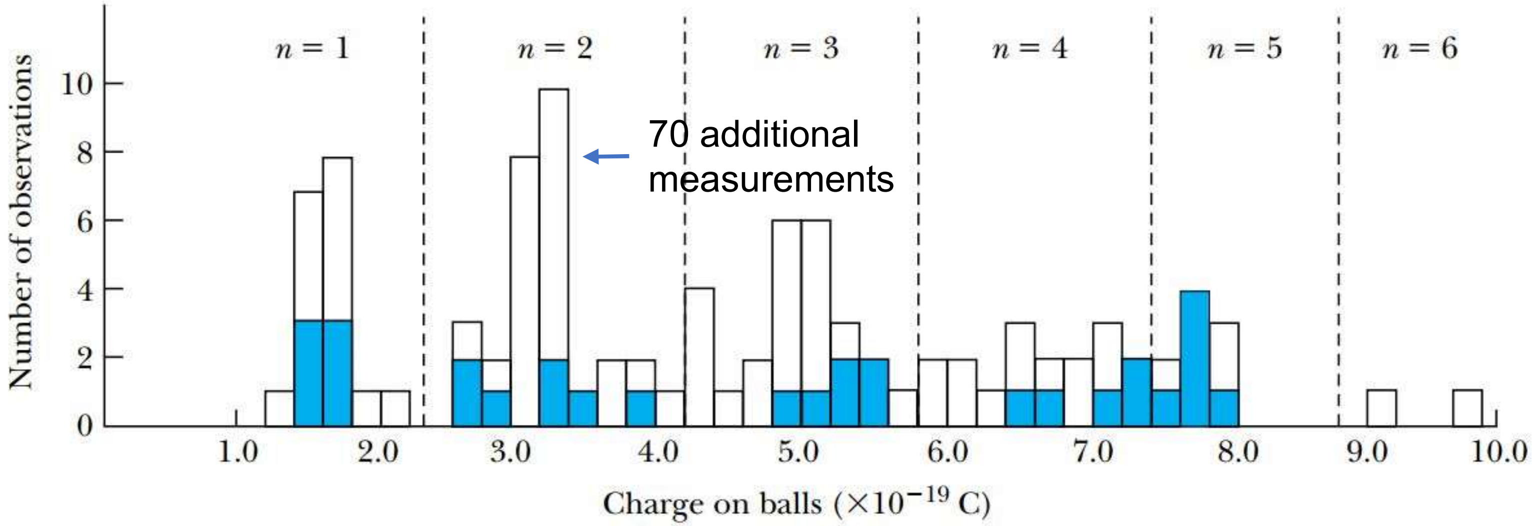


- Once the charge q of an electron is known, one can determine its mass m from the known q/m ratio.

Ex 3.2: We can make two changes in Millikan's procedure. First, we use plastic balls of about 1 micron diameter, for which we can measure the mass accurately. *This avoids the measurement of the oil drop's terminal velocity and the dependence on Stokes's law*. One then occasionally bombard balls with electrons from a radioactive source. We can determine whether the charges determined from q=mg/E=mgd/V are multiples of some basic charge unit.

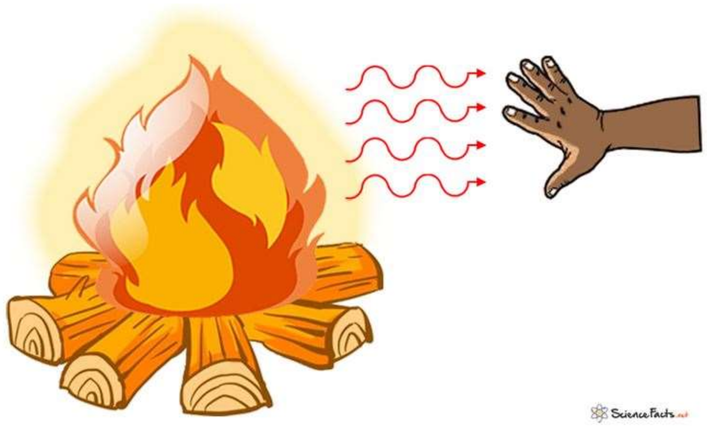
- In an actual undergraduate experiment the mass of the balls was m = 5.7 x 10^{-16} kg and the spacing between the plates was d = 4.0 mm.

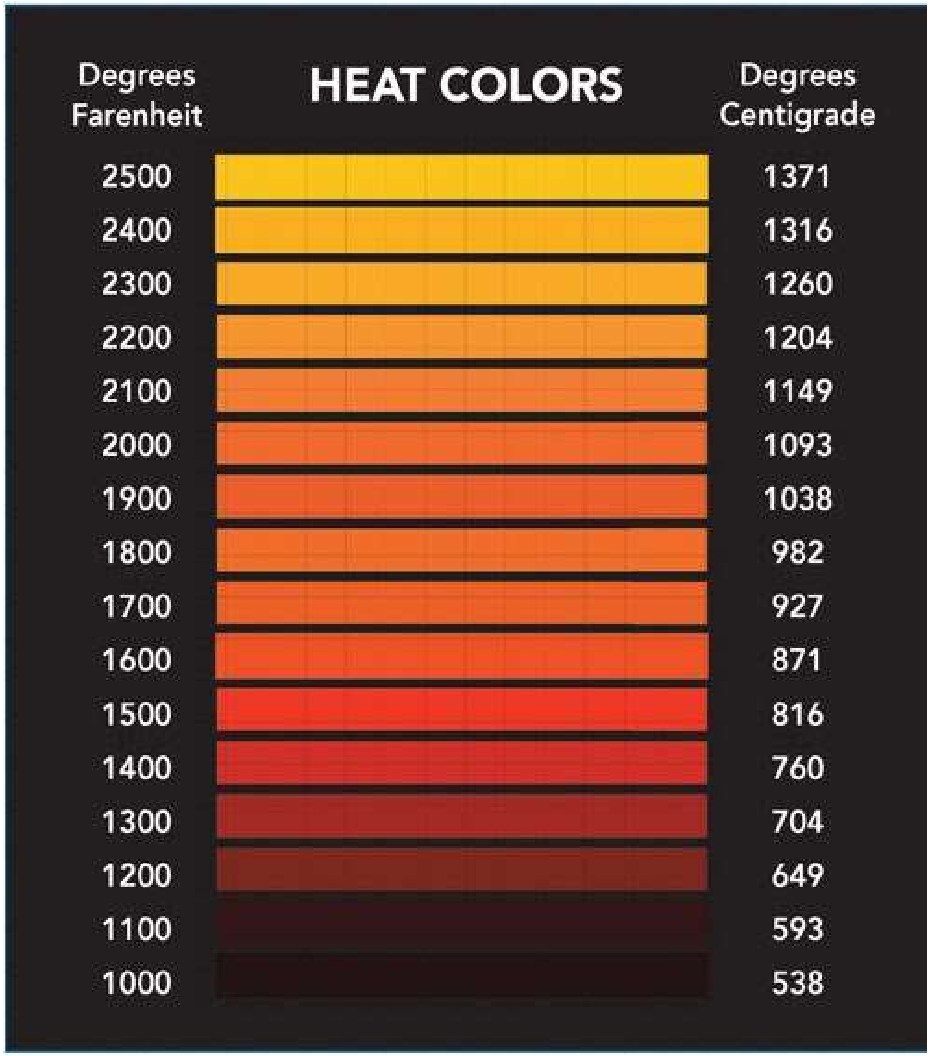
Actual data from student lab: $\Delta q = 0.2 \times 10^{-19}$ C



- Discovery of X-ray
- Discovery of electron
- Blackbody radiation
- Photoelectric effect
- Compton effect
- Pair production

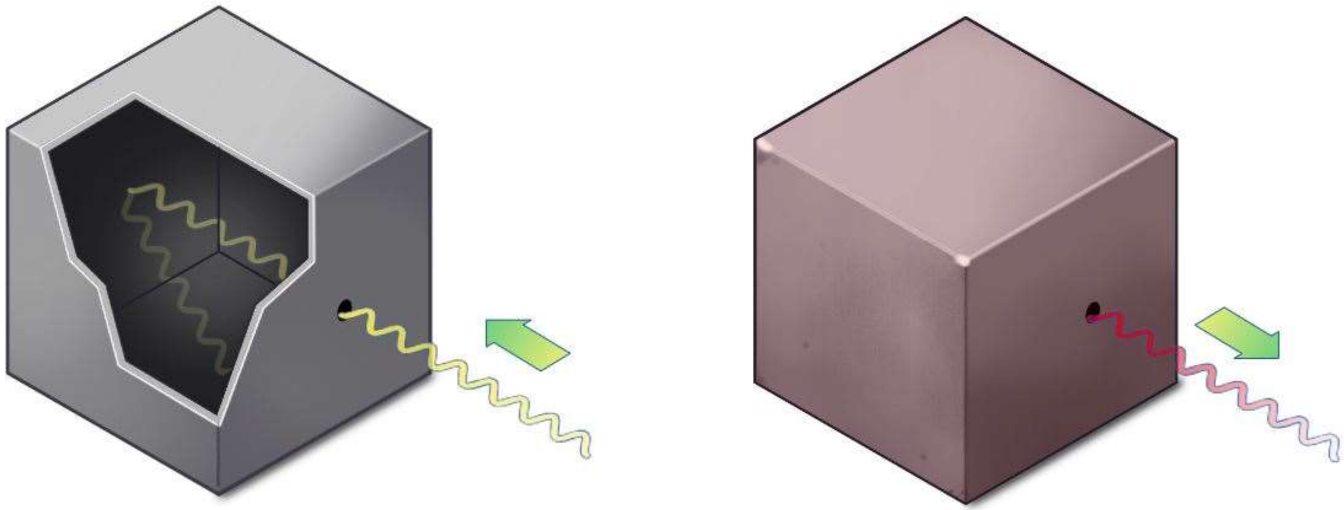
In the 19th century, people began to understand that thermal radiation are EM waves





Cavity radiation (in thermal equilibrium)

Also known as blackbody radiation 黑體輻射

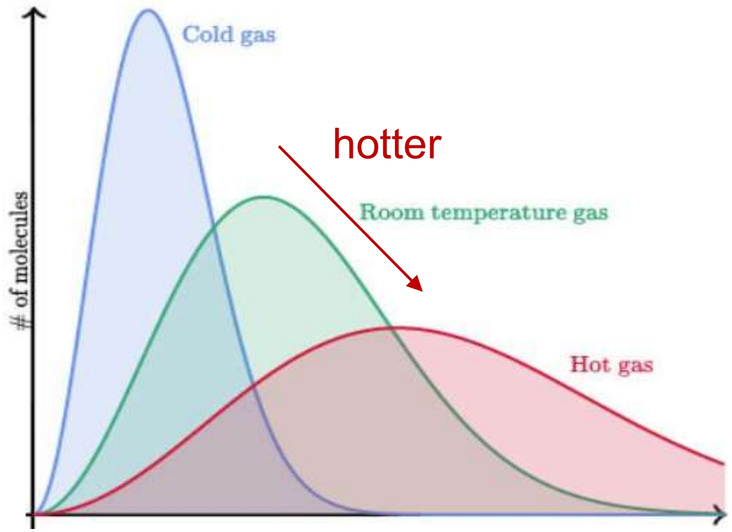


- A blackbody is a perfect absorber of radiation. It also emits the maximum amount of radiation by a surface at a given temperature.
- Cavity radiation does not depend on the nature of the wall.

  Two cavities at the same *T* should have the same spectral distribution of radiation. If not, then we can use a filter so that energy spontaneously flow from one to the other, thus violating the law of thermodynamics.

Recall

Maxwell speed distribution (1860)



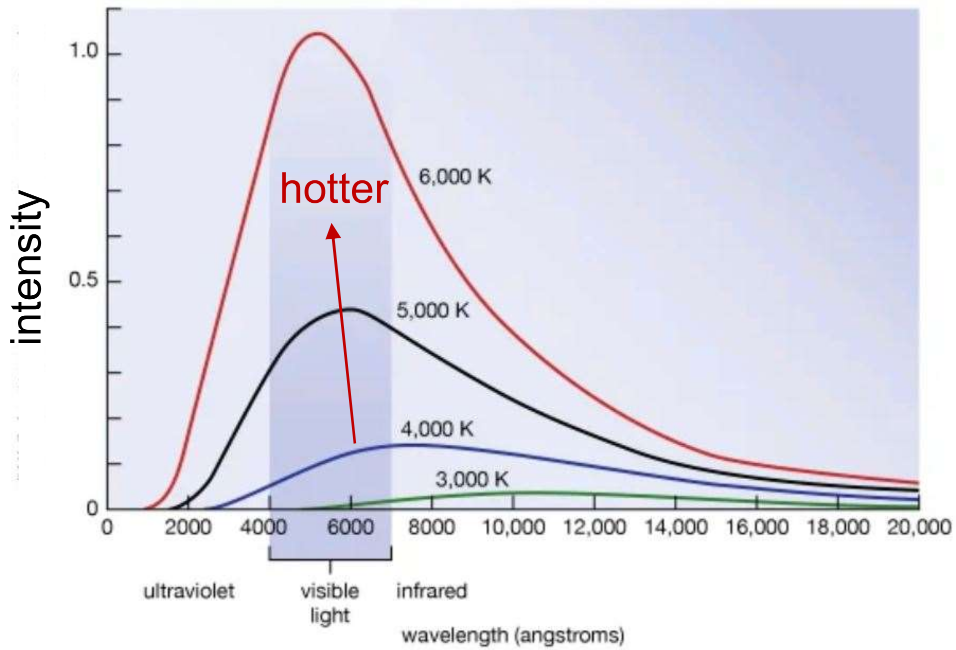
(the areas below these curves are the same)

$$f(v)\, d^3v = \left[\frac{m}{2\pi kT}\right]^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2kT}\right)\, d^3v,$$

Boltzmann's constant

$k = 1.381 \times 10^{-23}$ J/K

Spectral distribution of blackbody radiation



First, two basic properties:

1. The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.
2. The total power radiated increases with the temperature.

维恩位移定律

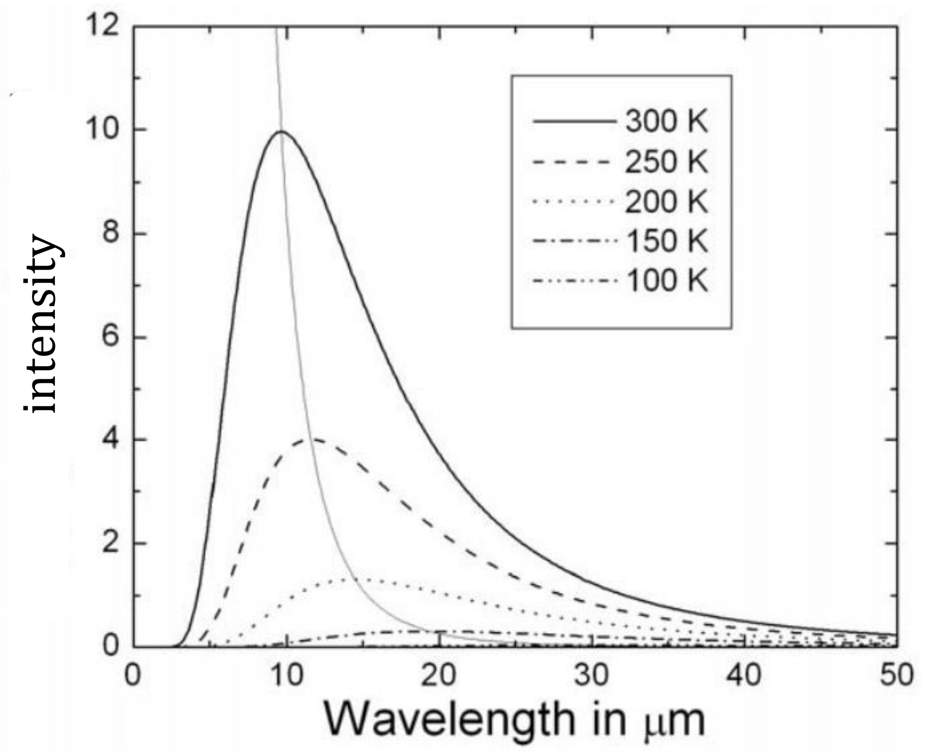
1. Wien's displacement law
(argued from thermodynamics)

$$\lambda_{\text{peak}} = \frac{b}{T}$$

$b \approx 2898$ μm·K.

e.g.,

- Skin temperature 300 K has a peak at around 10 μm (far infrared).
- Surface of the Sun T=5778 K has a peak at about 500 nm (green).



2. Stefan-Boltzmann law

(proposed by Stefan 1879, derived by Boltzmann 1884)

Power radiated per unit surface area:

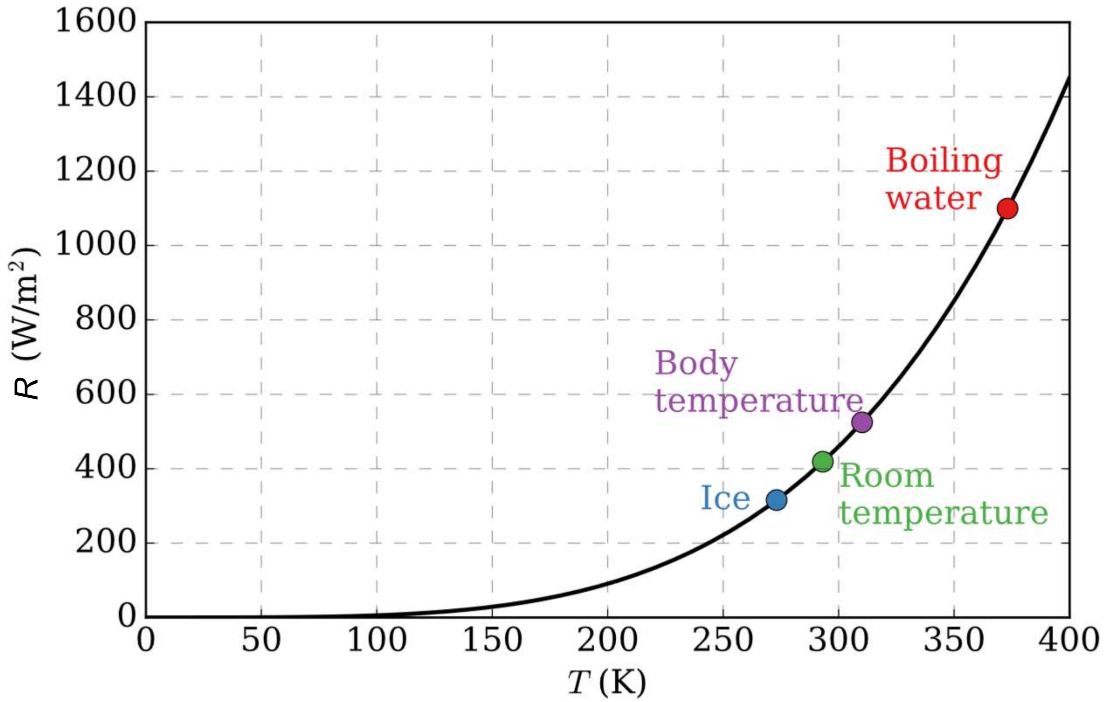
$$R(T) = \int_0^\infty d\lambda\, M_\lambda(T) = \sigma T^4$$

Thornton simply calls it "intensity" $\mathcal{I}(\lambda, T)$

the area below the spectral curve

Stefan–Boltzmann constant

$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}



Ex 3.5: The wavelength of maximum intensity of the sun's radiation is observed to be near 500 nm. Assume the sun to be a blackbody and calculate (a) the sun's surface temperature, (b) the power per unit area $R(T)$ emitted from the sun's surface, and (c) the energy received by the Earth each day from the sun's radiation.

Solution: (a) $$T_{\text{sun}} = \frac{2.898 \times 10^6}{500}\text{ K} = 5800\text{ K}$$

$r_s = 6.96 \times 10^8 m$

$r_E = 6.37 \times 10^6 m$

$R_{Es} = 1.49 \times 10^{11} m$

(b) $$R(T) = \sigma T^4 = 6.42 \times 10^7\text{ W/m}^2$$

(c) $$P_{\text{sun}} = 6.42 \times 10^7 \frac{\text{W}}{\text{m}^2}(6.09 \times 10^{18}\text{ m}^2) \qquad A_s = 4\pi r_s^2$$

$$= 3.91 \times 10^{26}\text{ W}$$

Fraction F of the sun's radiation received by Earth

$$F = \frac{\pi r_E^2}{4\pi R_{Es}^2} = \frac{(6.37 \times 10^6\text{ m})^2}{4(1.49 \times 10^{11}\text{ m})^2} = 4.57 \times 10^{-10}$$

➡ $$P_{\text{Earth}}(\text{received}) = (4.57 \times 10^{-10})(3.91 \times 10^{26}\text{ W})$$

$$= 1.79 \times 10^{17}\text{ W}$$

➡ Energy=1.55 x 10^{22} J

Early theories to fit the spectral distribution

$u_\nu d\nu$ =Energy density

- Wien's radiation law (1896) $u_\nu(T) = \alpha\nu^3 e^{-\beta\nu/T}$; α, β are constants

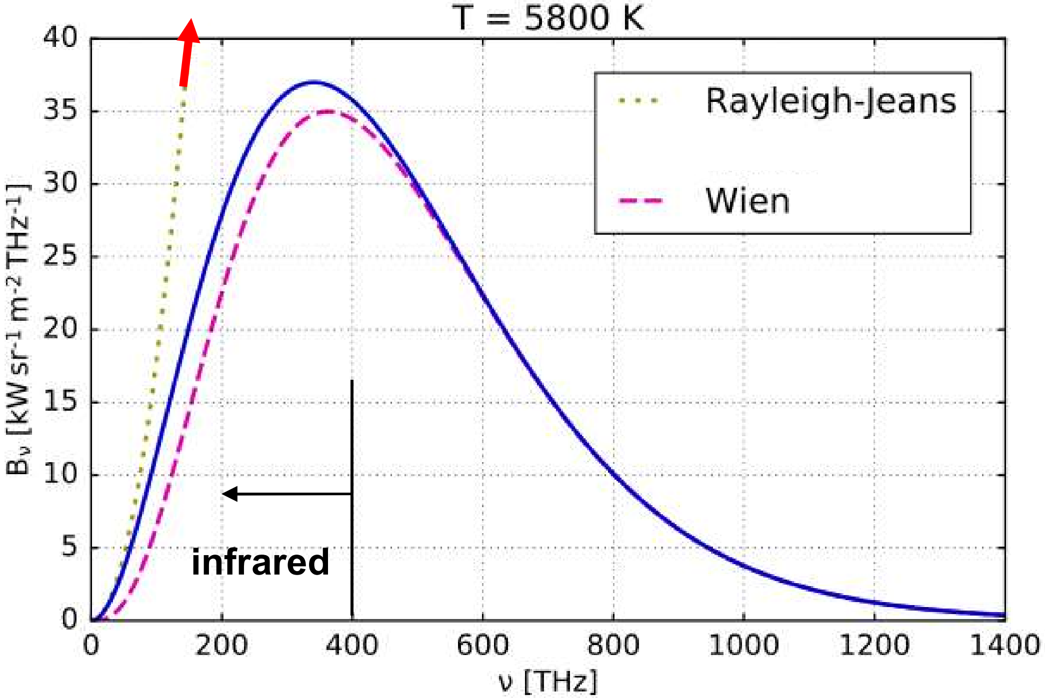
  Particle-like, good for high frequency

- Rayleigh-Jeans law (1900,1905) $u_\nu(T) = \frac{8\pi\nu^2}{c^3}kT$

  Wave-like, good for low frequency

Note: Thornton uses $M_\lambda(T)$, Eq.3.22

Blow up
(ultraviolet catastrophe)



Planck's discovery

- In 1899, Planck wrote a paper proving that Wien formula is the right answer.

  "*the limits of validity of this law, should there be any, therefore coincide with those of the second law of thermodynamics*."

- Oct 19, 1900: Kurlbaum presented the latest experimental data (with infrared), with Planck in the audience, which does not fit Wien's formula.
- To save his own reputation, Planck came up with a new formula in Oct., and found a "justification" in Dec.

Planck's formula (Energy density per frequency interval)

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

Note: Thornton uses $M_\lambda(T)$, Eq.3.23

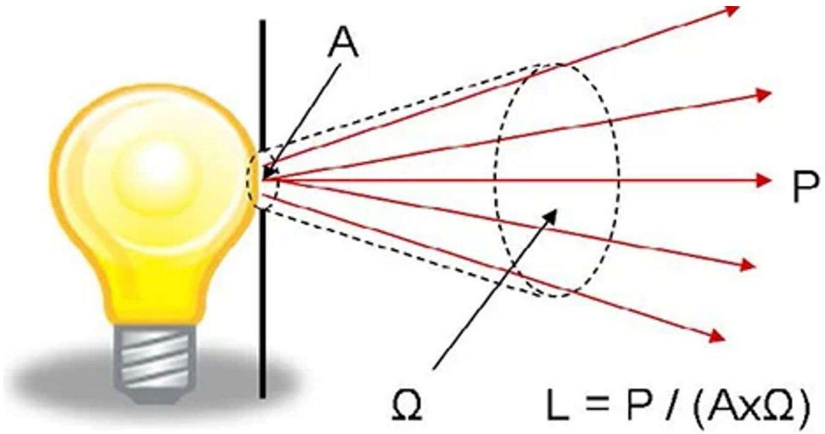
1. It reduces to Wien's and Rayleigh-Jean's results at two opposite limits.
2. It also leads to Wien's displacement law and Stefan-Boltzmann law.

Conflicted Genius: A New Perspective on Einstein's Science A.D. Stone

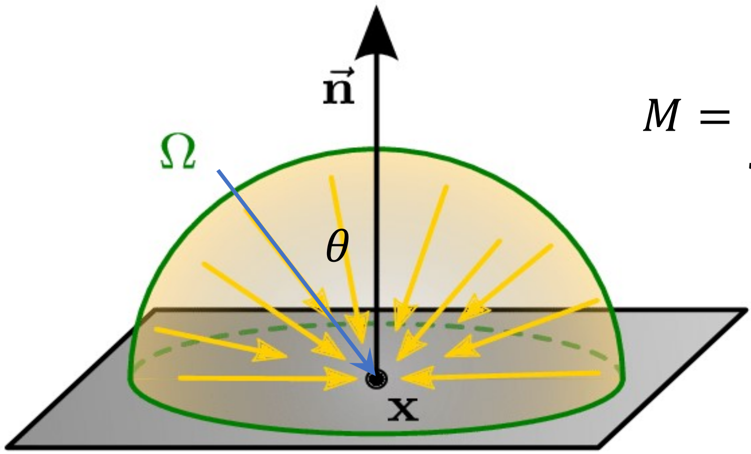
This math trick revolutionized physics J.S. Diaz

optional

輻射率(出射) • Radiance (L) of source is the Power (P) emitted from the source emitting Area (A) and propagated in the Solid Angle (Ω).



輻照度(出射/入射) • Irradiance (M), radiance integrated over a hemisphere



$$M = \int_0^1 L \cos\theta d\cos\theta \int_0^{2\pi} d\phi = \pi L$$

optional

Different Formulations of Planck's Law

	quantity	label	units	meaning	
L 輻射率(出射)	radiance	L	[W m^{-2} sr^{-1}]	spatial+directional power density	Steradian 球面度
M 輻照度	flux density (or irradiance)	$M = \pi L$	[W m^{-2}]	spatial power density	
u 能量密度	energy density	$u = \frac{4\pi}{c} L = \frac{4}{c} M$	[J m^{-3}]	spatial energy density	

In Terms of Frequency

radiance $L_\nu d\nu = dL$

$$L_\nu(\nu) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

Irradiance $M_\nu d\nu = dM$

$$M_\nu(\nu) = \frac{2\pi h\nu^3}{c^2} \cdot \frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

energy density $u_\nu d\nu = du$

$$u_\nu(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp(\frac{h\nu}{kT}) - 1}$$

This expression is the easiest to understand

$\Longleftrightarrow$

In Terms of Wavelength

$$L_\lambda(\lambda) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{\exp(\frac{hc}{\lambda kT}) - 1}$$

$$M_\lambda(\lambda) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{\exp(\frac{hc}{\lambda kT}) - 1}$$

$\mathscr{I}(\lambda, T)$ **In textbook**

$$u_\lambda(\lambda) = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\exp(\frac{hc}{\lambda kT}) - 1}$$

$$u_\nu d\nu = u_\lambda d\lambda$$

$$u_\nu(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp(\frac{h\nu}{kT}) - 1} \qquad M_\lambda(\lambda) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{\exp(\frac{hc}{\lambda kT}) - 1}$$

Ex 3.6: Show that Wien's displacement law follows from Planck's law.

Solution: $\frac{dM_\lambda(T)}{d\lambda} = 0$

➡ $xe^x = 5(e^x - 1)$ $\qquad x = \frac{hc}{\lambda_{\max} kT}$

This is a transcendental equation and can be solved numerically with the result $x \simeq 4.966$,

➡ $\lambda_{\max} T = \frac{hc}{4.966\ k} = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$

Ex 3.7: Show that the Stefan-Boltzmann law follows from Planck's law.

Solution:

$$R(T) = 2\pi c^2 h \int_0^\infty \frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}\, d\lambda$$

$$x = \frac{hc}{\lambda kT} \qquad = +2\pi c^2 h \left(\frac{kT}{hc}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1}\, dx$$

From Appendix 3D, $\int_0^\infty \frac{x^3}{e^x - 1}\, dx = \frac{\pi^4}{15}$

$$\Rightarrow \quad R(T) = 2\pi c^2 h \left(\frac{kT}{hc}\right)^4 \frac{\pi^4}{15}$$

$$= \boxed{\frac{2\pi^5 k^4}{15 h^3 c^2}} T^4$$

$$= 5.67 \times 10^{-8}\, T^4 \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$$

Stefan–Boltzmann constant σ

On Dec 14, 1900, Planck presented some explanation at a meeting in Berlin.

Planck's assumptions:

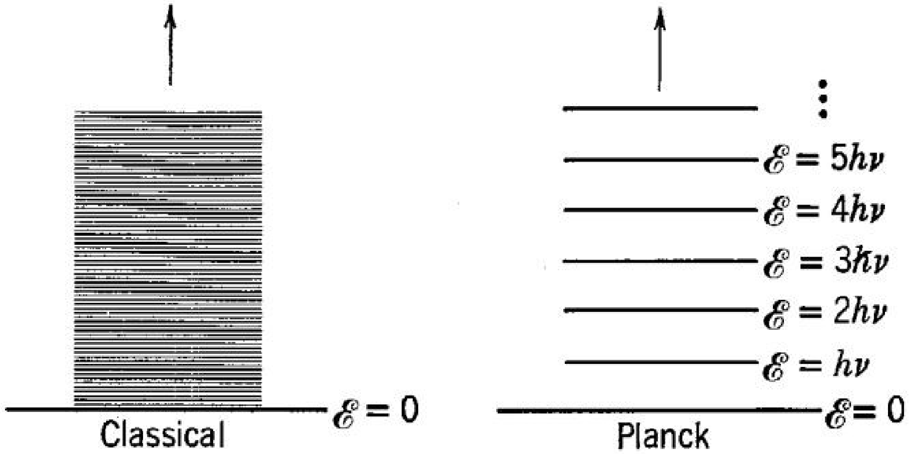
The thermal radiation is in equilibrium with oscillators on the wall. The oscillators can absorb or emit energy only in discrete multiples of the energy quantum hν .

- Below is Debye's argument, 1912 (Stone, Einstein and quantum).
  - Energy of EM wave oscillator: $\epsilon_n = nh\nu$ n=1,2,3…
  - Thermal distribution

    Boltzmann factor $P(E) = e^{-\frac{E}{kT}}$ the fraction of occupied states w.r.t. the E=0 state

The averaged energy of one *mode*:

$$\bar{\mathscr{E}} = \frac{\sum_{n=0}^{\infty} \mathscr{E} P(\mathscr{E})}{\sum_{n=0}^{\infty} P(\mathscr{E})} = \frac{\sum_{n=0}^{\infty} nh\nu\, e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$



Eisberg and Resnick

$$-\alpha \frac{d}{d\alpha} \ln \sum_{n=0}^{\infty} e^{-n\alpha} = \frac{\sum_{n=0}^{\infty} n\alpha e^{-n\alpha}}{\sum_{n=0}^{\infty} e^{-n\alpha}}$$

$$\Rightarrow \quad \bar{\mathscr{E}} = \frac{h\nu e^{-\alpha}}{1 - e^{-\alpha}} = \frac{h\nu}{e^{\alpha} - 1} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

After including the density of states of EM waves in a cavity $g(\nu) = \frac{8\pi}{c^3}\nu^2$, we will get Planck's law:

$$u_\nu(\nu) = g(\nu)\bar{\varepsilon}_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT} - 1}$$

Number of standing waves within d ν

Energy of standing wave with frequency ν

- By fitting experimental curves, Planck found two important constants:

Boltzmann const	$k = 1.346 \times 10^{-23}$ J/K	After 2019, it's a defined quantity	$K = 1.380649 \times 10^{-23}$ J/K
Planck constant	$h = 6.55 \times 10^{-27}$ J $\cdot$ sec	After 2019, it's a defined quantity	$h = 6.62607015 \times 10^{-34}$ J sec

Note: Planck is actually the first to introduce the Boltzmann constant *k*.
Boltzmann never used it in his works.

From *k* and *h*, other fundamental constants can be determined, such as

- From Molar gas const to Avogadro const

$$R = kN_A \rightarrow N_A = 6.17 \times 10^{23}$$

After 2019, it's a defined quantity	N_A=6.02214076×10^{23} particles

- From Faraday const to Electron charge

Faraday const: the electric charge of one mole of elementary carriers (e.g., protons)

$$F = eN_A \rightarrow e = 1.56 \times 10^{-19}\ C$$

After 2019, it's a defined quantity	*e*= 1.602176634×10^{-19} C

Note: There are seven "defining constants". Here we already see four.

(The others are C, caesium hyperfine frequency, and the luminous efficacy of a defined visible radiation)

“What I did can be simply described as an act of desperation”.

Afterwards, Planck stopped talking about this discovery for 5 years!

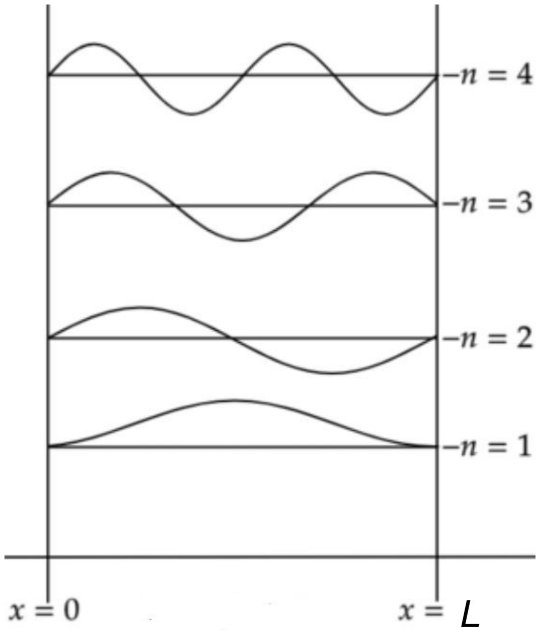
“… *the work was not ignored, but it was not at the center of attention. There were many spectacular discoveries at this time, and Planck himself was so diffident of the methods used that he spent years trying to explain his results in a less revolutionary way*.” Segre, From X-rays to quarks

Still, He was aware of the importance of his discovery from the beginning.

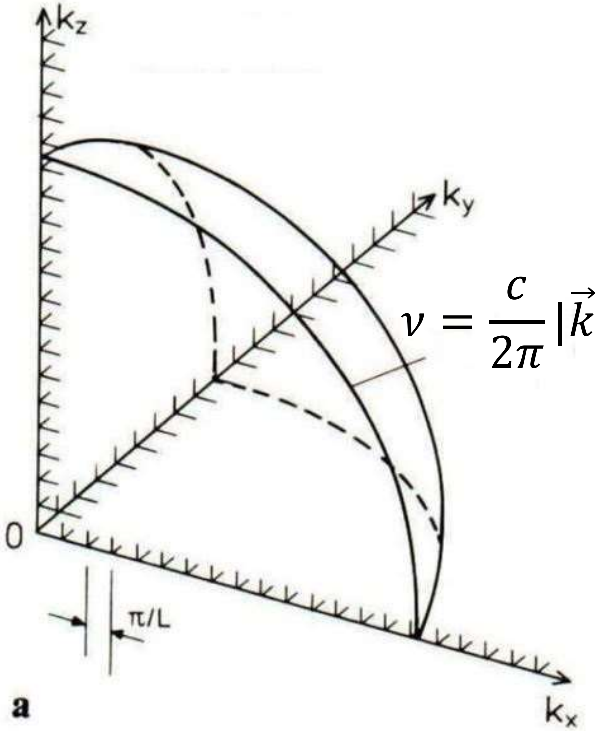
optional

Density of states $g(\nu)$ more in Chap 9

$g(\nu)d\nu =$ the number of standing waves (normal modes) within $[\nu, \nu+d\nu]$ (divided by volume)



$$k_x = n_x \frac{\pi}{L}, n_x = 1,2 \ ...$$
$$k_y = n_y \frac{\pi}{L}, n_y = 1,2 \ ...$$
$$k_z = n_z \frac{\pi}{L}, n_z = 1,2 \ ...$$



$$\nu = \frac{c}{2\pi}|\vec{k}|$$

Each wave vector ***k*** occupies $\Delta^3 k = \left(\frac{\pi}{L}\right)^3$

$$g(\nu)d\nu = 2 \cdot \frac{\frac{1}{8}4\pi k^2 dk}{\left(\frac{\pi}{L}\right)^3}\frac{1}{L^3} = \frac{8\pi\nu^2}{c^3}d\nu$$

Each *k* state has 2 polarizations

- Discovery of X-ray
- Discovery of electron
- Blackbody radiation
- Photoelectric effect
- Compton effect
- Pair production

Methods of electron emission:

1. Thermionic emission: Application of heat allows electrons to gain enough energy to escape. Vacuum tube
2. Secondary emission: The electron gains enough energy by transfer from a high-speed particle that strikes the material from outside.
3. Field emission: A strong external electric field pulls the electron out of the material. Crookes tube
4. Photoelectric effect: Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape.

Difficulty of knocking electrons out of materials: work function 功函數

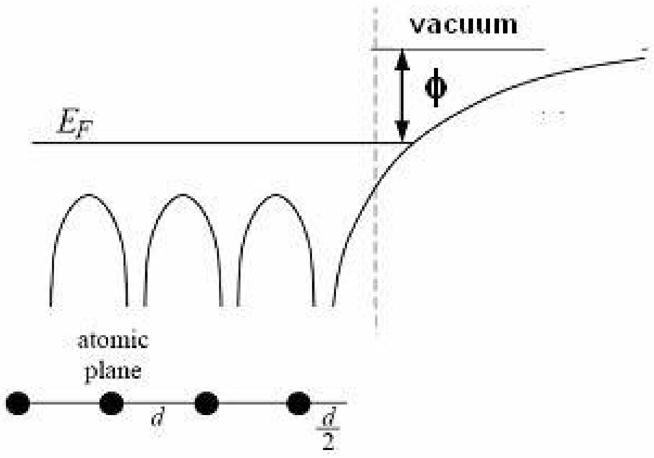


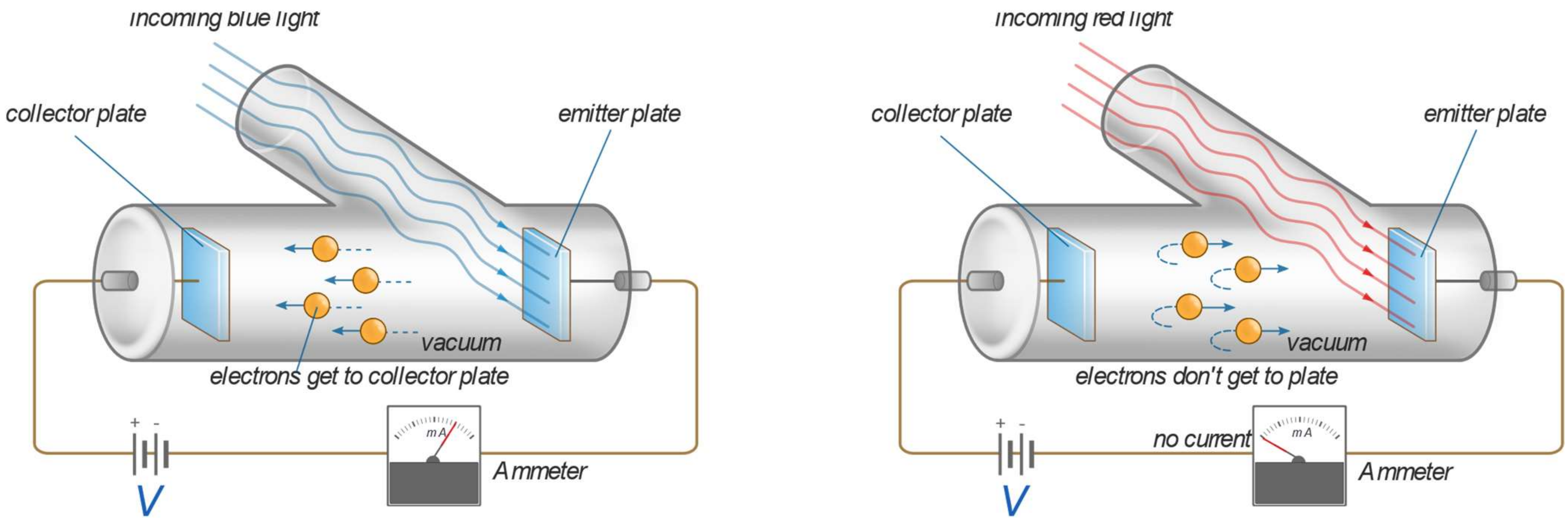
Table 3.3 Work Functions

Element	ϕ (eV)	Element	ϕ (eV)
Ag	4.64	K	2.29
Al	4.20	Li	2.93
C	5.0	Na	2.36
Cs	1.95	Nd	3.2
Cu	4.48	Ni	5.22
Fe	4.67	Pb	4.25

Photoelectric effect

- 1887, when Hertz was detecting EM wave. The sparks of the receiving gap was enhanced when the cathode was illuminated with ultraviolet light.
- 1898, Lenard measured the q/m ratio of the ejected current and found it to the the same as Thomson's value from cathode ray.
- 1902, Lenard found the surprising fact that the maximum velocity with which electrons are ejected by ultraviolet light is entirely independent of the **intensity** of light. This contradicts the physics then known, since classical EM predicts that the brighter the light, the larger the electron energy: $u_{EM} \simeq \varepsilon_0 \left|\vec{E}_0\right|^2$
- Also, classical theory predicts that for low light intensities, a long time (10^4 sec) would elapse before any electron ejection. However, the photoelectrons are ejected almost immediately.

Later, Millikan studied the dependence on light **frequency**



He found that

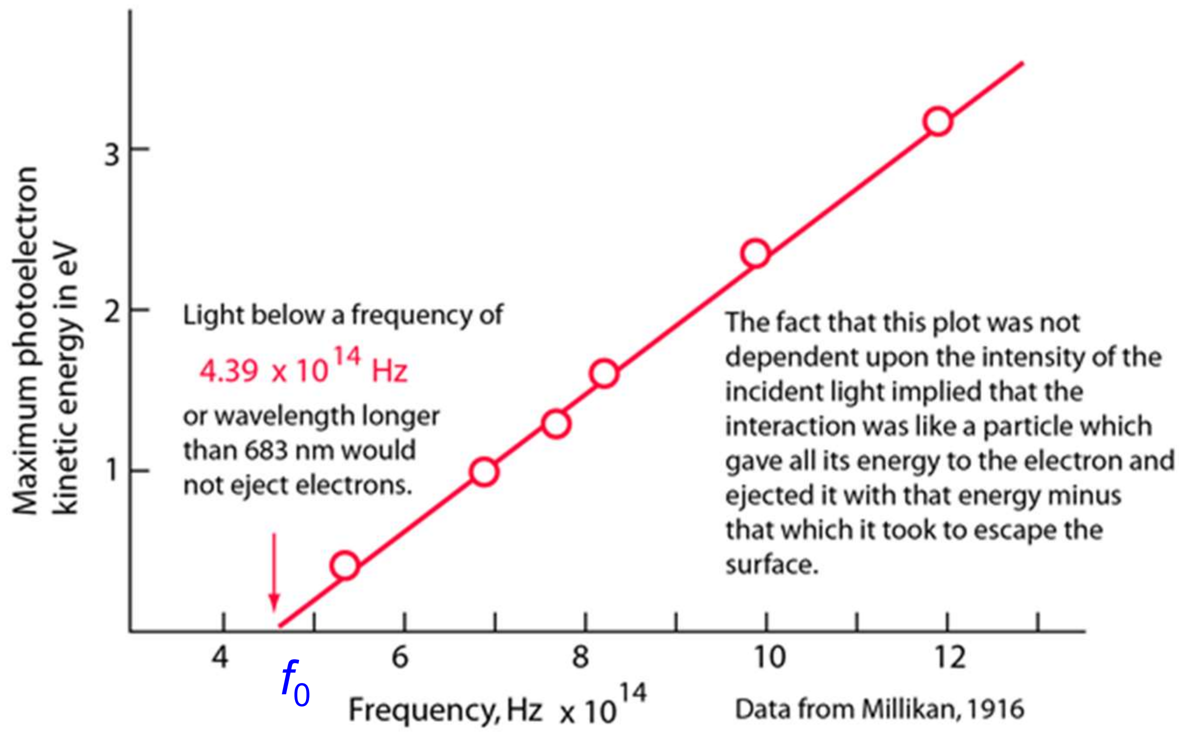
1. the kinetic energy of the electrons is linearly proportional to the **frequency** of the incident radiation above a threshold f_0 (no current is observed below f_0).

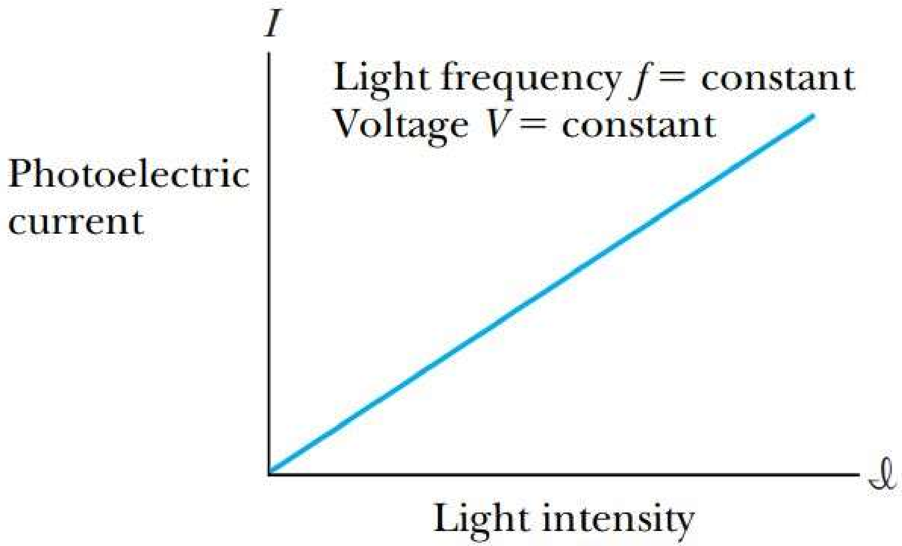
2. the number of electrons (i.e. the electric current) is proportional to the **intensity** and independent of the *frequency* of the incident radiation.

- Again neither can be understood by classical electromagnetism.

Source of figures

Properties of photoelectric effect



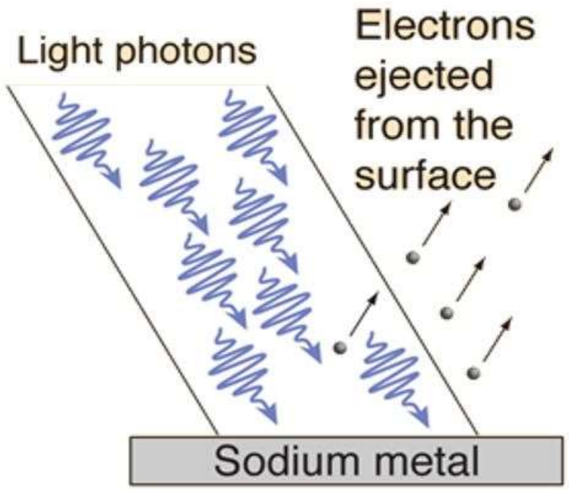


1905, Einstein assumed that

1. A light quantum delivers its entire energy to a single electron
2. The energy of a light quantum is related to the frequency, but not the amplitude of the EM wave,

$$E = hf$$

This is “The second coming of *h*” – A. Pais



hyperphys

According to these assumptions,

- The threshold frequency f_0 is required to overcome the work function ϕ:
$$hf_0 = \phi$$
- The kinetic energy of an electron depends only on the light frequency and the work function of the material.
$$\frac{1}{2}mv^2 = hf - \phi$$
- The number of light quanta, or the number of photoelectrons, is proportional to the intensity of the light.

Ex 3.10: Photoelectrons may be emitted from sodium ($\phi = 2.36$ eV) even for light intensities as low as 10^{-8} W/m^2. Calculate classically how much time the light must shine to produce a photoelectron of kinetic energy 1.00 eV.

Strategy We will assume that all of the light is absorbed in the first layer of atoms in the surface. Then we calculate the number of sodium atoms per unit area in a layer one atom thick. We assume that each atom in a single atomic layer absorbs equal energy, but a single electron in each of these atoms receives all the energy. We then calculate how long it takes these electrons to attain the energy (2.36 eV + 1.00 eV = 3.36 eV) needed for the electron to escape.

Solution: ......

➡ If energy is absorbed at the rate of 7.25×10^{-9} eV/s for a single electron, we can calculate the time t needed to absorb 3.36 eV:

$$t = \frac{3.36 \text{ eV}}{7.25 \times 10^{-9} \text{ eV/s}} = 4.63 \times 10^{8} \text{ s} = 14.7 \text{ years}$$

Ex 3.12: (a) What frequency of light is needed to produce electrons of kinetic energy 3.00 eV from illumination of lithium?
(b) Find the wavelength of this light and discuss where it is in the electromagnetic spectrum.

Solution: (a)

$$hf = \phi + \tfrac{1}{2} mv_{\text{max}}^2$$

$$= 2.93 \text{ eV} + 3.00 \text{ eV} = 5.93 \text{ eV}$$

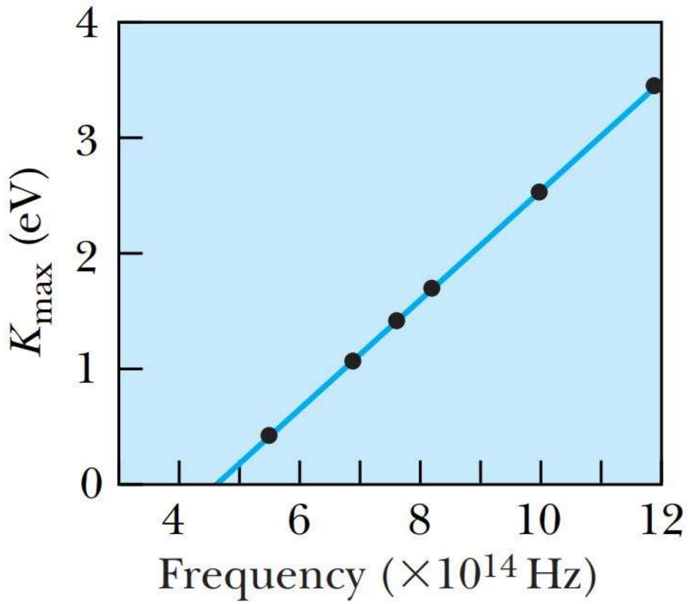
$$\Rightarrow f = \frac{E}{h} = 1.43 \times 10^{15} \text{ Hz}$$

(b)

$$\lambda = \frac{c}{f} = 210 \text{ nm}$$

This is ultraviolet light, because the wavelength 210 nm is below the range of visible wavelengths 380 to 750 nm

- Einstein's hypothesis of light quanta was not taken seriously for over fifteen years (and no good exp't data for comparison). The reasons are clear. It seemed to be an unnecessary rejection of the highly verified classical theory of radiation. How light quanta could possibly explain interference phenomena was always the central objection.
- As an unbeliever of Einstein's theory, Millikan spent more than 10 years trying to prove that Einstein is wrong, He ended up verifying Einstein's theory and getting a precise value of the Planck const *h*.



slope

$h = 6.57 \times 10^{-34}$ J·s

Millikan tests Einstein's Light Theory J.S. Diaz

Wheaton, B. R. (1983). *The tiger and the shark: Empirical roots of wave-particle dualism.*

Estimate of h from daily experience (Tomonaga, p.38 QM)

Since the wave length of red light is 8×10^{-5} cm its frequency is

$$\nu_{\text{red}} = \frac{c}{\lambda_{\text{red}}} = \frac{3 \times 10^{10}}{8 \times 10^{-5}} \cong 0.4 \times 10^{15} \text{ sec}^{-1} .$$

The observed fact that at about 500 °C it is a dim red light and becomes brilliant red at about 1000 °C suggests that kT at 500 °C is still too small compared with $h\nu_{\text{red}}$ to make the red oscillation very active, and that at 1000 °C kT becomes comparable to $h\nu_{\text{red}}$ thus sufficiently activating the red oscillation. Hence we can expect the following relation to hold,

$$h\nu_{\text{red}} \cong k\,(1000 + 273)$$

whence

$$h \cong 1273 \times \frac{k}{\nu_{\text{red}}} .$$

Using the above value of ν_{red} and Boltzmann's constant, we find

$$h = 4 \times 10^{-28} \text{ erg} \cdot \text{sec} .$$

1905, Einstein's letter to Habicht (May 18 or 25)

... I promise you four papers in return. The first deals with radiation and the energy properties of light and is very revolutionary, as you will see if you send me your work first. The second paper is a determination of the true size of atoms...The third proves that bodies on the order of magnitude 1/1000 mm, suspended in liquids, must already produce an observable, random motion ... who call it Brownian motion. The fourth paper is only a rough draft at this point, and is an electrodynamics of moving bodies which employs a modification of the theory of space and time.

Ex 3.13: For the light intensity of Example 3.10, $\mathcal{I} = 10^{-8}\ \text{W/m}^2$, a wavelength of 350 nm is used. What is the number of photons/$(\text{m}^2 \cdot \text{s})$ in the light beam?

Solution:

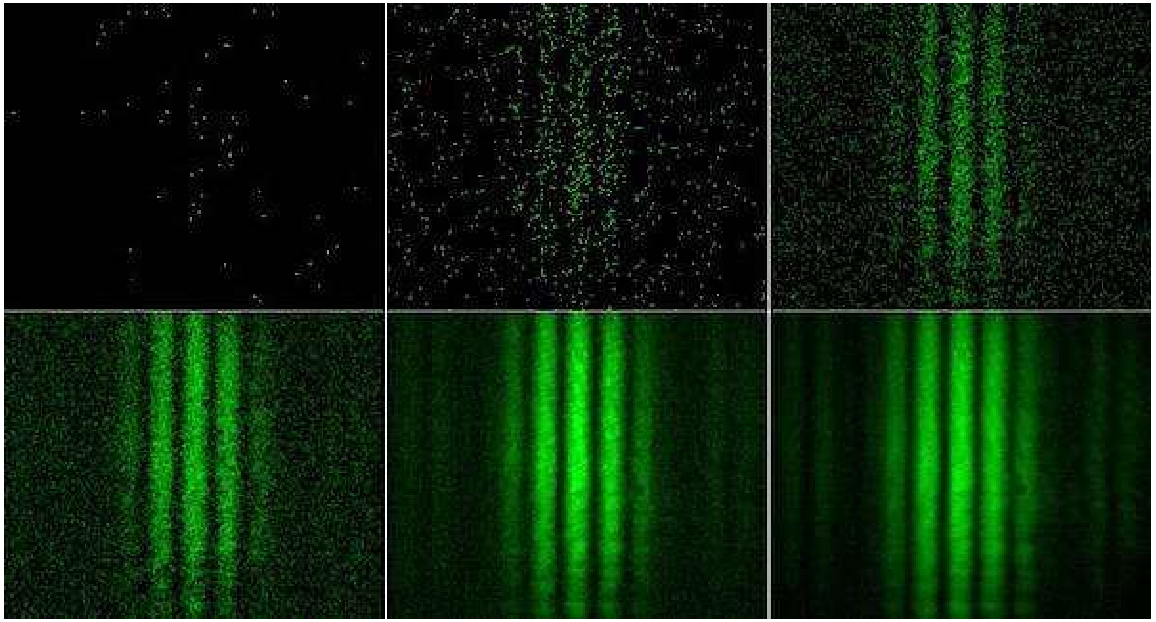
$$E = hf = \frac{hc}{\lambda} = 3.5\ \text{eV}$$

$$\text{Intensity } \mathcal{I} = \left[N\left(\frac{\text{photons}}{\text{m}^2 \cdot \text{s}}\right)\right]\left[E\left(\frac{\text{energy}}{\text{photon}}\right)\right]$$

$$= NE\left(\frac{\text{energy}}{\text{m}^2 \cdot \text{s}}\right)$$

➡ $$N = \frac{\mathcal{I}}{E} = 1.8 \times 10^{10}\ \frac{\text{photons}}{\text{m}^2 \cdot \text{s}}$$

Double-slit experiment
using single-photon source



wiki.anton-paar.com/en/double-slit-experiment/

Duality of light

Phenomenon	Can be explained in terms of waves.	Can be explained in terms of particles.
Reflection	∿∿∿ ✓	●→ ✓ ?
Refraction	∿∿∿ ✓	●→ ✓ ?
Interference	∿∿∿ ✓	●→ ⊗
Diffraction	∿∿∿ ✓	●→ ⊗
Polarization	∿∿∿ ✓	●→ ⊗
Photoelectric effect	∿∿∿ ⊗	●→ ✓

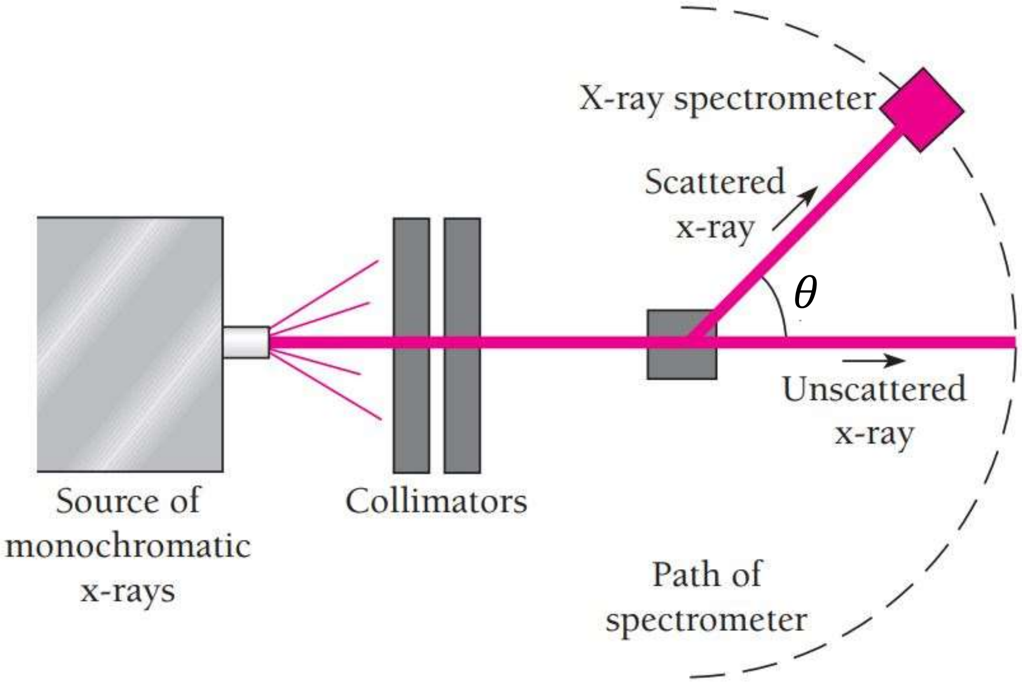
- Discovery of X-ray
- Discovery of electron
- Blackbody radiation
- Photoelectric effect
- Compton effect
- Pair production

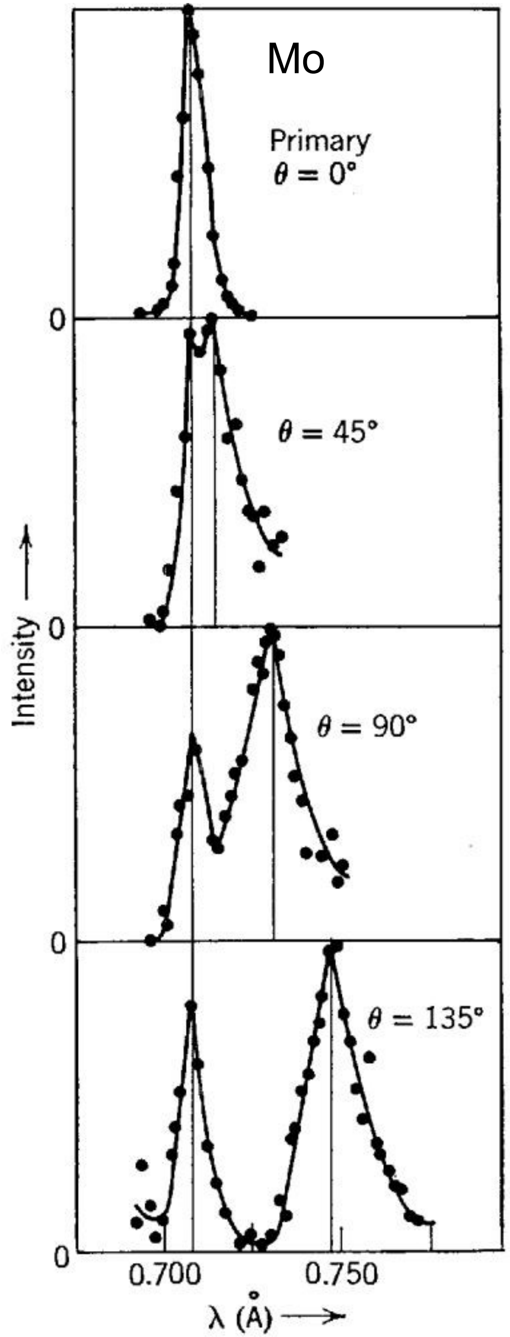
Note: From now on, we call a light quantum a "photon"

Compton effect (1923)

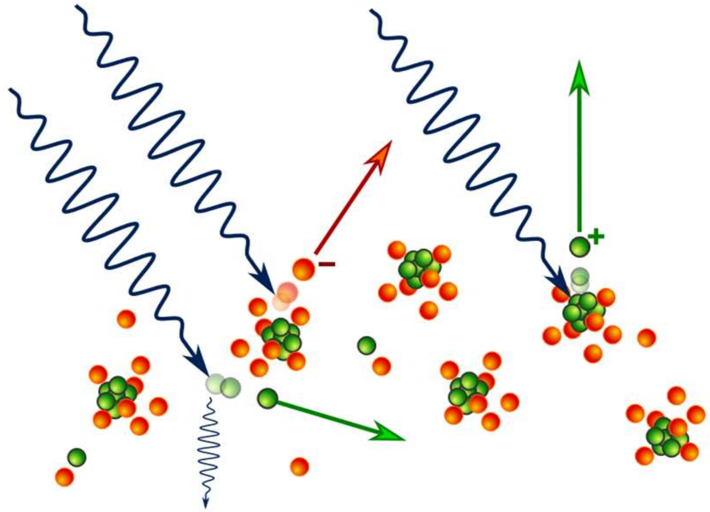
It is discovered when Compton was studying the scattering of X-rays by light elements

- Scattering of light by electrons alters the wavelength (energy) of the light





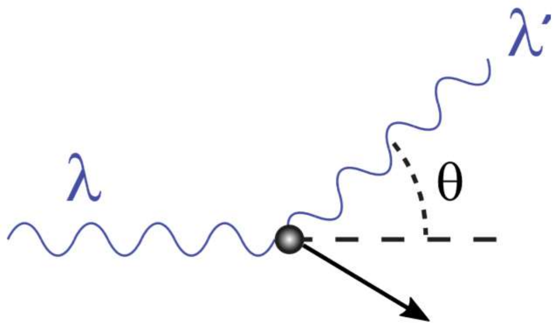
Compton effect



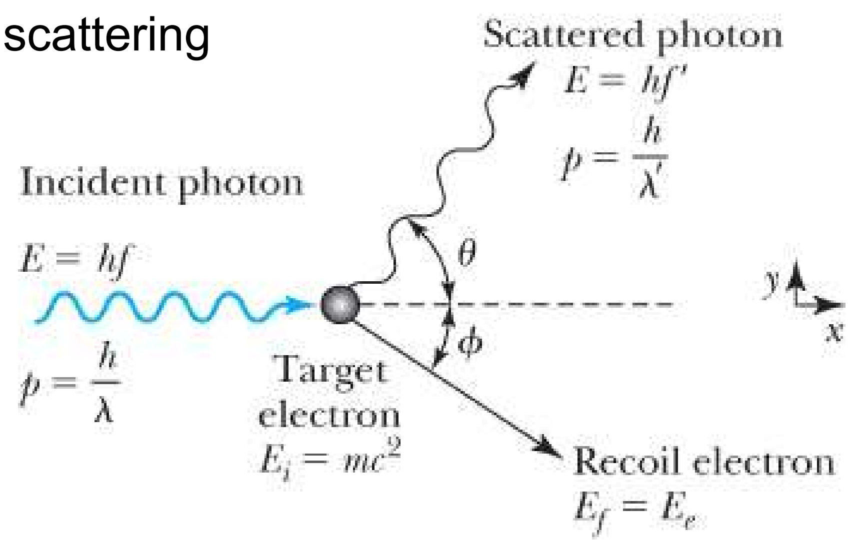
- The photon is scattered from only one electron, rather than from all the electrons in the material, and the laws of conservation of energy and momentum apply as in any elastic collision between two particles.

- Momentum of a photon

$$p = \frac{E}{c} = \frac{h\nu}{c}$$



Photon-electron scattering



Energy $$hf + mc^2 = hf' + E_e$$

p_x $$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + p_e\cos\phi$$

$$E_e^2 = (mc^2)^2 + p_e^2c^2$$

p_y $$\frac{h}{\lambda'}\sin\theta = p_e\sin\phi$$

$$\Rightarrow \quad \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

for electron

$$\lambda_C = \frac{h}{mc} = 2.426 \times 10^{-12}$$

Compton wavelength

Thus, this experiment verifies the formula of photon momentum

Ex 3.16: An x ray of wavelength 0.050 nm scatters from a gold target. (a) Can the x ray be Compton-scattered from an electron bound by as much as 62 keV? (b) What is the largest wavelength of scattered photon that can be observed? (c) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur?

Solution From Equation (3.35) the x-ray energy is

(a)

$$E_{\text{x ray}} = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{0.050 \text{ nm}} = 24{,}800 \text{ eV} = 24.8 \text{ keV}$$

Therefore, the x ray does not have enough energy

(b) Scattering may still occur from outer electrons, so we examine Equation (3.42) with the electron mass. The longest wavelength $\lambda' = \lambda + \Delta\lambda$ occurs when $\Delta\lambda$ is a maximum or when $\theta = 180°$.

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos 180°) = \lambda + \frac{2h}{m_e c}$$

$$= 0.050 \text{ nm} + 2(0.00243 \text{ nm}) = 0.055 \text{ nm}$$

(c) The energy of the scattered photon is then a minimum and has the value

$$E'_{\text{x ray}} = \frac{1.240 \times 10^3 \text{ eV} \cdot \text{nm}}{0.055 \text{ nm}} = 2.25 \times 10^4 \text{ eV} = 22.5 \text{ keV}$$

$$E_{\text{x ray}} = E'_{\text{x ray}} + \text{K.E. (electron)}$$

$$\Rightarrow \quad \text{K.E. (electron)} = E_{\text{x ray}} - E'_{\text{x ray}}$$

$$= 24.8 \text{ keV} - 22.5 \text{ keV} = 2.3 \text{ keV}$$

Pair production and annihilation (more in Chap 14)

- 1928, Dirac predicted the existence of electron with positive charge (called a positron). This is an anti-particle of an electron.

- 1932, Anderson observed a positively charged electron (e^+) in cosmic rays.

- A photon's energy can be converted entirely into an electron and a positron in a process called pair production.

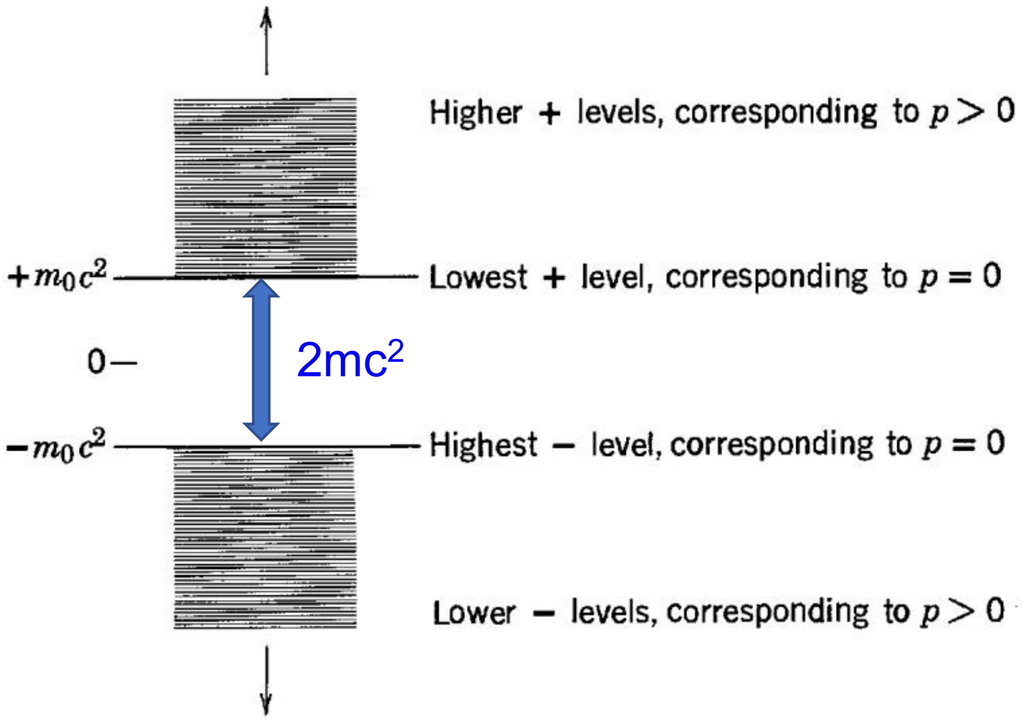
- Pair production

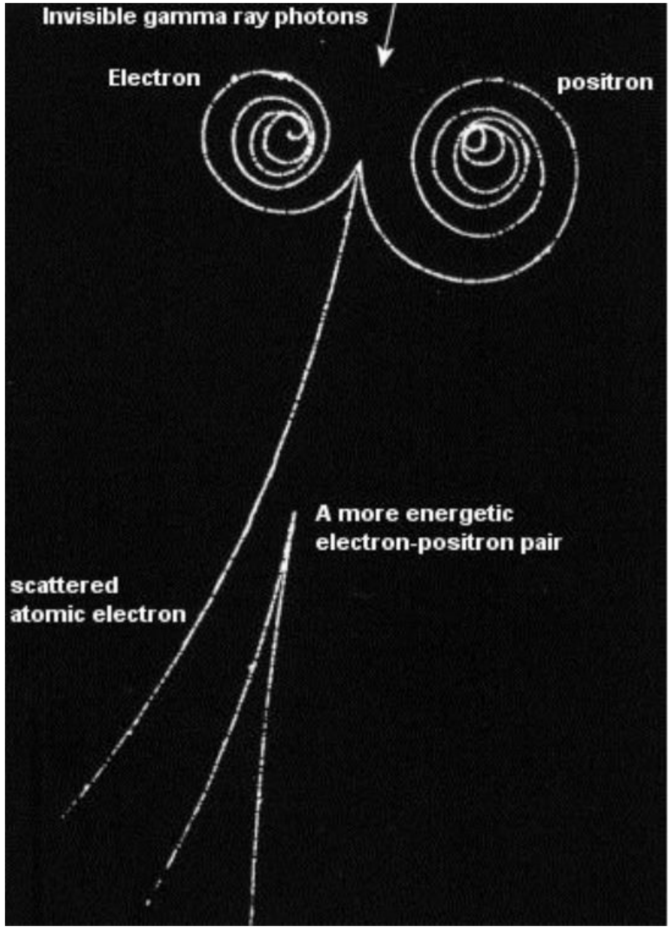
$$\gamma \rightarrow e^+ + e^-$$

- Pair annihilation

$$e^+ + e^- \rightarrow \gamma + \gamma$$

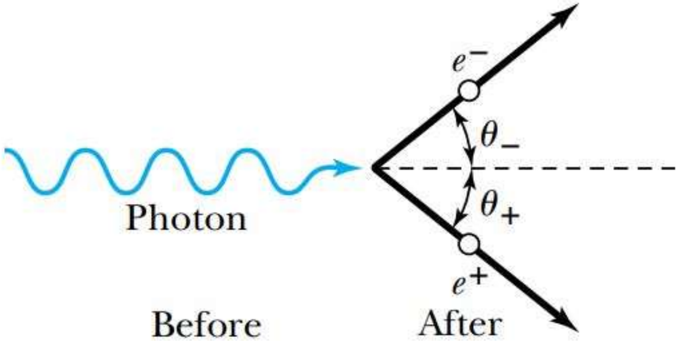
Energy levels of a free electron according to Dirac's theory



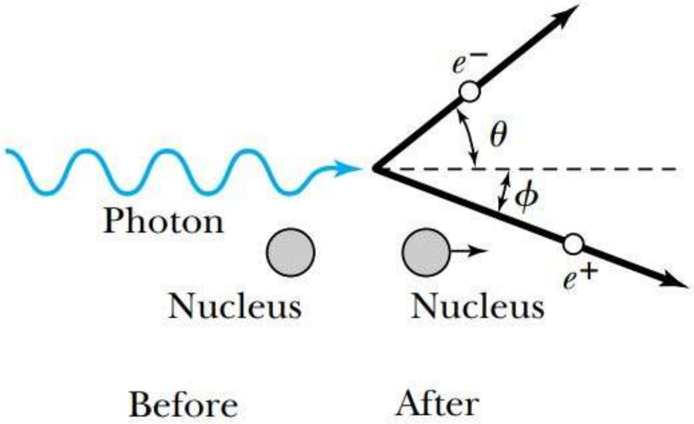


- Every particle has its anti-particle
- A photon with energy $E > 2mc^2$ can create a pair of particle-anti particle each with mass m (under the constraint of energy conservation and momentum conservation).

Ex 3.17: Show that a photon cannot produce an electron-positron pair in free space.



(a) Free space (**cannot occur**)



(b) Beside nucleus

Solution:

Energy $$hf = E_+ + E_-$$

Momentum, p_x $$\frac{hf}{c} = p_- \cos\theta_- + p_+ \cos\theta_+ \Rightarrow hf = p_- c \cos\theta_- + p_+ c \cos\theta_+$$

Momentum, p_y $$0 = p_- \sin\theta_- - p_+ \sin\theta_+$$

$$hf_{\max} = p_- c + p_+ c$$

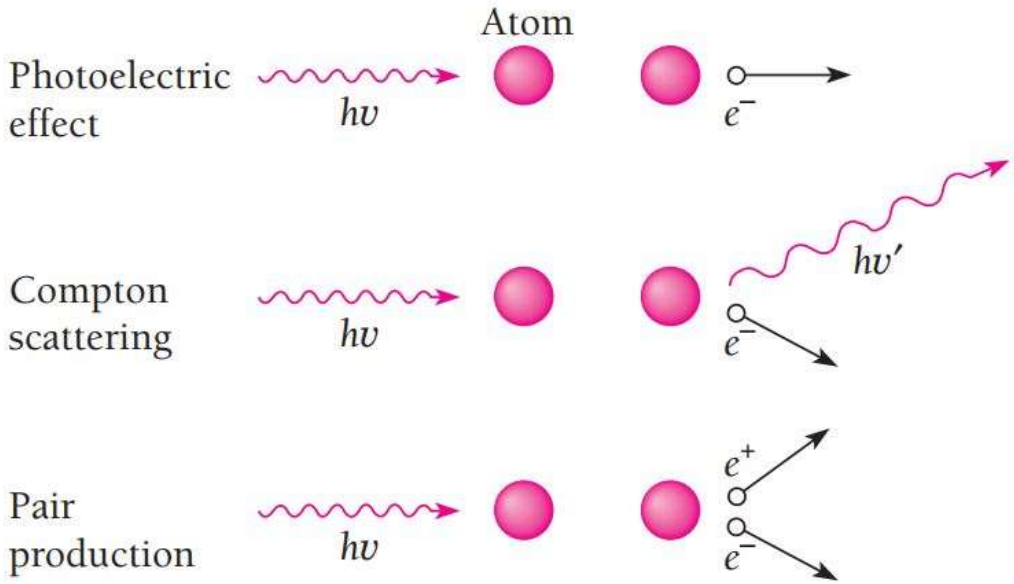
$$E_\pm^2 = p_\pm^2 c^2 + m^2 c^4$$

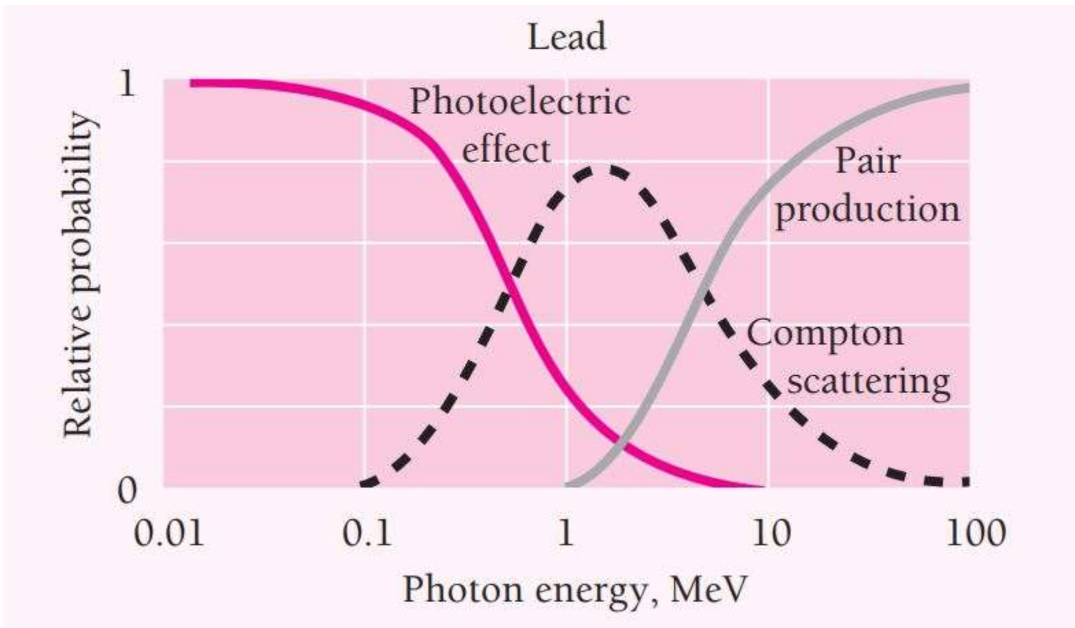
$$\Rightarrow hf = \sqrt{p_+^2 c^2 + m^2 c^4} + \sqrt{p_-^2 c^2 + m^2 c^4}$$

$$\Rightarrow hf > p_- c + p_+ c$$

i.e., energy and momentum cannot be simultaneously conserved.

A summary:


Atom
Photoelectric effect
hν
e⁻
Compton scattering
hν
hν′
e⁻
Pair production
hν
e⁺
e⁻


Lead
1
0
Relative probability
Photoelectric effect
Pair production
Compton scattering
0.01
0.1
1
10
100
Photon energy, MeV