Chap 2

Special theory of relativity

- Michelson-Morley experiment
- Einstein’s postulates
- Lorentz transformation
- Contraction of length
- Dilation of time
- Doppler effect
- Addition of velocities
- Momentum and energy

Traditional view (19th century) of EM wave:

1. Wave is a form of motion

2. All waves need a medium (water wave, sound wave …)

3. So EM wave also needs a medium (called ether. It fills space)

乙太

- Ether had to have such a low density that the planets could move through it without loss of energy.

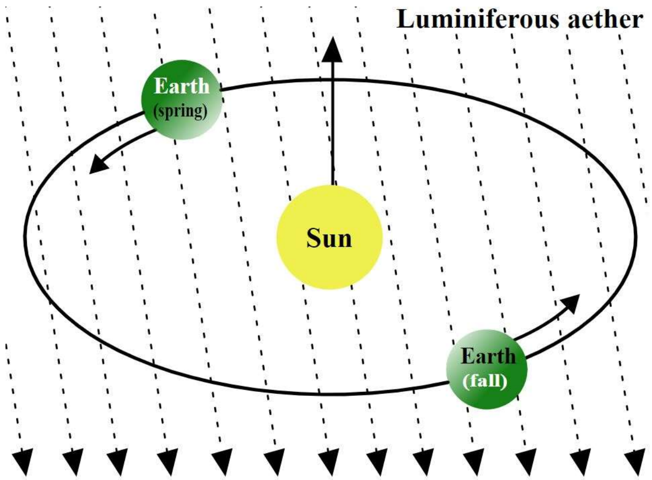
- It also had to have a high elasticity to support the high velocity of light waves.

Q: What is the velocity of the earth w.r.t. ether?

Many types of experiments tried to measure this velocity.

The most famous experiment was done by Michelson and Morley.

Michelson and Morley's experiment
(~1887)



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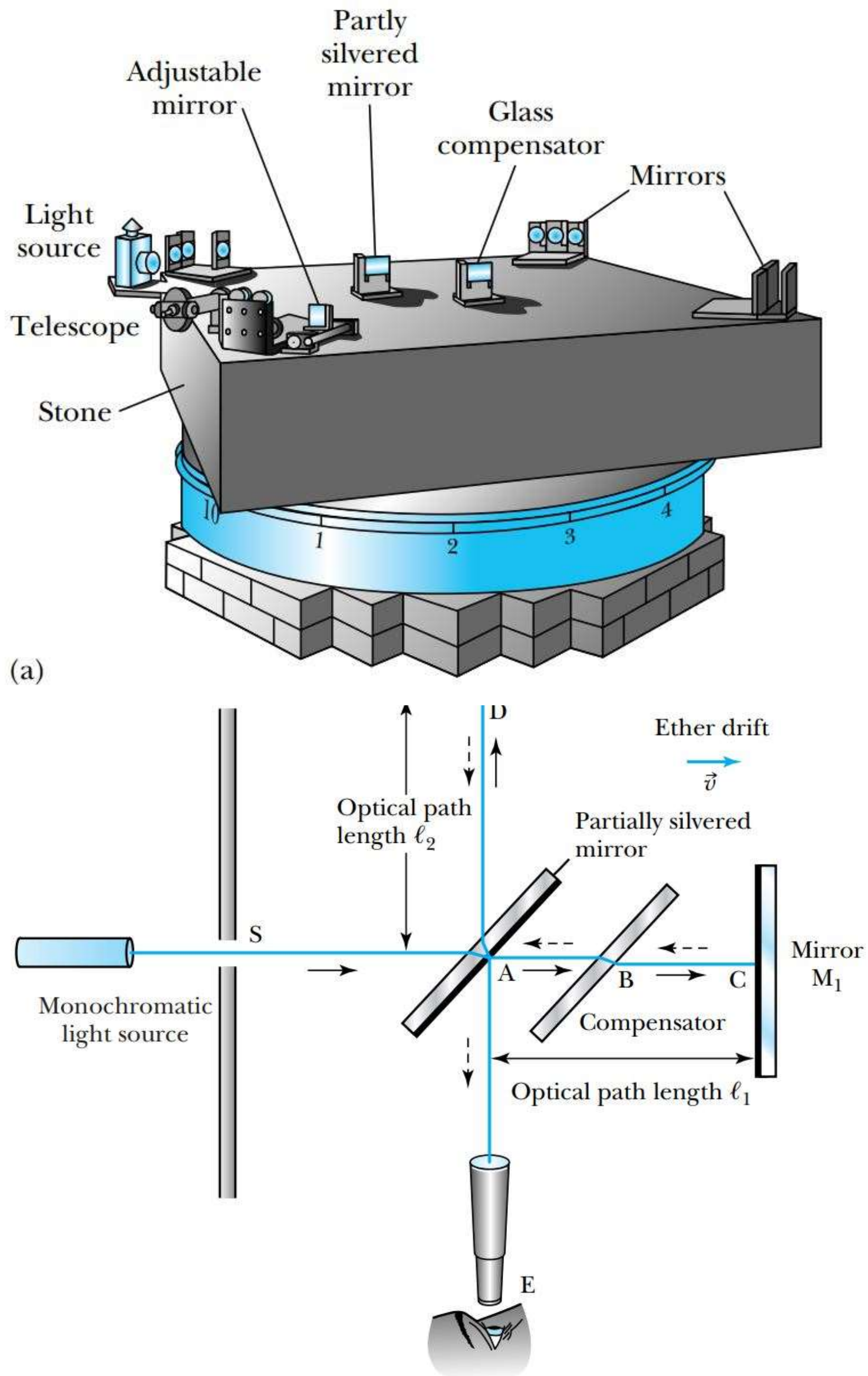
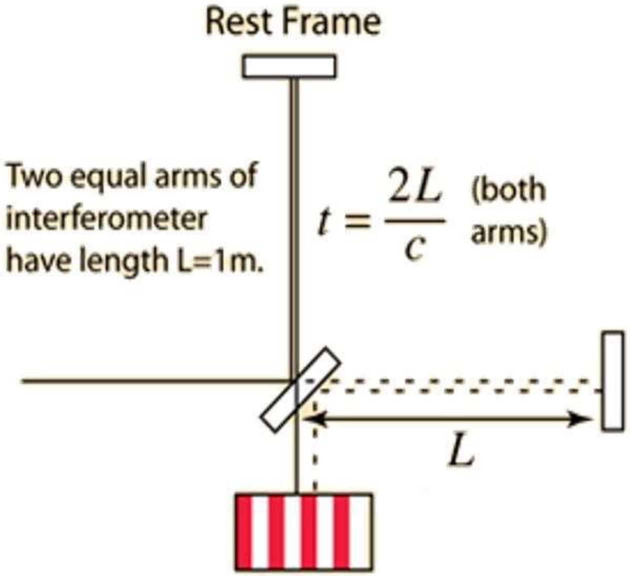
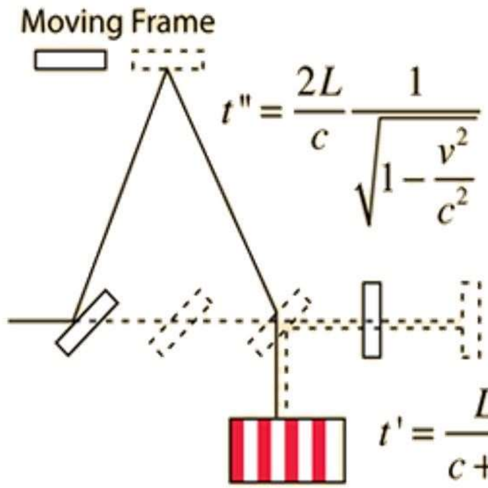


hyperphysics



Two equal arms of interferometer have length L=1m.

$$t = \frac{2L}{c} \quad \text{(both arms)}$$



$$t'' = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{L}{c+v} + \frac{L}{c-v}$$

$$t' = \frac{2L}{c}\frac{1}{1-\frac{v^2}{c^2}}$$

For velocity of Earth around Sun

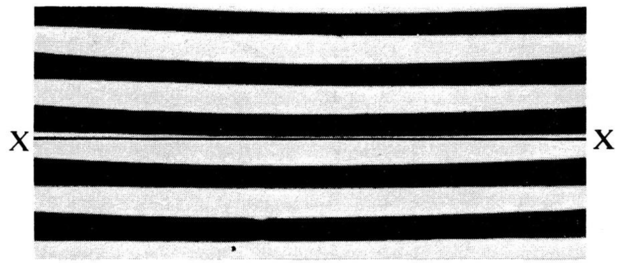
$$v = 3x10^4\, m/s$$

the time difference

$$t' - t'' = 3x10^{-17}\, s$$

and for $T = 10^{-15}\, s$

this is 3% of a fringe shift, and Michelson's instrument could detect that.

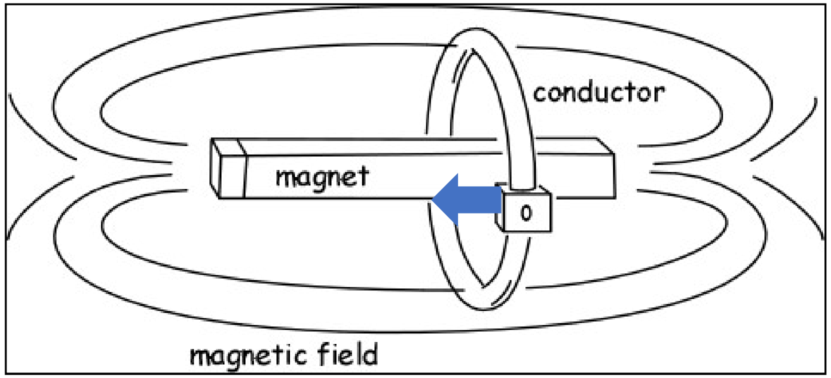


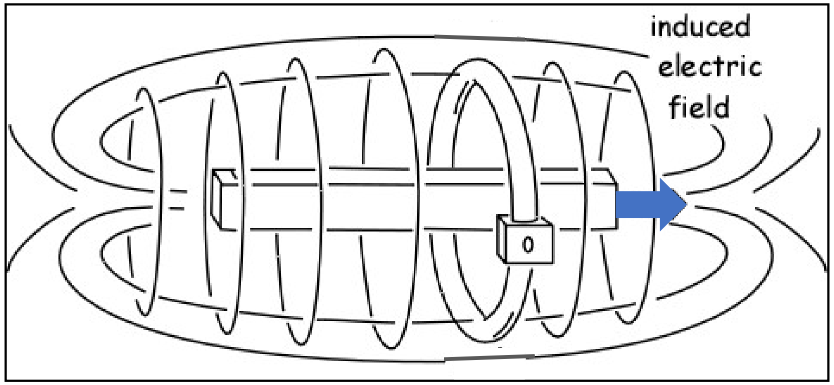
Earth around sun v ~ 30 km/s.
MM exp‘t found v < 5 km/s at any day of the year. Why?

Two puzzles that led Einstein to discover relativity

1. Failure to find the velocity of the earth w.r.t. ether

2. Some asymmetry in electrodynamics:





根據相對性原理，彼此相對等速運動的坐標系應具有相同的地位。所以，當磁鐵與線圈彼此等速運動時，就磁鐵的觀點而言，是線圈在移動；就線圈的觀點而言，是磁鐵在移動。前者在（靜止）磁鐵附近並無應電場；後者在（移動）磁鐵附近卻有應電場。這種描述上的差異似乎有違相對性原理。愛因斯坦年輕時曾被這個微妙而不明顯的問題所困擾。

Figs from J.D. Norton’s article

ON THE ELECTRODYNAMICS OF MOVING BODIES

BY A. EINSTEIN

June 30, 1905

From this two puzzles, Einstein built up a theory based on two postulates:

公設

A thing suggested or assumed as true as the basis for reasoning and discussion.

1. The laws of physics need be the same in all **inertial frames** of reference.
2. The speed of light c in empty space is a constant, independent of the relative motion of the source.

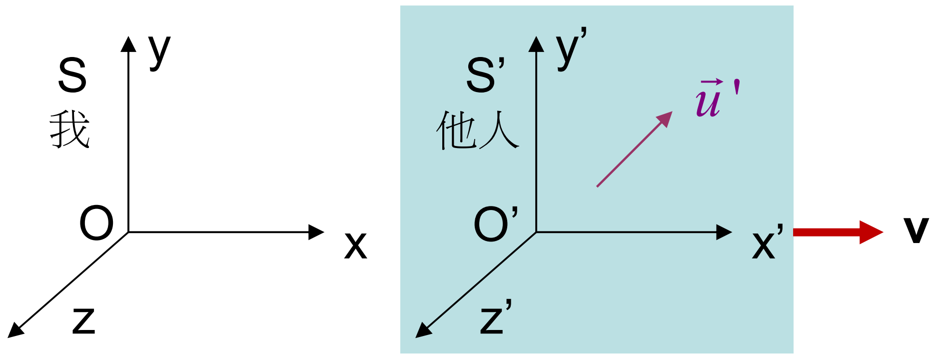
In the famous book in which he [Galileo] advocated the Copernican view of the solar system against the Ptolemaic, he argued that the vertical path of a falling object does not compel one to the conclusion that the earth is stationary.

(A.French, Special Relativity)

X: Science girl

Galileo's principle of relativity

2 **inertial frames** S, S'



At t=0, O'=O

$$\left.\begin{aligned}
x' &= x - vt \quad (\text{and } x = x' + vt)\\
y' &= y\\
z' &= z\\
t' &= t\\
u_x' &= u_x - v \quad (\text{and } u_x = u_x' + v)\\
a_x' &= a_x
\end{aligned}\right\} \text{Galilean transformation equations}$$

Newton: "*Absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external*."

絕對、真實而具有數學性質的時間，依其自身的本性，
平穩流動而不與外界的任何事物相關。

However, this cannot be consistent with Einstein's second postulate.

(2 observers moving w.r.t. each other will see the same light with different velocities)

Einstein's relativity (1905)

同時的相對性

- First, relativity of simultaneity:

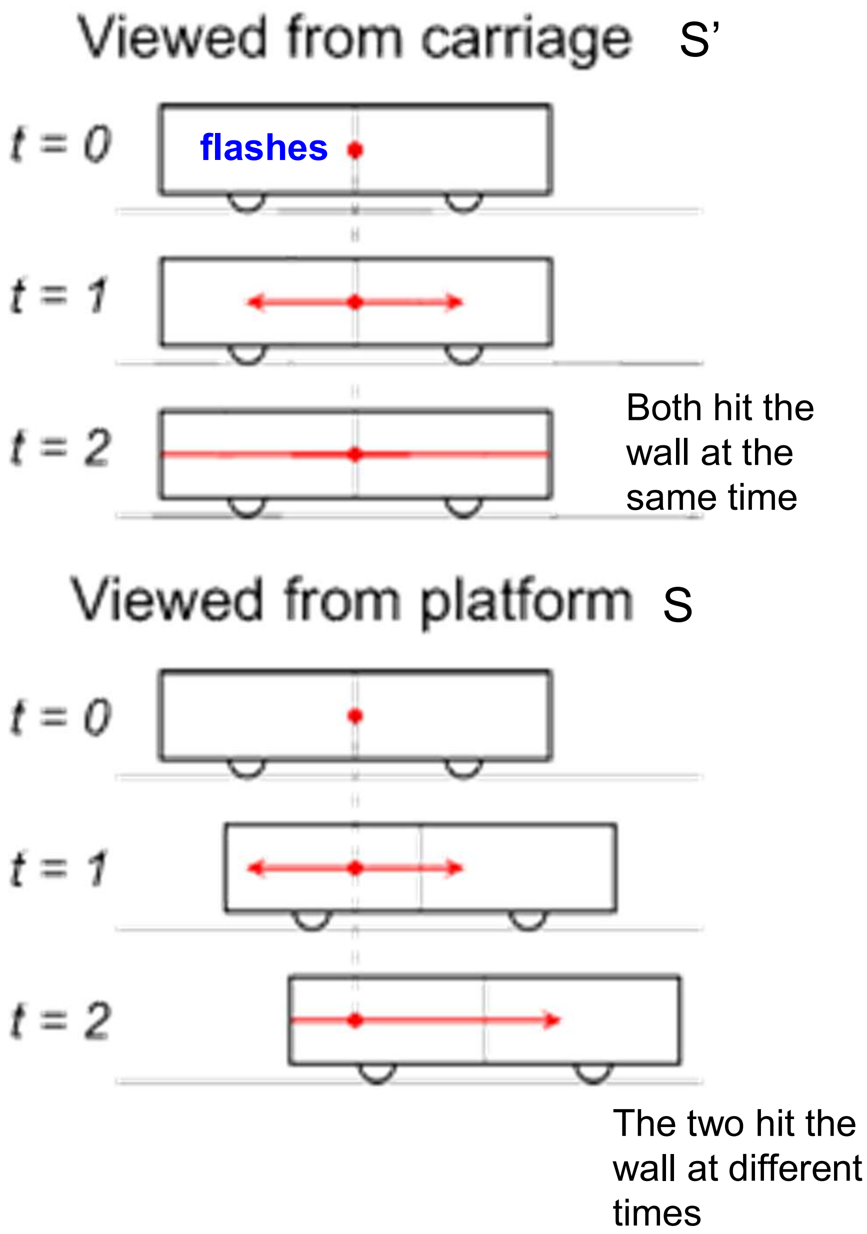
  The fact that two spatially separated events occur at the same time – is not absolute, but depends on the observer's reference frame.

  For example, a flash light in a train (see Figs)

- The ticking of time is not universal to all observers.

Q: How about throwing 2 balls (and using Galileo's principle?)

- Key difference: the velocity of light is the same for both observers here.



Bell's spaceship paradox 悖論 (似非而是的說法)

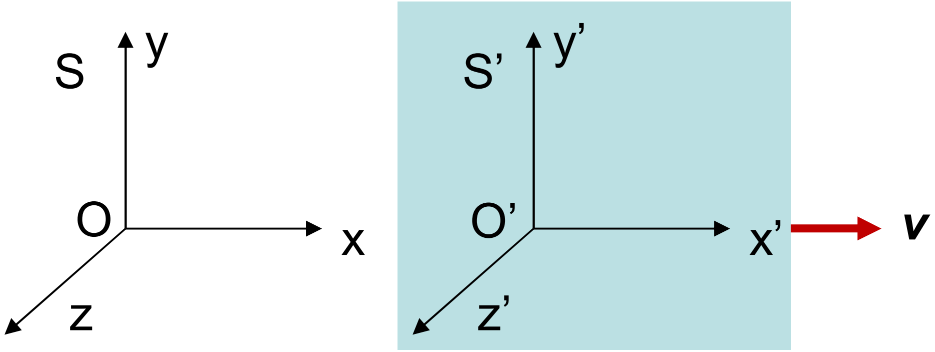
4–14 Two rockets are connected by an inextensible string of proper length l_0. At time $t = 0$ the rockets start out from rest with exactly equal constant accelerations as measured in S. At time $t = t_1$ the acceleration ceases and the rockets coast with equal constant velocities as measured in S. Why did the string break? [For further discussion see articles by E. Dewan and M. Beran, *Am. J. Phys.*, **27**, 517 (1959), and by E. Dewan, *Am. J. Phys.*, **31**, 383 (1963).]

(A. French, Special Relativity)

Transformation of space and time in Einstein's theory

(Note: All we need below is elementary algebra)

At t=t'=0, the origins coincide O=O'



Assume a linear transformation between coordinates.

$$\begin{cases} x' = \alpha x + \beta t \\ y' = y \\ z' = z \\ \boxed{t' = \gamma t + \delta x} \end{cases}$$

$$x' = 0, x = vt \rightarrow \beta = -\alpha v$$

(static in S')

$$x = 0, \quad \begin{cases} x' = -vt' = \beta t \\ t' = \gamma t \end{cases} \rightarrow \beta = -\gamma v$$

(static in S)

$$\gamma = \alpha$$

$$\Rightarrow \begin{cases} x' = \gamma(x - vt) \\ t' = \gamma t + \delta x \end{cases}$$

Consider a flash at $(x_0,t_0)= (x'_0,t'_0)=(0,0)$.

After time t, the wavefront is at $x=ct$ in S, $x'=ct'$ in S'.

➡ $x = ct, \qquad x' = ct'$ (both see the same velocity of light)

➡ $$\begin{cases} ct' = \gamma(ct - vt) \\ t' = \gamma t + \delta ct \end{cases}$$ ➡ $$\delta = -\frac{v}{c^2}\gamma$$

Inverse transformation

➡ $$\begin{cases} x' = \gamma(x - \frac{v}{c}ct) \\ y' = y \\ z' = z \\ ct' = \gamma(ct - \frac{v}{c}x) \end{cases} \qquad \begin{cases} x = \gamma(x' + \frac{v}{c}ct') \\ y = y' \\ z = z' \\ ct = \gamma(ct' + \frac{v}{c}x') \end{cases}$$

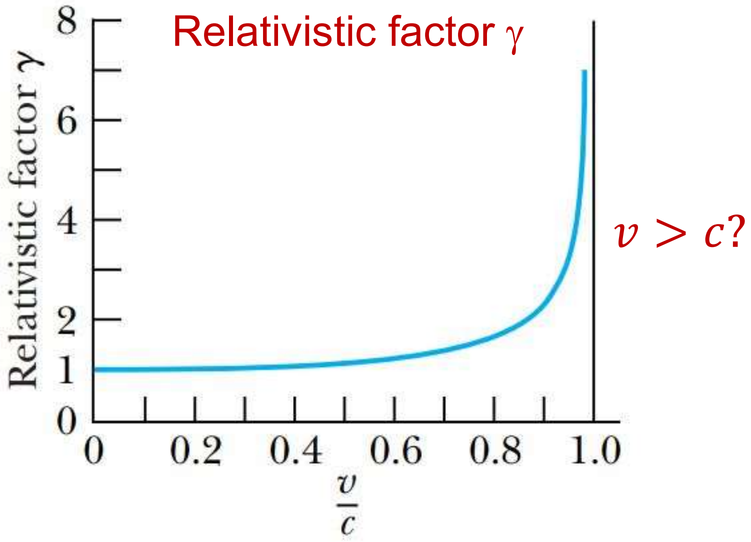
Q: What is γ?

Again consider a flash at $(x_0, t_0) = (x'_0, t'_0) = (0,0)$.

$$ct' = \gamma(ct - vt) \quad \text{or} \quad ct = \gamma(ct' + vt')$$ from inverse transformation

$$ct' = \gamma^2(c - v)\left(1 + \frac{v}{c}\right)t'$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$



Lorentz transformation (1892)

$$\boxed{\begin{aligned} x' &= \gamma\left(x - \frac{v}{c}ct\right) \\ y' &= y \\ z' &= z \\ ct' &= \gamma\left(ct - \frac{v}{c}x\right) \end{aligned}}$$

You may check that $x^2 + y^2 + z^2 = c^2t^2$ ← A sphere with radius ct in S

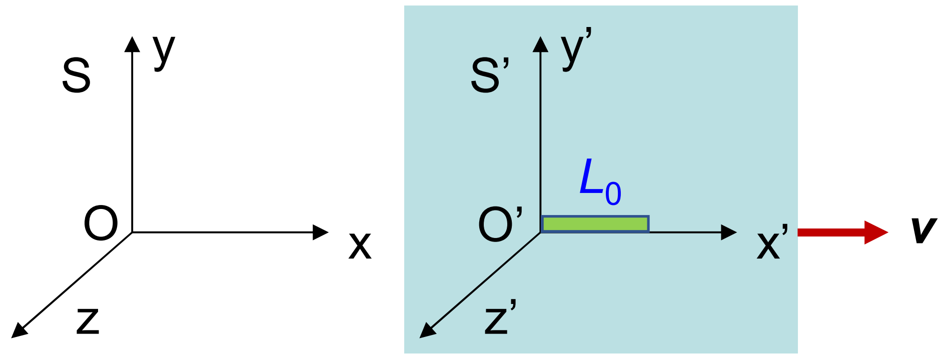
$x'^2 + y'^2 + z'^2 = c^2t'^2$ ← A sphere with radius ct' in S'

Contraction of space and Dilation of time

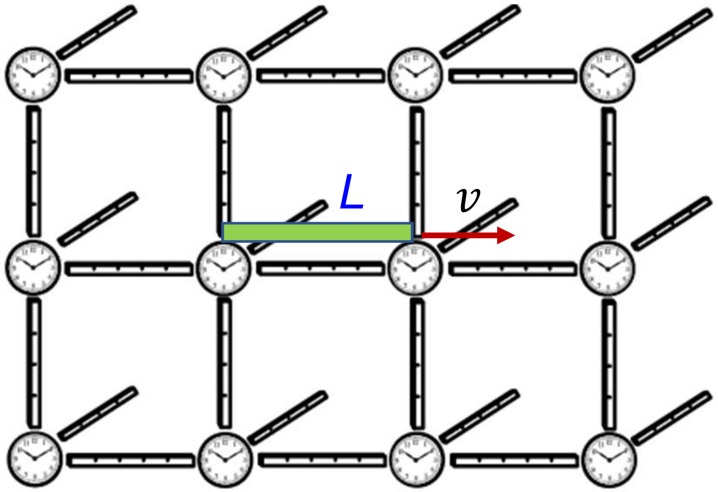
長度收縮 時間膨脹

1. Contraction of length

Consider a ruler static in S' (moving with velocity v in S)



Q: How to measure a flying ruler?



An array of synchronized clocks in S

To measure its length, we have to measure the coordinates at two ends simultaneously:

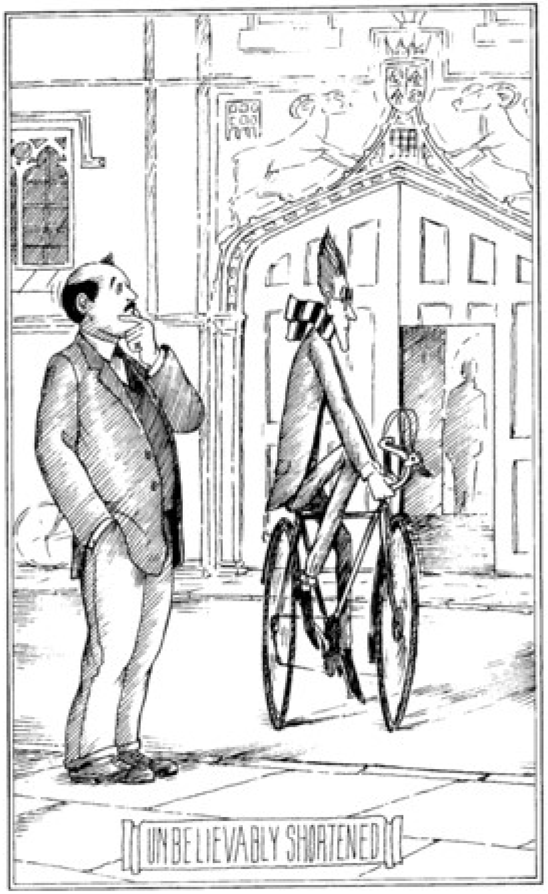
$$t_r - t_\ell = 0$$

Now, $$\overbrace{x'_r - x'_l}^{L_0} = \gamma[\overbrace{x_r - x_l}^{L} - v(t_r - t_l)]$$

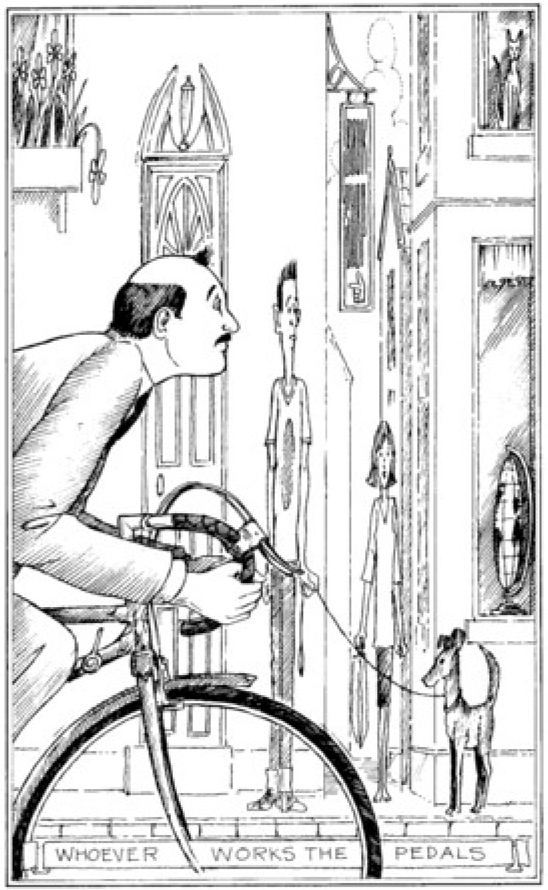
$$\Rightarrow \quad L = \frac{L_0}{\gamma} < L_0$$

Gamow,
Mr Tompkins
in wonderland

S

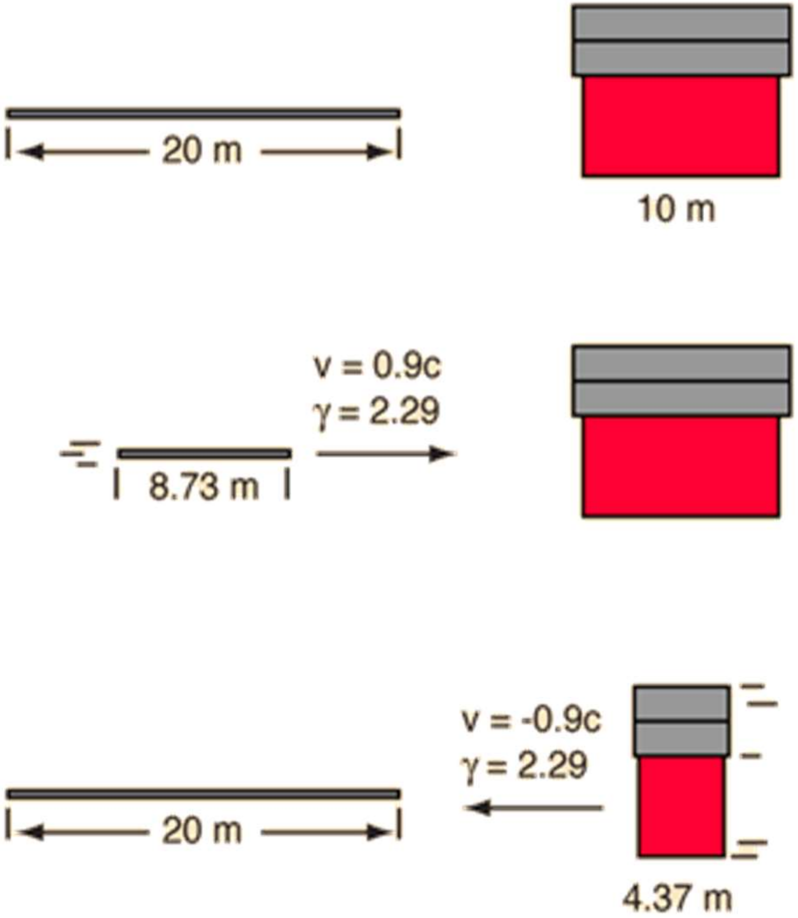


S'



竿與穀倉悖論

- The pole-barn paradox



A 20 meter pole and a 10 meter barn in their perspective when in the same rest frame.

If the pole has speed 0.9c, then it is length-contracted by a factor of 2.29 and short enough to fit momentarily within the barn, at least as seen by an observer in the barn.

But from the point of view of the pole, the barn is contracted and the pole will never fit inside it.

hyperphys

2. Dilation of time

Consider a clock static in S' (located at O') that is ticking with a period T_0

For an observer in S, the clock is moving with velocity v

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$$(t_2'-t_1') = \gamma[(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)]$$

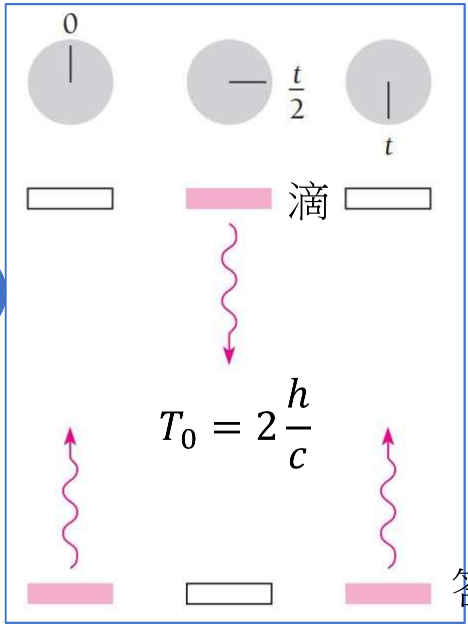
$$x_2 - x_1 = v(t_2 - t_1)$$

$$\Rightarrow \quad T_0 = \sqrt{1-\left(\frac{v}{c}\right)^2}\, T$$

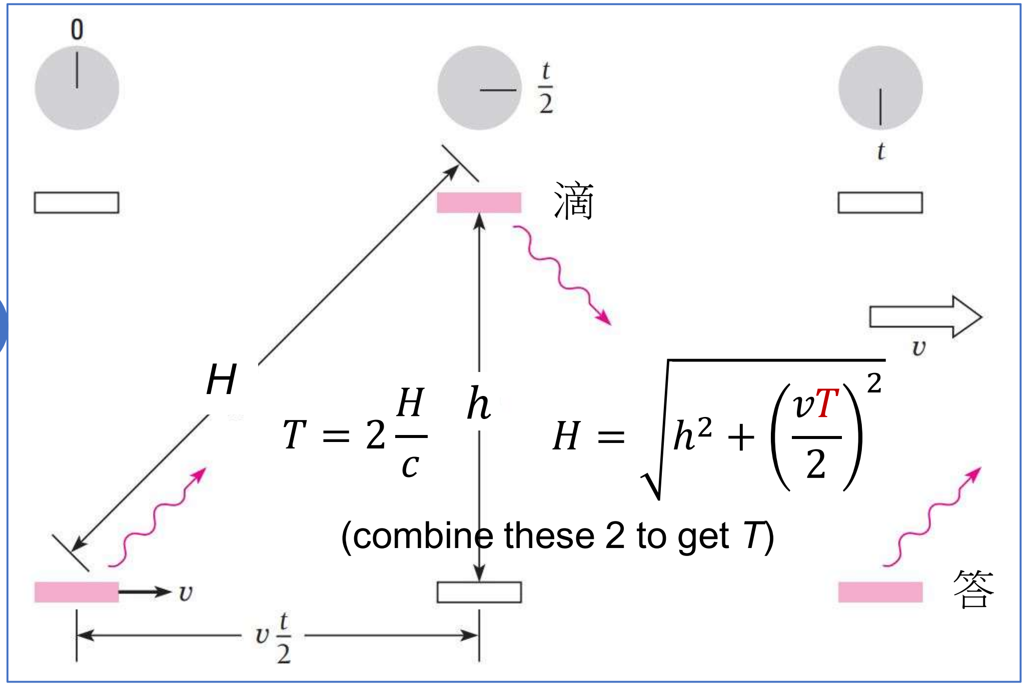
or $T = \gamma\, T_0 > T_0$

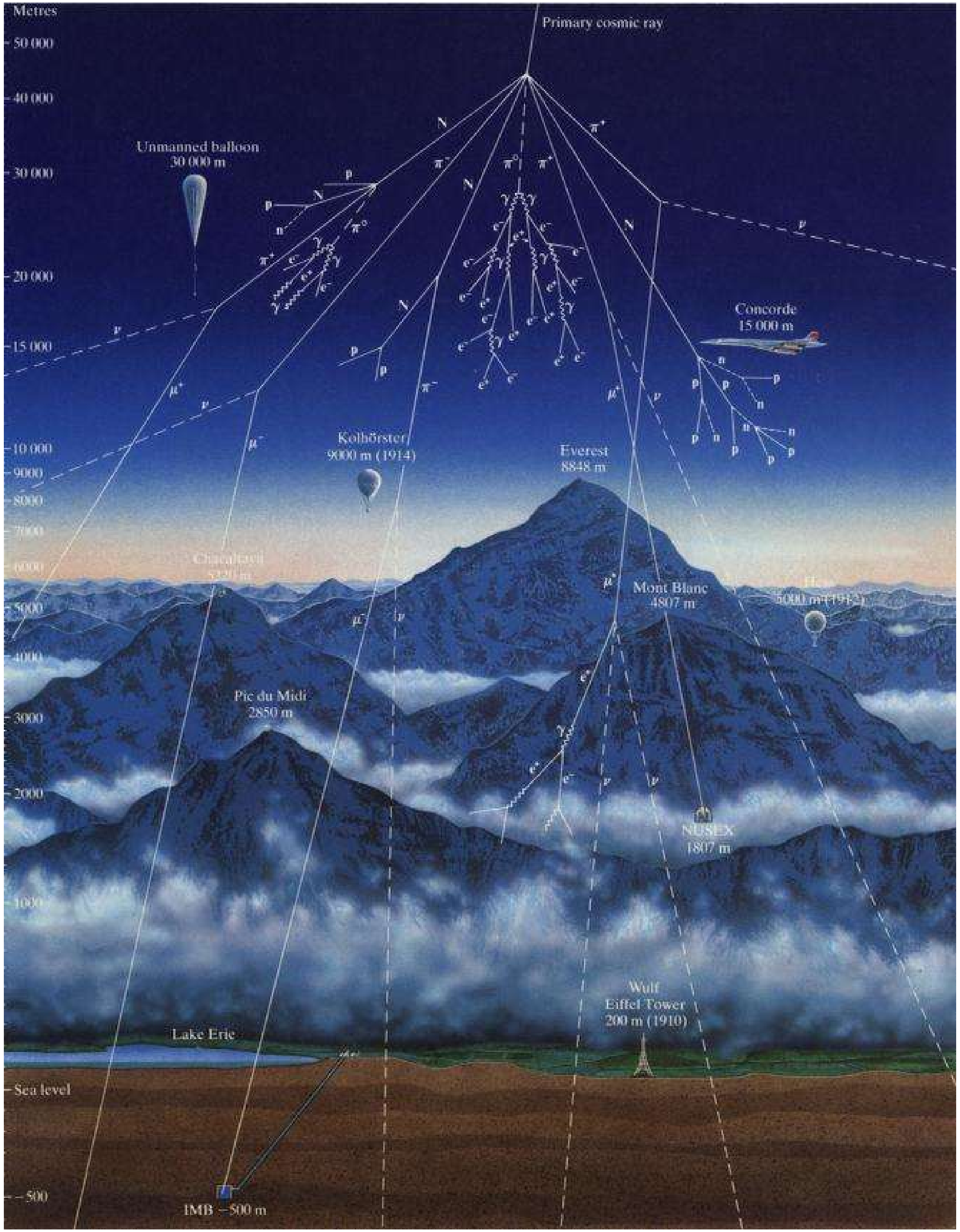
A simple explanation

S'



S



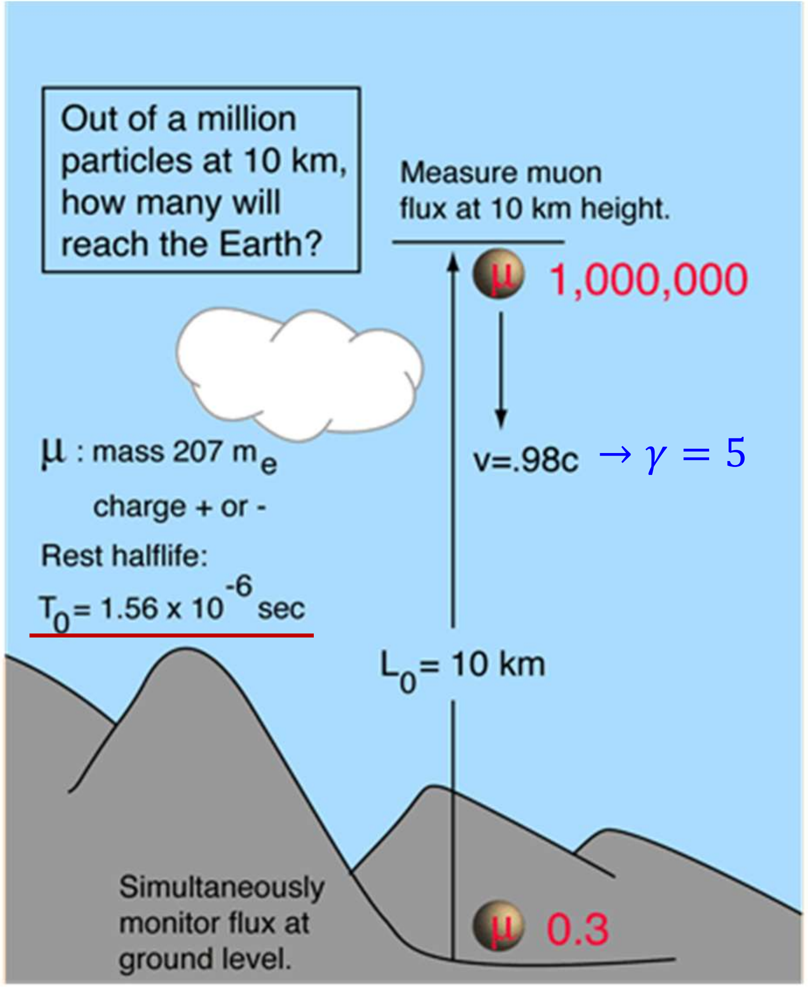


Cosmic ray

- Primary cosmic ray: About 99% of all particles are the nuclei of atoms (protons, He nuclei…). The rest are free electrons. They are from solar flares and stellar explosions.
- Secondary cosmic ray: protons, neutrons, pions, kaons…. Some of the pions and kaons decay into muons, neutrinos, gamma rays…

Example of time dilation:

The decay of muons in cosmic ray: $\mu \rightarrow e^{-} + \nu_{\mu} + \bar{\nu}_e$



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w/o time dilation

Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4\ \text{m}}{(0.98)(3 \times 10^8\ \text{m/s})}$

$T = 34 \times 10^{-6}\ \text{s} = 21.8$ halflives

Survival rate:

$$\frac{I}{I_0} = 2^{-21.8} = 0.27 \times 10^{-6}$$

Or only about 0.3 out of a million.

w/ time dilation

Distance: $L_0 = 10^4$ meters

Time: $T = \dfrac{10^4\ \text{m}}{(0.98)(3 \times 10^8\ \text{m/s})}$

$T = 34 \times 10^{-6}\ \text{s} = 4.36$ halflives

Survival rate:

$$\frac{I}{I_0} = 2^{-4.36} = 0.049$$

Or about 49,000 out of a million.

Q: Viewed from muon's frame of reference?

Ex 2.1: Show that an observer moving with S' frame will also see the clock in S frame slows down by the factor γ.

Solution

$$ct' = \gamma(ct - \frac{v}{c}x)$$

$$\Rightarrow \quad t_2' - t_1' = \gamma[t_2 - t_1 - \frac{v}{c^2}(x_2 - x_1)]$$

$$x_2 = x_1$$

$$\Rightarrow \quad T' = \gamma T_0$$

(T_0 is the local time for the clock)

Ex 2.2: The star closest to the sun is 4.3 light years away. In order to visit there and come back in 16 years (astronaut time). How fast the spaceship has to move. (neglect the time for acceleration, stay, and turn around)

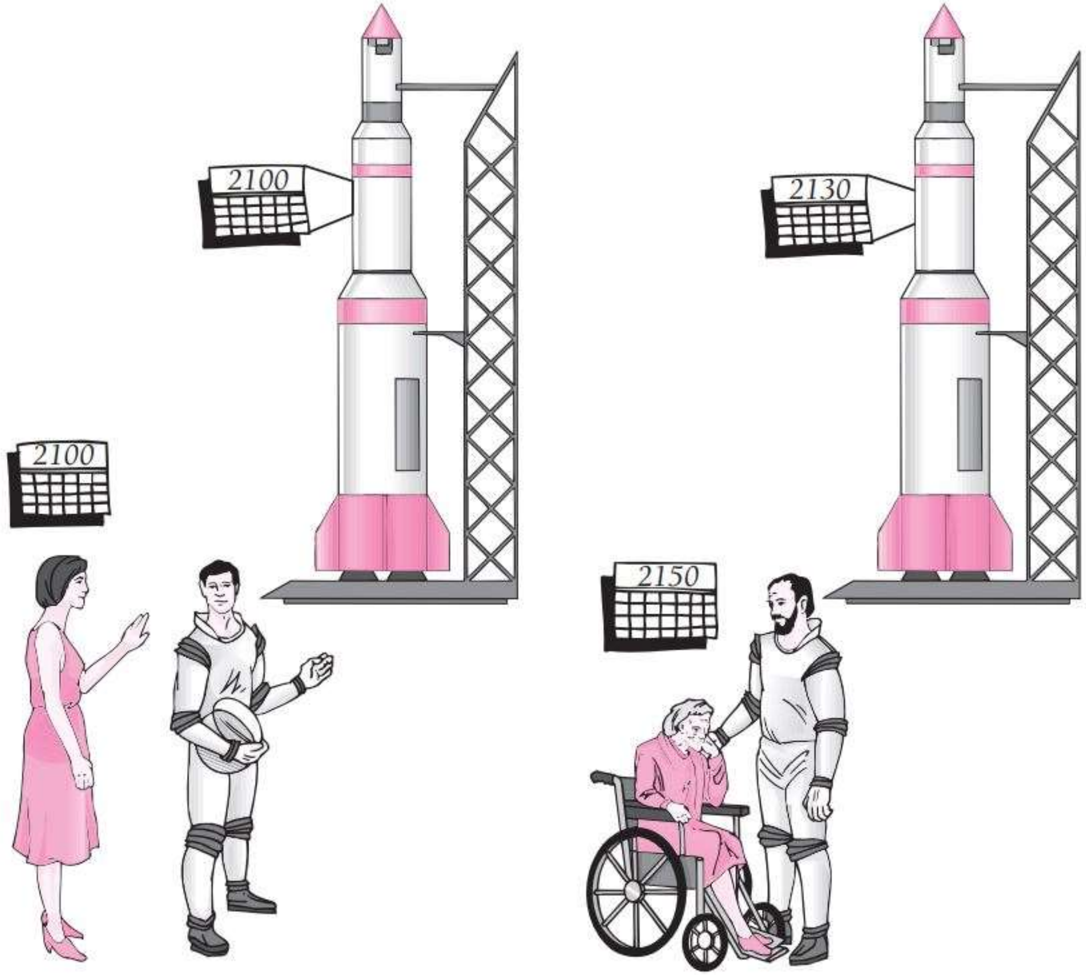
Solution

$$T = \frac{2L}{v} = \frac{T_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \quad \frac{2(4.30\ \text{ly})(9.46 \times 10^{15}\ \text{m/ly})}{v} = \frac{16\ \text{y}}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \quad v = 0.473c = 1.42 \times 10^8\ \text{m/s} \qquad (\gamma = 1.137)$$

Time dilation and **twin paradox** 孿生子悖論

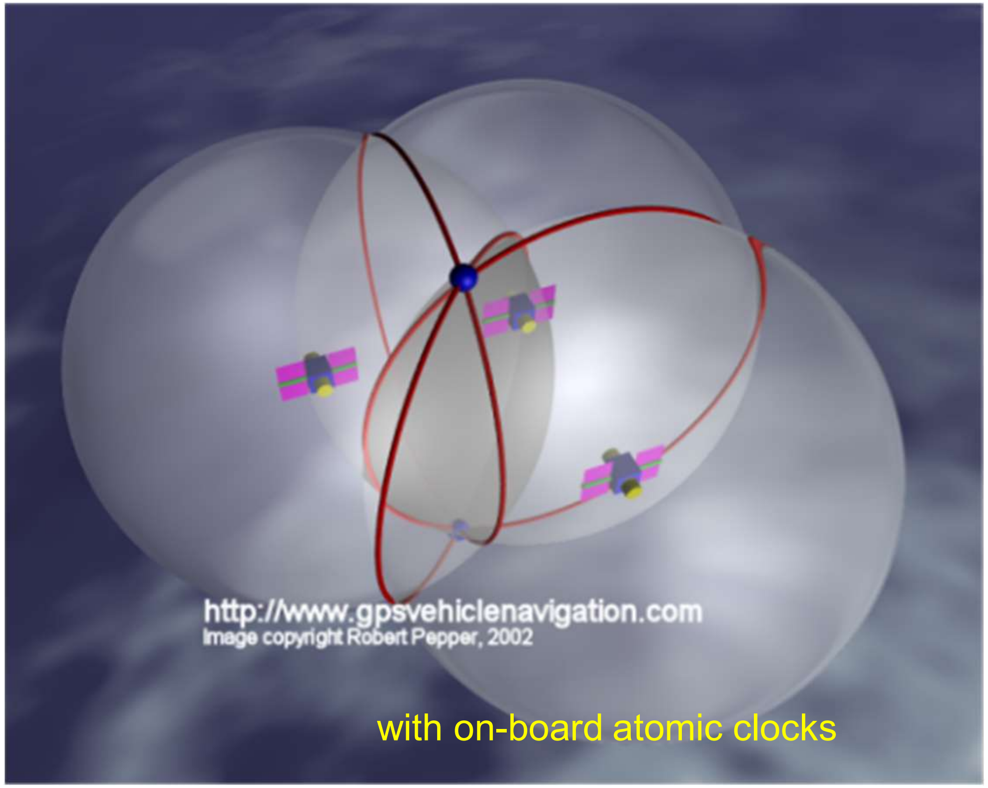


Q: Who has *really* been moving?

optional

Time dilation in GPS (Global Positioning System)

Velocity of satellite ~ 4 km/s



Accuracy: 30 ns ↔ 15 m

In addition to speed, gravity can also influence a clock

Due to relativistic effect, the time of a GPS clock is

Faster by $-7\ \mu s + 45\ \mu s$

(Special R) (General R)

$= 38\ \mu s$ per day

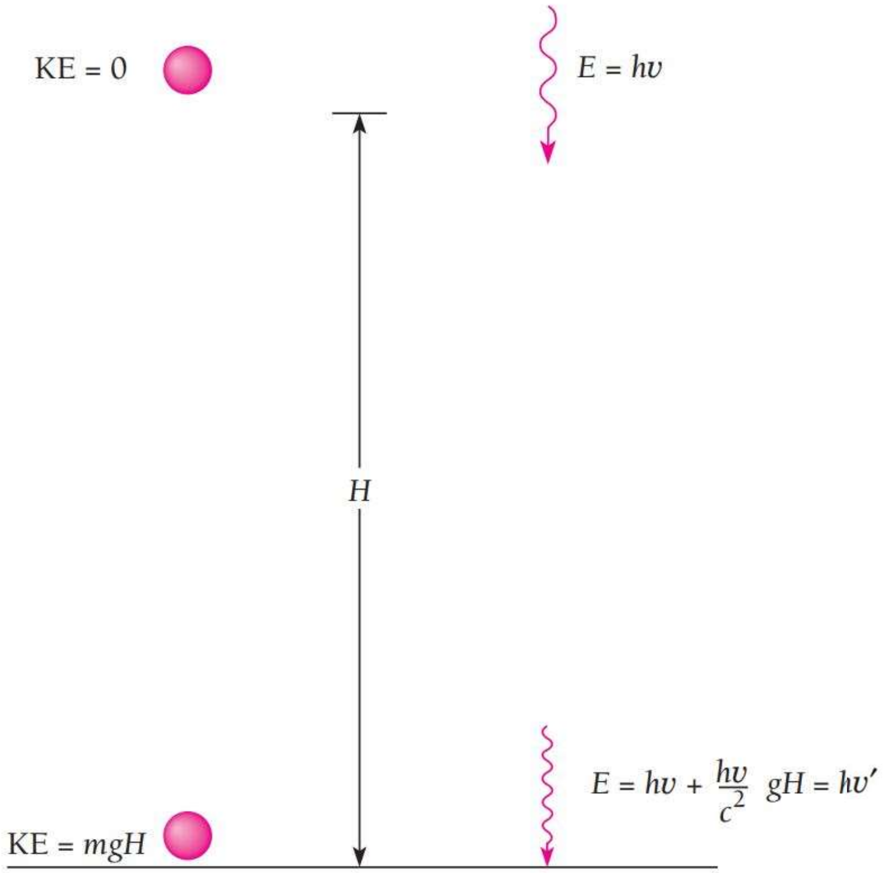
= an error of 10 km per day

optional

Photon and gravity

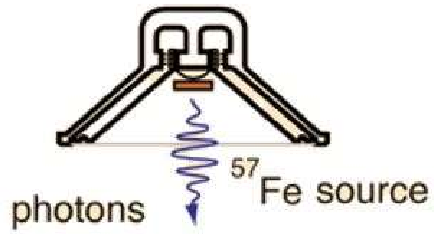
Photon "mass"

$E = cp = h\nu \rightarrow m = \frac{p}{v} = \frac{h\nu}{c^2}$



Photon energy after falling through height H $\quad h\nu' = h\nu\left(1 + \frac{gH}{c^2}\right)$

Pound and RebkaExperiment (1960)



14.4 keV

$$\Delta E = mgh = \frac{E}{c^2} gh = \frac{14.4keV}{c^2} g \cdot 22.6m$$

$$\Delta E = 3.5x10^{-11} eV$$

- Recent exp't: h=1 cm using optical lattice clock (2023)

Fig Interstellar

The time read on a rapidly moving clock

- Time dilation:

  S': A clock fixed at O'.

  S: 2 clocks at 2 locations to measure the interval of clock-S'

  (when S-S' clocks meet at these 2 locations)

- Now, What if there is only 1 clock at O and we look at a clock at O' moving away?
  - In the case above, consider a series of synchronized clocks in S.

    As the S' clock goes by, we have $t = \gamma t'$
  - But it takes time for the light from O' to reach O. When S reads a time t' at O', what is the time $\tilde{t}$ at O?

Observed time ➡ $$\tilde{t} = t + \frac{vt}{c} = \left(1 + \frac{v}{c}\right)\gamma t' = \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}\right)^{\frac{1}{2}} t'$$ Local time

- Note: If the clock at O' is approaching, then

$$\tilde{t} = t - \frac{vt}{c} = \left(1 - \frac{v}{c}\right)\gamma t' = \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}\right)^{\frac{1}{2}} t'$$

Doppler effect in light

Based on the result in previous page, we have

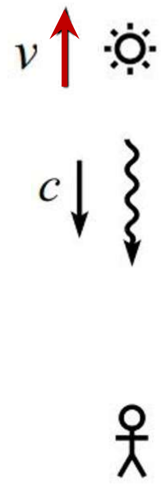
Longitudinal Doppler effect

- A receding source

$$\beta \equiv \frac{v}{c}$$

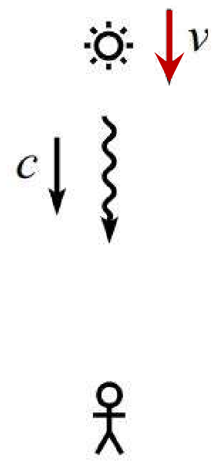
Observed time $\qquad T = \left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}} T_0 \qquad$ Local time

$$\text{Or} \quad f = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}} f_0 < f_0$$



- An approaching source

$$f = \left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}} f_0 > f_0$$



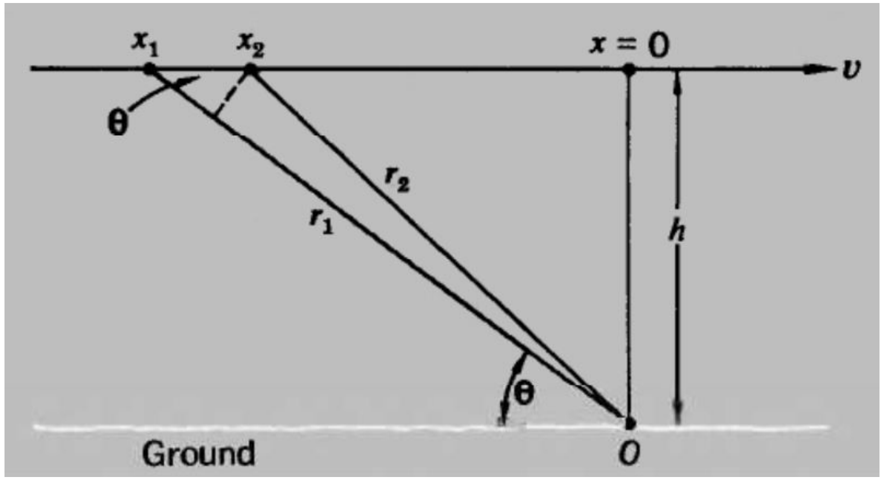
Q: Compare this with the Doppler effect in sound
(c is the speed of sound)

$$f = \left(\frac{c \pm v_{\mathrm{r}}}{c \mp v_{\mathrm{s}}}\right) f_0$$

Transverse Doppler effect

t = 0 marks the instant when
the satellite is directly overhead

(x, y)=(*vt*, *h*)



The emitter has frequency f_0 (period T_0) in S' frame

S frame: $$t_2 - t_1 = \gamma T_0$$ (time dilation over one period)

The pulses take times r_1/c and r_2/c respectively to reach 0, so that the measured time separation

$$\tilde{T} = t_2 + \frac{r_2}{c} - t_1 - \frac{r_1}{c}$$

$$= \gamma T_0 - \frac{r_1 - r_2}{c}$$

If $\Delta x \ll r_1$, then

$$r_1 - r_2 \simeq (x_2 - x_1)\cos\theta$$

$$= v\gamma T_0 \cos\theta$$

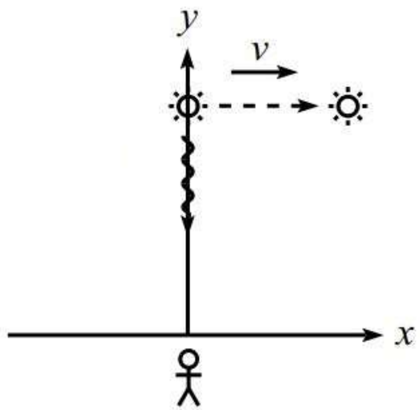
$$\Rightarrow \tilde{T} = \gamma T_0 \left(1 - \frac{v}{c}\cos\theta\right)$$

Observed frequency

$$\tilde{f} = \frac{f_0}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)}$$

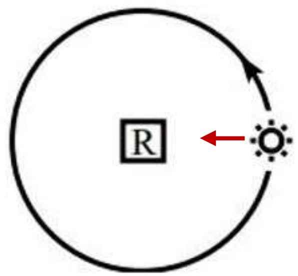
Special cases (also $\theta = 0, \theta = \pi$)

- **Case 1:**

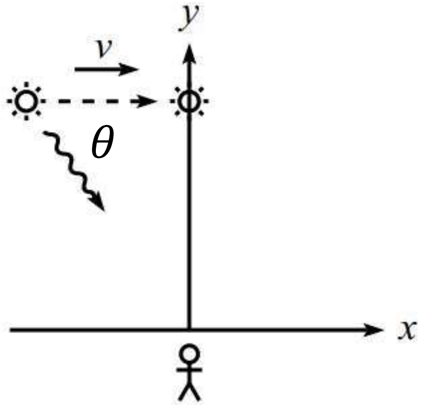


$$\theta = \frac{\pi}{2}$$

$$\tilde{f} = \frac{f_0}{\gamma} = \sqrt{1-\beta^2}\, f_0$$

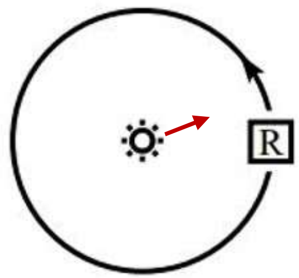


- **Case 2:**



$$\cos\theta = \frac{vt}{ct}$$

$$\tilde{f} = \frac{f_0}{\gamma\left(1-\frac{v^2}{c^2}\right)} = \frac{f_0}{\sqrt{1-\beta^2}}$$



因果律 Causality and maximum signal velocity

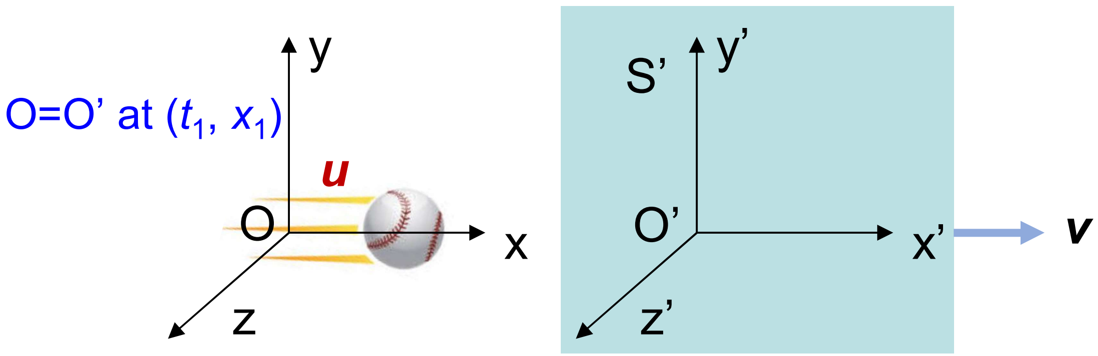
Different observers can disagree on simultaneity and interval of time. If one of them can travel faster then the speed of light, then they would also disagree on the order of event!

- First, simultaneity in Lorentz transformation,

$$c(t_2' - t_1') = \gamma[c(t_2 - t_1) - \frac{v}{c}(x_2 - x_1)]$$

When $t_2 = t_1$, in general $t_2' \neq t_1'$, unless 2 events happen at the same location

- Now, in S, some one throws a ball at (t_1=0, x_1=0) with velocity $\boldsymbol{u}$, which breaks a window at (t_2, $x_2 = ut_2$)

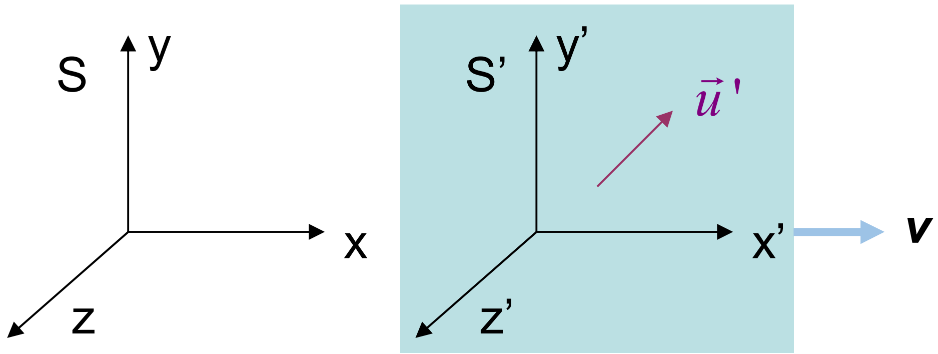


$$t_2' - t_1' = \gamma\left(t_2 - \frac{v}{c^2}ut_2\right), \qquad t_1' = 0$$

$$t_2' = \gamma\left(1 - \frac{uv}{c^2}\right)t_2 \ < 0 \text{ if } uv > c^2$$

The ball breaks the window before it's thrown out!

4. Addition of velocities



$$\begin{cases} u_x = \dfrac{dx}{dt}, u_y = \dfrac{dy}{dt}, u_z = \dfrac{dz}{dt} \\ u'_x = \dfrac{dx'}{dt'}, u'_y = \dfrac{dy'}{dt'}, u'_z = \dfrac{dz'}{dt'} \end{cases}$$

$$\begin{cases} dx = \gamma(dx' + vdt') \\ dt = \gamma(dt' + \dfrac{v}{c^2}dx') \end{cases}$$

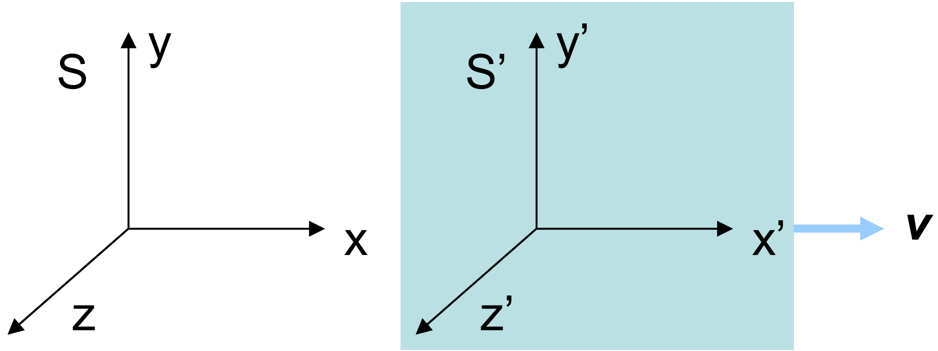
$$u_y = \frac{u'_y}{\gamma\left(1 + \frac{u'_x v}{c^2}\right)}$$

$$u_x = \frac{dx}{dt} = \frac{dx' + vdt'}{dt' + \frac{v}{c^2}dx'} = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}$$

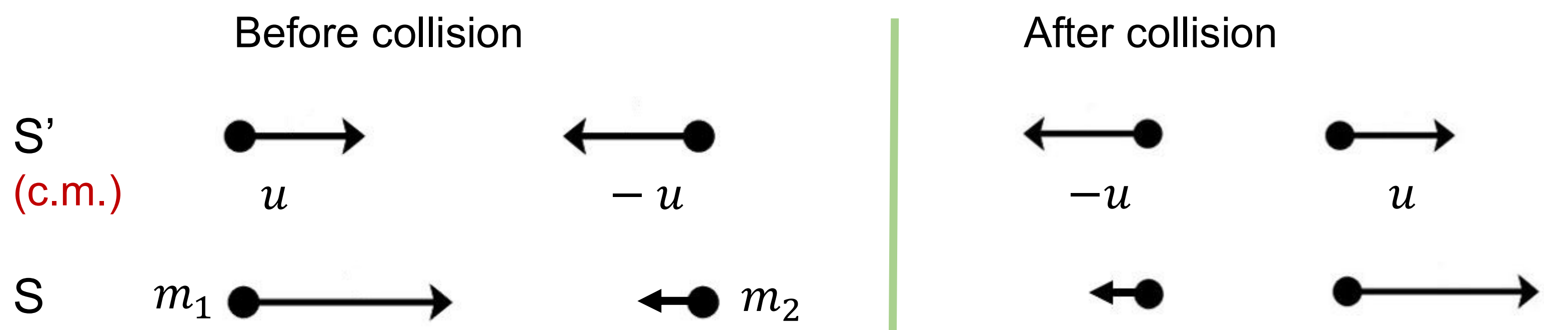
$$u_z = \frac{u'_z}{\gamma\left(1 + \frac{u'_x v}{c^2}\right)}$$

Special case: $u' = c$

5. Relativistic mass



Two particles with the same rest mass collide along x-axis:



Before collision:

$$u_1 = \frac{u+v}{1+\frac{uv}{c^2}} \qquad u_2 = \frac{-u+v}{1-\frac{uv}{c^2}}$$

After collision:

$$u_1 = \frac{-u+v}{1-\frac{uv}{c^2}} \qquad u_2 = \frac{u+v}{1+\frac{uv}{c^2}}$$

Conservation of momentum:

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$\to m_1 (u_1 - v) = m_2 (v - u_2)$$

$$u_1 - v = \frac{u}{\gamma^2}\frac{1}{1+\frac{uv}{c^2}} \qquad v - u_2 = \frac{u}{\gamma^2}\frac{1}{1-\frac{uv}{c^2}} \qquad \gamma \equiv \frac{1}{\sqrt{1-(v/c)^2}}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1+\frac{uv}{c^2}}{1-\frac{uv}{c^2}}$$

It can be shown that

$$1-\left(\frac{u_1}{c}\right)^2 = \frac{\left[1-\left(\frac{u}{c}\right)^2\right]\left[1-\left(\frac{v}{c}\right)^2\right]}{\left(1+\frac{uv}{c^2}\right)^2} \qquad 1-\left(\frac{u_2}{c}\right)^2 = \frac{\left[1-\left(\frac{u}{c}\right)^2\right]\left[1-\left(\frac{v}{c}\right)^2\right]}{\left(1-\frac{uv}{c^2}\right)^2}$$

Therefore,

$$\frac{m_1}{m_2} = \frac{\sqrt{1-\left(\frac{u_2}{c}\right)^2}}{\sqrt{1-\left(\frac{u_1}{c}\right)^2}} \quad \Rightarrow \quad m_1 = \frac{m_0}{\sqrt{1-\left(\frac{u_1}{c}\right)^2}}, \quad m_2 = \frac{m_0}{\sqrt{1-\left(\frac{u_2}{c}\right)^2}}$$

靜止質量

← rest mass

In general, for a particle moving with speed u

$$\boxed{m = \frac{m_0}{\sqrt{1-\left(\frac{u}{c}\right)^2}}}$$

Relativistic momentum

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 v \qquad m(v) = \gamma m_0$$

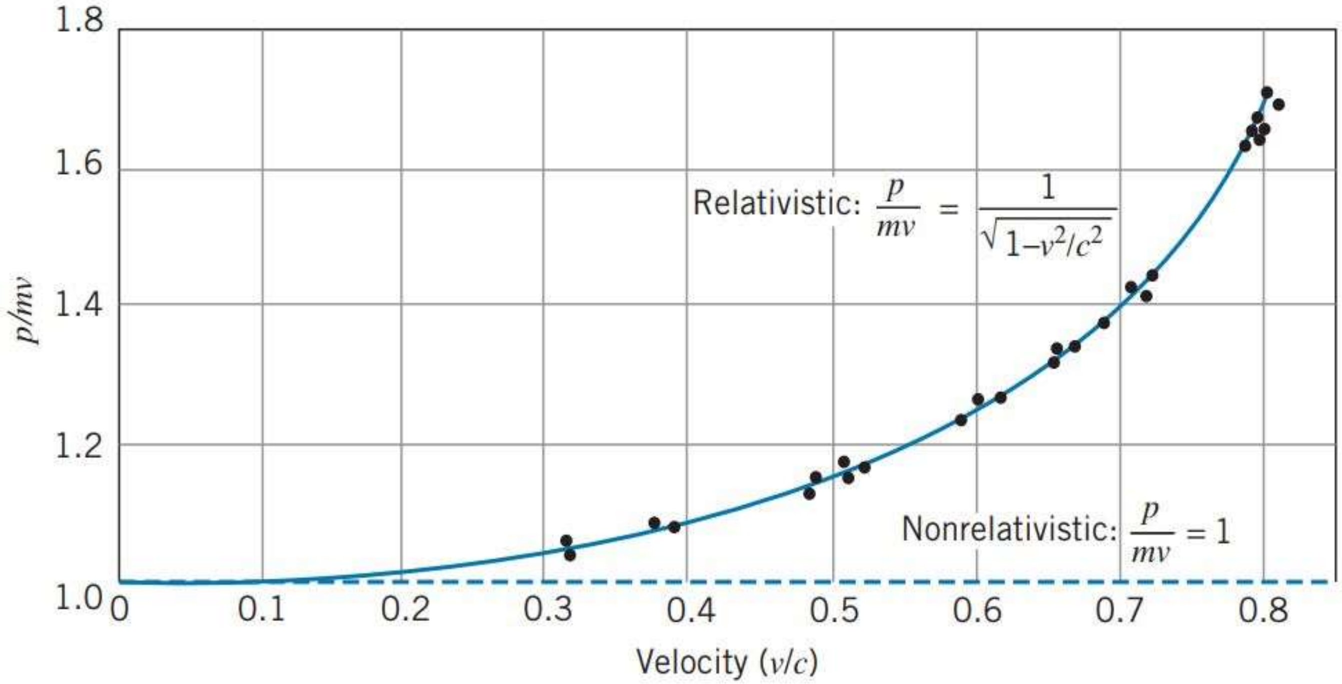


FIGURE 2.27 The ratio p/mv is plotted for electrons of various speeds. The data agree with the relativistic result and not at all with the nonrelativistic result ($p/mv = 1$).

6. Relativistic energy

$$\mathrm{KE} = \int_0^s F\, ds \Rightarrow \mathrm{KE} = \int_0^s \frac{d(\gamma m v)}{dt} ds = \int_0^{mv} v\, d(\gamma m v) = \int_0^v v\, d\left(\frac{mv}{\sqrt{1 - v^2/c^2}}\right)$$

$$= \frac{mv^2}{\sqrt{1 - v^2/c^2}} - m \int_0^v \frac{v\, dv}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2$$

The m here is m_0 (rest mass)

$\Rightarrow$ **Total energy** $\boxed{E = \gamma m c^2} = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}}$ $\qquad E(v)$

Low velocity $\quad \mathrm{KE} \approx \left(1 + \dfrac{1}{2}\dfrac{v^2}{c^2}\right) mc^2 - mc^2 \approx \dfrac{1}{2} m v^2$

Alternative expression $\quad \boxed{E = \sqrt{p^2 c^2 + (m_0 c^2)^2}} \qquad E(p)$

Special cases:

1. Rest mass energy $\quad E_0 = m_0 c^2$

2. Massless particle (e,g,photon) $\quad E = pc$

Ex 2.11: Electrons used to produce medical x rays are accelerated from rest through a potential difference of 25,000 volts before striking a metal target. Calculate the speed of the electrons and determine the error in using the classical kinetic energy result.

Solution

$$K = W = qV = (1.6 \times 10^{-19}\ \mathrm{C})(25 \times 10^3\ \mathrm{V})$$
$$= 4.0 \times 10^{-15}\ \mathrm{J}$$

$$K = (\gamma - 1)mc^2$$

$$mc^2 = 8.19 \times 10^{-14}\ \mathrm{J}$$

$$\gamma \simeq 1.049$$

$$\beta \simeq 0.3 \rightarrow u = 0.9 \times 10^8\ \mathrm{m/s}$$

$$K = \tfrac{1}{2} mu^2$$

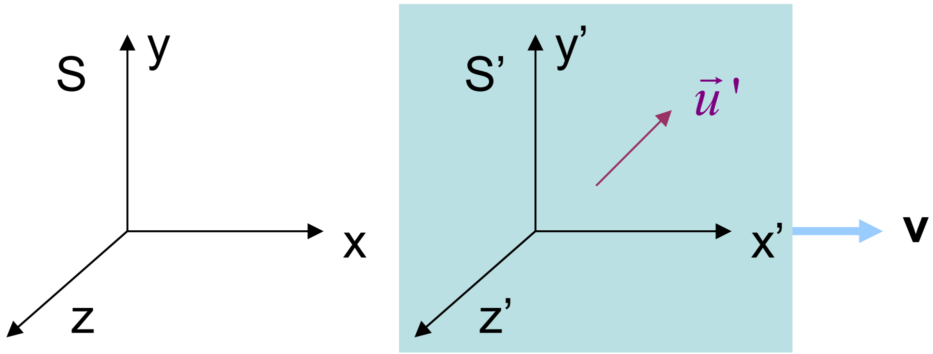
$$u = \sqrt{\frac{2(4.0 \times 10^{-15}\ \mathrm{J})}{9.11 \times 10^{-31}\ \mathrm{kg}}}$$

$$= 0.94 \times 10^8\ \mathrm{m/s} \quad \text{(nonrelativistic)}$$

about 4% error

optional

Transformation of space-time and energy-momentum



space-time

$$x = (x_{0,}\vec{x}) = (ct, \vec{x})$$

$$x'_0 = \gamma(x_0 - \beta x_1)$$
$$x'_1 = \gamma(x_1 - \beta x_0)$$
$$x'_2 = x_2$$
$$x'_3 = x_3$$

$$\beta \equiv v/c, \quad \gamma \equiv \frac{1}{\sqrt{1-(v/c)^2}}$$

energy-momentum

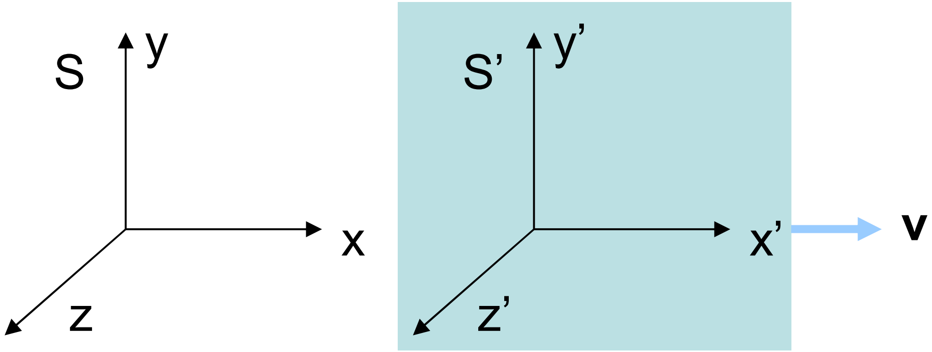
$$p = (p_{0,}\vec{p}) = (E/c, \vec{p})$$

$$E = \gamma_u mc^2, \ \vec{p} = \gamma_u m\vec{u}$$

$$p'_0 = \gamma_v(p_0 - \beta p_1)$$
$$p'_1 = \gamma_v(p_1 - \beta p_0)$$
$$p'_2 = p_2$$
$$p'_3 = p_3$$

optional

Transformation of charge-current and EM potentials



charge-current

$$J = (J_{0,}\vec{J}) = (c\rho, \vec{J})$$

$$J'_0 = \gamma(J_0 - \beta J_1)$$
$$J'_1 = \gamma(J_1 - \beta J_0)$$
$$J'_2 = J_2$$
$$J'_3 = J_3$$

EM potentials

$$A = (A_{0,}\vec{A}) = (\phi / c, \vec{A})$$

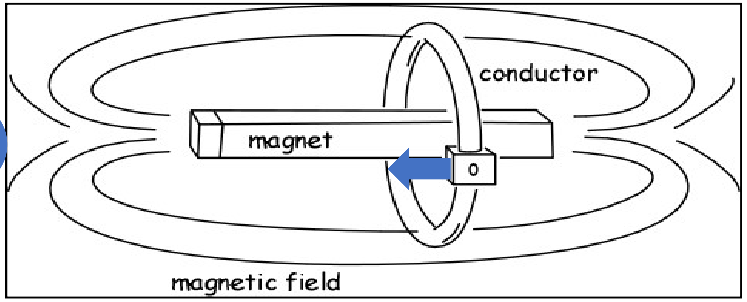
$$A'_0 = \gamma_v(A_0 - \beta A_1)$$
$$A'_1 = \gamma_v(A_1 - \beta A_0)$$
$$A'_2 = A_2$$
$$A'_3 = A_3$$

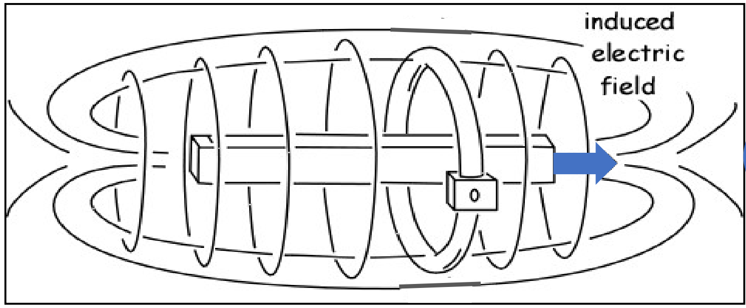
- The set $(v_0, \vec{v})$ that transforms according to the Lorentz transformation is called a 4-vector.
- Not all vectors transform like this. For example, velocity, acceleration, force, electric field and magnetic field are not 4-vectors.

optional

Back to Einstein's original puzzle about The asymmetry in electrodynamics

S'





S

- Transformation of EM fields in special relativity
  (see Lorrain and Corson for detailed derivation)

$$E_x = E'_x \qquad B_x = B'_x$$

$$E_y = \gamma\left(E'_y + \beta B'_z\right) \qquad B_y = \gamma\left(B'_y - \beta E'_z\right)$$

$$E_z = \gamma\left(E'_z - \beta B'_y\right) \qquad B_z = \gamma\left(B'_z + \beta E'_y\right)$$

Alternative form,

$$\vec{E}_{//} = \vec{E}'_{//} \qquad \vec{B}_{//} = \vec{B}'_{//}$$

$$\vec{E}_{\perp} = \gamma\left(\vec{E}'_{\perp} - \vec{\beta}\times\vec{B}'_{\perp}\right) \qquad \vec{B}_{\perp} = \gamma\left(\vec{B}'_{\perp} + \vec{\beta}\times\vec{E}'_{\perp}\right)$$

- When there is no ***E*** field in S' (moving with the magnet)

Induced ***E*** field in S, $\vec{E}_{\perp} = -\vec{\beta}\times\vec{B}'_{\perp} \quad \vec{B}_{\perp} = \vec{B}'_{\perp} \quad (\gamma \simeq 1)$