Classical electrodynamics 9:00 –11:30, 11/20/2008

1.[10%] A ring with radius R is placed above the origin (see Fig.1). It is uniformly charged with total charge Q. Using the spherical coordinate, write down the charge density $\rho(\boldsymbol{x})$ of this ring.

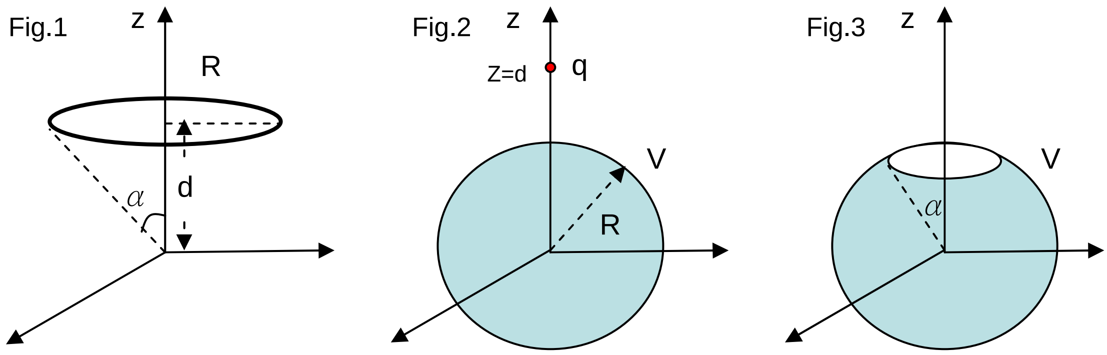
2. [40%] A conducting hollow sphere with radius R is located at the origin. The surface of the sphere is maintained at a potential V.

(a) Find out the potential ϕ_{in} inside the sphere.

(b) Find out the potential ϕ_{out} outside the sphere.

(c) Find out the surface charge distribution σ on the sphere.

(d) A point charge q is added to z=d outside sphere (see Fig.2). Find out the potential outside the sphere.



3. [20%] A conducting sphere with radius R has a hole at the north pole. The sphere is maintained at potential V (see Fig.3). We'll use Green's theorem to get the potential.

(a) First find out the Green's function G for this problem.

(b) Use the Green's theorem to find out the potential $\phi(z)$ along the positive z-axis.

4. [30%] A sphere is similar to the one in Fig.3. The only difference is that the hole is replaced by a spherical conducting cap with zero potential. The rest of the sphere has potential V.

(a) First solve the boundary value problem using the Laplace equation to find out the potential, then calculate the electric field E_{in} everywhere inside the sphere.

(b) What is the electric field E_{out} everywhere outside the sphere.

(c) Find out the distribution of surface charge σ on the sphere.

Note: $$\phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int dV'\rho(\vec{x}')G + \frac{1}{4\pi}\oint da'\left(G\frac{\partial\phi}{\partial n'} - \phi\frac{\partial G}{\partial n'}\right)$$

$$\frac{dP_{l+1}(x)}{dx} - \frac{dP_{l-1}(x)}{dx} - (2l+1)P_l(x) = 0$$