Classical electrodynamics 6:30 –9:30, 11/22/2007

1.[30%] (a) A point charge q is located at $\boldsymbol{x}_0$ outside of a grounded conducting sphere with radius a (centered at the origin). Find out the potential $\phi(\boldsymbol{x})$ outside the sphere using the method of image.
(b) What is the potential $\phi(\boldsymbol{x})$ outside if the sphere in (a) is not grounded, but maintained at a fixed potential V?
(c) Following (a), find out the attractive force between the charge and the sphere.

2.[40%] A conducting sphere with radius a is divided into 2 parts (see Fig.1). A cap near the north pole is maintained at potential V; the rest of the sphere is grounded.
(a) We will *use the Green function method* to solve for the potential $\phi(\boldsymbol{x})$ *inside* the sphere. First find out the appropriate Green function $G(\boldsymbol{x},\boldsymbol{x}')$ for this problem.
(b) Then find out the potential $\phi(\boldsymbol{x})$ along the z–axis *inside* the sphere.
(c) Find out the electric field $\boldsymbol{E}(\boldsymbol{0})$ at the origin.
Hint: $\phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int dV'\rho(\vec{x}')G + \frac{1}{4\pi}\oint da'\left(G\frac{\partial\phi}{\partial n'} - \phi\frac{\partial G}{\partial n'}\right)$

Fig.1

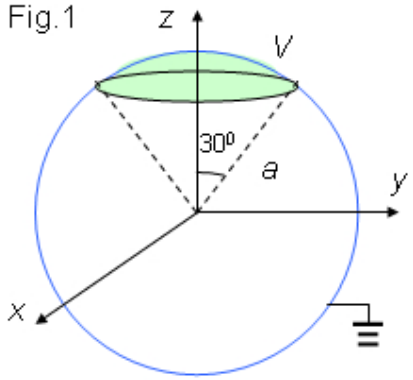
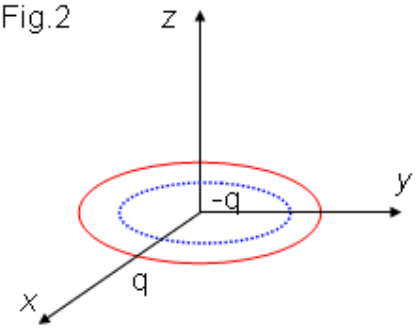


Fig.2



3.[30%] Two circular rings are lying on the x-y plane (see Fig.2). The larger ring with radius a has total charge q; the smaller ring with radius b has total charge $-q$. (The charges are distributed uniformly in the rings.)
(a) Write down the charge density $\rho(\boldsymbol{x})$ of this system using the Dirac delta functions (in spherical coordinate).
(b) Find out the dipole moment $\boldsymbol{p}$ (with respect to the origin).
(c) Find out the quadrupole moment Q_{ij} using the coordinate given. Write your answer as a 3 by 3 matrix.