1. [30%] Two infinitely large parallel conducting plates are separated by a distance *d*. One plate has a uniform surface charge density σ_0, the other is *grounded*.
   (a) Find the electric field *E* between the plates.
   (b) Find the induced surface charge density σ on the grounded plate.
   (c) What is the energy density *w* between the plates.

2. [30%] Two point charges (each has charge *Q*) are located at *z*=*R* and *z*=-*R*. In addition, a uniformly charged circular ring with charge –2*Q* and radius *R* is located on the *x*-*y* plane and centered at the origin, see Fig. 1.
   (a) Find out the potential $f(\vec{x})$ everywhere in space.
   (b) At large distance *r*, by comparing result (a) with the following asymptotic form of the potential, find out the electric dipole and quadruple of this system.

$$f(\vec{x}) \simeq \frac{1}{4pe_0}\left[\frac{q}{r}+\frac{\vec{p}\cdot\vec{x}}{r^3}+\frac{1}{2}\sum_{ij} Q_{ij}\frac{x_i x_j}{r^5}\right]$$

Fig.1

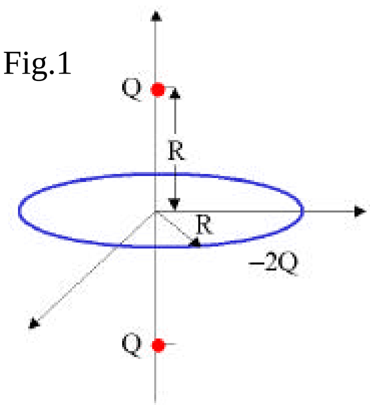
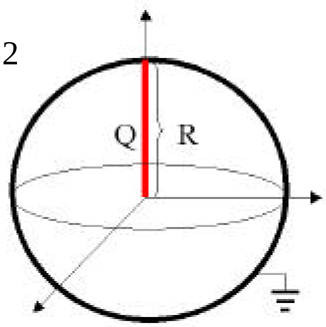


Fig.2



3. [40%] Inside a *grounded* conducting sphere with radius *R*, there is a uniformly charged rod (with total charge *Q*) along the positive *z*-axis (see Fig. 2).
(a) The charge density of the rod is of the following form, $r(\vec{x}) = cd(q)/r^2$ (*r*<*R*), where the factor $1/r^2$ is necessary to ensure that the charge density is *linearly* uniform. Find out the constant *c*.
(b) Find out the potential $f(\vec{x})$ everywhere *inside* the sphere.
(c) Find out the induced surface charge density $s(q)$ on the sphere.

$$P_l(1) = 1, P_{2n+1}(0) = 0, P_{2n}(0) = (-1)^n (2n-1)!!/(2^n n!)$$

$$Y_{lm}(q,f) = \sqrt{\frac{2l+1}{4p}\frac{(l-m)!}{(l+m)!}} P_l^m(\cos q) e^{imf}, \quad \frac{1}{|\vec{x}-\vec{x}'|} = \sum_{lm} \frac{4p}{2l+1}\frac{r_<^l}{r_>^{l+1}} Y_{lm}^*(q',f')Y_{lm}(q,f)$$

$$f(\vec{x}) = \frac{1}{4pe_0}\int dV' r(\vec{x}')G + \frac{1}{4p}\oint da'\left(G\frac{\partial f}{\partial n'} - f\frac{\partial G}{\partial n'}\right)$$