Classical electrodynamics 9:00 –**11:10**, 1/7/2010

1. (30%) In the multipole expansion of the vector potential, we have

$$\vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{1}{x}\int dV'\vec{J}(\vec{x}') + \frac{\mu_0}{4\pi}\frac{\vec{x}}{x^3}\cdot\int dV'\vec{x}'\vec{J}(\vec{x}')$$

(a) Prove that, for a localized current source, the monopole term is zero. Hint: $\nabla\cdot(x_i\vec{J})$

(b) The second term is related to magnetic moment ***m***. Write down the definition of ***m***, also write the second term in terms of ***m***. <u>No derivation is required for part (b).</u>

2. (40%) Inside a conducting media (with uniform ε, μ, and conductivity σ), there is an electric current ***J***, but no free charge ρ. Assume the electromagnetic fields in the conductor vary slowly in time. Based on these conditions, the divergences of ***E*** is zero, and

$$\nabla\times\vec{H} = \vec{J}, \quad \text{also} \quad \vec{J} = \sigma\vec{E}.$$

Since there is no free charge, the ***E*** field is generated by the time-dependent ***B*** field. We can neglect the scalar potential Φ and write $\vec{E} = -\partial\vec{A}/\partial t$.

(a) Use the Coulomb gauge, show that the vector potential obeys a diffusion equation,

$$\nabla^2\vec{A} = \alpha\,\partial\vec{A}/\partial t.$$

Find out the constant α.

(b) Assume the conductor in (a) fills the whole space above *x*-*y* plane. Outside the surface of the conductor, $\vec{H} = H_0 e^{-i\omega t}\hat{x}$; inside the conductor, $\vec{H}(z) = h(z)e^{-i\omega t}\hat{x}$. The ***H*** field is continuous across the surface. Assume *h*(z)=exp(*ikz*). Show that *h*(z) decays inside the conductor, *h*(z)=*f*(z) exp(-z/δ). Find out *f*(z) and the decay length δ.

3. (30%) A long coaxial cable consists of an inner conductor (radius *a*) and an outer conductor (radius *b*), separated by a medium with constant ε and μ (see Fig.).

(a) Because of the uniform charges (see Fig.), there is a radial electric field in the medium. Assume λ is the charges per unit length, *I* is the current in the conductor, find out the electric field ***E***(*r*) and the magnetic field ***B***(*r*) within *a*<*r*<*b*.

(b) Find out the Poynting vector ***S*** within the cable. Integrate ***S*** over a cross-section to get the power being transported by the cable.