Classical electrodynamics final 9:00 –**11:00**, 1/15/2009

1. (30%) There are 3 point charges on the *z*-axis. Two of them have charge *q*, the other has charge -2*q*. Their locations are shown in Fig.1.

(a) Write down the charge density $\rho(\boldsymbol{x})$ of this charge system.

(b) Find out the electric dipole $\boldsymbol{p}$.

(c) Find out the electric quadrupole Q_{ij}.

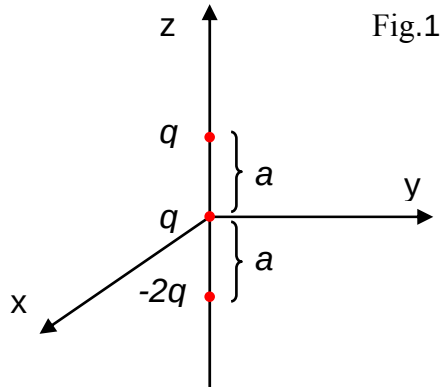

Fig.1

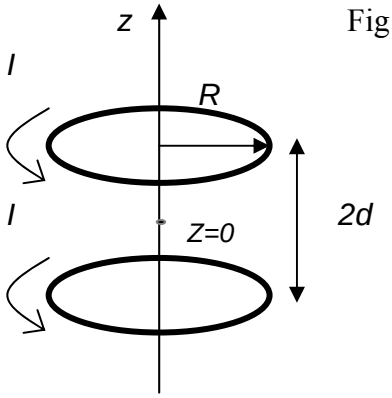

Fig.2

2. (40%) Two circular loops have the same radius *R* and carry the same current *I*. They are separated by a distance 2*d*, as shown in Fig. 2. The origin *z*=0 is in the middle.

(a) Find out the magnetic field *B*(*z*) along the z-axis.

(b) Show that *dB*(*z*)/*dz*=0 at *z*=0. Determine the value of *d* such that the second derivative of *B*(*z*) with respect to *z* is also zero at *z*=0.

3. (30%) (a) Write down the macroscopic Maxwell's equations for $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{B}$, and $\boldsymbol{H}$. You can write them either in differential form or in integral form.

(b) *Derive* the boundary conditions for $\boldsymbol{B}$ and $\boldsymbol{H}$, assuming the boundary between medium 1 and medium 2 has a surface current density $\boldsymbol{K}$.

(c) The field lines near the boundary between medium 1 and medium 2 are shown in Fig.3. Assuming surface current density $\boldsymbol{K}$=0, find out the relation between $\tan\theta_1$ and $\tan\theta_2$.

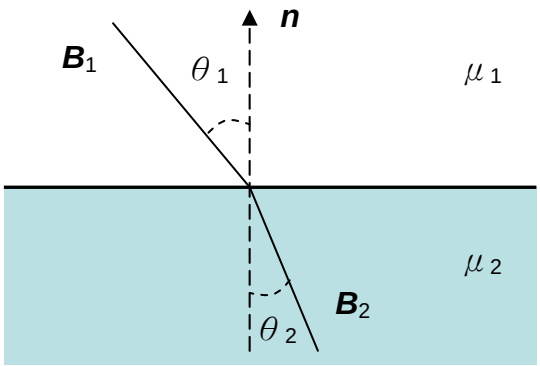

Fig.3

=== Note ===

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int dV' \frac{\boldsymbol{J}(\boldsymbol{x}')\times \boldsymbol{x}'}{x'^3}$$