Classical electrodynamics 9:00 –**11:00**, 1/15/2007

1. (40%) (a) There are two parallel wires (infinitely long) lying on a plane. The wires are separated by a distance d. The directions of the current are opposite (see Fig.1a). Find out the strength of the magnetic field at the point P between the wires.

   (b) A wire lying on a plane has the shape shown in Fig.1b. The curved part is a semi-circle with radius $d/2$. Find out the strength of the magnetic field at point P.

   (c) It is impossible to create a static magnetic field of the form $\boldsymbol{B}=B_0(x, y, 0)$ near the origin, why? If the first 2 components are fixed: $B_0(x, y, \_\_)$, what should be the z-component of the field?

Fig.1

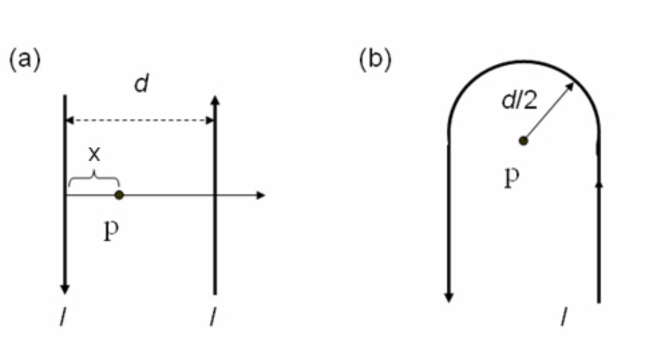
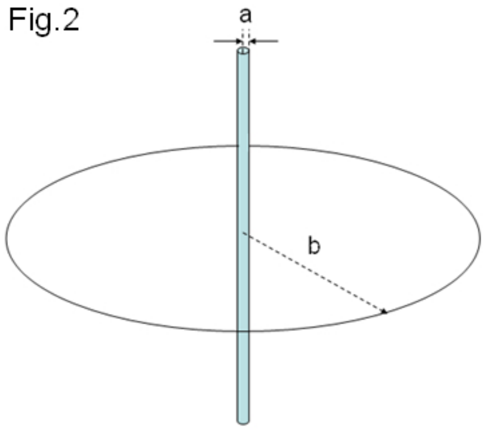


Fig.2



2. (30%) A solenoid with radius a and infinite length carries a current I (there are n turns of wire per unit length). There is a circular wire with radius b ($\gg a$) and resistance R co-axial with the solenoid (see Fig.2).

   (a) Assume the current in the solenoid is slowly reduced to 0 at a rate dI/dt, find out the induced current i in the circular ring.

   (b) Because of the changing flux, there is an induced electric field. There is also a magnetic field outside the solenoid *due to the circular loop*. Calculate the Poynting vector near (outside) the boundary of the solenoid. Then integrate over the surface of the whole solenoid to find out the *total power* (energy per unit time) flowing away from the solenoid. Write your answer in i and R.

3. (30%) In the Lorenz gauge, the scalar potential and vector potential are produced by retarded sources of charge and current. Consider an infinite straight wire along the z-axis. At time $t=0$, there is a current pulse $J(\boldsymbol{x}',t') = \delta(x')\delta(y')\delta(t')$ running up along the wire.

   (a) Find out the time-dependent vector potential $\boldsymbol{A}(\boldsymbol{x},t)$ at a distance d from the wire.

   (b) Find out the electric field $\boldsymbol{E}(\boldsymbol{x},t)$ and the magnetic field $\boldsymbol{B}(\boldsymbol{x},t)$ at a distance d from the wire.

$$\nabla \times \vec{A} = \hat{e}_\rho \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{e}_\phi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \hat{e}_z \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right)$$