Classical electrodynamics (close book exam) 9:00 –11:30, 5/6/2010

1. (30%) Answer the following questions about polarization briefly and clearly. Draw figures to clarify your answers. No mathematical derivation is required.
(a) An unpolarized EM wave becomes polarized after being reflected from a surface. Why? The reflected wave is more likely to be polarized along which direction?
(b) A point charge is oscillating along the *z*-axis. Draw the pattern of intensity of electric dipole radiation. An observer far away *on the x-y plane* is detecting the radiation. What is the direction of polarization s/he observed?
(c) An unpolarized EM wave becomes polarized after Rayleigh scattering from a dielectric sphere. If the sphere is located at the origin, and the wave is coming *from the z-axis below*, *where* can an observer detect the maximum polarization of the scattering wave? What's the direction of polarization s/he observed at those locations?

2. (30%) A point charge *e* is moving on a circle (in the *x*-*y* plane) with radius *R* and frequency ω. It has a time-dependent dipole $\vec{p}(t) = eR\left(\hat{x}\cos\omega t + \hat{y}\sin\omega t\right)$.

(a) Find out the electric dipole moment $\vec{p}$ that appears in $\vec{p}(t) = \mathrm{Re}\left[\vec{p}e^{-i\omega t}\right]$.

(b) Use $\vec{H}(\vec{x}) = \frac{ck^2}{4\pi}\hat{n}\times\vec{p}\frac{e^{ikr}}{r}$ and $\vec{E} = Z_0\vec{H}\times\hat{n}$;, find out the radiated electric field. *Describe* the polarizations of the EM wave along the *x*-axis, the *y*-axis, and the *z*-axis.

(c) Calculate the power distribution $\frac{dP}{d\Omega} = \frac{1}{2}\mathrm{Re}\left[r^2\hat{n}\cdot\vec{E}\times\vec{H}^*\right]$ and the total power *P*.

3. (40%) In this problem, we analyze the EM wave in a plasma with a magnetic field. The equation of motion for an electron in a plasma is $m\ddot{\vec{x}} = -e\dot{\vec{x}}\times\vec{B}_0 - e\vec{E}_0e^{-i\omega t}$, where $\vec{B}_0 = B_0\hat{z}$ is an uniform magnetic field, and $\vec{E}_0e^{-i\omega t}$ is the EM wave.

(a) Assume $\vec{x}(t) = \vec{x}_0e^{-i\omega t}$, find out the 3×3 matrix $\underset{\sim}{M}$ in the relation: $\underset{\sim}{M}\,\vec{x}_0 = -e\vec{E}_0$.

(b) Polarizability $\vec{P}_0 = N(-e\vec{x}_0)$, where *N* is the density of electrons. Based on the result in (a), the susceptibility matrix in $\vec{D}_0 = \underset{\sim}{\varepsilon}(\omega)\vec{E}_0$ can be written in the form,

$$\frac{\underset{\sim}{\varepsilon}}{\varepsilon_0} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}.$$

Find out ε_1, ε_2, and ε_3.

(c) Find out the eigenvalues and eigenvector of the matrix above (you only need to write the answers in the symbols $\varepsilon_1, \varepsilon_2, \varepsilon_3$). For an EM wave propagating along the *z*-axis, what are its possible polarizations?