Classical electrodynamics 9:00-11:30, 5/21/2009

1. [20%] Given the following charge density, $\rho(\vec{x},t)=\delta(x)\delta(y)\delta(z)\delta(t)$,
(a) Use the Coulomb gauge and find out the scalar potential $\phi(\vec{x},t)$.
(b) What is the scalar potential if the Lorenz gauge is used?

2. [30%] Recently, physicist found a material with a special electromagnetic property: the D and H in the Maxwell equations in Eq.(6.6) have to be modified as follows,

$$D=\varepsilon E \quad \rightarrow D=\varepsilon E+\alpha B$$
$$H=B/\mu \rightarrow H=B/\mu-\alpha E$$

Otherwise the form of the equations remain the same.
(a) Assuming there is no ρ and J near the surface of this material (the other side is vacuum), what is the boundary condition for ***D***, ***B***, ***E***, and ***H***? Are they different from the usual material with α=0? *Explain why.*
(b) For a plane wave propagating within such a material, do ***k***, ***E***, ***B*** remain orthogonal to each other? *Explain.*

3. [30%] A point charge *e* is moving on a circle (in the *x*-*y* plane) with radius *R* and frequency ω. The circle is centered at the origin.

(a) Find out the electric dipole moment $\vec{p}$ that appears in $\vec{p}(t)=\mathrm{Re}\left[\vec{p}e^{-i\omega t}\right]$.

(b) Use Eq.(9.16), $\vec{A}(\vec{x})=-\frac{i\mu_0\omega}{4\pi}\vec{p}\frac{e^{ikr}}{r}$, find out the radiated electric field (neglect $1/r^2$ terms). Describe the polarizations of the EM wave along the *x*-axis, the *y*-axis, and the *z*-axis.

4. [20%] (a) Assuming *N* gas molecules are *randomly* distributed on a straight line along the z-axis. Consider the Rayleigh scattering by these molecules, what would be the form factor $F(\vec{q})$ in Eq.(10.19)? Briefly explain your result.
(b) What is the form factor if these molecules form a one-dimensional *lattice* along the z-axis? The separation between neighboring molecules is *d*. How would this form factor change the behavior of scattered light (compared to (a))?