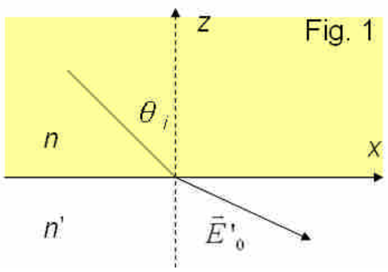
Classical electrodynamics 9:00-12:00, 5/6/2008

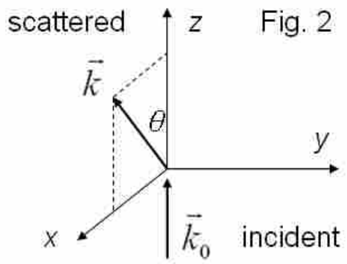
1. [30%] A plane wave is incident from a medium at $z>0$ to a medium at $z<0$. The refraction coefficients are n and n' ($n>n'$, see Fig.1). The space part of the *refracted* wave is of the form $\vec{E}'(\vec{x}) = \vec{E}'_0 e^{i\vec{k}'\cdot\vec{x}}$. When the incident angle θ_i is larger than a critical angle i_0, there will be total internal reflection.

(a) Find out the penetration depth $\delta=1/\kappa'$ of the evanescent wave as a function of θ_i and frequency ω, where κ' is the imaginary part of the refracted wave vector.

(b) The Poynting vector for the evanescent wave is $\vec{S} = (1/2)\,\mathrm{Re}(\vec{E}'_0 \times \vec{H}'^*_0)e^{2\kappa' z}$. Find out the energy being transmitted along the normal direction.

(Note: $k'_z = -\sqrt{k'^2 - k'^2_x}$, assuming $k'_y=0$. Also, $\vec{H}'_0 = (\vec{k}'/\omega)\times\vec{E}'_0$.)





2. [30%] For the scattering of an EM wave by a small dielectric sphere, the differential cross section is found to be $\dfrac{d\sigma}{d\Omega} = k^4 a^6 \left|\dfrac{\varepsilon_r - 1}{\varepsilon_r + 2}\right|^2 |\hat{\varepsilon}^* \cdot \hat{\varepsilon}_0|^2$.

(a) Assuming the polarization of the *incident* wave is not polarized, calculate the differential cross section $d\sigma_{//}/d\Omega$ and $d\sigma_{\perp}/d\Omega$ when the scattered polarization is parallel or perpendicular to the plane defined by $\boldsymbol{k}_0$ and $\boldsymbol{k}$ (see Fig. 2).

(b) If we *further* average over the polarizations of the scattered wave, what would be the differential cross section?.

3. [40%] A point charges e is moving on a circle (in the x-y plane) with radius R and frequency ω. The circle is centered at the origin.

(a) Find out the electric dipole moment $\vec{p}$ that appears in $\vec{p}(t) = \mathrm{Re}\left[\vec{p}e^{-i\omega t}\right]$.

(b) Starting from $\vec{A}(\vec{x}) = \dfrac{\mu_0}{4\pi}\dfrac{e^{ikr}}{r}\int \vec{J}(\vec{x}')dV'$, show that $\vec{A}(\vec{x}) = -\dfrac{i\mu_0\omega}{4\pi}\vec{p}\dfrac{e^{ikr}}{r}$.

(c) Based on the result in (b), find out the radiated electric field (neglect $1/r^2$ terms). Describe the polarizations of the EM wave along the x-axis, the y-axis, and the z-axis.

(d) Calculate the time-averaged power distribution $dP/d\Omega$ and the total power P.

(Note: $\vec{E} = Z_0\vec{H}\times\hat{n}$; $\dfrac{dP}{d\Omega} = \dfrac{1}{2}\mathrm{Re}\left[r^2\hat{n}\cdot\vec{E}\times\vec{H}^*\right]$)