Classical electrodynamics (open book exam) 9:10 –12:00, 5/24/2007

◎ If an equation or a formula needed in your calculation can be found in Jackson, then you can simply cite the equation. There is no need to re-derive it again.

1. Consider the TM field in a rectangular resonant cavity. The size and coordinate of the cavity is similar to the one in Fig. 8-5. The only difference is that now there are two walls at z=0 and z=d. (a) Find out the electric field $E_z(x,y,z)$ for the *mnp*-mode.

(b) Find out the resonant frequencies ω_{mnp}. What is the lowest resonant frequency?

2. If the polarization of an EM wave is parallel to the plane of incidence, then according to the Fresnel's formula, one has (the ratio of reflective field to incident field)

$$\frac{E_0''}{E_0}=\frac{\tan(i-r)}{\tan(i+r)}.$$

where i and r are the incident angle and the refractive angle.

(a) When i-r=π/2, with the help of the Snell's law, $k_{//}/k_{\perp}=\sin i/\cos i$, and $k_{//}^2+k_{\perp}^2=\omega^2/c^2$, show that $k_{\perp}=\frac{\omega}{c}\sqrt{\frac{1}{1+\varepsilon_r}}$, $k_{//}=\frac{\omega}{c}\sqrt{\frac{\varepsilon_r}{1+\varepsilon_r}}$,

where ε_r is the relative dielectric constant between the two media.

(b) If ε_r<-1, then $k_{//}$ is real, $k_{\perp}$ is imaginary, and we have the surface plasma wave. Assume $\varepsilon_r=1-\left(\omega_p/\omega\right)^2$, $(\varepsilon_r<-1)$, show that *at long wavelength*, the *low-frequency* surface plasma has the dispersion relation, $\omega\approx ck_{//}\sqrt{1-\frac{1}{2}\left(ck_{//}/\omega_p\right)^2}$.

3. An EM wave is scattered by a perfectly conducting sphere with radius a. Assume the incident field can only induce a magnetic dipole but *not* an electric dipole. Start from Eq.10.14 and use the same coordinate as the one in Fig. 10.1, find out $d\sigma_{//}/d\Omega$ and $d\sigma_{\perp}/d\Omega$ for an unpolarized incident wave.

4. There are two point charges (each with charge q) at the ends of a rigid rod with length d. The rod is lying on the x-y plane (the center of the rod is at the origin) and is spinning with angular frequency ω.

(a) Find out the electric dipole moment p(t), the magnetic dipole moment m(t), and the electric quadrupole moment Q(t). You'll find that p(t) and m(t) are independent of time.

(b) Calculate the radiating power $dP/d\Omega$ (assume ωd<c). Express your answer as a function of the variable θ in the spherical coordinate.