Persistent spin current

in

a spin ring

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A brief history

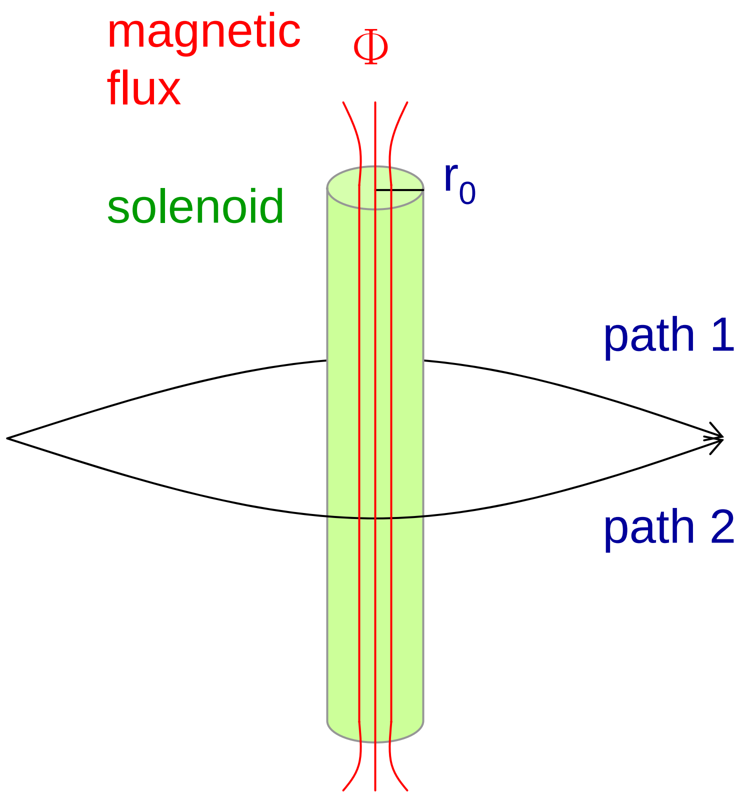
charge

- precursor: Hund, Ann. Phys. 1934
- persistent current in a metal ring
    - diffusive regime (Buttiker, Imry, and Landauer, Phys. Lett. 1983)
    - inelastic scattering (Landauer and Buttiker, PRL 1985)
    - the effect of lead and reservoir (Buttiker, PRB 1985 … etc)
    - the effect of e-e interaction (Ambegaokar and Eckern, PRL 1990)
- experimental observation (Levy et al, PRL 1990; Chandrasekhar et al, PRL 1991)
- electron charge and spin current
    - textured magnetic field (Loss, Goldbart, and Balatsky, PRL 1990)
    - spin-orbit coupling (Meir et al, PRL 1989; Aronov et al, PRL 1993 … etc)

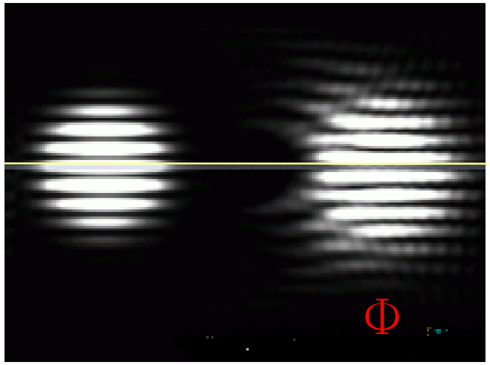
spin

- FM ring (Schutz, Kollar, and Kopietz, PRL 2003)
- AFM ring (Schutz, Kollar, and Kopietz, PRB 2003)
- this work: ferrimagnetic ring

Aharonov-Bohm (AB) effect (1959)



$$\vec{B} = 0 \ (\vec{A} \neq 0) \text{ for } r > r_0$$



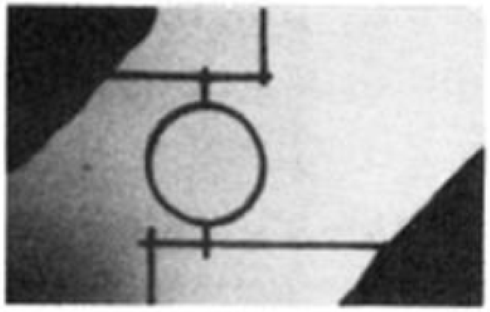
AB phase = $\dfrac{e}{\hbar}\oint \vec{A}\cdot d\vec{x}$

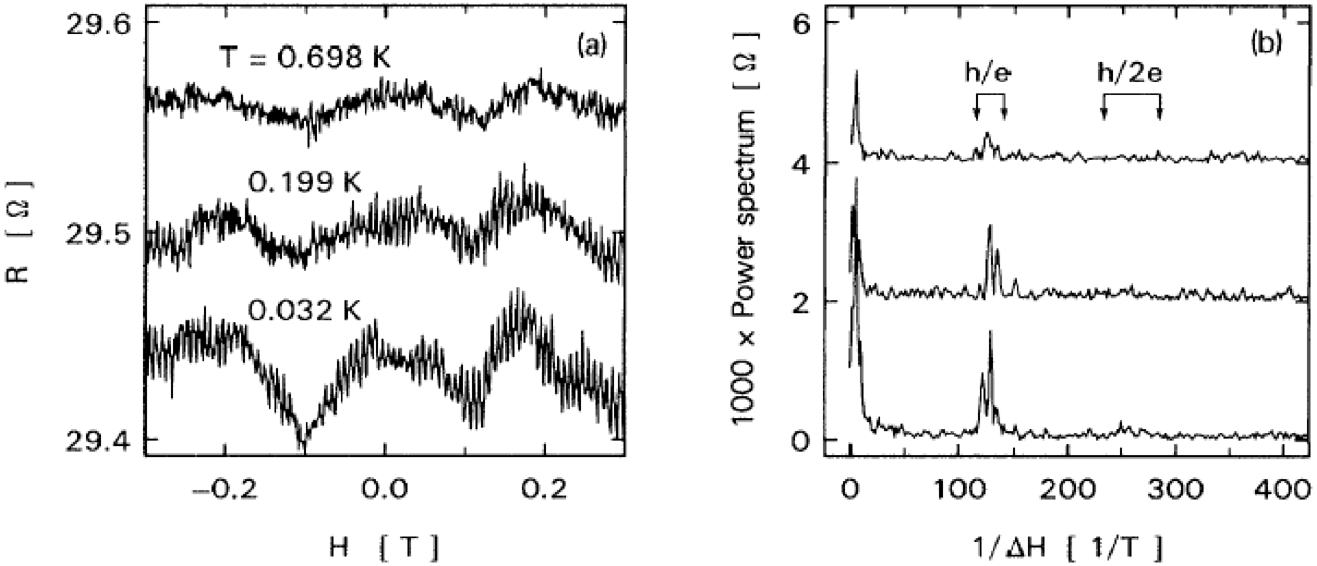
$= 2\pi\ \Phi/\Phi_0$

flux quantum Φ_0 = h/e

(0.4×10^{-6} Gauss-cm^2)

AB effect and resistance oscillation in a metal ring
(Webb et al, PRL 1985)

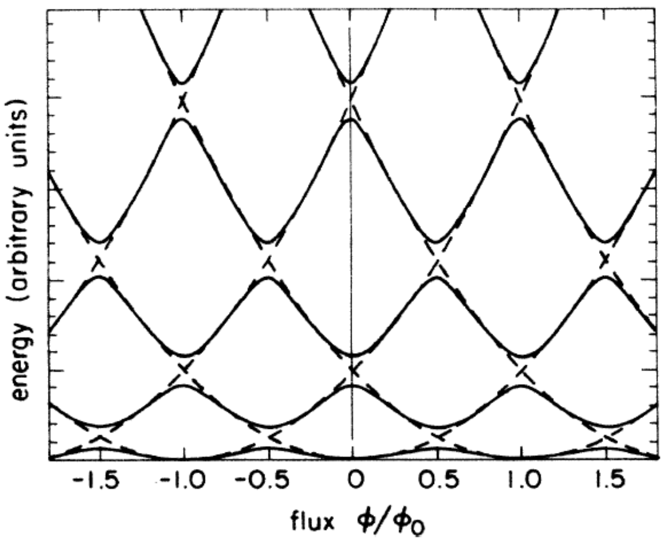




Persistent charge current in a metal ring

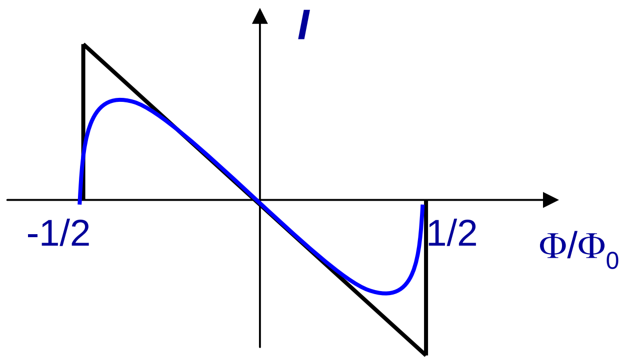
$$kL \rightarrow kL + 2\pi \frac{\Phi}{\Phi_0}$$

$$\varepsilon_n(\Phi) = \frac{\hbar^2}{2mR^2}\left(n + \frac{\Phi}{\Phi_0}\right)^2$$



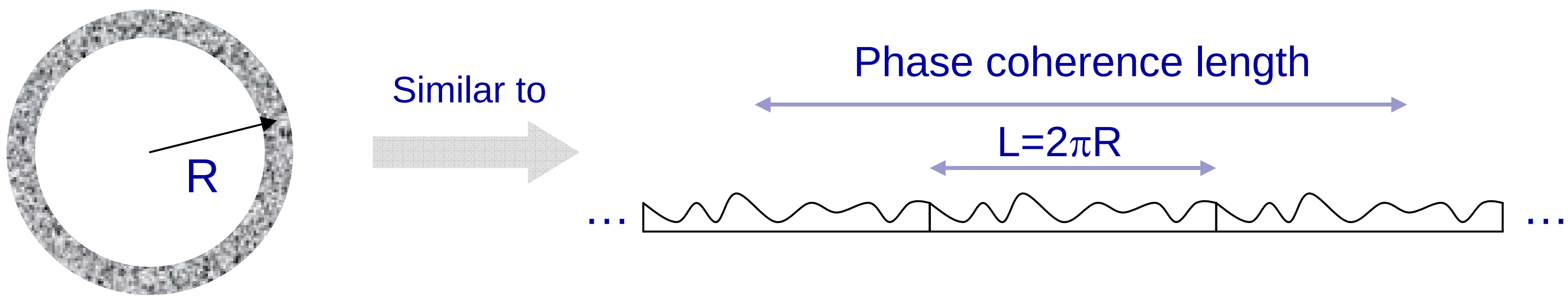
Persistent current

$$I_n = -\frac{e}{L} v_n = -\frac{e}{L}\frac{\partial E_n}{\hbar \partial k} = -\frac{\partial E_n}{\partial \Phi}$$

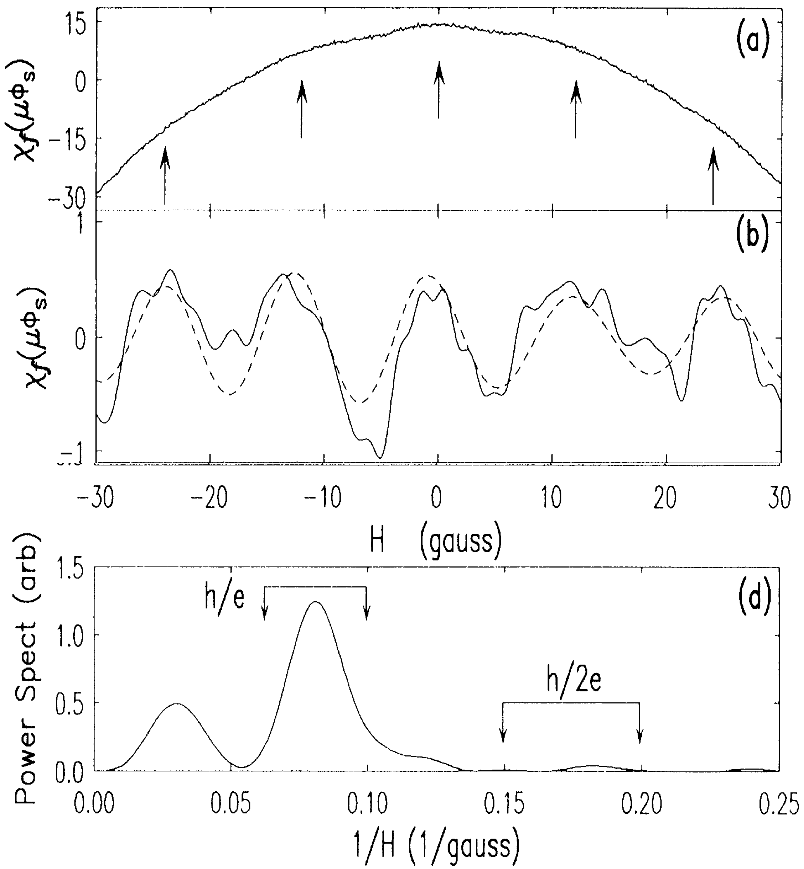


Smoothed by elastic scattering… etc

A ring with static disorder



Magnetic response of a gold ring
(Chandrasekhar et al, PRL 1991)

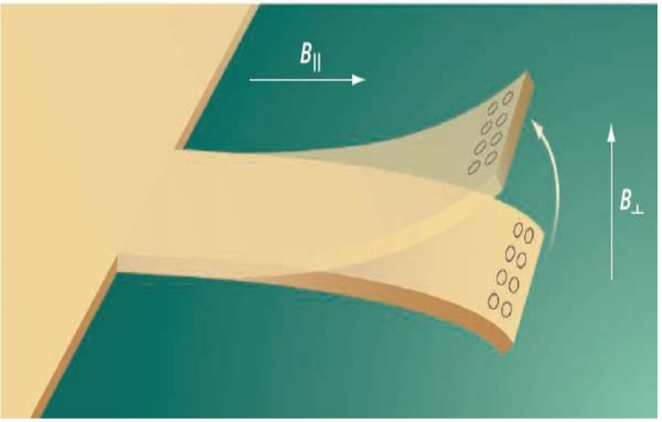


- magnitude of current 30 times larger than theoretical prediction

Update:

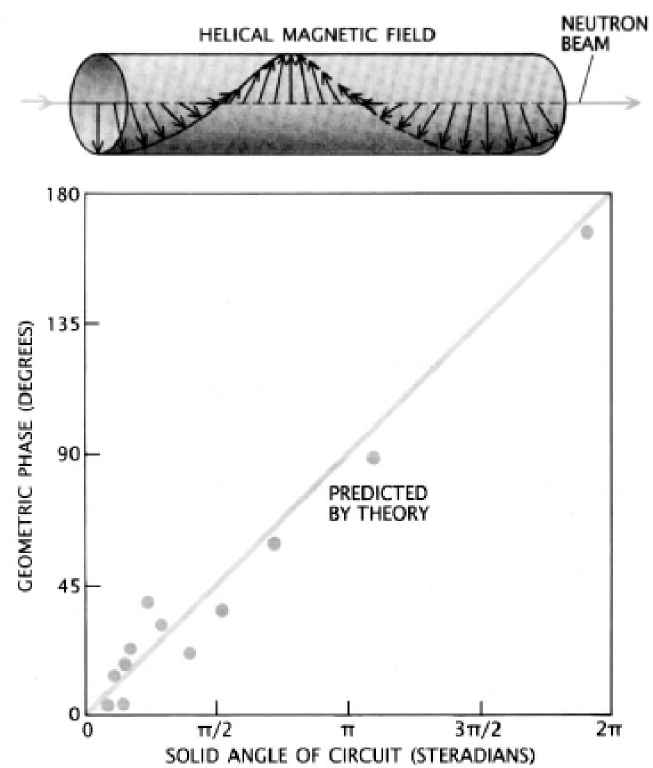
Bluhm et al, PRL 2009
(using scanning SQUID)

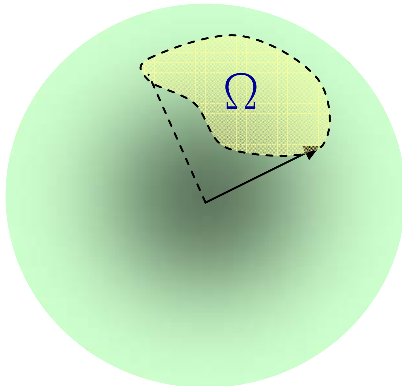
Bleszynski-Jayich et al, Science 2009 (using micro-cantilever)



Another phase accumulated over a cycle: Berry phase (1984)

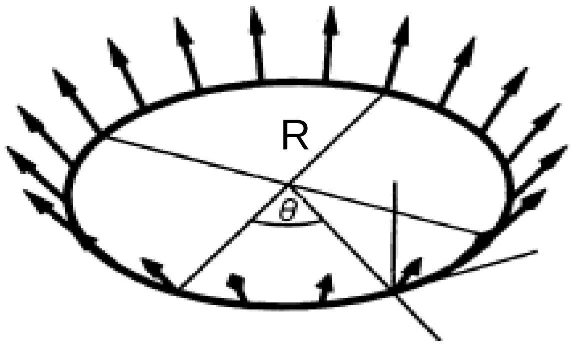
Neutron spin guided by a helical B field





Berry phase = spin$\times\Omega$

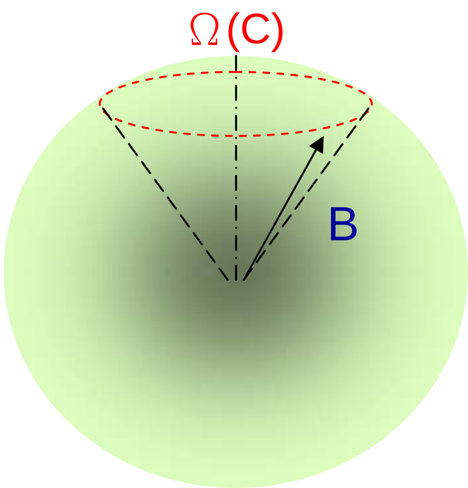
A metal ring in a textured B field (Loss et al, PRL 1990, PRB 1992)



$$H=\frac{1}{2m}\left(\frac{p_\theta}{R}+eA_\theta\right)^2+\mu_B\vec{B}\cdot\vec{\sigma}$$

After circling once, an electron acquires

- an AB phase $2\pi\,\Phi/\Phi_0$ (from the magnetic flux)
- a Berry phase $\pm$ (1/2) Ω (C) (from the "texture")



Electron energy:

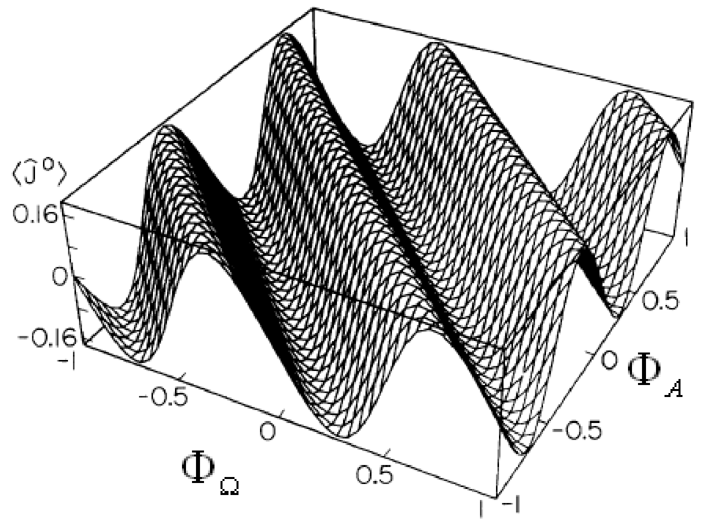
$$\varepsilon_{n\sigma}=\frac{\hbar^2}{2mR^2}\left(n+\frac{\Phi_A}{\Phi_0}+\sigma_z\frac{\Phi_\Omega}{\Phi_0}\right)^2+\sigma_z\mu_B B$$

$$\frac{\Phi_\Omega}{\Phi_0}\equiv\frac{\Omega}{4\pi}$$

Persistent charge and spin current
(Loss et al, PRL 1990, PRB 1992)

$$I = \frac{1}{L}\sum_{n\sigma}\langle n\sigma|(-e)v|n\sigma\rangle \frac{e^{-\beta\varepsilon_{n\sigma}}}{Z}$$

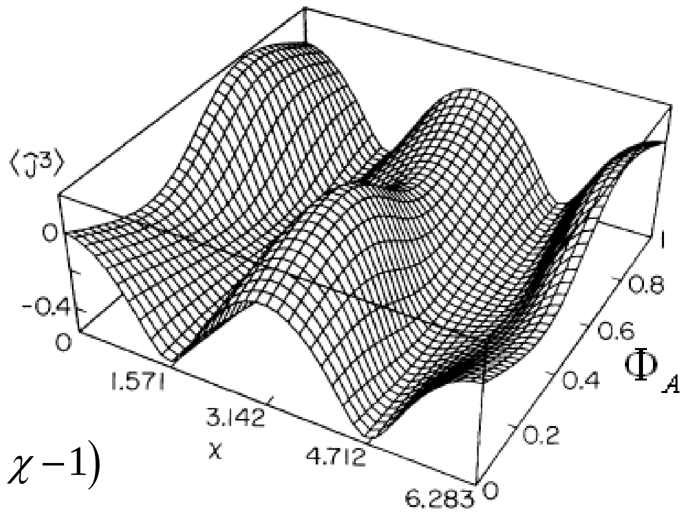
$$= \frac{1}{\beta}\frac{\partial}{\partial\Phi_A}\ln Z = -\frac{\partial F}{\partial \Phi_A}$$



$$\beta = 1/kT,\ F = -(1/\beta)\ln Z$$

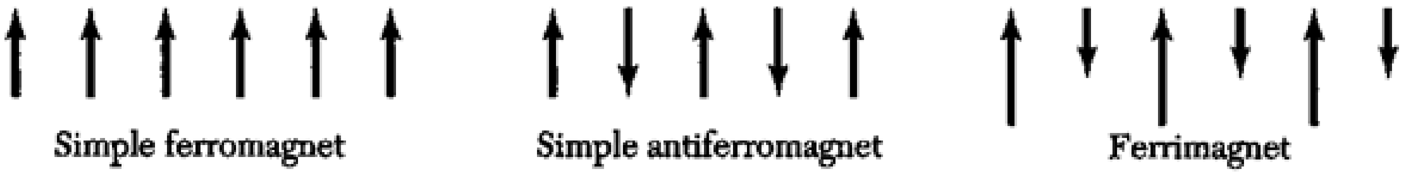
$$I_s = \frac{1}{L}\sum_{n\sigma}\langle n\sigma|s_z v|n\sigma\rangle \frac{e^{-\beta\varepsilon_{n\sigma}}}{Z}$$

$$= -\frac{\hbar}{2e\beta}\frac{\partial}{\partial\Phi_\Omega}\ln Z = \frac{\hbar}{2e}\frac{\partial F}{\partial \Phi_\Omega}$$

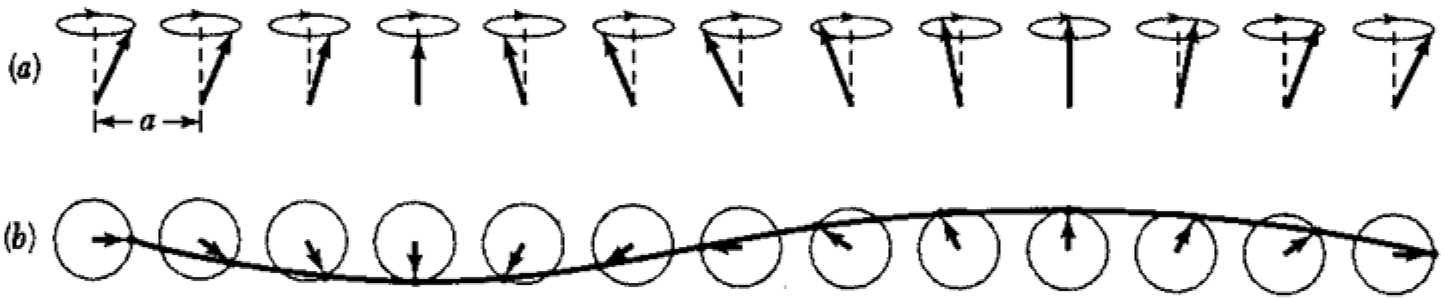


$$\Phi_\Omega = \frac{1}{2}(\cos\chi - 1)$$

Ferromagnet (FM), antiferromagnet (AFM), and ferrimagnet (FIM)



Spin wave in ferromagnet

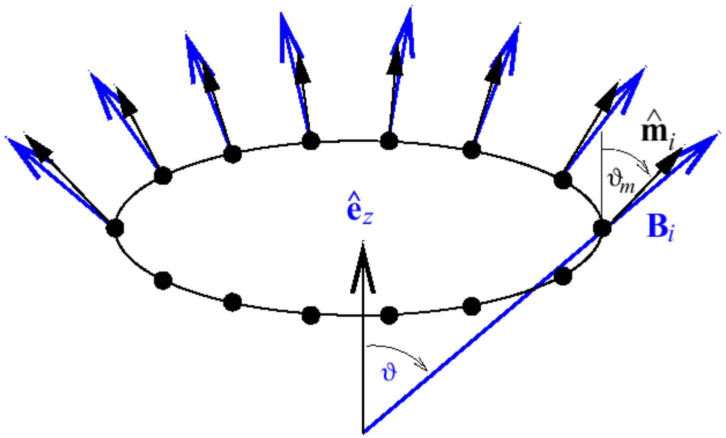


Quantum of spin-wave = magnon (boson)

Spin current = transport of magnons

Ferromagnetic Heisenberg ring in a non-uniform *B* field

(Schütz, Kollar, and Kopietz, PRL 2003)



$$\hat{H} = \frac{1}{2}\sum_{ij} J_{ij}\boldsymbol{S}_i\cdot\boldsymbol{S}_j - g\mu_B\sum_i \boldsymbol{B}_i\cdot\boldsymbol{S}_i$$

Expansion using a local triad

$$\vec{S}_i = S_i^{//}\hat{m}_i + \vec{S}_i^{\perp}$$

$$\vec{S}_i^{\perp} = S_{i1}\hat{e}_i^1 + S_{i2}\hat{e}_i^2$$

$$(\vec{h}_i \equiv g\mu_B\vec{B}_i)$$

Large spin limit, using

Holstein-Primakoff bosons:

$$S_i^{//} = S - b_i^+ b_i$$

$$S_i^+ = \sqrt{2S}b_i; \quad S_i^- = \sqrt{2S}b_i^+$$

$$\Rightarrow \quad H = \frac{1}{2}\sum J_{ij}\hat{m}_i\cdot\hat{m}_j S_i^{//}S_j^{//} - \sum \vec{h}_i\cdot\hat{m}_i S_i^{//} \qquad H^{//} = O(S^2+S)$$

$$+\frac{1}{2}\sum J_{ij}\vec{S}_i^{\perp}\cdot\vec{S}_j^{\perp} \qquad H^{\perp} = O(S)$$

$$+\sum_i \left(\sum_j J_{ij}S_j^{//}\hat{m}_j - \vec{h}_i\right)\cdot\vec{S}_i^{\perp} \qquad H' = O(\sqrt{S})$$

$//\,\hat{m}_i$ $(\perp \vec{S}_i^{\perp})$ with an error of order $O(1)$

Hamiltonian for spin wave (NN only, $J_{i,i+1} \equiv -J$)

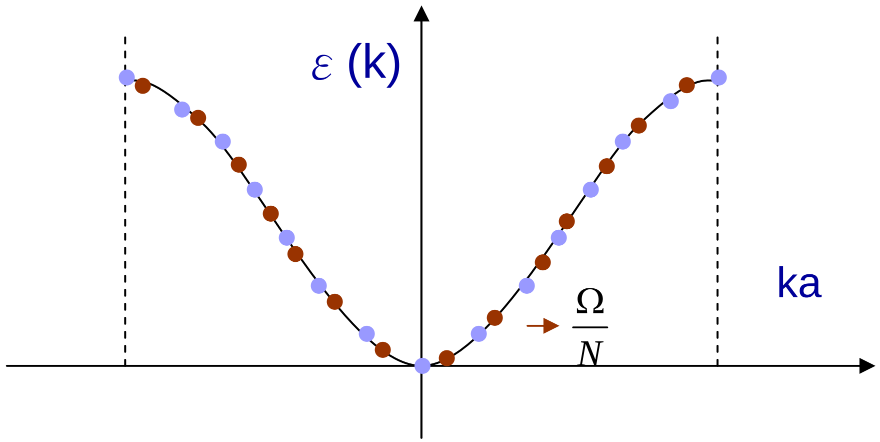
$$H_{SW} = H^{//} - H^{//}_{classical} + H^{\perp}$$

$$= 2JS\sum_i b_i^\dagger b_i + h\sum_i b_i^\dagger b_i - JS\sum_i \left\{ b_{i+1}^\dagger b_i \exp\left[i\Omega / N\right] + h.c. \right\}$$

$$\Rightarrow \quad H_{SW} = \sum_k (\varepsilon_k + h) b_k^+ b_k,$$

$$\varepsilon_k = 2JS\left(1 - \cos ka\right)$$

where $ka = \frac{2\pi}{N}\left(n + \frac{\Omega}{2\pi}\right)$



Persistent magnetization current

$$I_m = \frac{g\mu_B}{\hbar} I_s = -\frac{g\mu_B}{L}\sum_k \frac{v_k}{e^{(\varepsilon_k + h)/kT} - 1}$$

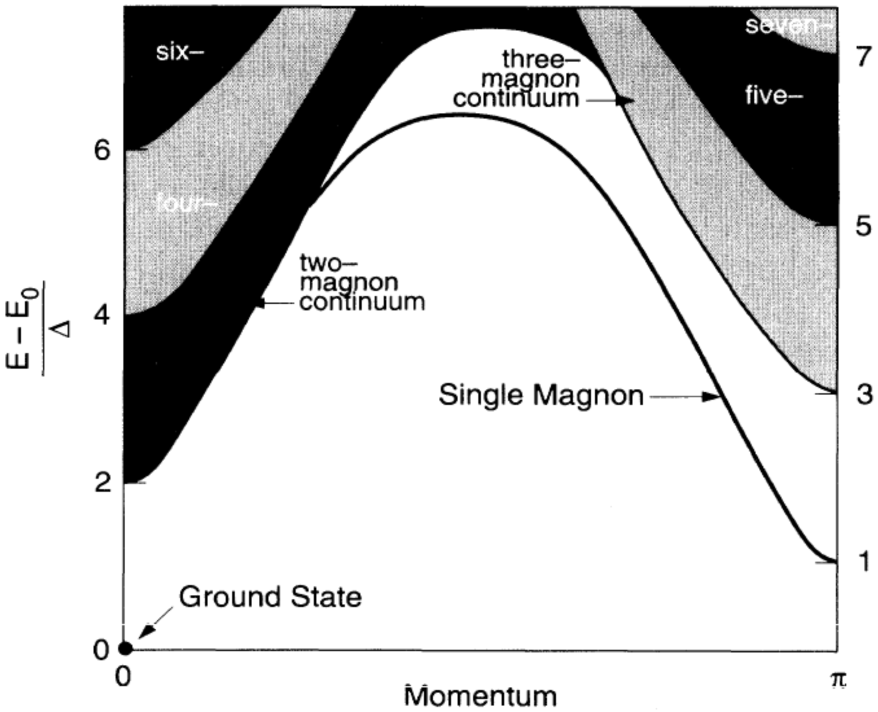
- I_m vanishes if T=0 (no magnon)
- I_m vanishes if N>>1

Antiferromagnetic Heisenberg chain

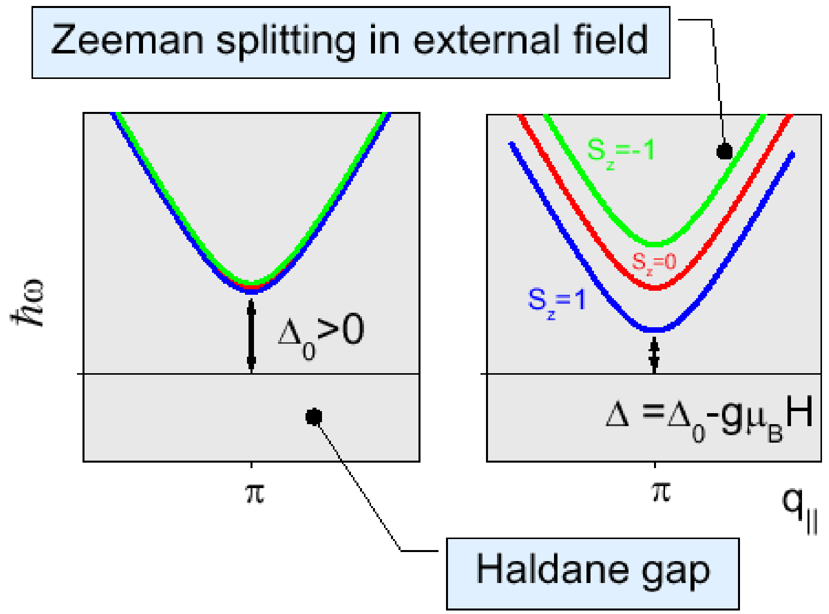
- AFM chain with half-integer-spins :

  low-energy excitation is spinon, not magnon
- so consider AFM chain with integer-spins

AFM spin chain with *S*=1



White and Huse, PRB 1993

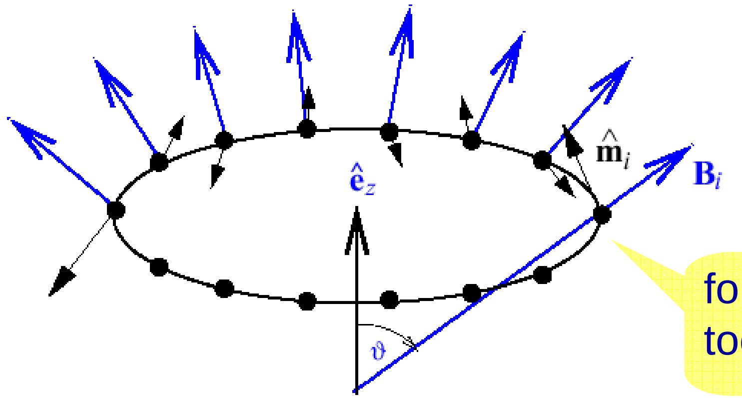


From Zheludev's poster

Antiferromagnetic Heisenberg ring in a textured B field

(Schütz, Kollar, and Kopietz, PRB 2004)

Large spin limit



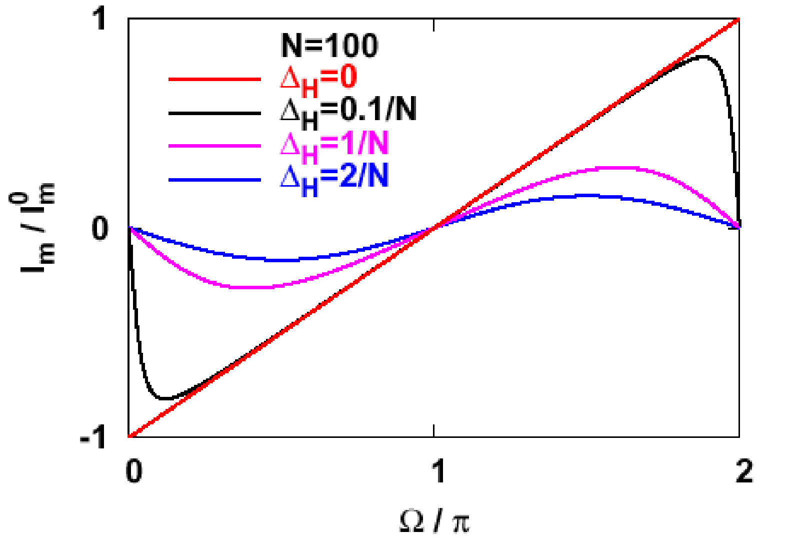
for a field not too strong

$$I_m(\Omega) = -\frac{2g\mu_B}{L}\sum_k v_k \left(n_k + \frac{1}{2}\right)$$

$$\xrightarrow{T=0} \quad \text{(and } \Delta_H = 0\text{)} \quad \boxed{I_m = I_m^0\left(1 - \frac{\Omega}{\pi}\right)}$$

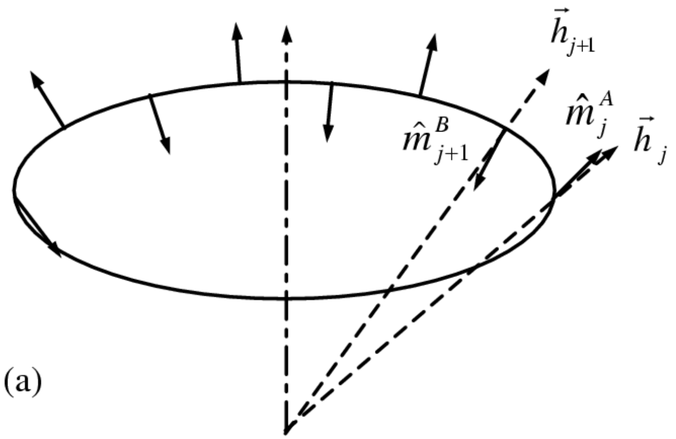
$$I_m^0 = \frac{g\mu_B}{\hbar}\frac{JS}{N}$$

Haldane gap $= 2JS\Delta_H \propto e^{-\pi S} \propto a/\xi$



Ferrimagnetic Heisenberg ring in a textured B field
(Wu, Chang, and Yang, PRB 2005)

- consider large spin limit, NN coupling only
- no need to introduce the staggered field



(a)

$\gamma \equiv S_B / S_A < 1$

Using HP bosons, plus Bogolioubov transformation, one has

$$H = \sum_k \left[\epsilon_k^- \left(\alpha_k^\dagger \alpha_k + \frac{1}{2} \right) + \epsilon_k^+ \left(\beta_k^\dagger \beta_k + \frac{1}{2} \right) \right] - \frac{NJS^A}{2}(1+\gamma)$$

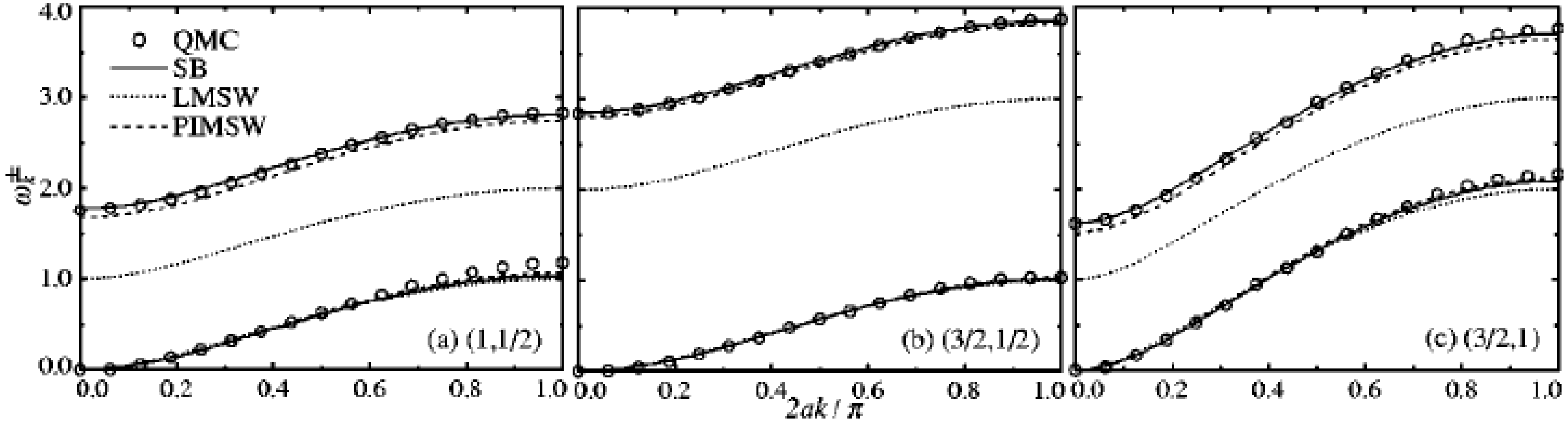
$$\epsilon_k^\pm = JS^A[\sqrt{(1-\gamma)^2 + 4\gamma \sin^2(ka)} \pm (1-\gamma)] \mp h_0$$

Gapless branch + gapful branch

where $k_n = \frac{2\pi}{L}\left(n + \frac{\Omega}{2\pi}\right)$

Ferrimagnetic Heisenberg chain

two separate branches of spin wave:



(S. Yamamoto, PRB 2004)

- Gapless FM excitation well described by linear spin wave analysis
- Modified spin wave qualitatively good for the gapful excitation

Persistent magnetization current

$\gamma = S_B / S_A < 1$

$$I_m = \frac{g\mu_B}{\hbar} I_s = -\frac{g\mu_B}{L} \sum_{k,\alpha=\pm} v_k^\alpha \left(n_k^\alpha + \frac{1}{2} \right)$$

$$v_k^\alpha = \frac{1}{\hbar} \frac{\partial \epsilon_k^\alpha}{\partial k} = \frac{2JS^A a}{\hbar} \frac{\gamma \sin(2ka)}{\sqrt{(1-\gamma)^2 + 4\gamma \sin^2(ka)}}$$

At T=0, the spin current remains non-zero

$$I_m = I_m^0 \sum_k \frac{\gamma \sin(2ka)}{\sqrt{(1-\gamma)^2 + 4\gamma \sin^2(ka)}}$$

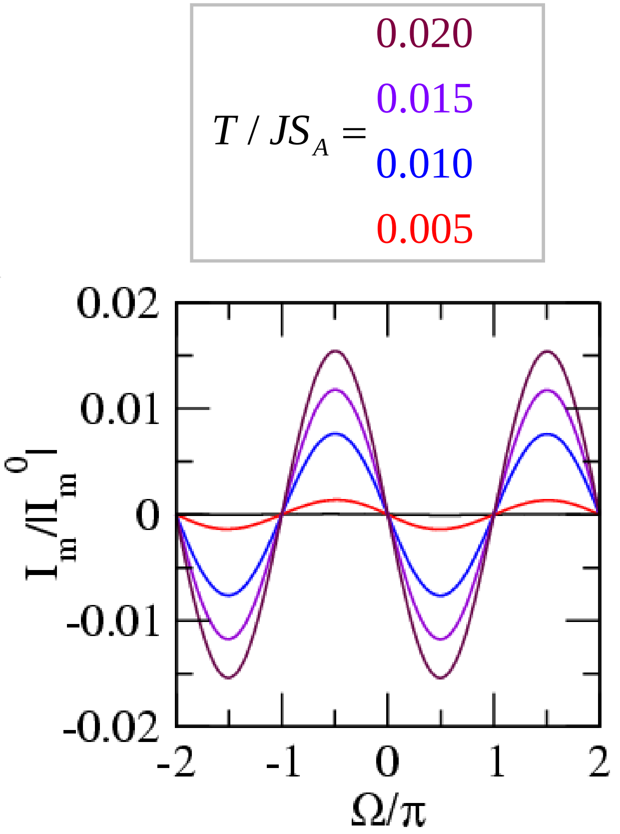
$$-\frac{2g\mu_B}{\hbar} \frac{JS_A}{N}$$

effective Haldane gap

$$\Delta_\gamma = \sqrt{\frac{1-\gamma^2}{4\gamma}}$$

$N = 100, \ \gamma = 0.8$

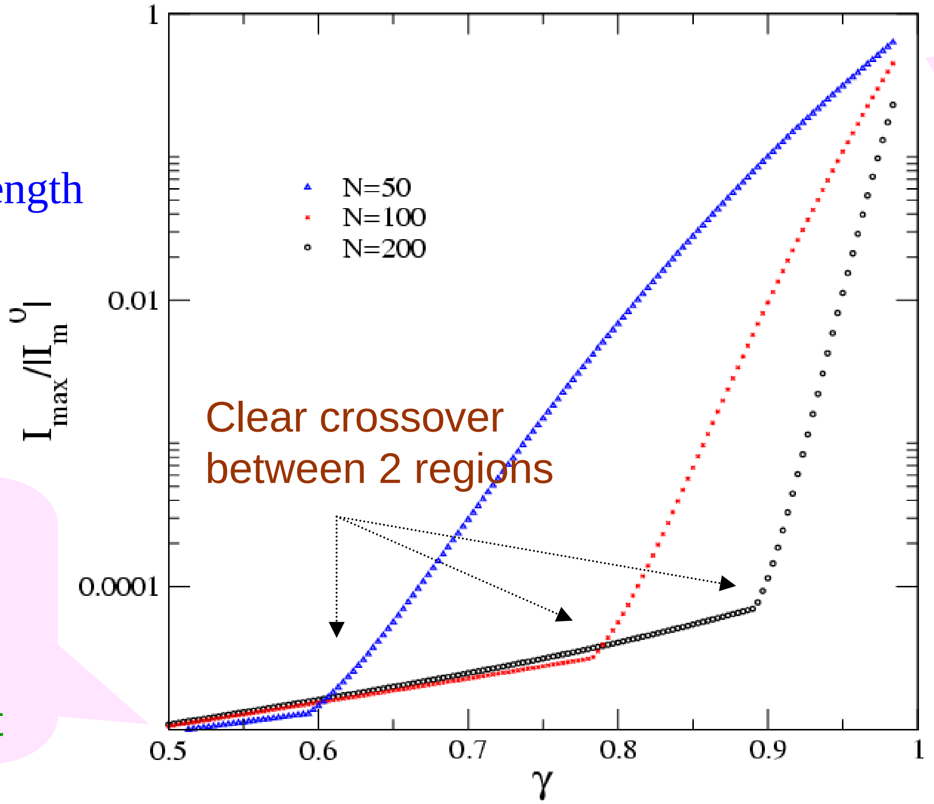
(nearly sinusoidal)



System size, correlation length, and spin current (T=0)

$$\Delta_\gamma = \sqrt{\frac{1-\gamma^2}{4\gamma}} \propto a/\xi$$

ξ : spin correlation length



AFM limit

$\gamma \approx 1, \quad \Delta_\gamma \ll 1$

$\xi \gg L$

Magnon current due to zero-point fluctuation

FM limit

$\gamma \ll 1, \quad \Delta_\gamma \gg 1$

$\xi \ll L$

no magnon current

Issues on the spin current

- Charge is conserved locally, and charge current density operator *J* is defined through the continuity eq.

The form of *J* is not changed for Hamiltonians with interactions.

- Spin current is defined in a similar way (if spin is locally conserved),

$$H = J\sum_{l=1}^{N} \vec{S}_l \cdot \vec{S}_{l+1} \qquad \frac{\partial S_l^z}{\partial t} + \nabla j_l^z = 0$$

$$j_l^z = J\left(S_l^x S_{l+1}^y - S_l^y S_{l+1}^x\right) = J\vec{S}_l^{\perp} \times \vec{S}_{l+1}^{\perp}$$

However,

- Even in the Heisenberg model, J_s is not unique when there is a non-uniform *B* field. (Schütz, Kollar, and Kopietz, E.Phys.J. B 2004).
- Also, spin current operator can be complicated when there are 3-spin interactions (P. Lou, W.C. Wu, and M.C. Chang, Phys. Rev. B 2004).
- Similar problems in spin-orbital coupled systems (e.g., Rashba).
- experimental measurement?

Related topics:

- spin ring with smaller spins
- spin ring with anisotropic coupling
- diffusive transport
- leads and reservoir
- itinerant electrons
- any application?

Thank you!