Spin Hall effect and related issues

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Basic spin interactions in semiconductors
(Dyakonov, cond-mat/0401369)

- **spin-orbit interaction**
  - required for optical spin orientation
  - major cause of spin relaxation

- **exchange interaction**
  - for ferromagnetic semiconductors
  - between FM/Semiconductor junction

- **hyper-fine interaction**
  - for materials with non-zero nuclei spin (e.g. GaAs)

- **dipole-dipole interaction** (usually very weak)
Spin-orbit interaction in an atom
(Eisberg and Resnick, Quantum Physics)

An electron moving in a static E field feels an effective B field

\[ \mathbf{B}_{\text{eff}} = \mathbf{E} \times \frac{\mathbf{v}}{c} \]

This B field couples with the electron spin

\[ H_{SO} = -\mathbf{\mu} \cdot \mathbf{B}_{\text{eff}} \]

\[ = -\left( -\frac{e}{mc} \mathbf{S} \right) \cdot \left( \mathbf{E} \times \frac{\mathbf{v}}{c} \right), \quad \mathbf{E} = -\hat{r} \frac{d\phi}{dr} \text{ for central force} \]

\[ = \left( \frac{1}{m^2 c^2} \frac{dV}{rdr} \right) \mathbf{S} \cdot \mathbf{L}, \quad V = -e\phi \]

(x 1/2 for Thomas precession)

• fine structure in atomic spectra
Spin-orbit interaction in semiconductor
(Kittel, Quantum Theory of Solids)

\[ H_{\text{SO}} = \frac{1}{2mc^2} \vec{S} \cdot \nabla V(\vec{x}) \times \vec{v} \]

(V(x) is the lattice potential energy)

- splitting of valence bands (GaAs, \( \Delta = 0.34 \) eV)
- change of g-factor (GaAs, \( g^* = -0.44 \))
- for materials without inversion symmetry, lift the spin degeneracy of energy bands (Dresselhaus, Rashba)
- skew scattering from impurities

For strong SO couplings, choose low-symmetry narrow-gap materials formed from heavy elements (\( g^* \approx -50 \) in InSb) (Rashba, cond-mat/0309441)
### Generation of spin in semiconductor using SO coupling (Rashba PRB 2004)

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Related issue

Hall effect in metal/semiconductor:

\[ \rho_{xy} = R_0 B \]

Anomalous Hall effect in ferromagnet:

\[ \rho_{xy} = R_0 B + R_S M \]

The Hall effect and its applications, by Chien and Westgate 1979

proposed mechanisms:

• Karplus-Luttinger’s mechanism (1954), revived by Jungwirth et al (PRL, 2002)  
  \text{Intrinsic, related to Berry curvature}

• Smit’s skew scattering mechanism (1955)

• Berger's side jump mechanism (1970)  
  \text{extrinsic}
Karplus-Luttinger mechanism (PRB, 1954)

\[
\begin{align*}
\hbar \frac{d\vec{k}}{dt} &= e\vec{E} \\
\frac{d\vec{x}}{dt} &= \frac{\partial E_\lambda(k)}{\hbar \partial k} - \frac{d\vec{k}}{dt} \times \Omega_\lambda(k)
\end{align*}
\]

Semiclassical EOM
(Chang and Niu, PRL 1995)

Anomalous velocity
due to Berry curvature

Berri curvature (1983)

\[
\Omega_{nz}(k) = i \left( \left\langle \frac{\partial u_n}{\partial k_x} \left| \frac{\partial u_n}{\partial k_y} \right| \right\rangle - \left\langle \frac{\partial u_n}{\partial k_y} \left| \frac{\partial u_n}{\partial k_x} \right| \right\rangle \right)
\]

Could be nonzero when (one of the following)

- time-reversal symmetry is broken (e.g. by a B field)
- lattice inversion asymmetry
- presence of SO interaction

Jungwirth et al, PRL 2002
Lee et al, Science 2004

one-band formulation
- Berry curvature as an effective
B field in k-space
- for multi-band generalization,
see Shindou and Imura, cond-mat/0411105,
Culcer et al, Nov, 2004

Semiclassical EOM
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Jungwirth et al, PRL 2002
Lee et al, Science 2004
Impurity (spinless) scattering with SO interaction

Skew scattering (Takahashi and Maekawa, PRL, 2002, Landau and Lifshitz, QM)

\[ H' = V(\vec{x}) + \lambda \vec{S} \cdot \nabla V(\vec{x}) \times \vec{v}, \quad \lambda = \frac{1}{2mc^2} \]

\[ \langle \vec{k}' s'| H' | \vec{k}s \rangle = \left[ \delta_{s's} + i\lambda \left( \frac{\hbar^2}{2m} \right) \vec{\sigma}_{s's} \cdot \vec{k}' \times \vec{k} \right] V_{\vec{k}'\vec{k}} \]

Transition rate:

\[ W_{\vec{k}s \rightarrow \vec{k}'s'} = \frac{2\pi}{\hbar} |\langle \vec{k}' s'| T | \vec{k}s \rangle|^2 \delta(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}) \]

\[ T = H' + \frac{1}{\varepsilon - H_0} H' + \cdots \]

(for \( \delta \) impurities, up to 2nd order Born approx.)

\[ W^{(AS)}_{\vec{k}s \rightarrow \vec{k}'s'} \approx \delta_{s's} n_i V_i^3 \left( \vec{\sigma}_{s's} \cdot \vec{k}' \times \vec{k} \right) \delta(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}'}) \]

Would modify the distribution function \( f_{\vec{k}\sigma'} \) through, eg., Boltzmann eq.
↑ Anomalous Hall effect in ferromagnet
- spin-polarized incident current
- charge-polarized outgoing current

↓ Spin Hall effect in semiconductor
- unpolarized incident current
- charge-unpolarized outgoing current
- but spin-polarized outgoing current
Hall effect (E.H. Hall, 1879)

[1] Spin Hall effect
(Dyakonov and Perel, JETP 1971,
J.E. Hirsch, PRL 1999)

skew scattering (due to SO coupling)
by spinless impurities:

transition rate,
\[ W_{\tilde{k}_s \rightarrow \tilde{k}'_s} \approx \lambda_{SO} \vec{\sigma}_{s's'} \cdot \tilde{k}' \times \tilde{k} \]

no magnetic field required

From spin accumulation to charge accumulation

L< spin coherence length \( \delta_s \)
\( \delta_s \approx 130 \ \mu m \) at 36 K for Al

(Johnson and Silsbee, PRL 1985)
Valence band of GaAs:

\[ H = \frac{1}{2m} \left[ \left( \gamma_1 + \frac{5}{2} \gamma_2 \right) \mathbf{k}^2 - 2\gamma_2 (\mathbf{k} \cdot \mathbf{S})^2 \right] \]

\[ \lambda = \hat{k} \cdot \mathbf{S} \] (helicity)

is a good quantum number

Luttinger Hamiltonian (1956)
(for \( j=3/2 \) valence bands)

Semiclassical EOM
(Chang and Niu, PRL 1995)

\[ \begin{cases} \hbar \frac{d\mathbf{k}}{dt} = e\mathbf{E} \\ \frac{d\mathbf{x}}{dt} = \frac{\partial E_\lambda (\mathbf{k})}{\hbar \partial k} - \frac{d\mathbf{k}}{dt} \times \mathbf{\Omega}_\lambda (\mathbf{k}) \end{cases} \]

Anomalous velocity
due to Berry curvature

[2] Intrinsic spin Hall effect in p-type semiconductor (I)
(Murakami, Nagaosa and Zhang, Science 2003)
Intrinsic spin Hall effect in p-type semiconductor (II)

(Non-Abelian) gauge potential

\[ A_{\lambda\lambda'}(\vec{k}) = i\langle\vec{k},\lambda|\frac{\partial}{\partial\vec{k}}|\vec{k},\lambda'\rangle \]

Berry curvature, due to monopole field in k-space
(Neglecting off-diagonal elements)

\[ \tilde{\Omega}_\lambda(\vec{k}) = -2\lambda\left(\lambda^2 - \frac{7}{4}\right)\frac{\vec{k}}{k^2} \]

Spin current

\[
\begin{align*}
\text{HH: } J^z_y &= \frac{\hbar}{3} \sum_{\lambda=\pm3/2,\vec{k}} \hat{y}s^z_{\lambda}(\vec{k}) = -\frac{k_F^H}{4\pi^2} eE_x, \\
\text{LH: } J^z_y &= \frac{\hbar}{3} \sum_{\lambda=\pm1/2,\vec{k}} \hat{y}s^z_{\lambda}(\vec{k}) = \frac{k_F^L}{12\pi^2} eE_x,
\end{align*}
\]

Spin Hall conductivity

\[ J^z_y = \sigma^z_{yx} E_x \]

\[ |\sigma^z_{yx}| = \frac{e}{12\pi^2} \left(3k_F^H - k_F^L\right) \quad \text{(semiclassical)} \]

\[ -\frac{e}{12\pi^2} \left(k_F^H + k_F^L\right) \quad \text{(Q correction)} \]

\[ = \frac{1}{6\pi^2} \left(k_F^H - k_F^L\right) \]

No magnetic field required
Applies to Si as well
Intrinsic spin Hall effect in 2 dimensional electron gas (2DEG) (I) (Sinova, Culcer, Niu, Sinitsyn, Jungwirth, and MacDonald, PRL 2004)

Semiconductor heterojunction

FIG. 3. Typical shape and cross section of a GaAs-Al$_x$Ga$_{1-x}$As heterostructure used for Hall-effect measurements. 

≈ triangular quantum well
Intrinsic spin Hall effect in 2DEG (II)

Structure Inversion Asymm (SIA)

Eigen-energies

\[ E_{\lambda}(k) = \frac{\hbar^2 k^2}{2m} + \lambda \alpha k, \quad \lambda = \pm 1 \]

Rashba Hamiltonian (1960)

\[ H = \frac{p^2}{2m} + \frac{\alpha}{\hbar} \vec{\sigma} \times \vec{p} \cdot \hat{z} \]

\( \lambda = (\vec{\sigma} \times \vec{p}) \cdot \hat{z} \) (helicity)

is a good quantum number

- no space inversion symmetry
- invariant under time reversal

Kramer degeneracy
Dynamics of spin under electric perturbation

\[ \frac{d\vec{S}}{dt} = \vec{\dot{S}} \times \vec{B}_{\text{eff}}(\vec{k}) + \gamma \vec{S} \times (\vec{S} \times \vec{B}_{\text{eff}}) \]

\[ \delta k = -eEt \parallel -x \]

\[ \delta B_{\text{eff}} \approx \lambda z \times \delta k \parallel -\lambda y \]

When both bands are filled, spin Hall conductivity:

\[ |\sigma_{yx}| = \frac{e}{8\pi} \]

- independent of \( \alpha \)!
- not so for non-parabolic bands
- only for clean system
- not related to Berry curvature (?)
Effect of disorder on the intrinsic spin Hall effect (I)

- Rashba system with short-range impurities
  - Dimitrova (2004)
  - Raimonde and Schwab (2004)

- Perturbative calculations for other systems
  - If $H(k) = H(-k)$, eg. Luttinger model
    then vertex correction is zero (Murakami, PRB 2004)
  - For systems with
    $$H(\vec{k}) = E_0(\vec{k}) + \sigma_x d_y(\vec{k}) - \sigma_y d_x(\vec{k})$$
    If $\partial E_0 / \partial \vec{k} \propto \vec{d}$, then perfect cancelation (eg. Rashba)
    otherwise $\sigma_s$ remains finite. (quoted from Murakami's talk)

Spin Hall effect is finite in general
Effect of disorder on the spin Hall effect in Rashba system (II)

- Numerical calculations: $\sigma_{\text{SH}}$ robust against weak disorder
  - Nikolic et al, cond-mat/0408693
  - Hankiewicz et al, PRB 2004
  - Sheng et al, PRL 2005
  - Nomura et al, PRB 2005
Spin Hall effect observed (I) (Kato et al, Science 2004)

- Local Kerr effect in strained n-type bulk GaAs/InGaAs, 0.03% polarization

The effect is independent of the direction of strain. Mostly likely extrinsic.
Spin Hall effect observed (II) (Wunderlich et al, to appear PRL 2005)

- spin LED in GaAs 2D hole gas, 1% polarization

might be intrinsic? (Bernevig and Zhang, cond-mat/0411457)
Beyond Rashba (clean): In a III-V quantum well

\[ H = \frac{p^2}{2m^*} + \frac{\alpha}{\hbar} (\sigma_x p_y - \sigma_y p_x) + \frac{\beta}{\hbar} (\sigma_x p_x - \sigma_y p_y) \]

Rashba

Dresselhaus (1955) [001] QW

Effective magnetic field:

BIA \quad SIA \quad BIA=SIA \quad BIA\neq SIA

Ganichev and Prettl, cond-mat/0304266
Rashba-Dresselhaus system in an in-plane magnetic field

\[ H = \frac{p^2}{2m^*} + \frac{\alpha}{\hbar} (\sigma_x p_y - \sigma_y p_x) + \frac{\gamma}{\hbar} (\sigma_x p_x - \sigma_y p_y) + \beta_x \sigma_x + \beta_y \sigma_y \]

Eigen-energies:

\[ E_\lambda(\vec{k}) = E_0(\vec{k}) + \lambda \sqrt{\left(\gamma k_x + \alpha k_y + \beta_x \right)^2 + \left(\alpha k_x + \gamma k_y - \beta_y \right)^2}, \quad \lambda = \pm \]

Distorted Fermi surfaces (generic cases):

Point of degeneracy

\[ \vec{k}_0 = \left(\frac{\gamma \beta_x + \alpha \beta_y}{\alpha^2 - \gamma^2}, -\frac{\alpha \beta_x + \gamma \beta_y}{\alpha^2 - \gamma^2}\right) \]

Parameters: \( \alpha \approx 1 \text{ eV} \cdot \text{A} \) (tunable by gate voltage)

\( \gamma \) of the same order

\( \beta = (g^* / 2) \mu_B B, \quad \mu_B \approx 0.06 \text{ meV/T} \)

\( k_F = \sqrt{2\pi n} \approx 10^2 / \text{A} \) for \( n \approx 10^{11} / \text{cm}^2 \)
Effect of in-plane magnetic field on spin Hall conductivity

Kubo formula

\[ \sigma_{\mu v}^{\eta} = \frac{1}{i\hbar} \sum_{k,\lambda,\lambda'} \frac{f_{k,\lambda} - f_{k,\lambda'}}{\omega_{\lambda\lambda'}(k)} \langle \bar{k},\lambda | j_{\mu}^{\eta} \bar{k},\lambda' \rangle \langle \bar{k},\lambda' | j_{v} \bar{k},\lambda \rangle, \]

\[ j_{\mu}^{\eta} = \frac{\hbar}{4} (\nu_{\mu} \sigma^{\eta} + \sigma^{\eta} \nu_{\mu}); \quad j_{v} = -e v_{\nu} \]

For \( \gamma = 0 \) (pure Rashba)

\( \sigma_{xy}^{z}(\vec{B}) \) could be changed by 100% simply by rotating the magnetic field
Spin Hall conductivity (electron density fixed) \[\sigma_{xy}^x = \sigma_{xy}^y = 0\]

Boundary of plateau \(E(k_\rho) = \mu\)

\[\beta_x^2 + 4\alpha\gamma\beta_x\beta_y + \beta_y^2 = \zeta \frac{(\alpha^2 - \gamma^2)^2}{\alpha^2 + \gamma^2}\]

M.C. Chang, PRB 2005
Acknowledgement: M.F. Yang
Existence of charge Hall effect?

Thouless formula (PRL 1982)

\[ \sigma_{xy} = \frac{e^2}{\hbar} \sum_{\tilde{k} \text{ filled}} \Omega_{\lambda}(\tilde{k}), \]

Berry curvature

\[ \Omega_{\lambda}(\tilde{k}) = i \sum_{\lambda \neq \lambda'} \frac{\langle \tilde{k}, \lambda | v_x | \tilde{k}, \lambda' \rangle \langle \tilde{k}, \lambda' | v_y | \tilde{k}, \lambda \rangle - \langle \tilde{k}, \lambda | v_y | \tilde{k}, \lambda' \rangle \langle \tilde{k}, \lambda' | v_x | \tilde{k}, \lambda \rangle}{\omega_{\lambda\lambda'}^2(\tilde{k})} = 0 \]

Berry phase

\[ \Gamma_{\lambda} = \oint \frac{d\tilde{k}}{i} \langle \tilde{k}, \lambda | \frac{\partial}{\partial \tilde{k}} | \tilde{k}, \lambda \rangle = \begin{cases} -\lambda \pi & \text{for } \alpha^2 > \gamma^2 \\ 0 & \text{for } \alpha^2 = \gamma^2 \\ +\lambda \pi & \text{for } \alpha^2 < \gamma^2 \end{cases} \]

\[ \Rightarrow \Omega_{\lambda}(\tilde{k}) = -\text{sgn}(\alpha^2 - \gamma^2)\lambda \pi \delta(\tilde{k} - \tilde{k}_0) \]

Hall conductivity is zero wherever the chemical potential is

\[ 0 + 0 = 0 \]
\[ (-\pi) + \pi = 0 \]
Issues on the use of SO coupling for spin injection:
(Rashba, cond-mat/0408119)

• spin current is not well defined
  (total spin not conserved)

• no experimental procedure to measure it directly
  (accumulation? Induced electric field?)

• existence of background spin current
  (in noncentrosymmetric materials) Rashba, PRB 2003

• …
Definition of spin current

\[ \frac{\partial}{\partial t} \sigma^\alpha + \nabla \cdot \mathbf{j}^\alpha = \text{Re} \psi^+ \lambda_0 \left[ \tilde{s} \times (\tilde{p} \times \nabla V) \right]^\alpha \psi, \]

where \( \sigma^\alpha \equiv \psi^+ s^\alpha \psi \)

\[ \mathbf{j}^\alpha \equiv \frac{1}{2} \text{Re} \psi^+ \left( s^\alpha \tilde{v} + \tilde{v} s^\alpha \right) \psi \]

Spin flux

\[ \tilde{\mathbf{j}}^\alpha \equiv s^\alpha \tilde{r} + s^\alpha \tilde{r} = \frac{d}{dt} \left( s^\alpha \tilde{r} \right) \]

Advantage:

- source term (RHS) vanishes under certain conditions
- Onsager relations rescued
- would remove most of the spin Hall insulators

<table>
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<tr>
<th>( \sigma_{xy}^s )</th>
<th>( k^3 \sigma_{xy}^s )</th>
<th>Luttinger</th>
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<tbody>
<tr>
<td>( e/8\pi )</td>
<td>(-9e/8\pi )</td>
<td>( e(\gamma_1 + 2\gamma_2)/12\pi^2\gamma_2 ) ( (k_H - k_L) )</td>
</tr>
<tr>
<td>(-e/4\pi )</td>
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<td>(- (e\gamma_1/12\pi^2\gamma_2) ) ( (k_H - k_L) )</td>
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most of the results on spin Hall conductivity need to be re-calculated (?)