Hysteresis effect in $\nu=1$ quantum Hall system under periodic electrostatic modulation

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Ferromagnet in $\nu = 1$ quantum Hall system

Zeeman splitting

exchange interaction

enhanced splitting

\[ \Sigma = (\pi/2)^{1/2} \frac{e^2}{\varepsilon \ell_B} \sqrt{\frac{\pi}{2}} = 125 \text{ K at 4 T} \]

$\rightarrow$ very stable ferromagnetic ordering
Dispersion of spin wave in $\nu = 1$ QHFM

(Bychkov, Iordanskii, and Eliashberg, JETP Lett. 1981

\[
E(k) = \frac{e^2}{\varepsilon \ell_B} \sqrt{\frac{\pi}{2}} \left[ 1 - e^{-k^2 \ell_B^2 / 4} I_0(k^2 \ell_B^2 / 4) \right]
\]
Periodically modulated QHS


Semi-classical picture

Dispersion of spin wave

\[ k = \frac{a}{2l_B^2} \]

\[ a = 15.7l_B, \quad V_0 = 0.58 \, \frac{e^2}{\varepsilon l_B} \]
Possibility of hysteresis

$V_0^*$

same $V_0^*$
1-dim electrostatic modulation

\[ H = \sum_{x, \alpha} \left( -\frac{\alpha}{2} \Delta_z^{(0)} - \mu \right)c_{x, \alpha}^\dagger c_{x, \alpha} + H_M + H_C, \]

\[ x = \text{guiding center coord.} \]

Modulation part (with LLL projection)

\[ H_M = \sum_{x, \alpha} V_0 e^{-\left(\frac{(G_0 t)^2}{4}\right)} \cos \left( G_0 X \right) c_{x, \alpha}^\dagger c_{x, \alpha}, \]

\[ G_0 = 2p/a \]

Electron-electron interaction part

\[ H_C = \frac{1}{2} \sum_{\{X_i\}, \alpha, \beta} \langle X_1, X_2 | \psi | X_3, X_4 \rangle c_{X_1, \alpha}^\dagger c_{X_2, \beta} c_{X_3, \beta} c_{X_4, \alpha} \]
Self-consistent Hartree-Fock calculation

\[ H_{\text{HF}} = \sum_{X, \alpha} (\varepsilon_{X, \alpha} - \mu) c_{X, \alpha}^{\dagger} c_{X, \alpha}, \]

\[ \varepsilon_{X, \alpha} = -\frac{\alpha}{2} \Delta_z^{(0)} + V_0 e^{-(G_0 t)^2/4} \cos(G_0 X) \]

\[ + \sum_{G_j} W_0^\alpha(G_j) e^{-iG_j X} \]

\[ W_0^\alpha(G) = \frac{e^2}{\kappa l} \sum_\beta \left[ H_0(G) - \delta_{\alpha, \beta} X_0(G) \right] \left\langle \rho_0^\beta(-G) \right\rangle \]

\[ H_0(G) = \frac{1}{|G|} e^{-(Gl)^2/2} (1 - \delta_{G,0}), \]

\[ X_0(G) = \sqrt{\frac{\pi}{2}} e^{-(Gl)^2/4} I_0 \left[ \frac{(Gl)^2}{4} \right], \]

\[ \left\langle \rho_0^\beta(-G) \right\rangle = \frac{1}{N_\varphi} \sum_X e^{iG X} \left\langle c_{X, \beta}^{\dagger} c_{X, \beta} \right\rangle, \]
Self-consistent band structure ($\gamma=1$)

(energy in units of $e^2/\text{el}$, period in units of $l$)

$V_0=0.5$, $a=10$

$V_0=0.6$, $a=10$
Hysteresis of spin polarization

\[ a = 10 \]

\[ a = 15.7 \]

\[ a = 10 \]
Changing the tilting angle of the magnetic field

Spin polarization (%)

1/cos(\(\phi\))

kT=0, kT=0.01
Hysteresis in a \( \nu = 1 \) modulated QHS


- Range of parameters (for GaAs with \( n = 3 \times 10^{11} \text{cm}^{-2} \), \( B = 12.4 \text{T}, \ l_B = 72.8 \text{A}, \ e^2/\epsilon l_B = 16 \text{meV} \))
  - Modulation period \( a = 15 \ l_B \approx 110 \text{nm} \)
  - Modulation amplitude \( V_0 = 0.5 \ e^2/\epsilon l_B \approx 8 \text{meV} \)

- What have been left out in this calculation
  - Landau level mixing (\( \hbar \omega_c = 21.4 \text{meV} \))
  - Screening and higher order correlations