Collective excitations
in
integer quantum Hall system

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Plasma excitation in 3DEG, NO magnetic field
$r_s=4$, sodium

$\omega_p=(4\pi\rho e^2/m)^{1/2}$

2DEG, NO magnetic field ? gapless excitation
Experimental observation

Al: 10.3 eV + 15.3 eV

10.3 eV due to surface plasmons
2DEG in a strong magnetic field
emergence of Landau levels

magnetoplasma modes: inter-LL excitations
can be Integer filling / non-integer filling
Inter-LL excitation/intra-LL excitation
Charge excitation / spin excitation
Fractional case, intra-LL charge excitations

Finite energy gap stabilizes the FQHL

Magnetoroton mode similar to rotons in superfluid

Indication of Wigner crystal instability?

(Girvin/MacDonald/Platzman 1985 PRL)
Integer case

classification of collective excitations

Intra-LL:

Spin flip excitations only

\[ 0? \quad 0? \quad 0? \]

Zeeman splitting

energy dispersion of spin wave

\[ E = (\pi/2)^{1/2} e^2/\varepsilon l \]
spin wave as coherent superposition of e-h pairs

\[ E_{\infty} = (\pi/2)^{1/2} \frac{e^2}{\epsilon l} \]

= the energy to flip the spin of an electron and take it far away from the hole

= the exchange energy of an electron in the LLL
Excitation energy

**Single particle:**
- cyclotron energy
- Zeeman energy

**Manybody effect:**
- exchange interaction
- polarization and screening
- excitonic attraction … etc

Methods of calculation

**Equation of motion method**

➡️ **Diagrammatic expansion**

- The pole of response function
- energy spectrum of collective excitation
- Time dependent HFA
Calculation of response function

Renormalized electron propagator

Bethe-Salpeter equation

ladder

bubble
Response function

\[ \chi(k, i\omega) \equiv -\int_0^\infty d\tau e^{i\omega\tau} \langle T_\tau \hat{\rho}(\vec{k}, \tau) \hat{\rho}(-\vec{k}, 0) \rangle. \]

\[ = \sum_{n_\alpha} \sum_{n_\beta} \langle n_\alpha, m_\alpha | e^{i\vec{k} \cdot \vec{r}} | n_\beta, m_\beta \rangle D_{n_\alpha n_\beta}(i\omega) \Gamma_{n_\alpha n_\beta}(k, i\omega) \]

inter/intra-LL indices

free 2-particle propagator

\[ D_{n_\alpha' n_\beta'}(i\omega) \equiv \int \frac{d\omega'}{2\pi} G_{n_\alpha'}(i\omega' - i\omega) G_{n_\beta'}(i\omega') \]

BS equation for the vertex

\[ \Gamma_{n_\alpha n_\beta}(k, i\omega) = \frac{i^{n_\alpha n_\beta}}{2\pi} e^{-k^2/2} g_{n_\alpha n_\beta}(-k) \]

\[ - \sum_{n_\alpha' n_\beta'} \left[ V_{n_\alpha n_\beta'}(k) - U_{n_\alpha n_\beta'}(k) \right] D_{n_\alpha' n_\beta'}(i\omega) \Gamma_{n_\alpha' n_\beta'}(k, i\omega) \]

ladder bubble
matrix form of the BS equation

\[
\sum_{n_{\alpha}, n_{\beta}} \left[ \delta_{n_{\alpha}, n_{\alpha}'} \delta_{n_{\beta}, n_{\beta}'} D_{n_{\alpha}, n_{\beta}}^{-1}(i\omega) + V_{n_{\alpha}, n_{\beta}'}(k) - U_{n_{\alpha}, n_{\beta}'}(k) \right] \Gamma_{n_{\alpha}, n_{\beta}'}(k, i\omega) = \frac{i^{n_{\alpha} - n_{\alpha}'} e^{-k^2/2g_{n_{\alpha}, n_{\beta}}(-\bar{k})}}{2\pi}.
\]

self-energy

bubble (Coulomb local field)

\[
U_{n_{\alpha}, n_{\beta}'}(k) = i^{n_{\alpha} - n_{\alpha}'} e^{-k^2/2g_{n_{\alpha}, n_{\beta}}(-\bar{k})} g_{n_{\alpha}, n_{\beta}'}(-\bar{k})
\]

ladder (exchange local field)

\[
V_{n_{\alpha}, n_{\beta}'}(k) = 2\pi \int d^2 r_1' \int d^2 r_2' \Phi_{n_{\alpha}, n_{\beta}}(r_1', r_2') v(r_1' - r_2') \Phi_{n_{\alpha}', n_{\beta}'}(r_1', r_2')
\]

exciton wave function
poles of the response function
= eigenvalues of the matrix eq.

consider \( n=1 \), neglect inter-mode coupling
one of the diagonal element

\[
M_{2,1}^{2,1}(k, \omega) = \omega - \omega_c - \frac{e^2}{\epsilon_l_0} (\tilde{\Sigma}_n^{1} = 0 - \Sigma_{n=1}) - \frac{e^2}{\epsilon_l_0} \tilde{U}_{2,1}^{2,1}(k) + \frac{e^2}{\epsilon_l_0} \tilde{V}_{2,1}^{2,1}(k) = 0.
\]

transition energy =
cyclotron energy + self energy correction (k independent)
+ Coulomb local field correction
+ exchange local field correction
General behavior of the manybody corrections

\[ k \to 0 \]

\[
\begin{align*}
\text{bubble correction} & \to 0 \\
\text{ladder correction} & \to \text{self-energy correction} \\
\text{Kohn theorem: the cyclotron energy gap is NOT renormalized by many-body correction}
\end{align*}
\]

\[ k \to \infty \]

\[
\begin{align*}
\text{bubble correction} & \to 0 \quad \text{exponentially} \\
\text{ladder correction} & \to 0 \quad \text{algebraically}
\end{align*}
\]

Coulomb interaction prohibits spin-flip

\[
\to \text{no bubble correction for spin-flip excitations} \\
\text{energy dispersion determined solely by} \\
\text{ladder correction}
\]
roton feature (spin triplet roton)