SUSY breaking by metastable states

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Symmetry has been an obsession of modern physics since Einstein!

Through this obsession, we indulged ourselves in talking and boasting about the beauty of Physics!
The **irony** is that artists, who are supposed to know beauty better than we do, has actually ...... moved on, to ....... breaking the symmetry.

Maybe it is also time for us to appreciate the thrust and the ecstasy of breaking a Grand Symmetry.
## Breaking of Supersymmetry

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SUSY breaking determines the masses of the superpartners and in principle is vital to electroweak symmetry breaking.

SUSY breaking is unlike any other symmetry breaking, just as SUSY is unlike any other symmetry.

It is really a Fearful Symmetry! Tony Zee

Tyger! Tyger! burning bright
In the forests of the night
What immortal hand or eye
Could frame thy fearful symmetry!

William Blake

It shackles you with a rigor that smells more mathematics than physics!

Whether it is a model of **elegance and beauty** in physics, or a **bad dream** you hope never realized, you decide.
SUSY Semi-Primer

The only non-trivial extension of symmetry in quantum field theory beyond Poincare symmetry and internal symmetry. It consists of symmetry with fermionic (anticommuting, spinorial) generators.

This *supersymmetry* identifies bosons and fermions!

\[ Q_\alpha |\text{bos}\rangle = |\text{ferm}\rangle_\alpha ; \quad Q_\alpha |\text{ferm}\rangle_\alpha = |\text{bos}\rangle \]

SUSY algebra

\[
\begin{align*}
[Q_\alpha, P^\rho] &= 0 \\
\{Q_\alpha, \bar{Q}_\beta\} &= 2(\sigma^\rho)_{\alpha\beta} P_\rho \\
[M^{\rho\sigma}, Q_\alpha] &= -i(\sigma^{\rho\sigma})_{\alpha}^\beta Q_\beta \\
\{Q_\alpha, Q_\beta\} &= \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0
\end{align*}
\]

Every Boson comes with a Fermion partner, which behaves identically, and vice versa.
What’s the motivation?

Quadratic Divergence

$$\Delta m^2 \propto \int d^4k \left( \frac{1}{k^2 - M^2} \right) \approx \Lambda^2$$

$$m_H^2 \approx m_0^2 + c\Lambda^2$$

We need a fine-tuning to get a small Higgs mass. $$m_H \ll \Lambda$$
SUSY forced the quadratic divergencies in the two loop diagrams to cancel.

Scalar self interaction is related to Yukawa coupling.
Coupling Unification

Forces Merge at High Energies

Particles

Supersymmetric "shadow" particles
String Theory

Extra Space Dimension

An acrobat can only move in one dimension along a rope...

...but a flea can move in two dimensions.
Chiral Superfield

It’s customary to organize SUSY multiplets by superfields: functions of $x^\mu$ and an imaginary superspace fermionic coordinates $\theta$.

\[
\begin{align*}
\{\theta_\alpha, \theta_\beta\} &= \{\theta_\alpha, \bar{\theta}_\beta\} = \{\bar{\theta}_\dot{\alpha}, \bar{\theta}_\dot{\beta}\} = 0 \\
\alpha, \beta &= 1, 2
\end{align*}
\]

Superfield can be expanded in powers of $\theta$. The expansion terminates soon. The components are various ordinary fields in a super-multiplet.

You put SUSY invariant constraints on superfield to get irreducible reps.

Chiral Superfield combines a scalar $\varphi$ and a left-handed Weyl spinor $\psi$

\[
\phi(x, \theta, \bar{\theta}) = \varphi(x) + \sqrt{2}\theta \psi(x) - i\theta \sigma^\mu \bar{\theta} \partial_\mu \varphi(x) + \frac{i}{\sqrt{2}} (\theta \bar{\theta})(\partial_\mu \psi(x) \sigma^\mu \bar{\theta})
\]

\[
- \frac{1}{4} (\theta \bar{\theta})(\bar{\theta} \bar{\theta}) \partial_\mu \partial^\mu \varphi(x) - (\theta \bar{\theta}) F(x)
\]

$F(x)$ is a auxiliary field and can be solved in terms other fields by EOM.
SUSY transformations can be realized as translations in the superspace $\theta$ (plus a translation in space time $x$).

Given a function of superfield $\Phi : W(\Phi)$

SUSY invariants can be obtained by integrating $W(\Phi)$ over $\theta$.

Funnily, the integration over $\theta$ acts just like differentiation.

$$\int d\theta^2 W(\phi) \supset -\frac{1}{2} \left( \frac{\partial^2 W(\varphi_i)}{\partial \varphi_i \partial \varphi_j} \right) \psi_i \psi_j - \frac{1}{2} \left( \frac{\partial^2 W^+(\varphi_i)}{\partial \varphi_i^\dagger \partial \varphi_j^\dagger} \right) \bar{\psi}_i \bar{\psi}_j$$

**Wess-Zumino Model**

General renormalizable SUSY model of Chiral superfields.

$$W(\phi) = \frac{1}{2} m\phi^2 + \frac{1}{3!} y\phi^3$$

Masses of fermions

Yukawa coupling between bosons and fermions

$W(\Phi)$ controls interactions and masses. **Superpotential**
Kinetic Energy term comes from: \[ \int d^4 \theta (\phi^+ \phi) \]

\[ \mathcal{L}_{\text{WZ}} = (\partial_\mu \phi_i) (\partial^\mu \phi_i)^\dagger + \frac{i}{2} \psi_i \sigma^\mu (\partial_\mu \bar{\psi}_i) - \frac{i}{2} (\partial_\mu \psi_i) \sigma^\mu \bar{\psi}_i - \sum_i |F_i|^2 \]

As a bonus, it also gives rise to scalar interactions.

Equation of Motion solves $F$ completely:

\[ F_i = \frac{\partial W(\phi_i)}{\partial \phi_i} \]

For Wess-Zumino

\[ F_i \approx m \phi + \frac{1}{2} y \phi^2 \]

\[ V = \sum_i |F_i|^2 = \sum_i \left| \frac{\partial W(\phi_i)}{\partial \phi_i} \right|^2 \]

\[ V \supset m^2 \phi^2 + \frac{1}{4} y^2 \phi^4 \]

Scalar mass and Fermion mass are degenerate.
Scalar self interaction is related to Yukawa coupling.
It is this scalar potential that will determine the vacuum or vacua.
Vector Superfield

Vector Superfield combines a vector $v$ and a left-handed Weyl spinor $\lambda$.

In Wess-Zumino gauge:

$$V_{WZ}(x, \theta, \bar{\theta}) = \theta \sigma^\mu \bar{\theta} v_\mu(x) + i(\theta \theta) \bar{\theta} \lambda(x) - i(\bar{\theta} \bar{\theta}) \theta \lambda(x) + \frac{1}{2} (\theta \theta)(\bar{\theta} \bar{\theta}) D(x)$$

$D(x)$ is a auxiliary field and can be solved in terms other fields by EOM.
The most general supersymmetric Lagrangian of a vector superfield

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a (F^a)^{\mu\nu} + \frac{i}{2} \lambda^a \sigma^{\mu} (D_\mu \bar{\lambda})^a - \frac{i}{2} (D_\mu \lambda)^a \sigma^{\mu} \bar{\lambda}^a + \frac{1}{2} \sum_a D_a^2$$

In the presence of a chiral field, we can solve the auxiliary $D$

$$D^a = g \varphi_i^+ T_{ij}^a \varphi_j$$

It gives a scalar potential

$$V = \frac{1}{2} \sum_a D_a^2$$
Under SUSY, F and D change by a total divergence.

\[ \delta_\xi F(x) \sim i\xi \bar{\sigma}^\mu \partial_\mu \psi(x) \]

For Abelian gauge theory, the D term of a vector superfield is both gauge invariant and supersymmetric.

We can add a D-term to the Lagrangian:

\[ \mathcal{L}_{\text{FI}} = kD \]

Fayet-Iliopoulos Term

In the presence of matter, we can solve the auxiliary D

\[ D = g_i \varphi_i^+ \varphi_i + k \]
Put everything together:

\[
\mathcal{L} = (D_\mu \varphi)_i \,(D^\mu \varphi)_i + \frac{i}{2} \psi_i \sigma^\mu (D_\mu \bar{\psi})_i - \frac{i}{2} (D_\mu \psi)_i \sigma^\mu \bar{\psi}_i \\
- \frac{1}{4} F_{\mu\nu}^a (F^a)^{\mu\nu} + \frac{i}{2} \lambda^a \sigma^\mu (D_\mu \bar{\lambda})^a - \frac{i}{2} (D_\mu \lambda)^a \sigma^\mu \bar{\lambda}^a \\
- \sqrt{2} i g \bar{\psi}_i \bar{\lambda}^a T^a_{ij} \varphi_j + \sqrt{2} i g \varphi_i \, T^a_{ij} \psi_j \lambda^a \\
- \frac{1}{2} \frac{\partial^2 W}{\partial \varphi_i \partial \varphi_j} \psi_i \psi_j - \frac{1}{2} \frac{\partial^2 W^\dagger}{\partial \varphi_i^\dagger \partial \varphi_j^\dagger} \bar{\psi}_i \bar{\psi}_j - V(\varphi_i, \varphi_j^\dagger)
\]

with the all (and the only) important scalar potential:

\[
V(\varphi_i, \varphi_j^\dagger) = F_i^\dagger F_i + \frac{1}{2} (D^a)^2 = \sum_i \left| \frac{\partial W}{\partial \varphi_i} \right|^2 + \frac{1}{2} \sum_a (g \, \varphi_i \, T^a_{ij} \, \varphi_j + k^a)^2
\]
SUSY vacuum

\[ Q_{\alpha} |0\rangle = 0 \]
\[ \bar{Q}_{\bar{\alpha}} |0\rangle = (Q_{\alpha})^\dagger |0\rangle = 0 \]

\[ \{ Q_{\alpha}, \bar{Q}_{\bar{\beta}} \} = 2 (\sigma^\rho)_{\alpha\bar{\beta}} P_\rho \]

\[ Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2 = 4 g^{0\mu} P_\mu = 4 P^0 \]

\[ \langle 0 | H | 0 \rangle = 0 \]

SUSY ground state has zero energy!

\[ F_i = 0, \quad D = 0 \]

\[ V(\varphi_i, \varphi_j^\dagger) = F_i^\dagger F_i + \frac{1}{2} (D^a)^2 \]
Spontaneous SUSY breaking

\[ Q_\alpha |0\rangle \neq 0 \]

\[ \langle 0 | H | 0 \rangle > 0 \]

Ground state energy is the order parameter.

\[ F_i \neq 0, \quad \text{or} \quad D \neq 0 \]

SUSY will be broken if all the auxiliary fields cannot be made zero simultaneously!
Another way to see it:

Spontaneous SUSY Breaking implies that under SUSY transformation:

\[ \langle 0 | \delta(\text{field}) | 0 \rangle \neq 0 \]

The transformation of components of a chiral superfield is

\[
\begin{align*}
\delta_\xi \tilde{\phi}(x) & \sim \xi \psi(x) \\
\delta_\xi \psi(x) & \sim i \sigma^\mu \bar{\xi} \partial_\mu \tilde{\phi}(x) + \xi F(x) \\
\delta_\xi F(x) & \sim i \bar{\xi} \sigma^\mu \partial_\mu \psi(x)
\end{align*}
\]

The only possible Lorentz invariant non-zero VEV at r.h.s. is that of \( F \).

\[ \langle F \rangle \neq 0 \]

Similar for vector superfield:

\[ \langle 0 | \delta_\xi \lambda(x) | 0 \rangle \propto \langle 0 | D | 0 \rangle \neq 0 \quad \langle D \rangle \neq 0 \]
There is a mass relation for the fields spontaneously breaking SUSY.

\[ \sum m^2_{\text{Bosons}} = \sum m^2_{\text{Fermions}} \]

Spontaneous SUSY breaking can’t be generated in SM or one of the squarks will be too light.

The mediation control the phenomenology. It could be gravity, gauge interaction, anomaly etc.
D-type SUSY breaking

Fayet-Illiopoulos mechanism (1974)

Assuming an Abelian Gauge Theory:

Two Chiral Superfield \( Q, \overline{Q} \) with opposite charge +1, -1

Introduce a non-zero mass for \( Q \) :

\[
W = mQ\overline{Q}
\]

and a non-zero FI term \( k \) for the Abelian gauge theory.

\[
D = Q^+Q - \overline{Q}^+\overline{Q} + k
\]

The scalar potential:

\[
V = |mQ|^2 + |m\overline{Q}|^2 + \frac{1}{8}|Q^+Q - \overline{Q}^+\overline{Q} + k|^2
\]

If \( m \) is large, the minimum is at \( Q = \overline{Q} = 0 \) U(1) gauge symmetry is unbroken.

At this vacuum:

\[
V = \frac{1}{8}k^2 \neq 0 \quad \text{SUSY is broken by a non-zero D term.}
\]
If mass is small \( m^2 < k \)

the minimum of \( V \) is at \( Q = 0, \, \bar{Q} = v \) with \( v^2 = m^2 - k \)

SUSY is broken by non-zero D term and F terms.

U(1) gauge symmetry is now broken

We expect a massive gauge boson and massless goldstino (mixture of gaugino and fermionic component of Q) of SUSY breaking.
F-type SUSY breaking

O’Raifeartaigh Type Model (OR)

There are as many F-terms as superfield.

In general, there will be a solution for all the F-terms to vanish unless the superpotential is special-designed.

Three chiral superfields: $X, \phi_2, \phi_1$

\[ W = X g_1(\phi_1) + \phi_2 g_2(\phi_1) \]

$X, \Phi_2$ don’t talk to each other.

\[ -F_X = \frac{\partial W}{\partial X} = g_1(\phi_1) \]
\[ -F_{\phi_2} = \frac{\partial W}{\partial \phi_2} = g_2(\phi_1) \]

Generically we can’t make both vanish.

SUSY is borken.
O’Raifeartaigh Model (OR) (1975)

\[ g_1(\phi) = \frac{1}{2} h \phi^2 + f, \quad g_2(\phi) = m\phi \]

\[ W = \frac{1}{2} h X \phi_1^2 + m\phi_1 \phi_2 + f X \]

\[ -F_X = \frac{\partial W}{\partial X} = g_1(\phi_1) \quad -F_{\phi_2} = \frac{\partial W}{\partial \phi_2} = g_2(\phi_1) \]

These two auxiliary fields are two distinct functions of just one field. They can’t be zero at the same time. SUSY is broken.
The vacuum (vacua) of OR model
minimum conditions

$$\frac{\partial V}{\partial \phi_i} = 0$$

$$V = |F_x|^2 + |F_{\phi_2}|^2 + |F_{\phi_1}|^2$$

$$= \left| \frac{1}{2} h\phi_1^2 + f \right|^2 + |m\phi_1|^2 + |hX\phi_1 + m\phi_2|^2$$

$$\left. \frac{\partial V}{\partial \phi_1} \right|_{\phi_1=0} = \left( \frac{1}{2} h\phi_1^2 + f \right) \cdot h\phi_1 + m^2\phi_1 = 0 \quad \Rightarrow \quad \phi_1 = 0$$

$$\frac{\partial V}{\partial X} , \frac{\partial V}{\partial \phi_2} \propto F_{\phi_1} = 0$$

$$- F_{\phi_1} = hX\phi_1 + m\phi_2 |_{\phi_1=0} = 0 \quad \Rightarrow \quad \phi_2 = 0, \ X \ \text{arbitrary}$$
\[ V = \left| \frac{1}{2} h \phi_1^2 + f \right|^2 + \left| m \phi_1 \right|^2 + \left| hX \phi_1 + m \phi_2 \right|^2 \]

\[ \phi_1 = 0 \quad \phi_2 = 0, \quad X \text{ arbitrary} \]

\[ V_{min} = \left| F_X \right|^2 = \left| f \right|^2 \]

SUSY is broken by a one complex dimensional space of degenerate non-SUSY vacua.

**Pseudo-Moduli Space of Vacua**

The degeneracy will be lifted by one-loop effective potential:

\[ V_{eff}(X) = \begin{cases} 
V_0 + m_X^2 |X|^2 + \mathcal{O}(|X|^4) & X \approx 0 \\
\left| f \right|^2 \left( 1 + \gamma_X \left( \log \left| \frac{hX}{M_{cutoff}} \right|^2 + \frac{3}{2} \right) + \mathcal{O} \left( h^4, \frac{\log |X|}{|X|^4} \right) \right) & X \to \infty 
\end{cases} \]

The minimum vacuum is at \[ |X| = 0 \]
At this vacuum \( |X| = 0 \)

We can calculate the masses of scalars and fermions.

\[
\begin{align*}
    m_s &= 0, 0, m^2, m^2, m^2 - hf, m^2 + hf \\
    m_f &= 0, m, m
\end{align*}
\]

Modulus Fields

SUSY breaking massless Goldstino
Dynamical SUSY Breaking

Both FI and OR model contains scales $k, f$ that are put in by hand. These scales generate SUSY breaking scale.

It is natural that we (with Witten) prefer a non-perturbative dynamic SUSY breaking mechanism where scale are generated by Dimensional Transmutation, just like $\Lambda$ in QCD.

This scale can be naturally small compared to cutoff scale:

$$M_s \approx e^{-\frac{8\pi^2}{b g_0^2}} M_{\text{cutoff}} << M_{\text{cutoff}}$$

$$\frac{\alpha(M_{\text{cutoff}})}{\alpha(M_{\text{cutoff}})} \approx -c \cdot \ln \frac{M_{\text{SUSY}}}{M_{\text{cutoff}}}$$

$$\alpha(M_{\text{SUSY}}) \approx 1$$

On the other hand, FI and OR seems to emerge as the low energy effective theory of a dynamical SUSY breaking mechanism.
$U(1)_R$ symmetry

O’Raifeartaigh Model (OR) has an unbroken $U(1)_R$ symmetry. This is a serious problem.

$U(1)_R : \theta \text{ charge } 1$

Boson and its superpartner have opposite charges.

Superpotential $W$ needs to be charge 2 to preserve $U(1)_R$

The R charges of the three chiral superfields:

$R(X) = R(\phi_2) = 2, \ R(\phi_1) = 0$

$W = Xg_1(\phi_1) + \phi_2g_2(\phi_1)$ is charge 2.

An unbroken $U(1)_R$ symmetry will prohibit Majorana gaugino masses and render model-building very difficult.
Generically it can be proven:

\[ U(1)_R \iff \text{SUSY Breaking Vacuum} \]

\[ U(1)_R \iff \text{SUSY Vacuum} \]

The issue of R symmetry is just one among several other strict constraints preventing SUSY breaking to appear easily.
Witten Index (1982)

\[
\text{Tr } (-1)^F = \sum_E n_B(E) - n_F(E) = n_B(0) - n_F(0)
\]

Every bosonic state of non-vanishing energy pair with a fermionic state.

If the Witten index is non-zero, there must be a state with zero energy and hence SUSY is unbroken!

\[
\text{Tr } (-1)^F \neq 0 \quad \Rightarrow \quad \text{SUSY is unbroken.}
\]

(The reverse is not true.)

Witten index is invariant under changes of the Hamiltonian that do not change the far away behavior of the potential!
It is possible to calculate Witten index at weak coupling while applying the conclusion to strong coupling.

Witten index is non-zero for pure SUSY Yang-Mills theory.

Gauge theories with massive vector-like matter, which flows to pure Yang-Mills at low energy, will also have a non-zero Witten indices.

For these two theories, SUSY is unbroken.

SUSY breaking seems to be a rather non-generic phenomenon.

“The issue of SUSY breaking has a topological nature: it depends only on asymptotics and global properties of the theory.”
Dynamical SUSY Breaking in Meta-Stable Vacua

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We point out that new model building avenues are opened up by abandoning the prejudice that models of dynamical supersymmetry breaking must have \textit{no} supersymmetric vacua. This prejudice is unnecessary, because it is a phenomenologically viable possibility that we happen to reside in a very long lived, false vacuum, and that there is a supersymmetric vacuum elsewhere in field space. Meta-stable supersymmetry breaking vacua
Modify OR model by adding a small mass term for $\phi_2$ (Deformation)

$$W = \frac{1}{2} hX \phi_1^2 + m\phi_1 \phi_2 + fX + \frac{1}{2} \epsilon m\phi_2^2 \quad \epsilon \ll 1$$

$$-F_x = \frac{1}{2} h\phi_1^2 + f \quad -F_\phi_2 = m\phi_1 + \epsilon m\phi_2 \quad -F_\phi_1 = hX\phi_1 + m\phi_2$$

Now 3 equations for 3 unknowns, a solution can be found:

$$\langle \phi_1 \rangle_{susy} = \pm \sqrt{-2f/h}, \quad \langle \phi_2 \rangle_{susy} = \mp \frac{1}{\epsilon} \sqrt{-2f/h}, \quad \langle X \rangle_{susy} = \frac{m}{h\epsilon}$$

This is a SUSY vacuum.

$U(1)_R$ has been broken by the small mass term as expected.

$$R(X) = R(\phi_2) = 2, \quad R(\phi_1) = 0$$
For small mass, the potential near the previous SUSY breaking minimum is not greatly modified.

\[ \phi_1 = 0, \phi_2 = 0, X = 0 \]

It will still be locally stable. Hence it becomes a metastable vacuum!

The universe can live in the metastable vacua with SUSY broken. Globally, there is a SUSY vacuum, hence ensuring \( U(1)_R \) is broken.

Using metastable state to break SUSY while keeping a SUSY ground state could help evade a lot of constraints such as Witten Index.

“Breaking SUSY by long-living metastable states is generic.”

Intrilligator, Seiberg, Shih (2006)
At A,
\[
\langle \phi_1 \rangle_{susy} = \pm \sqrt{-2f/h}, \quad \langle \phi_2 \rangle_{susy} = \mp \frac{1}{\varepsilon} \sqrt{-2f/h}, \quad \langle X \rangle_{susy} = \frac{m}{h\varepsilon}.
\]

At B, \( \phi_1 = 0, \phi_2 = 0, X = 0 \)  Metastable state breaking SUSY

As \( \varepsilon \) becomes smaller, SUSY vacuum A will be pushed further and further, diminishing the tunneling rate as small as you like, until disappear into infinity at \( \varepsilon = 0 \).

With a SUSY vacuum, R symmetry is explicitly broken.
Building a nest at tree level: Classical metastability and nontrivial vacuum structure in supersymmetric field theories

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It is becoming increasingly clear that metastable vacua may play a prominent role in supersymmetry breaking. To date, however, this idea has been realized complicates the analysis of metastability. In this paper, metastable vacua occur classically, i.e., at tree level, and by both $D$ terms and $F$ terms. All relevant dynamics is such which energies and lifetimes can be performed explicitly. Moreover, we rise to multiple nonsupersymmetric vacua which are deg models therefore suggests that they can provide a rich anatomy in supersymmetric field theories. Our results may also have and the string landscape.

Achieve a SUSY breaking metastable state perturbatively (tree level calculation).
The recipe is to put a Wess-Zumino and a Fayet-Iliopoulos together!

Three Chiral Superfields \( (\Phi_1, \Phi_2, \Phi_3) \)

A Wess-Zumino Superpotential \( W = \lambda \Phi_1 \Phi_2 \Phi_3 \)

Two Abelian U(1) with FI terms: \( U(1)_a, \xi_a, g_a \) \( U(1)_b, \xi_b, g_b \)

Massive vector matter with opposite charges in FI

\[ (\Phi_4, \Phi_5) \quad W = m\Phi_4 \Phi_5 \]

Together, you also need to assign appropriate charges to \( (\Phi_1, \Phi_2, \Phi_3) \)
\[ W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_4 \Phi_5 \]

**TABLE I.** The field content and charge assignment for the model under consideration.

<table>
<thead>
<tr>
<th>Field</th>
<th>$U(1)_a$</th>
<th>$U(1)_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1$</td>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>+1</td>
<td>−1</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>0</td>
<td>+1</td>
</tr>
<tr>
<td>$\Phi_4$</td>
<td>+1</td>
<td>+1</td>
</tr>
<tr>
<td>$\Phi_5$</td>
<td>−1</td>
<td>−1</td>
</tr>
</tbody>
</table>
The extrema are determined by the conditions:
\[
\frac{\partial V}{\partial \phi_i} = 0 \quad (i = 1, \ldots, 5)
\]

Solutions is a local minimum if the following mass matrix contains only positive eigenvalues! This is the hard part!

\[
\mathcal{M}^2 \equiv \left(\begin{array}{cc}
\frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} \\
\frac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i \partial \phi_j}
\end{array}\right)
\]
As an example, choose \((\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)\)

<table>
<thead>
<tr>
<th>Label</th>
<th>((v_1, v_3, v_5))</th>
<th>(V)</th>
<th>Stability</th>
<th>SUSY</th>
<th>(R) symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>((\sqrt{5}, 0, 0))</td>
<td>0</td>
<td>Stable</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>((0, 2, 2))</td>
<td>9/2</td>
<td>Metastable</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>C</td>
<td>((\sqrt{3}/2, \sqrt{7}/2, \sqrt{5}/2))</td>
<td>45/8</td>
<td>Unstable</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>D</td>
<td>((0, 0, \sqrt{2}))</td>
<td>17/2</td>
<td>Unstable</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>E</td>
<td>((0, 0, 0))</td>
<td>25/2</td>
<td>Unstable</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

A is a **SUSY true vacuum**, with \(R\) symmetry and a \(U(1)\) gauge symmetry.

B is a **SUSY breaking metastable local minimum**, with broken \(R\) symmetry and broken \(U(1)\) gauge symmetry.

<table>
<thead>
<tr>
<th>(R) symmetry</th>
<th>Gauge Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>(U(1)_b)</td>
</tr>
<tr>
<td>No</td>
<td>None</td>
</tr>
<tr>
<td>No</td>
<td>None</td>
</tr>
<tr>
<td>No</td>
<td>(U(1)_{a-b})</td>
</tr>
<tr>
<td>Yes</td>
<td>(U(1)_a \times U(1)_b)</td>
</tr>
</tbody>
</table>
Lifetime of the metastable state

The metastable state tunnels to the true vacuum through instanton transition.

The decay rate per unit volume is

$$\frac{\Gamma_{\text{inst}}}{\text{Vol}} = Ae^{-B}$$

$B$ is calculated from the distances in field space between barrier top (C) and metastable state (B) or true vacuum (A):

$$\Delta \phi_{\perp}$$

and the potential differences between similar combinations:

$$\Delta V_{\perp}$$
Under certain conditions:

\[ B = \frac{32\pi^2}{3} \frac{1 + c}{(\sqrt{1 + c} - 1)^4} \left( \frac{\Delta \phi^4_+}{\Delta V_+} \right) \]

In the example:

\((\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)\)

\[ B \approx 1300 \]

This is large enough for the metastable lifetime to exceed the age of the universe.
Our Model I: To simplify Dienes & Thomas Model

We throw away $U(1)_b$

$$W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_4 \Phi_5$$

As an example:

<table>
<thead>
<tr>
<th>Field</th>
<th>$U(1)_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>$+3$</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$\Phi_4$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$\Phi_5$</td>
<td>$-2$</td>
</tr>
</tbody>
</table>

$$V = |\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2 + |\phi_4|^2 + |\phi_5|^2 + \frac{1}{2}(5 - |\phi_1|^2 + 3|\phi_2|^2 - 2|\phi_3|^2 + 2|\phi_4|^2 - 2|\phi_5|^2)^2$$

We again find structures of minima:

<table>
<thead>
<tr>
<th>Label</th>
<th>$(\phi_1, \phi_2, \phi_3, \phi_4, \phi_5)$</th>
<th>$V$</th>
<th>Stability</th>
<th>SUSY</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$(\sqrt{5}, 0, 0, 0, 0)$</td>
<td>0</td>
<td>Stable</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>$(1, 0, \sqrt{2}/2, 0, \sqrt{2}/2)$</td>
<td>$\frac{15}{5}$</td>
<td>Metastable</td>
<td>No</td>
</tr>
<tr>
<td>C</td>
<td>$(0, 0, 0, 0, \frac{3}{2})$</td>
<td>$\frac{19}{8}$</td>
<td>Unstable</td>
<td>No</td>
</tr>
</tbody>
</table>
The metastable minimum is a bit shallow!
It will decrease the lifetime of B, but it turns out still OK.
Our Model II: we simplify our Model I even further:

We throw away one superfield and $U(1)_b$

$$W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_1 \Phi_4$$

As an example:

<table>
<thead>
<tr>
<th>Field</th>
<th>$U(1)_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>$+3$</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$\Phi_4$</td>
<td>$+1$</td>
</tr>
</tbody>
</table>

$$V = |\phi_1|^2 |\phi_2|^2 + |\phi_1|^2 |\phi_3|^2 + |\phi_2|^2 |\phi_3|^2 + |\phi_1|^2 + |\phi_4|^2$$

$$\quad + \frac{1}{2} (10 - |\phi_1|^2 + 3|\phi_2|^2 - 2|\phi_3|^2 + |\phi_4|^2)^2$$

We again find structures of minima:

<table>
<thead>
<tr>
<th>Label</th>
<th>$(\phi_1, \phi_2, \phi_3, \phi_4)$</th>
<th>$V$</th>
<th>Stability</th>
<th>SUSY</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$(0, 0, \sqrt{5}, 0)$</td>
<td>0</td>
<td>Stable</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>$(3, 0, 0, 0)$</td>
<td>$\frac{19}{2}$</td>
<td>Metastable</td>
<td>No</td>
</tr>
<tr>
<td>C</td>
<td>$(\sqrt{\frac{24}{5}}, 0, \sqrt{\frac{7}{5}}, 0)$</td>
<td>$\frac{72}{5}$</td>
<td>Unstable</td>
<td>No</td>
</tr>
</tbody>
</table>
We have constructed a model which is one field and one Abelian Gauge symmetry short of the Dienes Thomas Model, but achieves the same ground state structure.

The metastable local minimum is about as deep in DT and the distance between A,B is also about the same order. We expect the lifetime of metastable to exceed the age of the universe.
How about a model without mass terms?

\[ W = \lambda_1 \Phi_1^2 \Phi_2 + \lambda_2 \Phi_1 \Phi_3 \Phi_4 \]

<table>
<thead>
<tr>
<th>Field</th>
<th>$U(1)$</th>
<th>$U(1)_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_1$</td>
<td>-1</td>
<td>2/3</td>
</tr>
<tr>
<td>$\Phi_2$</td>
<td>+2</td>
<td>2/3</td>
</tr>
<tr>
<td>$\Phi_3$</td>
<td>-1</td>
<td>2/3</td>
</tr>
<tr>
<td>$\Phi_4$</td>
<td>+2</td>
<td>2/3</td>
</tr>
</tbody>
</table>

\[
V = 4|\phi_1|^2|\phi_2|^2 + |\phi_1|^2 + 4(|\phi_1|^2|\phi_3|^2 + |\phi_1|^2|\phi_4|^2 + |\phi_3|^2|\phi_3|^2) + 8\phi_1 \phi_2 \phi_3 \phi_4 + \frac{1}{2}(6 - |\phi_1|^2 + 2|\phi_2|^2 - |\phi_3|^2 + 2|\phi_4|^2)^2
\]

<table>
<thead>
<tr>
<th>Label</th>
<th>$(\phi_1, \phi_2, \phi_3)$</th>
<th>$V$</th>
<th>Stability</th>
<th>SUSY</th>
<th>R-symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$(0, 0, \sqrt{6}, 0)$</td>
<td>0</td>
<td>Stable</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>B</td>
<td>$(\sqrt{2}, 0, 0, 0)$</td>
<td>12</td>
<td>Metastable</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>C</td>
<td>$(\sqrt{\frac{24}{23}}, 0, \sqrt{\frac{18}{23}}, 0)$</td>
<td>$\approx 13.066$</td>
<td>Unstable</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
Summary

1. Breaking SUSY by metastable states and allowing at the same time a SUSY vacuum let model building escape from the stringent constraints posed by the global nature of SUSY breaking. It becomes generic and easy to build.
2. This proposal can be realized at the tree level as suggested by Dienes and Thomas, in a beautiful combination of Wess-Zumino model and Fayet-Illiopoulos Model. Both F-term and D-term acquire non-zero VEV at the metastable local minimum.
3. We simplify this model by reducing the number of U(1) gauge symmetry and superfield and find it works as in DT.
4. Further clarification of why they work and the essence of DT’s proposal is still under investigation.