1. A grounded conducting sphere of radius $a$ carries charge $q$. The dielectric constant outside the sphere varies with the radial distance from the center of the sphere,

$$\varepsilon(r) = \varepsilon_0 \left( 1 + \frac{b^2}{r^2} \right), \quad (r > a).$$

(a) Find the potential $\phi(r)$ in the region outside the sphere.

(b) What is the polarization surface charge density $\sigma_p = \mathbf{P} \cdot \hat{n}$ on the dielectric surface at $r=a$?

2. A circular ring with current $I$ and radius $a$ is located on the $x$-$y$ plane (see Figure).

(a) Find the vector potential $\mathbf{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{I d\ell'}{|\vec{x} - \vec{x}'|}$ at any point in space.

(b) Using the result in (a), find the vector potential near the $z$-axis by expanding $\theta$ to first order, then calculate the magnetic field $\mathbf{B}(z)$ on the positive $z$-axis.

3. The electromagnetic field momentum is $\mathbf{p}_{\text{field}} = \frac{1}{c^2} \int dV \mathbf{E} \times \mathbf{H}$.

(a) A localized charge distribution produces $\mathbf{E} = -\nabla \phi$. This field coexists with a $\mathbf{H}$ field that is generated by a localized charge density $\mathbf{J}$ (time-independent). Show that if $\phi \mathbf{H}$ falls off rapidly at large distance, then $\mathbf{p}_{\text{field}} = \frac{1}{c^2} \int dV \phi \mathbf{J}$.

(b) Based on the result in (a), show that if the $\mathbf{E}$ field changes little over the localized charge distribution, then $\mathbf{p}_{\text{field}} = \frac{1}{c^2} \mathbf{E}(0) \times \mathbf{m}$, where $0$ is the location within the current distribution, and $\mathbf{m}$ is the magnetic moment generated by the current.