

普物期中考

Apr 2024

1 atm = 1.01×10^5 Pa, $R = 8.31$ J/mol·K。

1. 在一空曠室內有一塊10.0 kg的冰塊，保持在 0°C 全部融化為液態水，已知冰塊周圍空氣溫度為 27°C ，而且房間夠大，以至空氣溫度一直沒有改變。在這個融化的過程中，冰水系統及它的環境的熵變化分別是多少 J/K？冰的溶解熱為： $L_f = 334 \times 10^3$ J/kg。注意正負號。
(20)

Consider an ice cube of mass 10.0 kg. It stays at 0°C but melts completely into water. The air temperature in the room stays at 27°C . Calculate the change of entropy for water-ice system and the environment during the melting process. The melting heat of ice is $L_f = 334 \times 10^3$ J/kg. Be careful to denote whether the change is positive or negative.

解答：冰的吸熱為： $Q = 334 \times 10^3 \times 10 = 3.3 \times 10^6$ J，冰的熵變化：

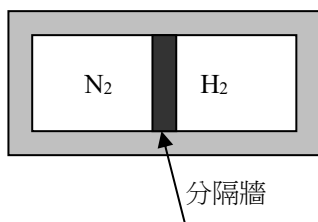
$$\Delta S = \frac{Q}{T} = \frac{3.3 \times 10^6}{273} = 1.2 \times 10^4 \text{ J/K}$$

。空氣放出同樣大小的熱，但符號為負，溫度不同：

$$\Delta S = \frac{Q}{T} = \frac{-3.3 \times 10^6}{300} = -1.1 \times 10^4 \text{ J/K}$$

。總熵是增加的。

2. 考慮一個孤立固定容器，中央以一牆分隔，左右兩室體積相等。左室內充滿氮氣 N_2 一莫耳，溫度為 $T_{\text{N}_2} = 227^\circ\text{C}$ ，壓力為 $P_{\text{N}_2} = 1$ atm，右室內則充滿氫氣 H_2 三莫耳。將中央的固定分隔牆移除，使兩種氣體反應產生氨氣： $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$ 。當反應結束後，氨氣 NH_3 的溫度為 $T_{\text{NH}_3} = 127^\circ\text{C}$ ，本題中所有氣體皆可視為理想氣體。H 原子量為 1，N 的原子量是 14。(25)



- A. 反應後氨氣 NH_3 的壓力 P_{NH_3} 是多少 atm? 提示: NH_3 的體積是 N_2 的體積的兩倍。
- B. 反應後氨氣 NH_3 氣體分子的平均動能, 是反應前氮氣 N_2 分子的平均動能的多少倍?
- C. 我們通常可以用方均根速率 $v_{\text{rms}} \equiv \sqrt{\langle v^2 \rangle}$ (速率平方的平均值的平方根) 來近似氣體分子的平均速率。反應後氨氣 NH_3 氣體分子的 v_{rms} , 是反應前氮氣 N_2 分子的 v_{rms} 的多少倍?

A container is isolated and separated in the middle by a wall into two rooms with identical volume. In the left room, there is 1 mol of nitrogen gas N_2 at temperature $T_{\text{N}_2} = 227^\circ\text{C}$ and its pressure is $P_{\text{N}_2} = 1$ atm. In the right room, there is 3 mol of hydrogen gas H_2 . We then remove the wall and the gases mix with the other. A process occurs generating ammonia $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$. At the end of the process, the temperature of NH_3 is $T_{\text{NH}_3} = 127^\circ\text{C}$. The atomic masses of H and N are 1 and 14 respectively. Consider both gases as ideal gas.

- A. What is the pressure of the ammonia gas P_{NH_3} at the end of the process? Hint: The volume of NH_3 is twice that of N_2 .
- B. What is the ratio of the average kinetic energy of NH_3 gas molecules at the end of the process versus the average kinetic energy of N_2 gas molecules before the process?
- C. We usually estimate the speed of the gas molecules by the root-mean-squared speed $v_{\text{rms}} \equiv \sqrt{\langle v^2 \rangle}$. What is the ratio of the v_{rms} of NH_3 gas molecules at the end of the process versus the v_{rms} of N_2 gas molecules before the process?

解答:

$$\text{A. } V_{\text{NH}_3} = \frac{nRT_{\text{NH}_3}}{P_{\text{NH}_3}} = \frac{2R \cdot 400}{P_{\text{NH}_3}} = 2V_{\text{N}_2} = 2 \frac{nRT_{\text{N}_2}}{P_{\text{N}_2}} = 2 \frac{1R \cdot 500}{1}$$

$$P_{\text{NH}_3} = 0.8 \text{ atm}$$

B. $\langle \frac{1}{2}mv^2 \rangle = \frac{3}{2}kT$, 平均動能的比就是溫度比:

$$\frac{T_{\text{NH}_3}}{T_{\text{N}_2}} = \frac{400}{500} = 0.8$$

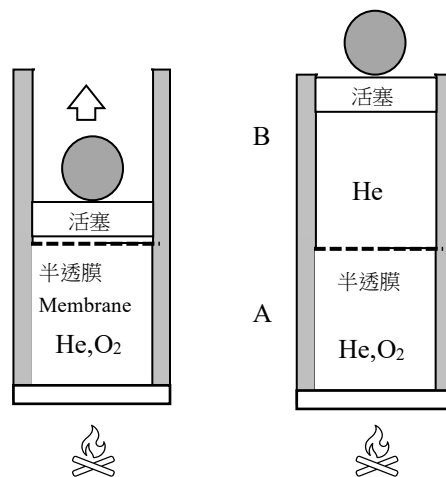
C. 已知:

$$v_{\text{rms}} = \sqrt{\langle v^2 \rangle} \propto \sqrt{\frac{T}{m}}$$

v_{rms} 的比就是 $\sqrt{\frac{T}{m}}$ 的比：

$$\frac{v_{\text{rmsNH}_3}}{v_{\text{rmsN}_2}} = \frac{\sqrt{\frac{T_{\text{NH}_3}}{m_{\text{NH}_3}}}}{\sqrt{\frac{T_{\text{N}_2}}{m_{\text{N}_2}}}} = \frac{\sqrt{\frac{400}{17}}}{\sqrt{\frac{500}{28}}} = \sqrt{1.31} = 1.14$$

3. 考慮一個長方體容器，以一固定的半透膜（圖中虛線）分割為 A 室及 B 室，B 室中有一可上下移動的活塞。起始時，活塞在半透膜上方幾乎與之重合。A 室內含可視為理想氣體的氦氣 He 與氧氣 O₂ 各 1.0 mol，半透膜只能讓氦氣通過，氧氣無法通過。已知活塞上的壓力保持固定為 P_0 。因為氧氣無法透過半透膜，因此對活塞上的壓力沒有貢獻，所以 P_0 就是氦氣的壓力。氧氣與氦氣莫耳數相同，因此 A 室中氧氣的壓力也是 P_0 。



A. 設 $V_A = V_0$ ，此時 A 室內溫度是多少？以已知的 P_0, V_0 及常數 R 表示。

現在於 A 室下方慢慢加熱，使活塞向上移動，過程中一直維持熱平衡，直到活塞下方 B 室的體積與 A 室相等為止，過程活塞保持定壓。

B. 此時 B 室中氦氣溫度是多少？過程中所有氦氣吸收了多少熱量？A 室中氧氣溫度是多少？過程中氦氣吸收了多少熱量？以已知的 P_0, V_0 及常數 R 表示。(30)

提示：因為氦氣可以透過半透膜，所以對氦氣來說，A，B 兩室其實是聯通。

Consider a rectangular container, separated by a membrane into two rooms A and B as shown in the diagram. There is a piston in room B which can move freely up and down. At the beginning, a piston sits right above the membrane and almost coincides with it in room B. Room A contains 1.0 mol of ideal helium gas He and 1.0 mol of ideal oxygen O_2 . The membrane only allows helium gas to pass freely but not the oxygen gas. The pressure on the piston is maintained at P_0 during the process. Since oxygen gas cannot pass the membrane, it does not contribute to the pressure on the piston. Hence P_0 is the pressure of helium. And the oxygen and helium gas have the same mole number. The pressure of oxygen is also P_0 . The volume of room A is $V_A = V_0$.

What is the temperature in room A? The answer can be expressed in terms of P_0, V_0 and R .

The gases are heated from the bottom of room A. The piston move slowly up so that the gasses maintain equilibrium during the process while the pressure is fixed. It stops when the volume of room B below piston is equal to the volume of room A. What is the temperature of the helium gas in room B? How much heat does it absorb in the process? What is the temperature of the oxygen gas in room A? How much heat does it absorb in the process?

提示：因為氦氣可以透過半透膜，所以對氦氣來說，A，B 兩室其實是聯通。

Hint: Since the Helium gas can pass freely through the membrane, for the helium gas the two rooms are connected.

解答：理想氣體滿足狀態方程式： $T = \frac{PV}{nR}$ 。

因為氧氣無法透過半透膜，因此對活塞上的壓力沒有貢獻，因此 P_0 完全是氦氣壓力，加熱

前，A 室內 1 莫耳氦氣壓力為 P_0 ， $T_A = \frac{P_0 V_0}{nR} = \frac{P_0 V_0}{R}$ 。

加熱後，氦氣壓力一直維持在 P_0 ，氦氣可以自由通過膜，因此兩室的氦氣可以合起來看

待，且溫度相等，因此體積為 $2V_0$ ，溫度為： $T_{He} = \frac{P_0 2V_0}{R}$ 。氦氣經歷定壓過程，因此吸收熱

量等於 $nc_p\Delta T = \frac{5}{2}R \frac{P_0V_0}{R} = \frac{5P_0V_0}{2}$ 。而氧氣與氦氣在 A 室內達到熱平衡因此溫度相等：

$$T_{O_2} = T_{He} = \frac{2P_0V_0}{R}。氧氣經歷定容過程，因此吸收熱量等於 $nc_v\Delta T = \frac{5}{2}R \frac{P_0V_0}{R} = \frac{5P_0V_0}{2}$ 。$$

4. 考慮一大空間，在中央置一球型黑體熱源 A，熱源球的半徑為 b ， b 遠小於空間的大小。熱源的溫度為 T_1 且一直保持不變，其熱輻射是球對稱的。在距熱源 A 距離 a 處，置一小的黑體 B，其形狀為一正方形薄片，邊長為 d ，厚度很小可以忽略，已知 $d, b \ll a$ ，平面方向垂直於熱源與薄片間的距離。假設空間的牆面只吸熱，且溫度極低，空間中的空腔輻射及牆面放熱都可以忽略（即可完全忽略牆面）(25)
- A. 在小黑體 B 吸收的輻射功率是多少？
- B. 當小黑體 B 達成熱平衡後，溫度是多少？。

答案以 a, b, d, T_1, σ 表示。黑體熱輻射的公式為 $P = \sigma AT^4$ 。

Consider a large room with a blackbody spherical heat source A in the middle. The radius of A is b , much smaller than the size of the room. The temperature of A remains constant at T_1 . The radiation is spherically symmetric. At a distance of a from A is a small blackbody B, with the shape of a thin square sheet. The length of a side of the square is d . The thickness of sheet is so small that it could be ignored. $d, b \ll a$. The plane of the square sheet is perpendicular to its distance from the source A. Assume that the walls only absorb heat and the radiation from it can be ignored (ie. We can ignore the wall all together).

- A. What is the radiation power the blackbody B absorbs?
- B. When B reaches thermal equilibrium, what is its temperature?

Answer in terms of a, b, d, T_1, σ . The blackbody radiation formula is $P = \sigma AT^4$.

解答：熱源的總輻射 $P = \sigma AT^4 = \sigma 4\pi b^2 T_1^4$ ，平均分配至距球心 a 處的每單位面積功率

為： $P \times \frac{1}{4\pi a^2} = \sigma T_1^4 \frac{b^2}{a^2}$ 。

小黑體的截面積為 d^2 ，故吸收 $\sigma T_1^4 \frac{b^2}{a^2} d^2$ 。

此能量必須等於小黑體的輻射總量： $\sigma \cdot 2d^2 T_2^4$ ：

$$\sigma \cdot 2d^2 T_2^4 = \sigma T_1^4 \frac{b^2}{a^2} d^2$$

$$T_2 = \sqrt[4]{\frac{1}{2} \frac{b}{a}} T_1$$