手徵異常 **Chiral anomaly** (Nielsen and Ninomiya, Phys Lett B 1983)

Weyl semimetal in magnetic field

$$\mathsf{H} = \chi v \boldsymbol{\sigma} \cdot \mathbf{p}. \qquad \chi = \pm$$

+B $\quad \vec{p} \rightarrow \vec{\pi} = \vec{p} + e\vec{A}$

$$\Rightarrow \quad \mathsf{H} = \chi v(\sigma_x \pi_x + \sigma_y \pi_y) + \chi v \sigma_z p_z,$$

$$[\pi_x, \pi_y] = \frac{\hbar e B}{i}.$$

Define creation and annihilation operator

$$\begin{cases} a &= \frac{1}{\sqrt{2\hbar e B}}(\pi_x - i\pi_y) \\ a^\dagger &= \frac{1}{\sqrt{2\hbar e B}}(\pi_x + i\pi_y) \end{cases},$$

$$[a, a^\dagger] = 1.$$

$$\Rightarrow \quad \mathsf{H} = \chi \hbar\omega(\sigma_+ a + \sigma_- a^\dagger) + \chi v \sigma_z \hbar k_z \qquad \hbar\omega \equiv v\sqrt{2\hbar e B}$$

$$= \chi \begin{pmatrix} v\hbar k_z & \hbar\omega a \\ \hbar\omega a^\dagger & -v\hbar k_z \end{pmatrix},$$

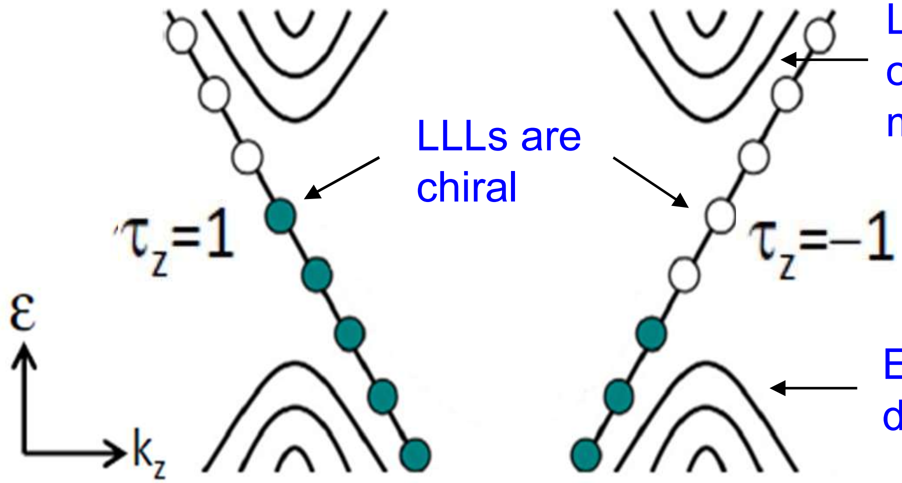
Solving eigenenergy

$$\mathrm{H}\Psi_n = \varepsilon_n \Psi_n,$$

$$\text{with} \quad \Psi_n = u_n \begin{pmatrix} 1 \\ 0 \end{pmatrix} |n-1\rangle + v_n \begin{pmatrix} 0 \\ 1 \end{pmatrix} |n\rangle$$

$$\begin{cases} \varepsilon_0^{\chi} = -\chi v \hbar k_z . \\ \varepsilon_{n\pm}^{\chi} = \pm \chi \hbar \omega \sqrt{n + (v k_z/\omega)^2}, n \geq 1. \end{cases}$$

n=0: the LLL is chiral



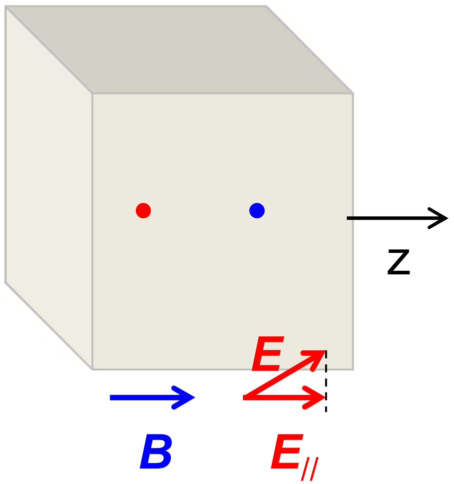
LLs disperse along B only (electron cannot move ⊥ B)

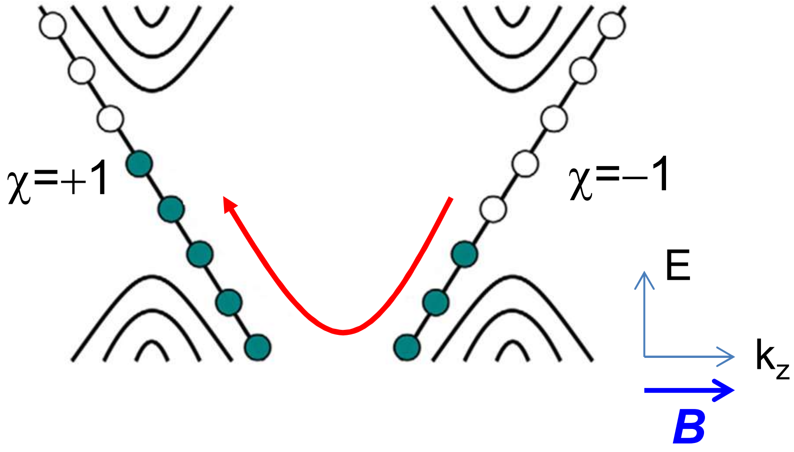
Each LL is highly degenerate

$$D = \frac{\phi_{samp}}{\phi_0}, \quad \phi_0 \equiv \frac{h}{e}$$

(Kittel, chap 9)

Charge transport between valleys





Hosur and Qi, Comptes Rendus Phys 2013

1. LL degeneracy $\propto B$
2. LLs disperse only along B !
3. 0-th Landau levels are chiral !
4. $E_{//}$ field drives the current

Current $\propto \quad (\vec{E}\cdot\hat{B})B = \vec{E}\cdot\vec{B}$

Chiral charge
$$\frac{d\rho_\pm}{dt} = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B}$$
(next page)

Same as the *chiral anomaly* in particle physics:

Adler, Bell, and Jackiw, 1969
$$\partial_\mu j^\mu_\pm = \pm\frac{e^3}{h^2}\frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$
Covariant form

- Difference: the Dirac sea has no bottom

The simplest way to derive the chiral anomaly

- 1D charge transport

$$\frac{dQ^z_\chi}{dt} = (-e)\chi\frac{\frac{\Delta k_z}{2\pi/L_z}}{\Delta t}$$
$$= -e\chi\frac{\dot{k}_z}{2\pi/L_z}, \quad \hbar\dot{k}_z = -eE_z$$
$$= e^2\chi\frac{E_\parallel L_z}{h}.$$

- Degeneracy of a LL

$$D = \frac{\phi_{\mathrm{tot}}}{\phi_0} = \frac{A_{\mathrm{samp}}B}{h/e}$$

- 3D charge transport

$$\frac{dQ^{3D}_\chi}{dt} = \frac{AB}{h/e}\frac{dQ^z_\chi}{dt}$$
$$= \chi\frac{e^3}{h^2}AL_zBE_\parallel$$

$$\frac{\partial\rho_\chi}{\partial t} = \chi\frac{e^3}{h^2}\mathbf{E}\cdot\mathbf{B}$$

Same result can be obtained without LLs
(Son and Spivak, PRB 2013)

Chiral anomaly and magnetoresistance $\rho(B)$

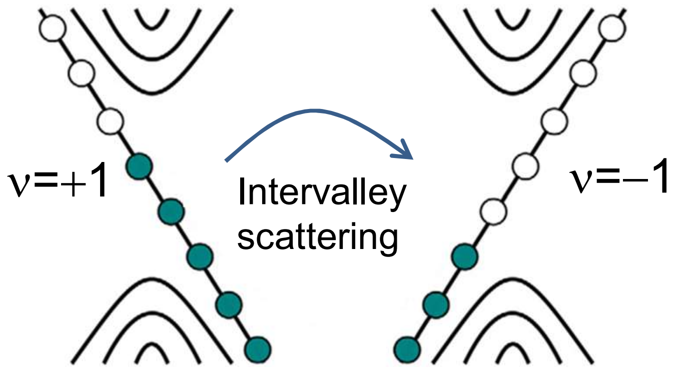
- Chiral anomaly (charge pumping between Weyl nodes)

$$\frac{d\rho_\pm}{dt} = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B} - \frac{\rho_\pm}{\tau_V}$$

Inter-valley relaxation time

Steady state $\rightarrow$ $\rho_\pm = \pm\frac{e^3}{h^2}\vec{E}\cdot\vec{B}\,\tau_V$

$\rightarrow$ $\Delta\mu \equiv \mu_+ - \mu_- \propto \vec{E}\cdot\vec{B}\,\tau_V$



- Chiral magnetic effect (later)

$$\vec{j} = \frac{e^2}{h^2}\Delta\mu\vec{B}$$

- conductivity enhanced by B

$\rightarrow$ $\vec{j} \propto \left(\vec{E}\cdot\vec{B}\right)\vec{B}\,\tau_V$

Positive magneto-conductance, or **negative** magneto-resistance

max when $\vec{E}//\vec{B}$ $\quad \sigma(B) = \sigma_0 + \left(\frac{e^2B}{2\pi}\right)^2 \frac{\tau_V}{g(\varepsilon_F)} \approx B^2$

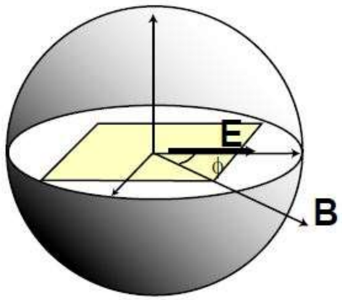
Fukushima, Kharzeev, and Warringa Phys. Rev. D 2008

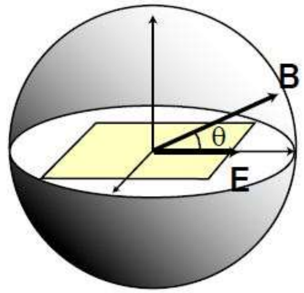
Li et al, Nature Phys 2016

Negative Magneto-Resistance in Weyl semimetal

- MR is usually positive (i.e. resistivity increases with B)
- An exception is a system with *weak localization*

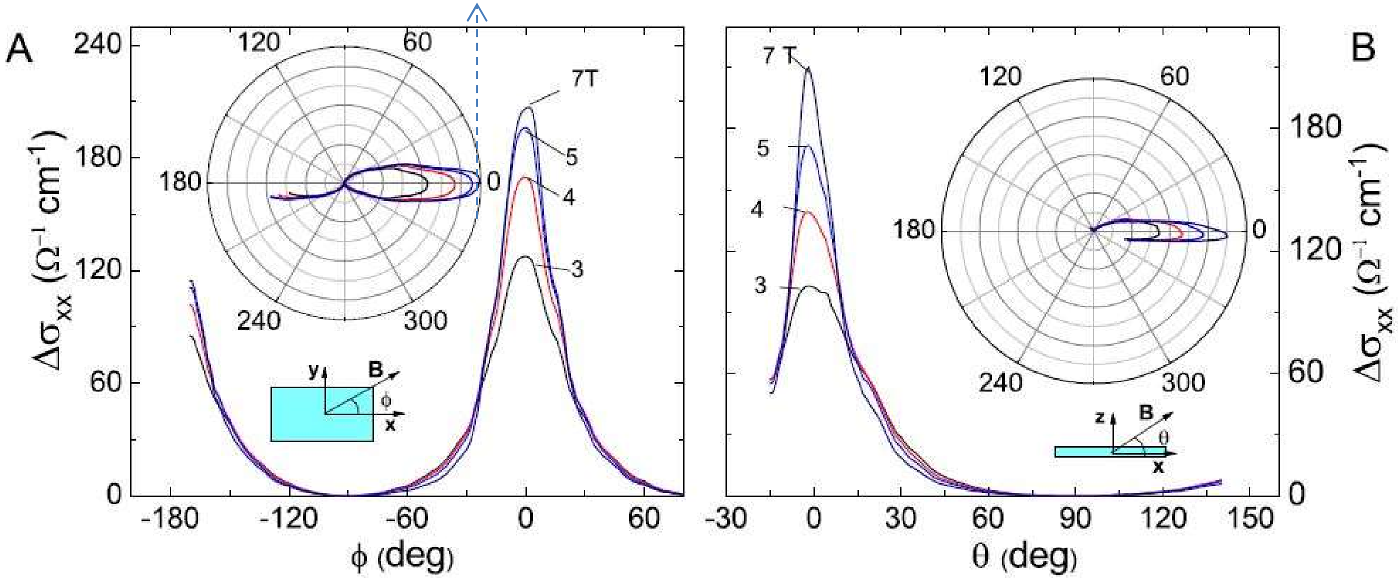
  (also, a few other materials)





Na_3Bi
(Dirac node
split by B field)

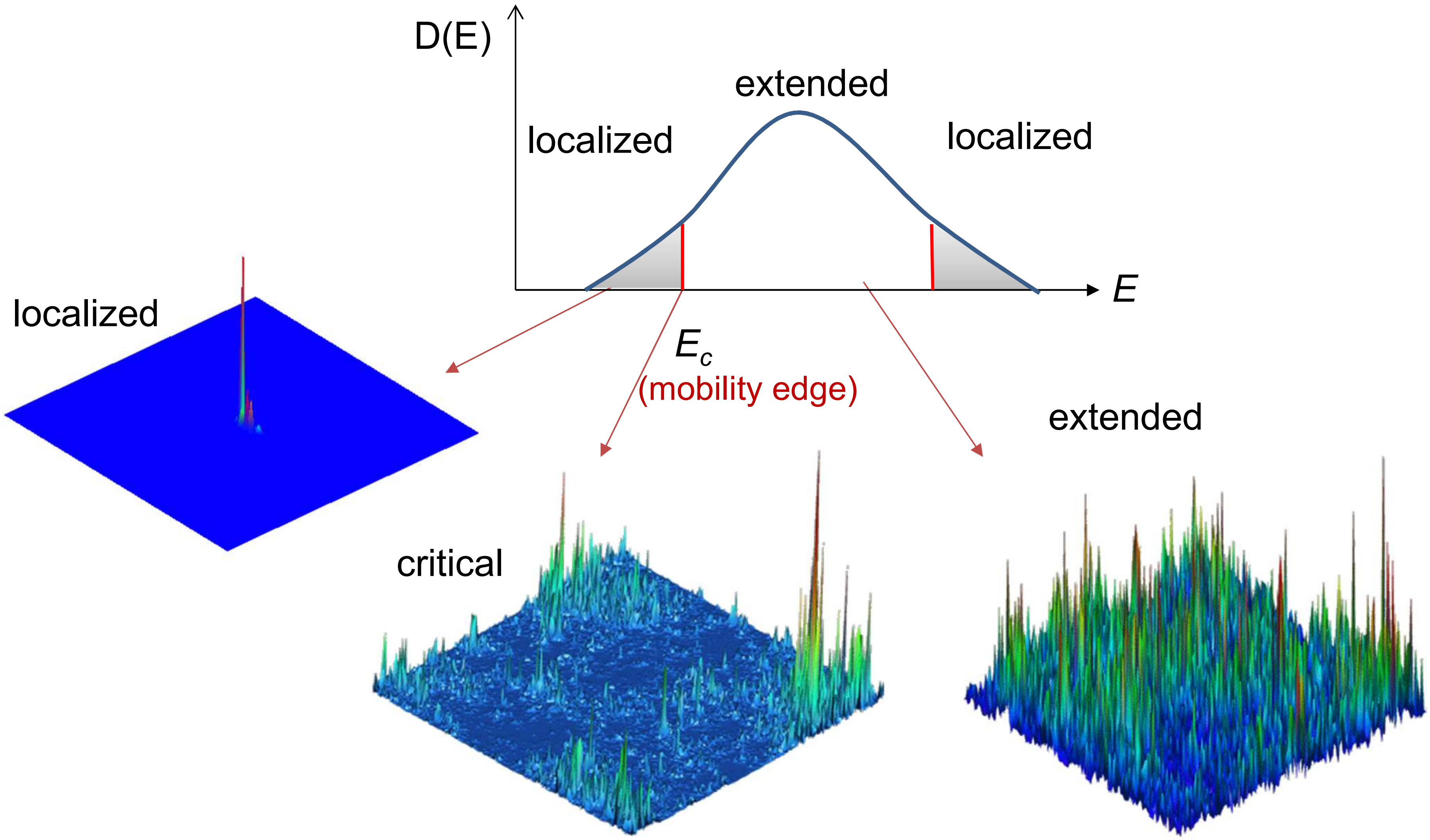
- Locking of the current max to B (not pinned to crystal axis)



Xiong et al, 1503.08179
Xiong et al, Science express 2015

Short notes on Localization:

1. Anderson localization in disordered system (Anderson 1958)



From Furusaki's ppt

2. Weak localization (Gorkov et al, 1979)

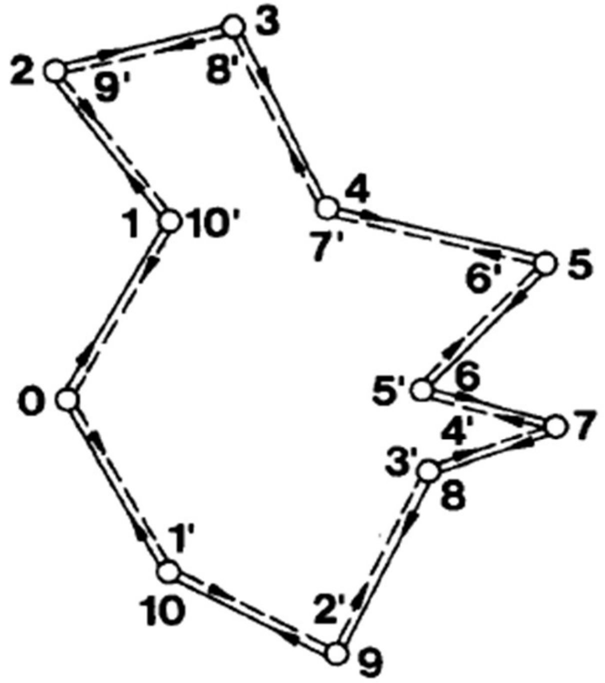


Fig. 2.5. Diffusion path of the conduction electron in the disordered system. The electron propagates in both directions (full and dashed lines). In the case of quantum diffusion the probability to return to the origin is twice as great as in classical diffusion since the amplitudes add coherently.

Bergmann Phys Rep 1984

Transition amplitude

$$A_{a\to b} = \sum_i A_{path\ i}$$

$$|A_{a\to b}|^2 = \sum_i |A_i|^2 + \sum_{i\neq j} A_i A_j^*$$

$$\left\langle \sum_{i\neq j} A_i A_j^* \right\rangle \neq 0 \quad \text{for time-reversed path } A_i = \overline{A}_j$$

$$(\text{possible only when } a = b)$$

$$\to \quad |A_{a\to a}|^2 = 2\sum_i |A_i|^2$$

coherent back-scattering

(Constructive interference)

Note: elastic scattering (static disorder) does not destroy quantum coherence (inelastic: phonons, other electrons… etc)

B field destroys coherent back-scattering

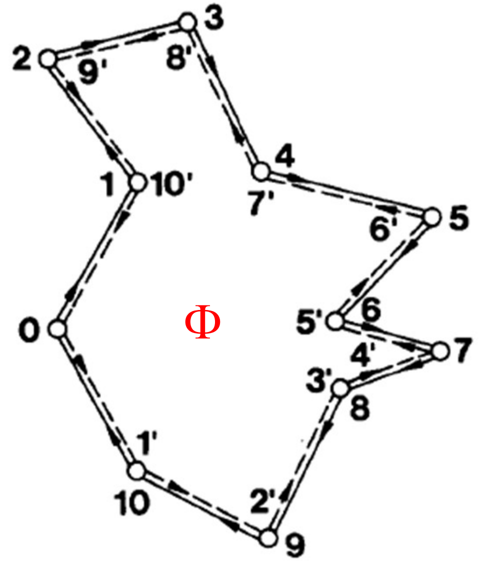
$$A_{a\to b} = \sum_i A_{path\ i}$$

$$|A_{a\to b}|^2 = \sum_i |A_i|^2 + \sum_{i\neq j} A_i A_j^*$$

$$\sum_{i\neq j} A_i A_j^* = \sum_{i\neq j} |A_i||A_j| e^{i(\phi_i-\phi_j)}$$

for time-reversed path $A_i = \overline{A}_j$

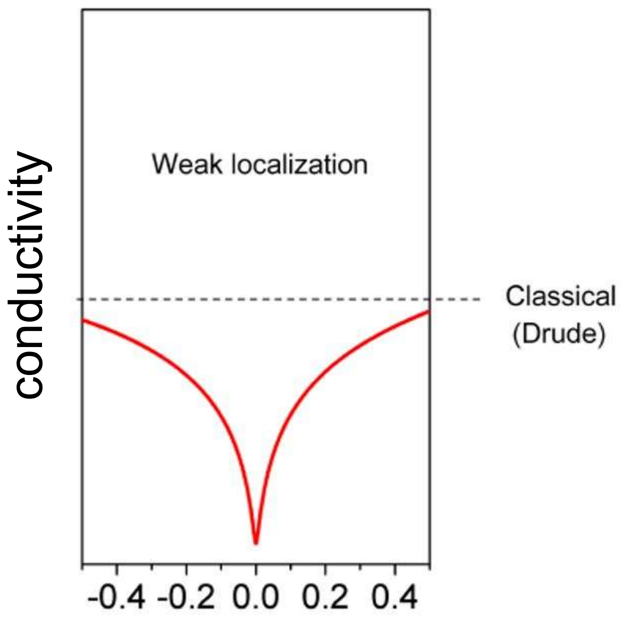
Bohm-Aharonov phase $\quad \phi_i - \phi_j = 2\frac{e}{\hbar}\int_C \vec{A}\cdot d\vec{\ell} = 4\pi\frac{\Phi}{\Phi_0}$



constructive
interference
⇓
enhanced
backscattering
⇓
decreased
conductivity

$$\approx -\frac{e^2}{h}\ln\frac{\ell_\phi}{\ell_e}$$

Negative magnetoresistance



ㄴu ㅂZ’s ppt

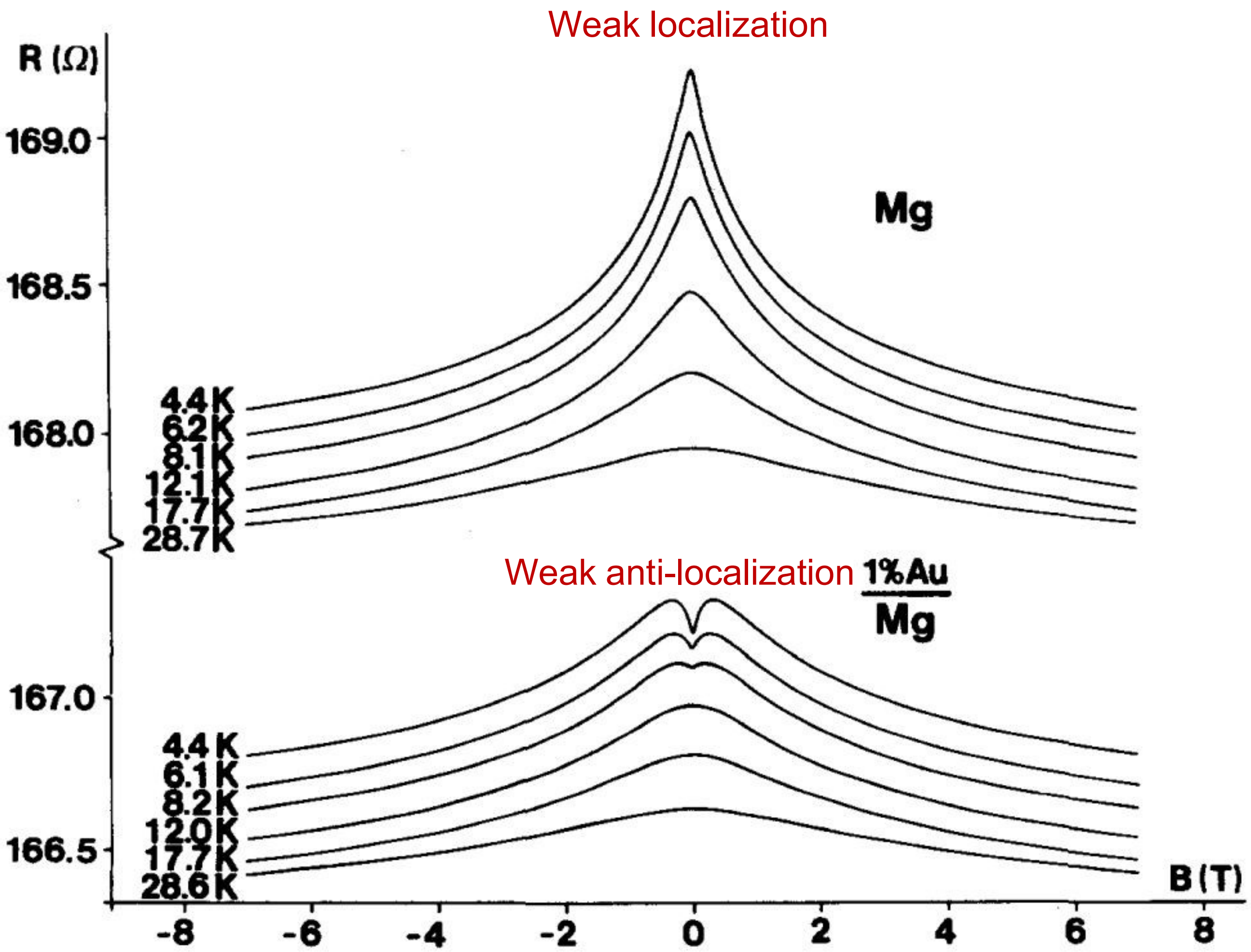


Fig. 2.9. The magneto-resistance curves of a thin Mg-film (upper set of curves). After a superposition with 1/100 atomic layer of Au the magneto-resistance changes drastically. The Au introduces a rather pronounced spin–orbit scattering which rotates the spins of the complementary scattered waves. This changes the interference from constructive to destructive.

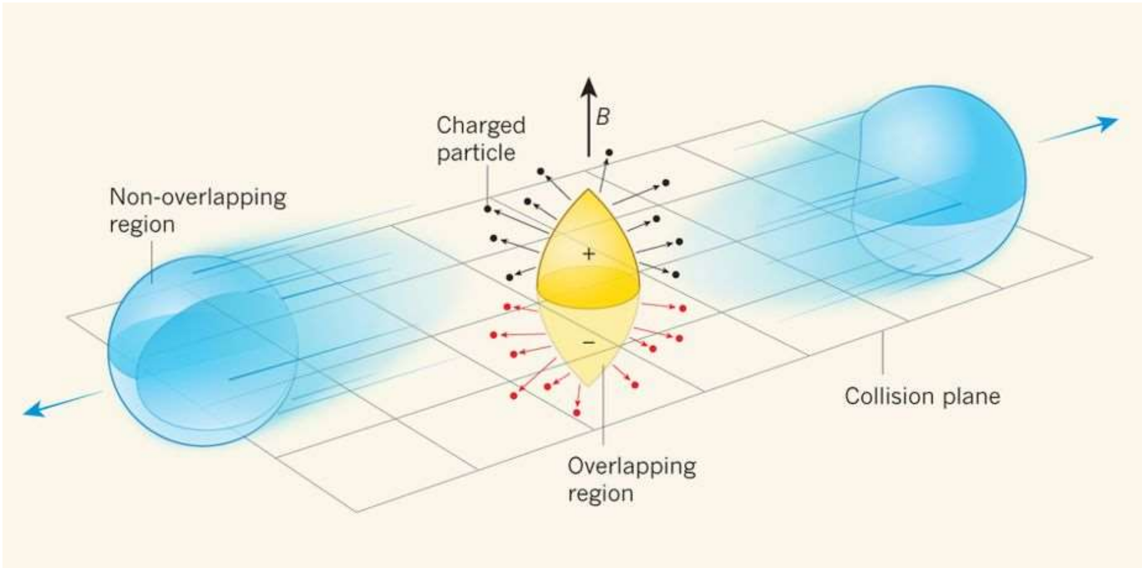
Bergmann Phys Rep 1984

Chiral magnetic effect (Vilenkin, Phys Rev D **22**, 3080, 1980)

$$\boxed{\vec{J}_{CME} = -\alpha_B \vec{B}}$$

- Quark-gluon plasma in heavy ion collision
- Relativistic plasma in astrophysics
- Weyl semimetal
- …

Note: need to break space-inversion symmetry



Figs from Dobrin Nature 2017, Chernodub arXiv 1002.1473, Vazifeh PRL 2013

https://www.bnl.gov/newsroom/news.php?a=119062

Kharzeev and Liao, Nat Rev Phys 2020

Symmetry in HE and CME

Hall effect

$$J_y = \sigma_H E_x$$

Space	–	–
Time	–	+

Chiral magnetic effect

$$\boldsymbol{J} = -\alpha_B \boldsymbol{B}$$

Space	–	+
Time	–	–

- $J_y = \sigma_H E_x$, needs to break TRS
- $\boldsymbol{J} = -\alpha_B \boldsymbol{B}$, needs to break SIS (hard in high-energy experiment)

However, an argument against (static) CME

(Basar, Kharzeev, and Yee, PRB 2014)

$$\vec{J} = -\alpha_B \vec{B}$$

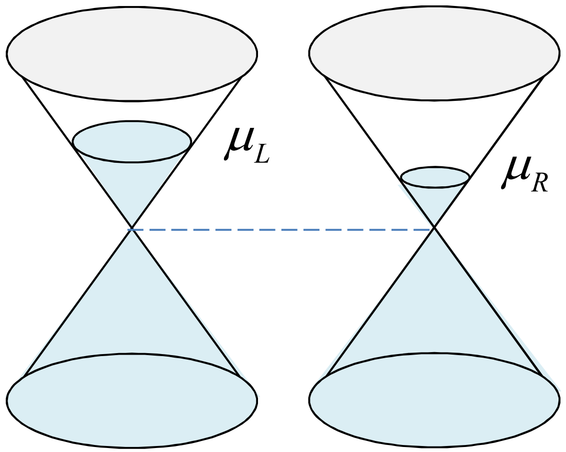
- Work done by field on charge carriers

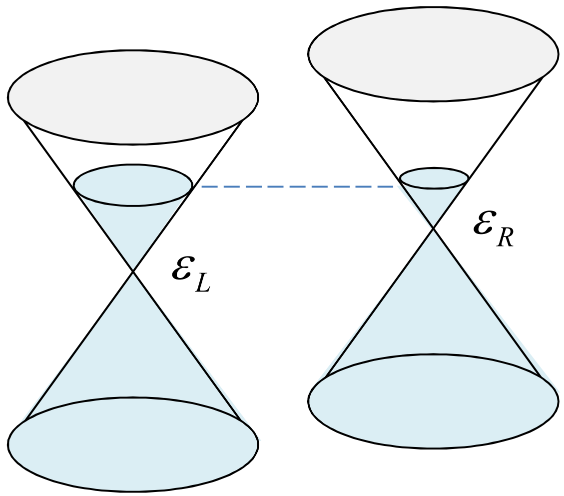
$$\frac{dP}{dt} = \vec{J} \cdot \vec{E} \sim \vec{E} \cdot \vec{B}$$ can be > 0 or < 0

Can extract energy out of equilibrium state!

More detailed analysis (semiclassical, linear response) also shows that CME could not exist.

Beyond static CME





- **Different chemical potentials**

$$\vec{J} = \frac{e^2}{h^2}\Delta\mu\vec{B} \quad \text{(non-equilibrium)}$$

Magnitude: $J \sim 0.01$ (A/mm^2)

if $\Delta\mu$=0.01 meV, B=0.1 T

→ Negative MR mentioned earlier

Zhang et al, Nature Comm 2015

- **Same chemical potential**
  - Static *B* field: no current
  - Dynamic *B* field (non-equilibrium):

    can have CME current 旋光效應

    related to natural gyrotropic effect

    (no B field required)

Basar et al, Phys Rev B 2014
Yang and Chang, PRB 2015
Zhong et al, Phys Rev Lett 2016

High-energy physics	Low energy physics (solid state physics)
• Magnetic monopole	Degenerate point
• Non-Abelian gauge theory (Yang-Mills theory)	Non-abelian Berry phase
• Skyrmion	Skyrmion in QHS, magnetic materials
• Axion	Axion in Weyl SM
• Dirac	in Graphene
Weyl	in Weyl SM
Majorana	in TI-SC hybrid structure
• Chiral anomaly	in Weyl SM
Chiral magnetic effect	in Weyl SM