Topology in solid state physics

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Curvature

extrinsic curvature $K$ vs intrinsic curvature $G$ (Gaussian curvature)

Positive and negative Gaussian curvature

- $G > 0$
- $G = 0$
- $G < 0$
• Gauss-Bonnet theorem (for a 2D closed surface)

\[ \int_M da \ G = 2\pi \chi(M), \quad \chi = 2(1 - g) \]

Euler characteristic

- \( g = 0 \)
- \( g = 1 \)
- \( g = 2 \)

• Gauss-Bonnet theorem (for a 2D surface with boundary)

\[ \int_M da \ G + \int_{\partial M} ds \ k_g = 2\pi \chi(M, \partial M) \]

• Can be generalized to higher (even) dimension.

Marder, Phys Today, Feb 2007
Condensed matter physics is physics of dirt - Pauli

- Flux quantization
- Quantum Hall effect
- …

Often protected by topology
Quantum Hall effect (von Klitzing, PRL 1980)

2-dim electron gas (2DEG)
\(\rho_{xy}\) deviates from \((h/e^2)/n\) by less than 3 ppm on the very first report.

- This result is independent of the shape/size of sample.
- Different materials lead to the same effect (Si MOSFET, GaAs heterojunction…)

\[ h/e^2 = 25.81202 \text{ k-ohms} \]

\[ \alpha^{-1} = h/e^2c = 137.036 \text{ (no unit)} \]

\[ \rightarrow \text{a very convenient resistance standard (1990).} \]
Quantization of Hall conductance
Thouless et al’s argument (1982), bulk description

- Berry connection

\[ \vec{A}_n(\vec{k}) \equiv i \langle u_n | \nabla_{\vec{k}} | u_n \rangle \]

- Berry curvature (for n-th band)

\[ \vec{\Omega}_n(\vec{k}) = i \langle \nabla_{\vec{k}} u_n | \times | \nabla_{\vec{k}} u_n \rangle \]
\[ = \nabla_{\vec{k}} \times \vec{A}_n(\vec{k}) \]

- From linear response theory, the Hall conductivity for the n-th band is

\[ (\sigma_H)_n = \frac{e^2}{h} \left[ \frac{1}{2\pi} \int_{BZ} d^2k \, \Omega_{nz}(\vec{k}) \right] \]

Gauss-Bonnet theorem

\[ \int_{BZ} d^2k \, \Omega_{nz}(\vec{k}) = 2\pi C_1 \]
\[ \Rightarrow \int_M da \, G = 2\pi \chi \]

1st Chern number
(an integer for a filled band)

\( C_1 \) is not changed as long as the band does not touch the other bands
Bulk-edge correspondence

Different topological classes

Semiclassical (adiabatic) picture: energy levels must cross (otherwise topology won’t change).

→ gapless states bound to the interface, which are protected by topology.

Edge states in quantum Hall system

• Classical picture
  Chiral edge state (skipping orbit)

• Robust against disorder (no back-scattering)
• classical Hall effect

• anomalous Hall effect (in magnetic materials)

• spin Hall effect (transverse spin current)

No magnetic field required!
(extrinsic) Spin Hall effect

Local Kerr effect in strained n-type bulk InGaAs, 0.03% polarization

- skew scattering by spinless impurities
- no magnetic field required
Quantum version of spin Hall effect?

Spin current is no longer quantized when we turn on the spin-orbit coupling.

• 2006: Kane and Mele, S.C. Zhang et al found Quantum spin Hall insulator
  (aka 2D topological insulator)
Topological insulators in real life

band inversion induced by spin-orbit coupling

2D

• HgTe/CdTe QW (Bernevig, Hughes, and Zhang, Science 2006)
• Bi bilayer (Murakami, PRL 2006, Drozdov et al Nat Phys 2014 [exp])
• Silicine (Liu et al, PRL 2011)
• Tin thin film (Xu et al, PRL 2013)
• BiX/SbX monolayer (X=H,F,Cl,Br) (Song et al 2014)

• Bi$_{1-x}$Sb$_x$, α-Sn ... (Fu, Kane, Mele, PRL, PRB 2007)
• Bi$_2$Te$_3$ (0.165 eV), Bi$_2$Se$_3$ (0.3 eV) ... (Zhang, Nature Phys 2009)
• The half Heusler compounds (LuPtBi, YPtBi ...) (Lin, Nature Material 2010)
• thallium-based III-V-VI$_2$ chalcogenides (TlBiSe$_2$, Lin, PRL 2010; BiTeCl [strong SIA], Chen Nat Phys 2014)

3D

• Ge$_n$Bi$_{2m}$Te$_{3m+n}$ family (GeBi$_2$Te$_4$ ...)
• BaBiO$_3$ [SC] (0.7 eV, Yan, Nat Phys 2013 [theo])
• ...

...
The topology behind the topological insulator

• Insulators are characterized by (Fu and Kane 2006)

\[ \nu = \frac{1}{2\pi} \left[ \int_{EBZ} d^2k \, \Omega - \int_{\partial(EBZ)} dk \, A \right] \mod 2 = \begin{cases} 
0 & \text{Ordinary insulator} \\
1 & \text{Quantum spin Hall insulator} 
\end{cases} \]

\( \nu \) does not change as long as the band gap remains open

\sim \text{Gauss-Bonnet theorem for a 2D surface with boundary}

\[ \int_M da \, G + \int_{\partial M} ds \, k_g = 2\pi \chi(M, \partial M) \]

\( \chi = 2 \quad \chi = 0 \quad \chi = 1 \)
Hall effects, their **quantum** companions, and edge states

3D Topological insulator
Edge state: *Quantum Hall insulator vs topological insulator*

**Robust chiral edge state**

**Robust helical edge state**

Real space

Energy spectrum

2 robust edge channels, but spin transport is not quantized.

• Dirac point at TRIM
• Dirac cone with spin texture
Angle-Resolved Photo-Emission Spectroscopy (ARPES)
Band inversion, parity change, and
spin-momentum locking (Dirac cone with spin texture)