

1. (20%) Answer the following questions briefly. Use formulas and figures if that helps your explanation.

(a) What is the *order of magnitude* of Fermi wave vector for alkali metal? What is the order of magnitude of Fermi temperature for alkali metal?

(b) What is the Wiedemann-Franz law?

2. (30%) (a) In 2-dim free electron model, we have a Fermi circle (instead of a Fermi sphere). The radius k_F of the Fermi circle depends on the density of free electrons $n=N/A$ (N is the total number of electrons, A is the area of the sample). Find out the connection between k_F and n .

(b) For a 2-dim electron gas (instead of 3-dim), what would be the temperature dependence of electron specific heat $C_e(T)$ at low T ? Briefly explain how you obtain your answer.

3. (20%) (a) A crystal has N_1, N_2, N_3 lattice points along each side of the sample. The primitive vectors of the direct and the reciprocal lattices are $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ and $(\vec{b}_1, \vec{b}_2, \vec{b}_3)$.

Suppose we impose Periodic Boundary Condition on the crystal. What are the positions of \mathbf{k} points in k -space? You can simply write down the answer directly.

(b) In the textbook example explaining the connection Bragg diffraction and energy gap, a lattice potential $V(x)=V\cos(2\pi x/a)$ is used. If the potential is $V(x)=V_1\cos(2\pi x/a) + V_2\cos(4\pi x/a)$, where will be the energy gaps? (show it in a figure)

4. (30%) In the reduced zone scheme, energy bands are plotted in the first Brillouin zone (BZ).

(a) Consider free electrons in an (empty) square lattice with lattice constant a . Plot all of the energy bands below energy $5(\hbar^2/2m)(\pi/a)^2$ along the ΓX direction (see below). Also, write down the energies of the curves at points Γ and X.

(b) If we have a rectangular lattice with lattice constant a and $2a$ (along x and y directions). What would the curves in (a) be changed? Re-plot the figure and write down the energies of the curves at points Γ and X.

