**Introduction to Solid State Physics** 9:10-11:00 2007/5/21

◎ Each problem is 15 points. There is a 10-point bonus.

◎ Use words, equations, and/or figures to help you clarify an explanation.

1. The intrinsic carrier concentration in a Si is $n_i$=1.5×10$^{10}$atoms/cm$^3$. After doping, the hole and electron concentrations become $p$ and $n$. Assume the Si is doped with $N_d$=10$^{17}$ As atoms/cm$^3$, use the mass-action law to find out the hole concentration. (In this heavily doped case, $N_d \gg n_i$, one can approximate $n=N_d$)

2. (a) Roughly draw the distributions of magnetic field $B$(r) and superconducting current density $n_s$(r) near the center of a vortex. (b) Assume the penetration length and the coherence length of a type-II superconductor is λ and ξ. Estimate the critical magnetic fields $H_{c1}$ and $H_{c2}$. Briefly explain.

3. Explain, as clearly as you can, the AC Josephson effect. Also explain why it is a very sensitive sensor for tiny change of voltage.

4. Assume that $\vec{k}\cdot\vec{E}\neq 0$, then from Gauss' law, the equation of continuity, and Ohm's law, show that $4\pi\sigma(\omega)\rho(\omega)=i\omega\rho(\omega)$. Explain why this leads to $\varepsilon$ = 0. This shows that the longitudinal component of an electromagnetic wave can exist only when $\varepsilon$ = 0.

5. Explain, briefly but clearly, with the help of figures or equations, why the electron-electron scattering rate in metal is proportional to $T^2$.

6. The electric susceptibility for a collection of bounded oscillators is

$$\chi(\omega)=\frac{1}{V}\sum_j \frac{e^2}{m(\omega_j^2-\omega^2-i\eta_j\omega)}.$$

In order to apply this result to a metal, we can consider the case with *identical* oscillators, and take the limit with $\omega_j$ = 0, $\eta$ being very small. With the help of the susceptibility, write down the electric conductivity σ(ω), then verify the Ferrel-Glover sum rule *($\omega_p$* is the plasma frequency):

$$\int_0^\infty \sigma'(\omega)d\omega=\frac{\omega_p^2}{8}.$$

Note: $\varepsilon = 1 + 4\pi i\sigma/\omega$